Ρομποτική Ι

Δεύτερη Σειρά Ασκήσεων

Αναστασία Χριστίνα Λίβα

03119029

Άσκηση 1

$$A_1^0(q_1) = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & c_1 & -s_1 & l_0 \\ 0 & s_1 & c_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2^1(q_2) = \begin{bmatrix} c_2 & -s_2 & 0 & -l_2 s_2 \\ s_2 & c_2 & 0 & l_2 c_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2(q_3) = \begin{bmatrix} c_3 & -s_3 & 0 & -l_3 s_3 \\ s_3 & c_3 & 0 & l_3 c_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

α) Έχω

$$A_2^0(q_1, q_2) = A_1^0(q_1) A_2^1(q_2) = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & c_1 & -s_1 & l_0 \\ 0 & s_1 & c_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & -l_2 s_2 \\ s_2 & c_2 & 0 & l_2 c_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} c_2 & -s_2 & 0 & -l_2 s_2 + l_1 \\ c_1 s_2 & c_1 c_2 & -s_1 & c_1 c_2 l_2 + l_0 \\ s_1 s_2 & c_2 s_1 & c_1 & s_1 c_2 l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^0 = A_2^0 A_3^2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_1 - s_2 l_2 \\ c_1 s_2 & c_1 c_2 & -s_1 & l_0 + c_1 c_2 l_2 \\ s_1 s_2 & c_2 s_1 & c_1 & l_2 c_2 s_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & -l_3 s_3 \\ s_3 & c_3 & 0 & l_3 c_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} c_2 c_3 - s_2 s_3 & -c_2 s_3 - s_2 c_3 & 0 & -l_3 s_3 c_2 - s_2 l_3 c_3 + l_1 - s_2 l_2 \\ c_1 c_3 s_2 + s_3 c_1 c_2 & -s_3 c_1 s_2 + c_3 c_1 c_2 & -s_1 & -l_3 s_3 s_2 c_1 + c_1 c_2 c_3 l_3 + l_0 + c_1 c_2 l_2 \\ s_1 s_2 c_3 + s_1 c_2 s_3 & -s_1 s_2 s_3 + s_1 c_2 c_3 & c_1 & -s_1 s_2 s_3 l_3 + c_2 c_3 s_1 l_3 + l_2 c_2 s_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} c_{23} & -s_{23} & 0 & l_1 - s_2 l_2 - l_3 s_{23} \\ c_1 s_{23} & c_1 c_{23} & -s_1 & l_0 + l_2 c_2 c_1 + c_1 c_{23} l_3 \\ s_1 s_{23} & s_1 c_{23} & c_1 & s_1 c_2 l_2 + l_3 s_1 c_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$b_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad b_1 = \begin{bmatrix} 0 \\ -s_1 \\ c_1 \end{bmatrix} \qquad b_2 = \begin{bmatrix} 0 \\ -s_1 \\ c_1 \end{bmatrix}$$

$$\begin{bmatrix} r_{03} \\ 0 \end{bmatrix} = A_3^0[1:3,4] - A_1^0[1:3,4] = \begin{bmatrix} r_{1,3} \\ 0 \end{bmatrix} = \begin{bmatrix} -l_2 s_2 - l_3 s_{23} \\ l_2 c_1 c_2 + l_3 c_1 c_{23} \\ s_1 s_2 c_2 + s_1 l_3 c_{23} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} r_{2,3} \\ 0 \end{bmatrix} = A_3^0[1:3,4] - A_2^0[1:3,4] = \begin{bmatrix} -l_3 s_{23} \\ l_3 c_1 c_{23} \\ l_3 s_1 c_{23} \\ 0 \end{bmatrix}$$

$$[J_{L_1}] = [b_0 \times r_{0,3}] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} -l_2 s_2 - l_3 s_{23} \\ + c_1 c_2 l_2 + c_1 c_{23} l_3 \\ s_1 c_2 l_2 + s_1 l_3 c_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ -l_2 c_2 s_1 - l_3 s_1 c_{23} \\ l_0 + l_2 c_1 c_2 + l_3 c_1 c_{23} \end{bmatrix}$$

$$[J_{L_2}] = [b_1 \times r_{1,3}] = \begin{bmatrix} 0 \\ -s_1 \\ c_1 \end{bmatrix} \times \begin{bmatrix} -l_2 s_2 - l_3 s_{23} \\ c_1 c_2 l_2 + c_1 c_{23} l_3 \\ s_1 c_2 l_2 + s_1 c_{23} l_3 \end{bmatrix} = \begin{bmatrix} -l_2 c_2 - l_3 c_{23} \\ -l_2 s_2 c_1 - l_3 c_1 s_{23} \\ -l_2 s_2 s_1 - l_3 s_{23} s_1 \end{bmatrix}$$

$$[J_{L_3}] = [b_2 \times r_{2,3}] = \begin{bmatrix} 0 \\ -s_1 \\ c_1 \end{bmatrix} \times \begin{bmatrix} -l_3 s_{23} \\ l_3 c_1 c_{23} \\ l_3 s_1 c_{23} \end{bmatrix} = \begin{bmatrix} -l_3 c_{23} \\ -c_1 l_3 s_{23} \\ -s_1 l_3 s_{23} \end{bmatrix}$$

$$J[A_1] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad J[A_2] = \begin{bmatrix} 0 \\ -s_1 \\ c_1 \end{bmatrix} \quad J[A_3] = \begin{bmatrix} 0 \\ -s_1 \\ c_1 \end{bmatrix}$$

Άρα

$$J = \begin{bmatrix} 0 & -l_2 c_2 - l_3 c_{23} & -c_{23} l_3 \\ -l_2 s_1 c_2 - l_3 s_1 c_{23} & c_1(-l_2 s_2 - l_3 s_{23}) & -c_1 l_3 s_{23} \\ c_1 c_2 l_2 + c_1 c_{23} l_3 & s_1(-l_2 s_2 - s_{23} l_3) & -s_1 s_{23} l_3 \\ 1 & 0 & 0 \\ 0 & -s_1 & -s_1 \\ 0 & c_1 & c_1 \end{bmatrix}$$

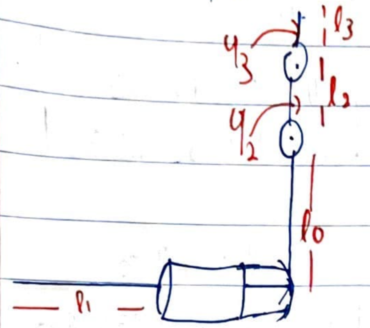
β) Θέτω $\det(J(q)) = 0 \Rightarrow$

$$\begin{vmatrix} 0 & -l_2 c_2 - l_3 c_{23} & -c_{23} l_3 \\ -l_2 s_1 c_2 - s_1 c_{23} l_3 & c_1(-l_2 s_2 - l_3 s_{23}) & -c_1 l_3 s_{23} \\ l_0 + c_1 c_2 l_2 + c_1 c_{23} l_3 & s_1(-l_2 s_2 - s_{23} l_3) & -s_1 s_{23} l_3 \end{vmatrix} = 0$$

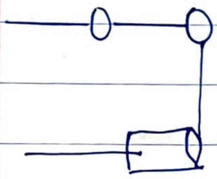
Θέτω $\left.\begin{matrix} -l_2 c_2 - c_{23} l_3 = 0 \\ -l_3 c_{23} = 0 \end{matrix}\right\} \Rightarrow$ $c_2 = 0$ άρα $q_2 = 90°$ ή $270°$

$c_2 c_3 - s_2 s_3 = 0 \Rightarrow s_3 = 0 \Rightarrow q_3 = 0°$ ή $q_3 = 180°$

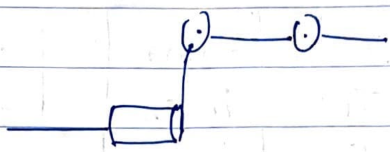
Εχω τρεις ιδιομορφες διατάξεις για $q_2 = 90^\circ$ και $q_3 = 0^\circ$ ή 180°, $q_2 = 270^\circ$ και $q_3 = 0^\circ$



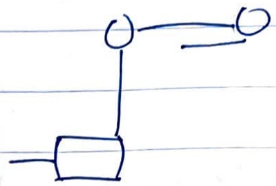
$q_2 = 270^\circ$ $q_3 = 0^\circ$



$q_2 = 90^\circ$ $q_3 = 0^\circ$



$q_2 = 90^\circ$ $q_3 = 90^\circ$



Άσκηση 2

a) $b_0 = \hat{z}_0$

	d	θ	a_i	α_i
q1	q_1	0	0	0
q2	l_1	q_2	0	$\pi/2$
q3	0	q_3	0	$-\pi/2$
q4	l_2	q_4	0	0

$$A_1^0(q_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2^1(q_2) = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^0 = A_1^0 A_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & l_1+q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2(q_3) = \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^0 = A_2^0 A_3^2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & l_1+q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_2c_3 & -s_2 & -c_2s_3 & 0 \\ s_2c_3 & c_2 & -s_2s_3 & 0 \\ s_3 & 0 & c_3 & l_1+q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^3(q_4) = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^0 = \begin{bmatrix} C_2C_3C_4 - S_2S_4 & -C_2C_3S_4 - C_1S_4 & -C_2S_3 & -l_2C_2S_3 \\ S_2C_3C_4 + C_2C_4 & -S_2C_3S_4 + C_2C_4 & -S_2S_3 & -l_2S_2S_3 \\ S_2C_4 & -S_3S_4 & C_3 & l_1+q_1+l_2C_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^0 = \begin{bmatrix} C_2C_3C_4 - S_2S_4 & -C_2C_3S_4 - S_2C_1 & -C_2S_3 & -l_2C_2S_3 \\ S_2C_3C_4 + C_2C_4 & -S_2C_3S_4 + C_2C_4 & -S_2S_3 & -l_2S_2S_3 \\ S_3C_4 & -S_3S_4 & C_3 & l_1+q_1+l_2C_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Πρισματική q_1: $\begin{bmatrix} J_{L1} \\ J_{A1} \end{bmatrix} = \begin{bmatrix} b_0 \\ 0_{(3\times3)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Στροφική q_2: $\begin{bmatrix} J_{L2} \\ J_{A2} \end{bmatrix} = \begin{bmatrix} b_1 \times r_{1,E} \\ b_1 \end{bmatrix}$

$r_{1,E} = r_{0,E} - r_{0,1} = \begin{bmatrix} -l_2C_2S_3 \\ -l_2S_2S_3 \\ l_1 + l_2C_3 \end{bmatrix}$ $\quad b_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$b_1 \times r_{1,E} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -l_2C_2S_3 \\ -l_2S_2S_3 \\ l_1 + l_2C_3 \end{bmatrix} = \begin{bmatrix} -l_2S_2S_3 \\ l_2C_2S_3 \\ 0 \end{bmatrix}$

Οπότε $\begin{bmatrix} J_{L2} \\ J_{A2} \end{bmatrix} = \begin{bmatrix} -l_2S_2S_3 \\ l_2C_2S_3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Στροφική q_3 $\begin{bmatrix} J_{L3} \\ J_{A3} \end{bmatrix} = \begin{bmatrix} b_2 \times r_{2,E} \\ b_2 \end{bmatrix}$ $r_{2,E} = r_{0,E} - r_{0,2} = \begin{bmatrix} -l_2 C_2 S_3 \\ -l_2 S_2 S_3 \\ l_2 C_3 \end{bmatrix}$ $b_2 = \begin{bmatrix} S_2 \\ -C_2 \\ 0 \end{bmatrix}$

$$b_2 \times r_{2,E} = \begin{bmatrix} S_2 \\ -C_2 \\ 0 \end{bmatrix} \times \begin{bmatrix} -l_2 C_2 S_3 \\ -l_2 S_2 S_3 \\ l_2 C_3 \end{bmatrix} = \begin{bmatrix} -l_2 C_2 C_3 \\ -l_2 S_2 C_3 \\ -l_2 S_3 \end{bmatrix}$$

Άρα $\begin{bmatrix} J_{L3} \\ J_{A3} \end{bmatrix} = \begin{bmatrix} -l_2 C_2 C_3 \\ -l_2 S_2 C_3 \\ -l_2 S_3 \\ S_2 \\ -C_2 \\ 0 \end{bmatrix}$

Στροφική q_4 $\begin{bmatrix} J_{L4} \\ J_{A4} \end{bmatrix} = \begin{bmatrix} b_3 \times r_{3,E} \\ b_3 \end{bmatrix}$

$r_{3,E} = r_{0,E} - r_{0,3} = \begin{bmatrix} -l_2 C_2 S_3 \\ -l_2 S_2 S_3 \\ l_2 C_3 \end{bmatrix}$ $b_3 = \begin{bmatrix} -C_2 S_3 \\ -S_2 S_3 \\ C_3 \end{bmatrix}$

$$b_3 \times r_{3,E} = \begin{bmatrix} -C_2 S_3 \\ -S_2 S_3 \\ C_3 \end{bmatrix} \times \begin{bmatrix} -l_2 C_2 S_3 \\ -l_2 S_2 S_3 \\ l_2 C_3 \end{bmatrix} = l_2 \begin{bmatrix} -C_2 S_3 \\ -S_2 S_3 \\ C_3 \end{bmatrix} \times \begin{bmatrix} -C_2 S_3 \\ -S_2 S_3 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

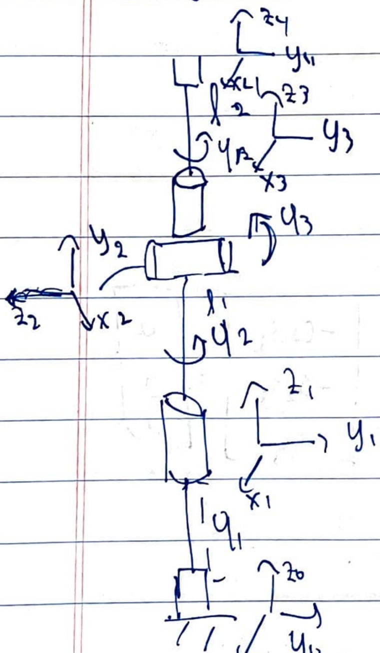
Άρα $\begin{bmatrix} J_{L4} \\ J_{A4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -C_2 S_3 \\ -S_2 S_3 \\ C_3 \end{bmatrix}$ $J = \begin{bmatrix} J_{L1} & J_{L2} & J_{L3} & J_{L4} \\ J_{A1} & J_{A2} & J_{A3} & J_{A4} \end{bmatrix} \Rightarrow$

$$
J = \begin{bmatrix}
0 & -l_2 s_2 s_3 & -l_2 c_2 c_3 & 0 \\
0 & l_2 c_2 s_3 & -l_2 s_2 c_3 & 0 \\
1 & 0 & -l_2 s_3 & 0 \\
0 & 0 & s_2 & -c_2 s_3 \\
0 & 0 & -c_2 & -s_2 s_3 \\
0 & 1 & 0 & c_3
\end{bmatrix}
$$

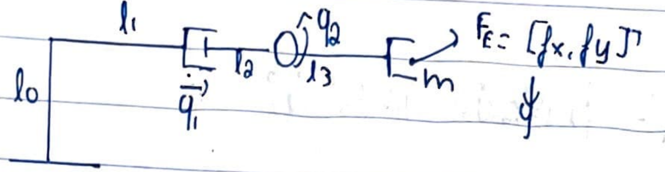
β1 Η q_1 δεν επηρεάζει τη γωνιακή ταχύτητα του end effector οπότε οι ιδιόμορφες διατάξεις βρίσκονται από τη μήτρα J_A'

$$
J_A' = \begin{bmatrix}
0 & s_2 & -c_2 s_3 \\
0 & -c_2 & -s_2 s_3 \\
1 & 0 & c_3
\end{bmatrix}
\qquad \det(J_A') = -s_2^2 s_3 - c_2^2 s_3 = -s_3
$$

$\det(J_A') = 0 \Rightarrow s_3 = 0 \Rightarrow q_3 = \pm k\pi$ ευθυγραμμισμένα joints



Άσκηση 2



Για το m: $p_x = l_1 + q_1 + l_2 + c_2 l_3$ $\quad p_y = l_0 + s_2 l_3$ $\quad p_z = 0$

$$v_x = \dot q_1 - l_3 s_2 \dot q_2 \quad v_y = c_2 \dot q_2 l_3 \quad v_z = 0 \quad w_z = \dot q_2$$

$$v_q = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(\dot q_1 - l_3 s_2 \dot q_2)^2 + c_2^2 (\dot q_2)^2 l_3^2} =$$

$$\sqrt{(\dot q_1)^2 + l_3^2 s_2^2 (\dot q_2)^2 - 2\dot q_1 \dot q_2 l_3 s_2 + c_2^2 (\dot q_2)^2 l_3^2} \Rightarrow$$

$$v_q^2 = (\dot q_1)^2 - 2\dot q_1 \dot q_2 l_3 s_2 + l_3^2 (\dot q_2)^2 \quad w_{oz}^2 = (\dot q_2)^2$$

Οπότε

$$K = \frac{1}{2} m \left((\dot q_1)^2 - 2\dot q_1 \dot q_2 l_3 s_2 + l_3^2 (\dot q_2)^2\right)^2 + \frac{1}{2} I m (\dot q_2)^2$$

$$P = mg(l_0 + l_3 s_2)$$

$$\tau_i = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot q_1}\right) - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i}$$

Για $i=1$:

$$\frac{\partial K}{\partial q_1} = 0 \quad \frac{\partial P}{\partial q_1} = 0 \quad \frac{\partial K}{\partial \dot q_1} = \frac{1}{2} m (2\dot q_1 - 2 l_3 s_2 \dot q_2)$$

$(\dot q_1)^2$

$$\frac{\partial}{\partial t}\left(\frac{\partial K}{\partial \dot q_1}\right) = \frac{1}{2} m (2\ddot q_1 - 2 l_3 c_2 (\dot q_2)^2 - 2 l_3 s_2 \ddot q_2) = m \ddot q_1 - m l_3 c_2 (\dot q_2)^2 - l_3 s_2 \ddot q_2 m$$

Για i=2:

$$\frac{\partial K}{\partial q_2} = \frac{1}{2} m(-2\dot{q}_1 l_3 c_2 \dot{q}_2) = -m\dot{q}_1 l_3 c_2 \dot{q}_2$$

$$\frac{\partial P}{\partial q_2} = mg l_3 c_2$$

$$\frac{\partial K}{\partial \dot{q}_2} = \frac{1}{2} m\left(-2\dot{q}_1 l_3 s_2 + 2l_3^2 \dot{q}_2\right) + I_m \dot{q}_2$$

$$\frac{\partial}{\partial t}\left(\frac{\partial K}{\partial \dot{q}_2}\right) = \frac{1}{2} m\left(-2\ddot{q}_1 l_3 s_2 - 2\dot{q}_1 l_3 c_2 \dot{q}_2 + 2l_3^2 \ddot{q}_2\right) + I_m \ddot{q}_2 =$$

$$m\left(-\ddot{q}_1 l_3 s_2 - \dot{q}_1 l_3 c_2 \dot{q}_2 + l_3^2 \ddot{q}_2\right) + I_m \ddot{q}_2$$

$$T = -J^T F \qquad J = \begin{bmatrix} 1 & -l_3 s_2 \\ 0 & l_3 c_2 \end{bmatrix} \qquad -J^T = \begin{bmatrix} -1 & 0 \\ l_3 s_2 & -l_3 c_2 \end{bmatrix}$$

$$T_1 = -F_x$$

$$T_2 = l_3 s_2 F_x - l_3 c_2 F_y$$

Άρα

$$T_1 = m\ddot{q}_1 - m l_3 s_2 \ddot{q}_2 - m l_3 c_2 (\dot{q}_2)^2 - f_x$$

$$T_2 = m(-\ddot{q}_1 l_3 s_2 + l_3^2 \ddot{q}_2) + I_m \ddot{q}_2 + mg l_3 c_2 +$$

$$l_3 s_2 F_x - l_3 c_2 F_y$$

Τελικά:

$$T_1 = m\ddot{q}_1 - m l_3 s_2 \ddot{q}_2 - m l_3 c_2 (\dot{q}_2)^2 - f_x$$

$$T_2 = m(-\ddot{q}_1 l_3 s_2 + l^3 \ddot{q}_2) + I_m \ddot{q}_2 + mg l_3 c_2 + l_3 s_2 F_x - l_3 c_2 f_y$$