In mathematics, **Fourier analysis** (English pronunciation: /ˈfʊərieɪ/) is the study of the way general functions may be represented or approximated by sums of simpler trigonometric functions. Fourier analysis grew from the study of Fourier series, and is named after Joseph Fourier, who showed that representing a function as a sum of trigonometric functions greatly simplifies the study of heat transfer.

Today, the subject of Fourier analysis encompasses a vast spectrum of mathematics. In the sciences and engineering, the process of decomposing a function into oscillatory components is often called Fourier analysis, while the operation of rebuilding the function from these pieces is known as **Fourier synthesis**. For example, determining what component frequencies are present in a musical note would involve computing the Fourier transform of a sampled musical note. One could then re-synthesize the same sound by including the frequency components as revealed in the Fourier analysis. In mathematics, the term *Fourier analysis* often refers to the study of both operations.

The decomposition process itself is called a Fourier transformation. Its output, the Fourier transform, is often given a more specific name, which depends on the domain and other properties of the function being transformed. Moreover, the original concept of Fourier analysis has been extended over time to apply to more and more abstract and general situations, and the general field is often known as harmonic analysis. Each transform used for analysis (see list of Fourier-related transforms) has a corresponding inverse transform that can be used for synthesis.

Fourier analysis has many scientific applications – in physics, partial differential equations, number theory, combinatorics, signal processing, digital image processing, probability theory, statistics, forensics, option pricing, cryptography, numerical analysis, acoustics, oceanography, sonar, optics, diffraction, geometry, protein structure analysis, and other areas.

This wide applicability stems from many useful properties of the transforms:

• The transforms are linear operators and, with proper normalization, are unitary as well (a property known as Parseval's theorem or, more generally, as the Plancherel theorem, and most generally via Pontryagin duality) (Rudin 1990).

• The transforms are usually invertible.

• The exponential functions are eigenfunctions of differentiation, which means that this representation transforms linear differential equations with constant coefficients into ordinary algebraic ones (Evans 1998). Therefore, the behavior of a linear time-invariant system can be analyzed at each frequency independently.

• By the convolution theorem, Fourier transforms turn the complicated convolution operation into simple multiplication, which means that they provide an efficient way to compute convolution-based operations such as polynomial multiplication and multiplying large numbers (Knuth 1997).

• The discrete version of the Fourier transform (see below) can be evaluated quickly on computers using Fast Fourier Transform (FFT) algorithms. (Conte & de Boor 1980)

In forensics, laboratory infrared spectrophotometers use Fourier transform analysis for measuring the wavelengths of light at which a material will absorb in the infrared spectrum. The FT method is used to decode the measured signals and record the wavelength data. And by using a computer, these Fourier calculations are rapidly carried out, so that in a matter of seconds, a computer-operated FT-IR instrument can produce an infrared absorption pattern comparable to that of a prism instrument.[1]

Fourier transformation is also useful as a compact representation of a signal. For example, JPEG compression uses a variant of the Fourier transformation (discrete cosine transform) of small square pieces of a digital image. The Fourier components of each square are rounded to lower arithmetic precision, and weak components are eliminated entirely, so that the remaining components can be stored very compactly. In image reconstruction, each image square is reassembled from the preserved approximate Fourier-transformed components, which are then inverse-transformed to produce an approximation of the original image.

**Applications in signal processing**[edit]

When processing signals, such as audio, radio waves, light waves, seismic waves, and even images, Fourier analysis can isolate narrowband components of a compound waveform, concentrating them for easier detection or removal. A large family of signal processing techniques consist of Fourier-transforming a signal, manipulating the Fourier-transformed data in a simple way, and reversing the transformation.[2]

Some examples include:

• Equalization of audio recordings with a series of bandpass filters;

• Digital radio reception without a superheterodyne circuit, as in a modern cell phone or radio scanner;

• Image processing to remove periodic or anisotropic artifacts such as jaggies from interlaced video, strip artifacts from strip aerial photography, or wave patterns from radio frequency interference in a digital camera;

• Cross correlation of similar images for co-alignment;

• X-ray crystallography to reconstruct a crystal structure from its diffraction pattern;

• Fourier transform ion cyclotron resonance mass spectrometry to determine the mass of ions from the frequency of cyclotron motion in a magnetic field;

• Many other forms of spectroscopy, including infrared and nuclear magnetic resonance spectroscopies;

• Generation of sound spectrograms used to analyze sounds;(frequance) • Passive sonar used to classify targets based on machinery noise.