The **travelling salesman problem** (**TSP**) asks the following question: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?" It is an NP-hard problem in combinatorial optimization, important in operations research and theoretical computer science.

TSP is a special case of the travelling purchaser problem and the vehicle routing problem.

In the theory of computational complexity, the decision version of the TSP (where, given a length *L*, the task is to decide whether the graph has any tour shorter than *L*) belongs to the class of NP-complete problems. Thus, it is possible that the worst-case running time for any algorithm for the TSP increases superpolynomially (but no more than exponentially) with the number of cities.

The problem was first formulated in 1930 and is one of the most intensively studied problems in optimization. It is used as a benchmark for many optimization methods. Even though the problem is computationally difficult, a large number of heuristics and exact algorithms are known, so that some instances with tens of thousands of cities can be solved completely and even problems with millions of cities can be approximated within a small fraction of 1%(graph,crimonology)The TSP has several applications even in its purest formulation, such as planning, logistics, and the manufacture of microchips. Slightly modified, it appears as a sub-problem in many areas, such as DNA sequencing. In these applications, the concept *city* represents, for example, customers, soldering points, or DNA fragments, and the concept *distance* represents travelling times or cost, or a similarity measure between DNA fragments. The TSP also appears in astronomy, as astronomers observing many sources will want to minimize the time spent moving the telescope between the sources. In many applications, additional constraints such as limited resources or time windows may be imposed.

The travelling salesman problem was mathematically formulated in the 1800s by the Irish mathematician W.R. Hamilton and by the British mathematician Thomas Kirkman. Hamilton’s Icosian Game was a recreational puzzle based on finding a Hamiltonian cycle.[3] The general form of the TSP appears to have been first studied by mathematicians during the 1930s in Vienna and at Harvard, notably by Karl Menger, who defines the problem, considers the obvious brute-force algorithm, and observes the non-optimality of the nearest neighbour heuristic:

We denote by *messenger problem* (since in practice this question should be solved by each postman, anyway also by many travelers) the task to find, for ﬁnitely many points whose pairwise distances are known, the shortest route connecting the points. Of course, this problem is solvable by finitely many trials. Rules which would push the number of trials below the number of permutations of the given points, are not known. The rule that one first should go from the starting point to the closest point, then to the point closest to this, etc., in general does not yield the shortest route. [4]

It was first considered mathematically in the 1930s by Merrill Flood who was looking to solve a school bus routing problem.[5] Hassler Whitney at Princeton University introduced the name *travelling salesman problem* soon after.[6]

In the 1950s and 1960s, the problem became increasingly popular in scientific circles in Europe and the USA after the RAND Corporation in Santa Monica offered prizes for steps in solving the problem.[5] Notable contributions were made by George Dantzig, Delbert Ray Fulkerson and Selmer M. Johnson from the RAND Corporation, who expressed the problem as an integer linear program and developed the cutting plane method for its solution. They wrote what is considered the seminal paper on the subject in which with these new methods they solved an instance with 49 cities to optimality by constructing a tour and proving that no other tour could be shorter. Dantzig, Fulkerson and Johnson, however, speculated that given a near optimal solution we may be able to find optimality or prove optimality by adding a small amount of extra inequalities (cuts). They used this idea to solve their initial 49 city problem using a string model. They found they only needed 26 cuts to come to a solution for their 49 city problem. While this paper did not give an algorithmic approach to TSP problems, the ideas that lay within it were indispensable to later creating exact solution methods for the TSP, though it would take 15 years to find an algorithmic approach in creating these cuts.[5] As well as cutting plane methods, Dantzig, Fulkerson and Johnson used branch and bound algorithms perhaps for the first time.[5]

In the following decades, the problem was studied by many researchers from mathematics, computer science, chemistry, physics, and other sciences. In the 1960s however a new approach was created, that instead of seeking optimal solutions, one would produce a solution whose length is provably bounded by a multiple of the optimal length, and in doing so create lower bounds for the problem; these may then be used with branch and bound approaches. One method of doing this was to create a minimum spanning tree of the graph and then double all its edges, which produces the bound that the length of an optimal tour is a most twice the weight of a minimum spanning tree.[5]

Christofides made a big advance in this approach of giving an approach for which we know the worst-case scenario. His algorithm given in 1976, at worst is 1.5 times longer than the optimal solution. As the algorithm was so simple and quick, many hoped it would give way to a near optimal solution method. However, until 2011 when it was beaten by less than a billionth of a percent, this remained the method with the best worst-case scenario.[7]

Richard M. Karp showed in 1972 that the Hamiltonian cycle problem was NP-complete, which implies the NP-hardness of TSP. This supplied a mathematical explanation for the apparent computational difficulty of finding optimal tours.

Great progress was made in the late 1970s and 1980, when Grötschel, Padberg, Rinaldi and others managed to exactly solve instances with up to 2392 cities, using cutting planes and branch-and-bound.

In the 1990s, Applegate, Bixby, Chvátal, and Cook developed the program *Concorde* that has been used in many recent record solutions. Gerhard Reinelt published the TSPLIB in 1991, a collection of benchmark instances of varying difficulty, which has been used by many research groups for comparing results. In 2006, Cook and others computed an optimal tour through an 85,900-city instance given by a microchip layout problem, currently the largest solved TSPLIB instance. For many other instances with millions of cities, solutions can be found that are guaranteed to be within 2-3% of an optimal tour.

• An equivalent formulation in terms of graph theory is: Given a complete weighted graph (where the vertices would represent the cities, the edges would represent the roads, and the weights would be the cost or distance of that road), find a Hamiltonian cycle with the least weight.

• The requirement of returning to the starting city does not change the computational complexity of the problem, see Hamiltonian path problem.

• Another related problem is the Bottleneck traveling salesman problem (bottleneck TSP): Find a Hamiltonian cycle in a weighted graph with the minimal weight of the weightiest edge. The problem is of considerable practical importance, apart from evident transportation and logistics areas. A classic example is in printed circuit manufacturing: scheduling of a route of the drill machine to drill holes in a PCB. In robotic machining or drilling applications, the "cities" are parts to machine or holes (of different sizes) to drill, and the "cost of travel" includes time for retooling the robot (single machine job sequencing problem).[9]

• The generalized travelling salesman problem, also known as the "travelling politician problem", deals with "states" that have (one or more) "cities" and the salesman has to visit exactly one "city" from each "state". One application is encountered in ordering a solution to the cutting stock problem in order to minimize knife changes. Another is concerned with drilling in semiconductor manufacturing, see e.g., U.S. Patent 7,054,798. Noon and Bean demonstrated that the generalized travelling salesman problem can be transformed into a standard travelling salesman problem with the same number of cities, but a modified distance matrix.

• The sequential ordering problem deals with the problem of visiting a set of cities where precedence relations between the cities exist.

• A common interview question at Google is how to route data among data processing nodes; routes vary by time to transfer the data, but nodes also differ by their computing power and storage, compounding the problem of where to send data.

• The travelling purchaser problem deals with a purchaser who is charged with purchasing a set of products. He can purchase these products in several cities, but at different prices and not all cities offer the same products. The objective is to find a route between a subset of the cities, which minimizes total cost (travel cost + purchasing cost) and which enables the purchase of all required products.