

comparing two proportions using Bayes factors and posterior probabilities

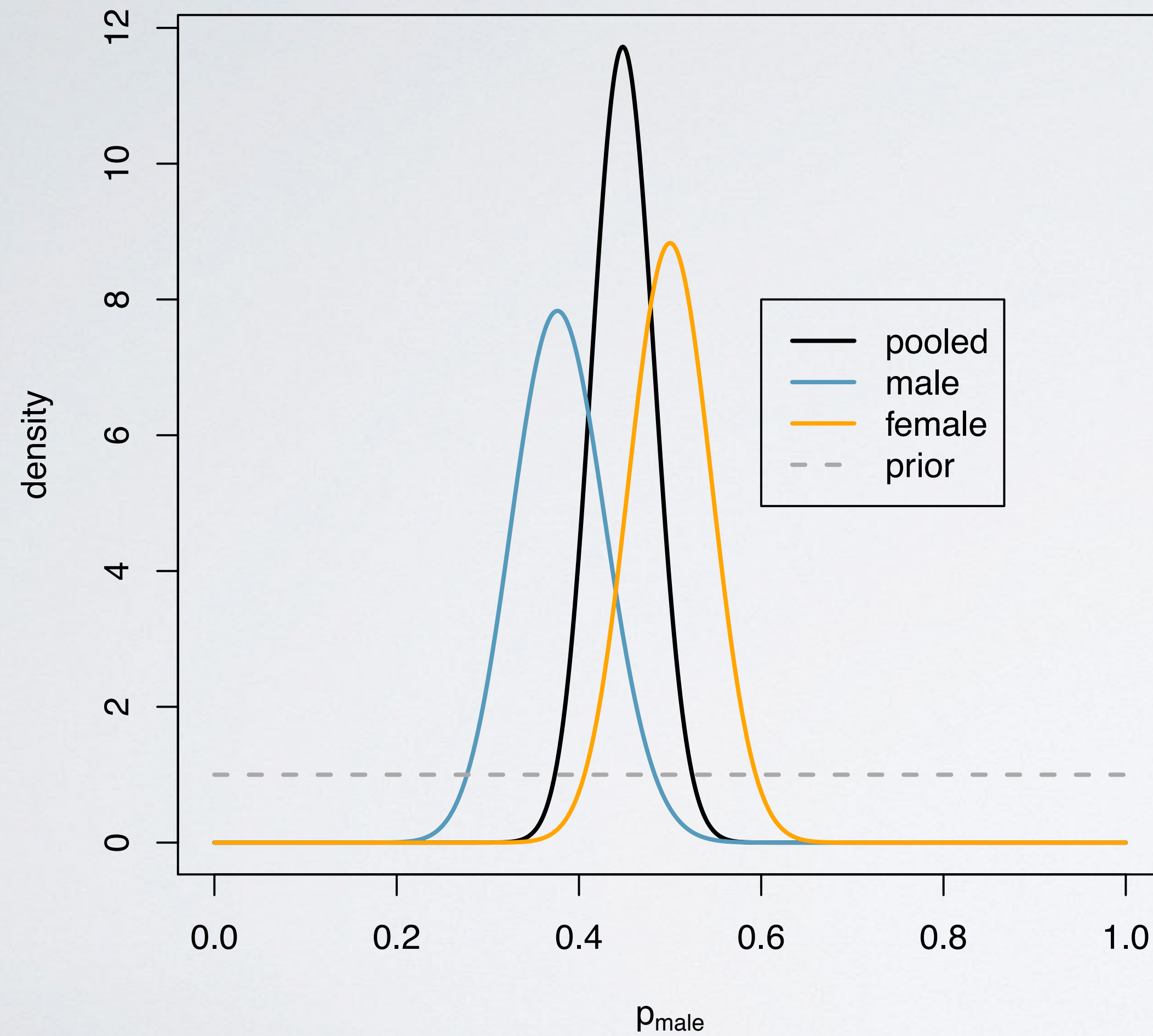
recap



$$H_1: p_{\text{male}} = p_{\text{female}}$$

$$H_2: p_{\text{male}} \neq p_{\text{female}}$$

pooled prior and posterior distributions under H_1



posterior probability of H_1

$$p(H_1 \mid \text{data}) = \frac{p(\text{data} \mid H_1)p(H_1)}{p(\text{data} \mid H_1)p(H_1) + p(\text{data} \mid H_2)p(H_2)} = \frac{PO[H_1:H_2]}{PO[H_1:H_2] + 1}$$

- ▶ posterior odds = Bayes factor \times prior odds

$$PO[H_1 : H_2] = BF[H_1 : H_2] \times O[H_1 : H_2]$$

- ▶ Bayes factor to compare H_1 to H_2

$$BF[H_1 : H_2] = \frac{p(\text{data} \mid H_1)}{p(\text{data} \mid H_2)}$$

- ▶ prior odds $O[H_1 : H_2] = \frac{p(H_1)}{1 - p(H_1)}$

Bayes factor for comparing two proportions

$$BF[H_1 : H_2] = \frac{B(a_m + a_f + R_m + R_f, b_m + b_f + n_m + n_f - (R_m + R_f))}{B(a_m + a_f, b_m + b_f)} \div$$
$$\frac{B(a_m + R_m, b_m + n_m - R_m)}{B(a_m, b_m)} \times \frac{B(a_f + R_f, b_f + n_f - R_f)}{B(a_f, b_f)}$$

- ▶ $B(a, b)$ is a special function called the Beta function

SurveyUSA example

- ▶ Bayes factor for comparing H_1 to H_2 is 2.93
- ▶ slight evidence in favor of H_1 but not “worth a bare mention”
- ▶ posterior probability of H_1 with equal prior odds

$$P(H_1 \mid \text{data}) = \frac{2.93}{1+2.93} = 0.75$$

- ▶ p-value ≈ 0.08

summary

- ▶ default prior distribution for testing
- ▶ posterior distributions under H_2 and H_1
- ▶ Bayes factors and posterior probabilities
- ▶ different prior distributions for p_{male} , p_{female} and p and $p(H_1)$
- ▶ inference with paired or matched normal samples