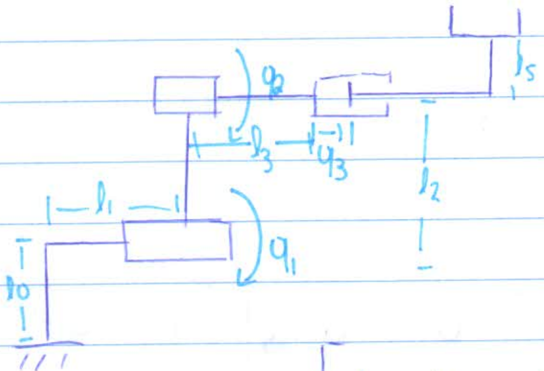
Ρομποτική Ι 2022 - 2023

Πρώτη Σειρά Αναλυτικών Ασκήσεων

Αναστασία Χριστίνα Λίβα
03119024

Άσκηση 1



α) Βάσει του σχήματος:

$$A_E^0 = [Tra(z,l_0)\cdot Tra(x,l_1)]\,[Rot(x,q_1)\,Tra(z,l_2)]$$
$$[Rot(x,q_2)\,Tra(x,l_3)]\,[Tra(x,q_3+l_4)\,Tra(z,l_5)]$$

$$Tra(z,l_0)\,Tra(x,l_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot(x,q_1)\,Tra(z,l_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_1 & -s_1 & 0 \\ 0 & s_1 & c_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_1 & -s_1 & -l_2 s_1 \\ 0 & s_1 & c_1 & l_2 c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot(x, q_2)\,Tra(x, l_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Tra(x, q_3+l_4)\,Tra(z, l_5) = \begin{bmatrix} 1 & 0 & 0 & q_3+l_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & q_3+l_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_1 & -s_1 & -l_2 s_1 \\ 0 & s_1 & c_1 & l_2 c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & c_1 & -s_1 & -l_2 s_1 \\ 0 & s_1 & c_1 & l_2 c_1 + l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & C_1 & -S_1 & -l_2 S_1 \\ 0 & S_1 & C_1 & l_2 C_1 + l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & C_2 & -S_2 & 0 \\ 0 & S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & l_1 + l_3 \\ 0 & C_1 C_2 - S_1 S_2 & -C_1 S_2 - S_1 C_2 & -S_1 l_2 \\ 0 & S_1 C_2 + C_1 S_2 & -S_1 S_2 + C_1 C_2 & l_2 C_1 + l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & l_1 + l_3 \\ 0 & C_1 C_2 - S_1 S_2 & -C_1 S_2 - S_1 C_2 & -S_1 l_2 \\ 0 & S_1 C_2 + C_1 S_2 & -S_1 S_2 + C_1 C_2 & l_2 C_1 + l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & q_3 + l_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & q_3 + l_4 + l_1 + l_3 \\ 0 & C_1 C_2 - S_1 S_2 & -C_1 S_2 - S_1 C_2 & l_5 (-C_1 S_2 - S_1 C_2) - S_1 l_2 \\ 0 & S_1 C_2 + C_1 S_2 & -S_1 S_2 + C_1 C_2 & l_5 (-S_1 S_2 + C_1 C_2) + l_2 C_1 + l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & q_3 + l_1 + l_3 + l_4 \\ 0 & C_{12} & -S_{12} & -l_5 S_{12} - S_1 l_2 \\ 0 & S_{12} & C_{12} & l_5 C_{12} + l_2 C_1 + l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b)

$$\bar{P}_E = \begin{bmatrix} P_{Ex} \\ P_{Ey} \\ P_{Ez} \end{bmatrix} = \begin{bmatrix} q_3 + l_1 + l_3 + l_4 \\ -S_{12} l_5 - S_1 l_2 \\ C_1 l_2 + l_0 + C_{12} l_5 \end{bmatrix} \Rightarrow \begin{cases} P_{Ex} = q_3 + l_1 + l_3 + l_4 & (1) \\ P_{Ey} = -l_5 \sin(q_1 + q_2) - \sin q_1 l_2 & (2) \\ P_{Ez} = \cos q_1 l_2 + l_0 + l_5 \cos(q_1 + q_2) & (3) \end{cases}$$

(1) $\Rightarrow q_3 = p_{Ex} - (l_1 + l_3 + l_4)$

(2) $\Rightarrow p_{Ey}^2 = l_5^2 \sin^2(q_1+q_2) + \sin^2(q_1)\, l_2^2 + 2 l_2 l_5 \sin(q_1+q_2)\sin q_1$ (4)

(3) $\Rightarrow p_{Ez}^2 = \cos^2 q_1\, l_2^2 + 2\cos q_1\, l_0 l_2 + l_0^2 + l_5^2 \cos^2(q_1+q_2) + 2(\cos(q_1 l_2 + l_0))\, l_5 \cos(q_1+q_2)$ (5)

(4)+(5): $p_{Ey}^2 + p_{Ez}^2 = l_5^2 + 2l_2 l_5 c_1 c_{12} + 2 l_0 l_5 c_{12} + 2 l_0 l_2 c_1 + l_2^2 c_1^2 + l_0^2 +$
$l_2^2 s_1^2 + 2 l_2 l_5 s_1 s_{12} = l_2^2 + l_5^2 + 2 l_2 l_5 c_2 + l_0^2 + 2 l_0 l_5 c_{12} + 2 l_0 l_2 c_1$ (6)

(6) $- 2l_0$ (3) $\Rightarrow p_{Ey}^2 + p_{Ez}^2 - 2 l_0 p_{Ez} = l_2^2 + l_5^2 + 2 l_2 l_5 c_2 + l_0^2 + 2 l_0 \cancel{l_5} c_{12} +$
$2 l_0 \cancel{l_2 c_1} - 2 l_0 \cancel{l_5} c_{12} - 2 \cancel{l_0} l_2 c_1 - 2 l_0^2 \Rightarrow$

$p_{Ey}^2 + p_{Ez}^2 - 2 l_0 p_{Ez} + l_0^2 = l_2^2 + l_5^2 + 2 l_5 l_2 c_2 \Rightarrow$

$$c_2 = \frac{p_{Ey}^2 + (p_{Ez} - l_0)^2 - l_2^2 - l_5^2}{2 l_2 l_5}$$

$$q_2 = \pm \arccos\left(\frac{p_{Ey}^2 + (p_{Ez} - l_0)^2 - l_2^2 - l_5^2}{2 l_2 l_5}\right) \Rightarrow s_2 = \pm\sqrt{1 - c_2^2} \Rightarrow$$

$q_2 = \operatorname{atan2}(s_2, c_2)$

Από (2),(3):

$$\left.\begin{aligned} p_{Ey} &= -l_5 s_{12} - s_1 l_2 \\ p_{Ez} &= c_1 l_2 + l_0 + l_5 c_{12} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} -p_{Ey} &= l_5 (s_1 c_2 + c_1 s_2) + s_1 l_2 \\ p_{Ez} &= c_1 l_2 + l_0 + l_5 (c_1 c_2 - s_1 s_2) \end{aligned}\right\} \Rightarrow$$

$(l_2 + l_5 c_2) s_1 + c_1 s_2 l_5 = -p_{Ey}$

$-l_5 s_1 s_2 + (l_2 + l_5 c_2) c_1 = p_{Ez} - l_0$

$$\text{Έστω} \quad \left.\begin{aligned} d_1 &= l_2 + c_2 l_5 \\ d_2 &= l_5 s_2 \\ d &= \sqrt{d_1^2 + d_2^2} \\ a &= \operatorname{atan2}(d_2, d_1) \end{aligned}\right\} \Rightarrow \begin{aligned} d_1 &= d\cos a = d c_a \\ d_2 &= d \sin a = d s_a \\ a &= \operatorname{atan2}(l_2 + l_5 c_2, l_5 s_2) \end{aligned}$$

Προκύπτει:

$$d c_a s_1 + d s_a c_1 = -p_{Ey} \Rightarrow \begin{cases} d\cos(a+q_1) = -p_{Ey} \\ d\sin(a+q_1) = p_{Ez} - l_0 \end{cases}$$
$$-d s_a s_1 + d c_a c_1 = p_{Ez} - l_0$$

$$q_1 = \mathrm{atan2}(p_{Ez} - l_0, -p_{Ey}) - a$$

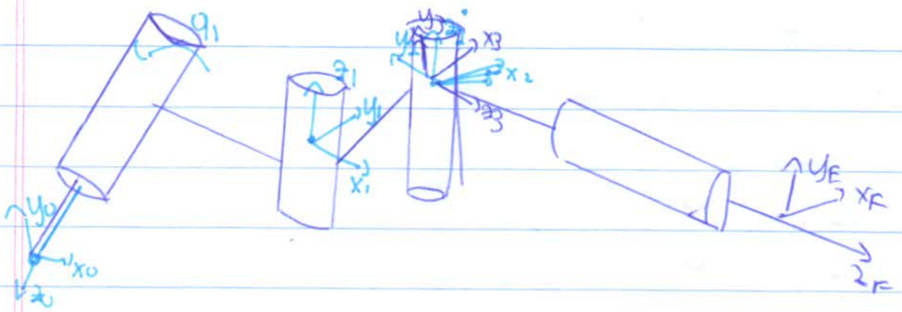
Τελικά έχω τις εξής δύο λύσεις

$$\begin{cases} q_{1+} = \mathrm{atan2}(p_{Ez} - l_0, -p_{Ey}) - \mathrm{atan2}(l_5 + l_5 c_2, l_2\sqrt{1-c_2^2}) \\ q_{2+} = \arccos\left(\dfrac{p_{Ey}^2 + (p_{Ez} - l_0)^2 - l_2^2 - l_5^2}{2 l_2 l_5}\right) \\ q_3 = p_{Ex} - l_1 - l_3 - l_4 \end{cases}$$

$$\begin{cases} q_{1-} = \mathrm{atan2}(p_{Ez} - l_0, -p_{Ey}) - \mathrm{atan2}(l_5 + l_5 c_2, -l_2\sqrt{1-c_2^2}) \\ q_{2-} = -\arccos\left(\dfrac{p_{Ey}^2 + (p_{Ez} - l_0)^2 - l_2^2 - l_5^2}{2 l_2 l_5}\right) \\ q_3 = p_{Ex} - l_1 - l_3 - l_4 \end{cases}$$

$$c_2 = \frac{p_{Ey}^2 + (p_{Ex} - l_0^2) - l_2^2 - l_5^2}{2 l_2 l_5}$$

Ασκηση 2



a) i) Ο αξονας z ειναι ο αξονας περιστροφης για καθε περιστροφικη αρθρωση

ii) Ο αξονας x πρεπει να ειναι καθετος στο δικο του αξονα z και στον προηγουμενο

iii) Ο αξονας y καθοριζεται απο τους αξονες x και z συμφωνα με το δεξιοστροφο συστημα συντεταγμενων

iv) Ο αξονας x πρεπει να τεμνει τον αξονα z του προηγουμενου πλαισιου

Προκυπτει ο εξης πινακας DH

	θ_i	d_i	α_i	a_i
1	q_1	$-l_1$	l_2	$-\pi/2$
2	$q_2+\pi/2$	0	l_3	0
3	q_3	0	0	$\pi/2$
4	0	$q_4+l_4+l_5$	0	0

θ_i: γωνια x_{i-1}, x_i ($\sim z_{i-1}$)

d_i: αποσταση x_{i-1}, x_i ($\sim z_{i-1}$)

a_i: γωνια z_{i-1}, z_i ($\sim x_i$)

α_i: αποσταση z_{i-1}, z (x_i)

b)

$$A_1^0 = \begin{bmatrix} c_1 & -s_1 - \cos(-\pi/2) & s_1 \sin(-\pi/2) & c_1 l_2 \\ s_1 & c_1 \cos(-\pi/2) & -c_1 \sin(-\pi/2) & s_1 l_2 \\ 0 & \sin(-\pi/2) & \cos(-\pi/2) & -l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

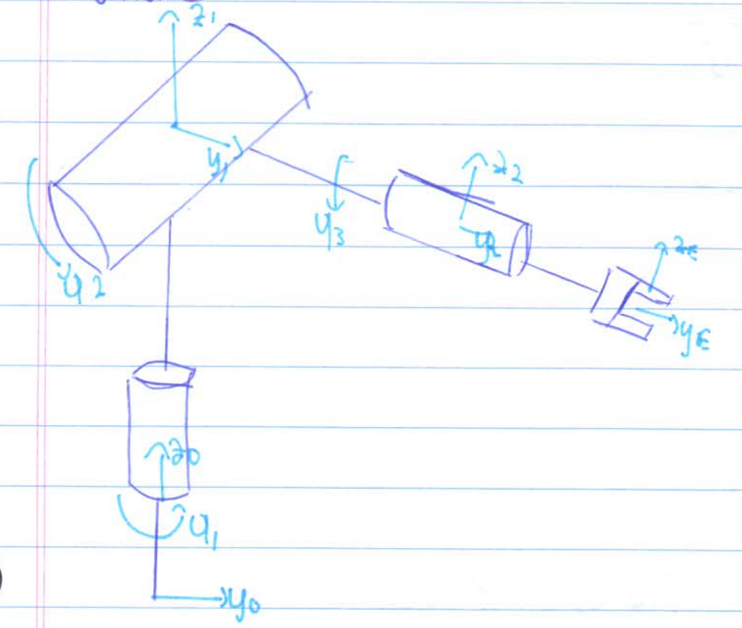
$$
\begin{bmatrix}
c_1 & 0 & -s_1 & l_2 c_1 \\
s_1 & 0 & c_1 & l_2 s_1 \\
0 & -1 & 0 & -c_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
A_2^1 = \begin{bmatrix}
-s_2 & -c_2 & 0 & -l_3 s_2 \\
c_2 & -s_2 & 0 & c_2 l_3 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

Τελικά:

$$
A_2^0 = A_1^0 A_2^1 = \begin{bmatrix}
-c_1 s_2 & -c_1 c_2 & -s_1 & -l_3 c_1 s_2 + l_2 c_1 \\
-s_1 s_2 & -s_1 c_2 & c_1 & -l_3 s_1 s_2 + l_2 s_1 \\
-c_2 & s_2 & 0 & -l_3 c_2 - l_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

Άσκηση 3



$$R_E^0(q_1, q_2, q_3) = R_z(q_1) R_x(q_2) R_y(q_3)$$

Άρα $R_E^0(q_1, q_2, q_3) = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{bmatrix} \begin{bmatrix} c_3 & 0 & s_3 \\ 0 & 1 & 0 \\ -s_3 & 1 & c_3 \end{bmatrix} =$

$$\begin{bmatrix} c_1 & -s_1 c_2 & s_1 s_2 \\ s_1 & c_1 c_2 & -c_1 s_2 \\ 0 & s_2 & c_2 \end{bmatrix} \begin{bmatrix} c_3 & 0 & s_3 \\ 0 & 1 & 0 \\ -s_3 & 0 & c_3 \end{bmatrix} =$$

$$\begin{bmatrix} c_1 c_3 - s_1 s_2 s_3 & -s_1 c_2 & c_1 s_3 + s_1 s_2 c_3 \\ s_1 c_3 + c_1 s_2 s_3 & c_1 c_2 & s_1 s_3 - c_1 s_2 c_3 \\ -c_2 s_3 & s_2 & c_2 c_3 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} \hat{n} & \hat{o} & \hat{a} \end{array} \\ \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix} \end{array} = R_E^0$$

Εν γενει για [illegible] με στροφικες αρθρωσεις εχω: $q_i = atan2(s_i, c_i)$

$\hat{o} = \begin{bmatrix} -s_1 c_2 \\ c_1 c_2 \\ s_2 \end{bmatrix}$ η $\quad c_2 = \pm\sqrt{o_x^2 + o_y^2}$

$\qquad s_2 = o_z$

Ετσι εχω δυο λυσεις για το q_2.

$$q_{2+} = \operatorname{atan2}(o_z, \sqrt{o_x^2 + o_y^2})$$
$$q_{2-} = \operatorname{atan2}(o_z, -\sqrt{o_x^2 + o_y^2})$$

Έχοντας βρει το c_2 από $[o_x, o_y]^T$ έχω

$$\left.\begin{aligned} s_1 c_2 &= -o_x \\ c_1 c_2 &= o_y \end{aligned}\right\} \quad \begin{aligned} s_1 &= -o_x/c_2 \\ c_1 &= o_y/c_2 \end{aligned}$$

Οπότε:

$$q_{1+} = \operatorname{atan2}(-o_x, o_y)$$ Για $c_2 = 0$ $\nexists$ λύση
$$q_{1-} = \operatorname{atan2}(o_x, -o_y)$$

Για να βρω q_3:

$$\begin{aligned} n_z &= -c_2 s_3 \\ a_z &= c_2 c_3 \end{aligned} \Rightarrow \begin{aligned} s_3 &= -n_z/c_2 \\ c_3 &= a_z/c_2 \end{aligned}$$

Άρα $q_3 = \operatorname{atan2}\left(\frac{-n_z}{c_2}, \frac{a_z}{c_2}\right)$

$$q_{3+} = \operatorname{atan2}(-n_z, a_z)$$
$$q_{3-} = \operatorname{atan2}(n_z, -a_z)$$