

# comparing two proportions using Bayes factors: assumptions

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# parent's perceptions on bullying



	male	female
yes	34	61
no	52	61
not sure	4	0
total	90	122

Note: in the following analyses, “no” and “not sure” are combined

$$H_1: p_{\text{male}} = p_{\text{female}}$$

$$H_2: p_{\text{male}} \neq p_{\text{female}}$$



# assumptions for inference about probabilities

- ▶ independence within groups
- ▶ independence between groups
- ▶ common response rate between groups under  $H_2$

$$P(\text{Response}_{i,\text{male}} = \text{Yes} | p_{\text{male}}, H_2) = p_{\text{male}}$$

$$P(\text{Response}_{i,\text{female}} = \text{Yes} | p_{\text{female}}, H_2) = p_{\text{female}}$$

- ▶ common response rate within groups under  $H_1$

$$P(\text{Response}_{i,\text{male}} = \text{Yes} | p, H_1) = p$$

$$P(\text{Response}_{i,\text{female}} = \text{Yes} | p, H_1) = p$$

- ▶ do not need to check sample size!

# conjugate prior and posterior distributions for beta-binomial

$$p_{\text{male}} \sim \text{Beta}(a_m, b_m)$$

$a_m$  prior number of males reporting bullying

$b_m$  prior number of males not reporting bullying

$a_m + b_m$  prior sample size

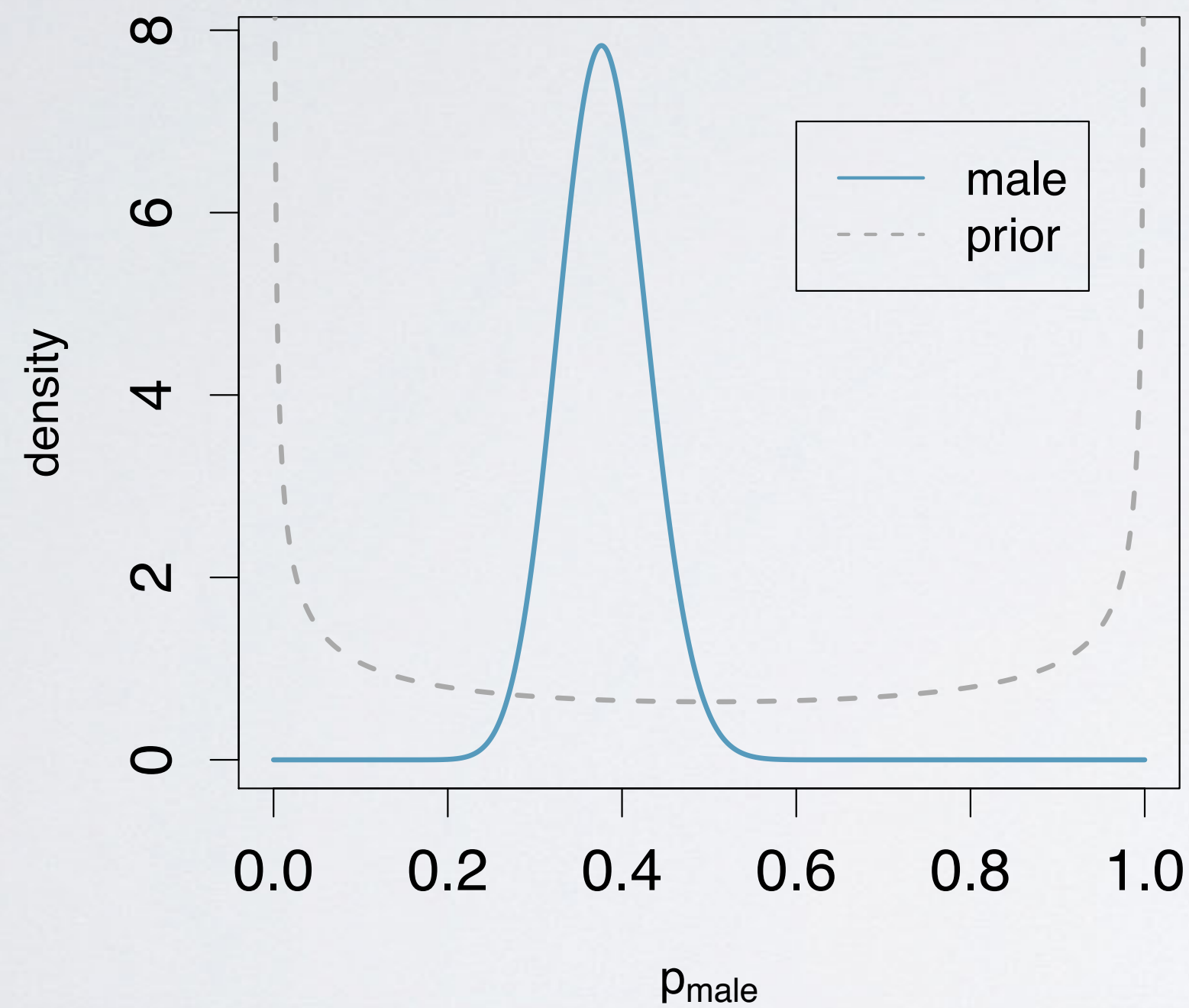
$$\Rightarrow p_{\text{male}} \mid \text{data} \sim \text{Beta}(R_m + a_m, n_m - R_m + b_m)$$



# prior and posterior distributions for males under $H_2$

prior  $p_{\text{male}} \sim \text{Beta}(1/2, 1/2)$

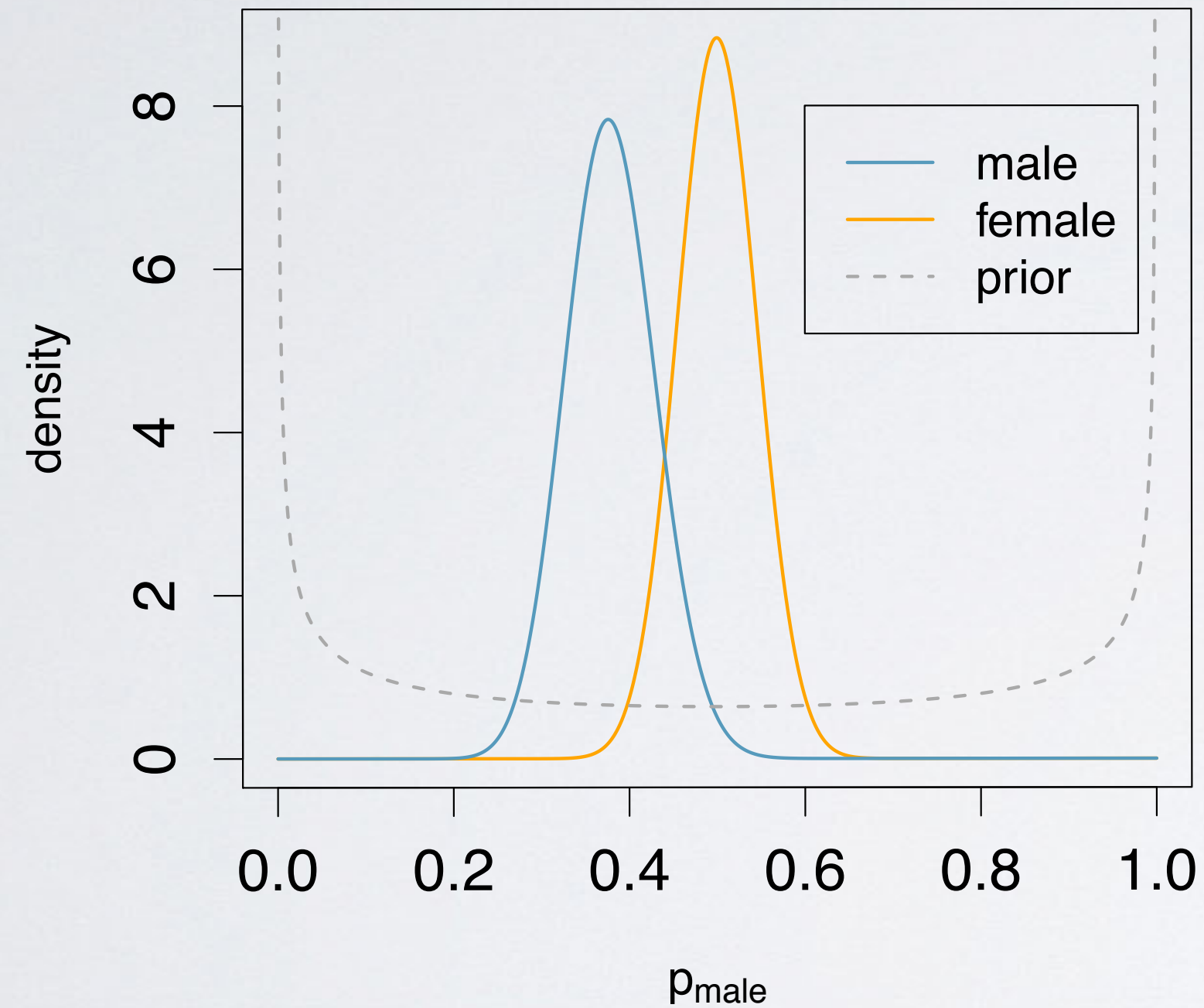
posterior  $p_{\text{male}} \mid \text{data} \sim \text{Beta}(34 + 1/2, 56 + 1/2)$



# prior and posterior distributions for females under $H_2$

prior  $p_{\text{female}} \sim \text{Beta}(1/2, 1/2)$

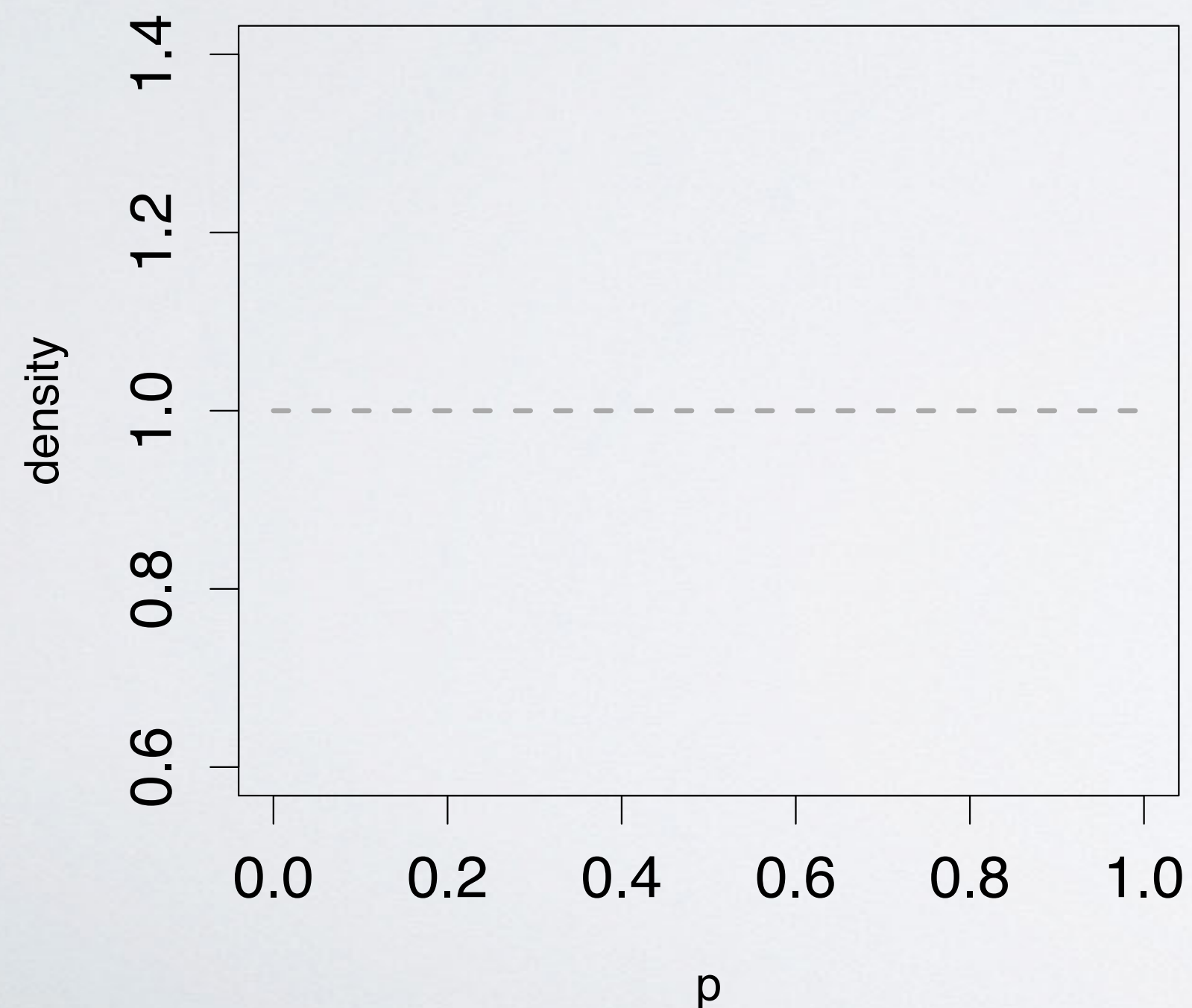
posterior  $p_{\text{female}} \mid \text{data} \sim \text{Beta}(61 + 1/2, 61 + 1/2)$





# pooled prior distributions under $H_1$

$$\begin{aligned} p(p) &\propto p^{a_m-1} (1-p)^{b_m-1} p^{a_f-1} (1-p)^{b_f-1} \\ &= p^{a_m+a_f-1} (1-p)^{b_m+b_f-1} \\ \Rightarrow p &\sim \text{Beta}(a_m + a_f, b_m + b_f) \\ p &\sim \text{Beta}(1/2 + 1/2, 1/2 + 1/2) \end{aligned}$$



# pooled prior and posterior distributions under $H_1$

	male	female
yes	34	61
no+not sure	56	61
total	90	122

prior  $p \sim \text{Beta}(1, 1)$

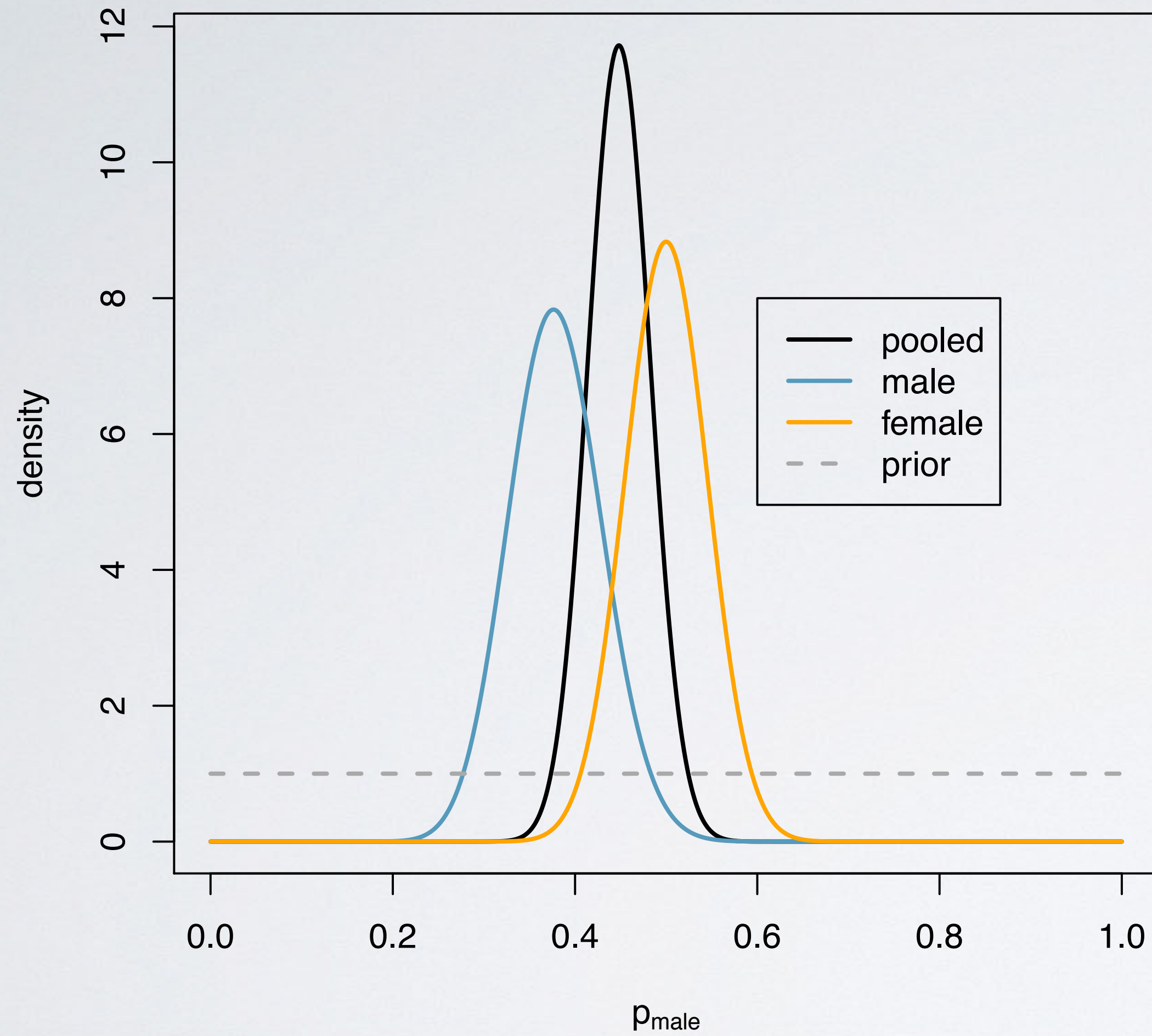
likelihood  $R_{\text{male}} \sim \text{Bin}(90, p)$   $R_{\text{female}} \sim \text{Bin}(122, p)$

posterior  $\propto p^{34}(1 - p)^{56} p^{61}(1 - p)^{61} p^{1-1}(1 - p)^{1-1}$

$p \mid \text{data} \sim \text{Beta}(95 + 1, 117 + 1)$



# pooled prior and posterior distributions under $H_1$





## summary

- ▶ inference with paired or matched normal samples
- ▶ default prior distributions under the two hypotheses
- ▶ posterior distributions for proportions under  $H_2$  and  $H_1$

next:

- ▶ Bayes factors and posterior probability