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Chain Rule Assignment

1. Given $f(z) = \log_e(1+z)$

where $z = x^T x$, $x \in \mathbb{R}^d$

$\Rightarrow$ if $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$

then, $x^T = [x_1 \, x_2, \ldots x_d]$

$x^T x = [x_1^2 + x_2^2 + \ldots + x_d^2]$

Applying chain rule,

$$\frac{db}{dx} = \frac{db}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}\left(\log(1+z)\right) \cdot \frac{d}{dx}\left(x^T x\right)$$

$$= \frac{1}{1+z}\left(2x_1 + 2x_2 + \cdots + 2x_d\right)$$

$$= \frac{2}{1+z} \sum_{i=1}^{d} x_i$$

(Ans.)

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2. $f(z) = e^{-z/2}$, where $z = g(y)$,

$g(y) = y^T \cdot S^{-2} y$, $y = h(x)$

$h(x) = x - \mu$

Using chain rule

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

here, $\frac{df}{dz} = \frac{d}{dz}\left(e^{-z/2}\right) = -\frac{e^{-z/2}}{2}$

$$\frac{dz}{dy} = \frac{d}{dy}\left(y^T S^{-1} y\right)$$

$$= \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) S^{-1} (y+h) - y^T \cdot S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{(y^T S^{-1} + h S^{-1})(y+h) - y^T S^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{y^T \cdot S^{-1} y + y^T S^{-1} h + h S^{-1} y + h^2 S^{-1} y}{h}$$

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$$= \lim_{h \to 0} \left(g^T S^{-1} + S^{-1} g + h S^{-1} \right)$$

$$= g^T S^{-1} + S^{-1} y$$

$$\frac{dy}{dx} = \frac{d(x-\mu)}{dx} = 1$$

$$\therefore \frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= -\frac{e^{-z/2}}{2} \left(g^T S^{-1} + S^{-1} g \right) \cdot 1$$

$$= -\frac{e}{2} \cdot \frac{1}{S} \left(g^T + g \right)$$

(Ans.)

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