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# N-body simulations: On the study of a triple star system

## Project III - Computational Astronomy

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**ABSTRACT**

*Aims.* We aim to study the dynamics of multiple body gravitational systems, namely the orbital stability for a case in which N=3, and how close encounters disturb a gravitationally bound configuration.
*Methods.* An algorithm is developed and applied to the Alpha Centauri system, formed by the binary star $\alpha$ Centauri A+B together with Proxima Cebtauri, for which coordinates are found in literature. Projectiles of different masses are sent towards a short distance of the system. Trajectories and energies are studied. The code is validated using the solar system as a first example.
*Results.* Four different energetic regimens were found. The orbits and energies for each are analyzed, and the stability of Proxima around the binary system discussed.

**Key words.** N-body systems – orbital stability – alpha centauri

## 1. Introduction

N-body simulations are one of the main tools to investigate the dynamics of multiple particle systems ruled by gravity. It is used to model different types of astrophysical phenomena, from planetary formation to black-hole binaries.

As simple as this problem may seem to an unattentive observer, its exact solution is not easy to calculate for most cases. While the two-body problem (N=2) was analytically solved by Johann Bernoulli in the 17th century, the three-body problem posed a greater challenge. It was only in the 20th century that major progress was made. Numerical investigations started in the 60s, with the pioneering effort of von Hoerner (Aarseth (2003)).

One particular application of a N-body algorithm is the study of orbital stability. There are systems that are shown to be surprisingly stable, namely those with hierarchical mass configurations (Kiseleva et al. (1994)), for which the stars can be divided into subgroups, each of which orbits around the system's barycenter.

The visual triple star comprising $\alpha$ Centauri A, B and Proxima, is a good example of such system. The first two are of simillar mass of around $M_\odot$, are at distance of $\sim 10AU$ from the center of mass and orbit each other with period of $\sim$ 80 years. Proxima is considerably less massive, $0.1M_\odot$, has an orbital period of 500,000 years around the system's barycenter, at a distance of $1.3\text{x}10^4AU$. This work aims to analyze Proxima's orbit around the binary system. We aim to study its stability, disturbing the initial configuration (projecting a body next/through to it), and looking at system's orbit and energy.

## 2. N-body simulations

### 2.1. Quick description of the problem

To study the trajectory of a particle that belongs in a gravitationally bound system for which no other types of forces have significance, one must obtain the force acting on it. This is given by the resulting interaction with the remaining N-1 particles.

$$\boldsymbol{F}_i = -\sum_{j\neq i}^{N} G m_i m_j \frac{\boldsymbol{r}_i - \boldsymbol{r}_j}{\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|^3} \tag{1}$$

The acceleration is calculated as usual, shown in 2.

$$\boldsymbol{a}_i \equiv \frac{\partial \boldsymbol{v}_i}{\partial t} = \frac{\partial^2 \boldsymbol{r}_i}{\partial t^2} = \frac{\boldsymbol{F}_i}{m_i} \tag{2}$$

This translates to $N$ second order differential equations, describing how each particle's trajectory evolves with time. We can rearrange this into a system of 6x(2N) first order differential equations (equation 3):

$$\frac{\mathrm{d}\boldsymbol{r}_i}{\mathrm{d}t} = \boldsymbol{v}_i \qquad \frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} = \sum_{j\neq i}^{N} \boldsymbol{a}_{i,j} \tag{3}$$

With $\boldsymbol{a}_{i,j} \equiv -G m_j \frac{\boldsymbol{r}_i - \boldsymbol{r}_j}{\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|^3}$

### 2.2. Dimensionless, barycentric vectors

In order to remove the dimensions of the system of equations one must introduce dimensionless vectors,

$$m_i = m_r q_i \tag{4}$$

$$\boldsymbol{r}_i = r_r \boldsymbol{x}_i \tag{5}$$

$$\boldsymbol{v}_i = v_r \boldsymbol{y}_i \tag{6}$$

$$\boldsymbol{a}_{i,j} = a_r \boldsymbol{b}_{i,j} \tag{7}$$

Where the coefficients are given, as in (Monteiro (2019)) by

$$a_r = \frac{G m_r}{r_r^2} \tag{8}$$

$$v_r^2 = \frac{G m_r}{2 r_r} \tag{9}$$

$$m_r = max(m_i) \tag{10}$$

$$r_r = max(|\boldsymbol{r}_i|) \tag{11}$$

Switching to the center of mass reference frame, setting

$$\mathbf{r}_{\mathbf{CM}} = \frac{1}{M} \sum_{i=1}^{n} m_i \mathbf{r}_i \tag{12}$$

we have our final position vector, now adimensional and relative to the center of mass given by $x_{1,C} = x_i - r_{CM}$. The same is done for the velocityvector.

### 2.3. Numerical integration

The equations presented above do not allow collisions to take place, since when two particles are very close to each other a numerical singularity takes place ($\boldsymbol{x}_i - \boldsymbol{x}_j \simeq 0$). In order to add numerical stability, one bay avoid collisions altogether, doing the following approximation:

$$\frac{\boldsymbol{x}_{i1} - \boldsymbol{x}_{j1}}{\left|\boldsymbol{x}_{i1} - \boldsymbol{x}_{j1}\right|^3} \rightarrow \frac{\boldsymbol{x}_{i1} - \boldsymbol{x}_{j1}}{\left|\boldsymbol{x}_{i1} - \boldsymbol{x}_{j1}\right|^3 + \varepsilon_c} \tag{13}$$

The algorithm has the following steps:

1. Establishing initial conditions. This is one of the most import important steps of this work. Adequate initial conditions for the position and velocity of each particle must be set, $\boldsymbol{r}_i(t = t_0)$ and $\boldsymbol{v}_i(t = t_0)$. Correct values for mass and orbital period are also crucial. Since this study only concerns gravitational interactions, having proper values is a determinant factor of the quality of the results.
2. Estimating force between particles, through equation (1).
3. Evolving the system, solving the system of differential equations.

## 3. Code validation

The algorithm is validated using the Sun+Earth+Mars system. We plot the trajectories over the course of approximately 100 years, and the velocity over 1, at an arbitrary plane (here denoted xy). Data used is found on (Monteiro (2019)).

The results are shown in figures (1) and (2) and are according to expected: circular orbits (here a circular orbit approximation is done) and sinusoidal velocity profiles.

A true validation of the code is to check how energy varies with time. Is there an artificial dissipation of energy, thanks to numerical errors? One may plot the energy over time to check if the method is precise enough, and assure if the step of integration is properly chosen.

We can see in figure (3) that the algorithm allows conservation of energy, so one may proceed with the actual study.

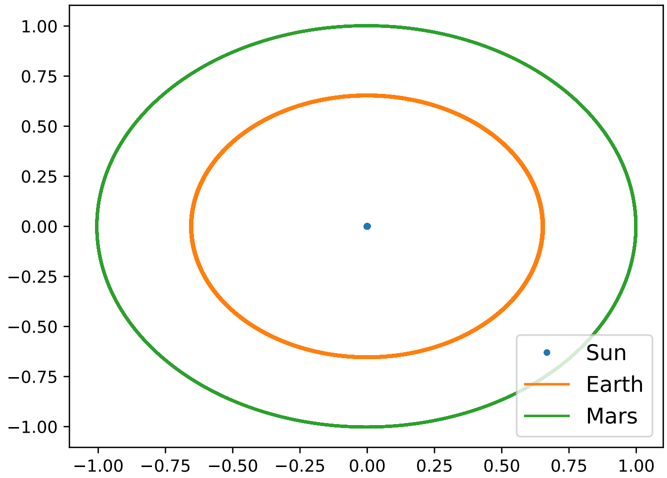


**Fig. 1.** Sun+Earth+Mars system: trajectories on the xy plane

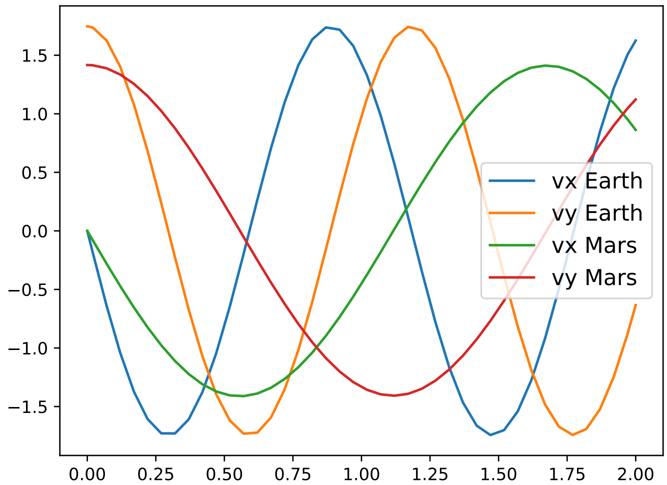


**Fig. 2.** Sun+Earth+Mars system: velocities on the xy plane over time

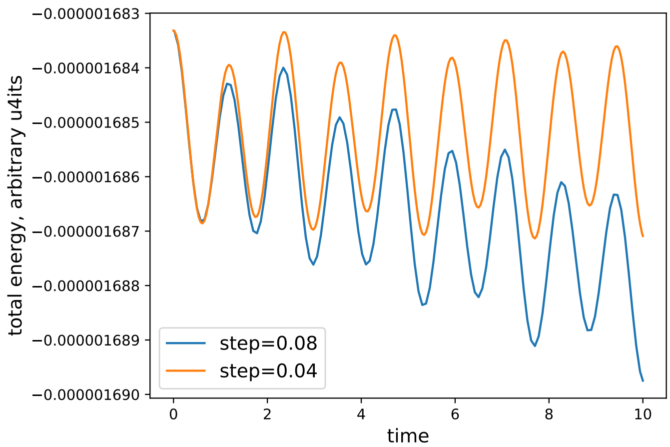


**Fig. 3.** Sun+Earth+Mars system: Energy over 5 periods. We can see that for a step of integration 0.04, the system's energy is conserved, which indicates the code presented works properly. A step smaller than this was assured to be used throughout this work.

**Table 1.** Physical parameters regarding the stars studied. $M_T$ refers to the total mass of the system.

| $Star$ | $M/M_{\odot}$ | $R/R_{\odot}$ |
|---|---|---|
| $\alpha$CenA | $1.1055 \pm 0.0039^a$ | $1.2234 \pm 0.0053^b$ |
| $\alpha$CenB | $0.9373 \pm 0.0033^a$ | $0.8632 \pm 0.0037^b$ |
| $\alpha$CenA + B | $2.0429 \pm 0.0072^a$ | – |
| Proxima | $0.1221 \pm 0.0022^c$ | $0.1542 \pm 0.0045^c$ |
| $M_T$ | 2.1650 | – |

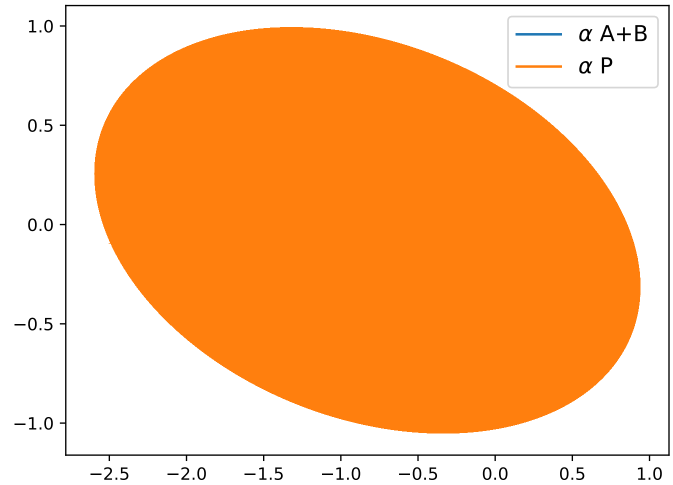


**Fig. 4.** Proxima's orbit around the AB binary: arbitrary plane, adimensional, normalized coordinates on the center of mass reference frame.

## 4. Application: Proxima's orbit around the $\alpha$ Cen binary system

As mentioned in section (1), the case studied was the $\alpha$ Centauri system. One of the goals is to study Proxima's stability around the AB binary.

The data used concerning this system is from (Meng et al. (2019)). Stellar parameters such as mass are found in table (1), and heliocentric coordinates for Proxima and the center of mass of $\alpha$ A+B can be found in the appendix.

### 4.1. Proxima's orbit around the binary system

Proxima is known to be gravitationally bound to $\alpha$ Cen A+B and to have a modetarely eccentric and very long-period, long distance orbit. It is reasonable, then, to reduce the $\alpha$ Centauri A+B binary to its center of mass.

The initial step is to study this system in equilibrium, simply with the data provided above. This allowed for trajectory and energy profiles as shown in figures (4) to (5).

### 4.2. System perturbations and the study of instability

A third body is then added, with the purpose of disturbing the initial configuration.

Initial conditions for this body were set as planar, in relation to the center of mass of the system, as shown in table (2). The choice was made so as to allow for a close encounter.

The initial conditions chosen for the test body allows the object to pass close to the system but not intercept it, as shown in figure (6). This is because our algorithm does not take into account collisions, due to the approximation made in equation (13),

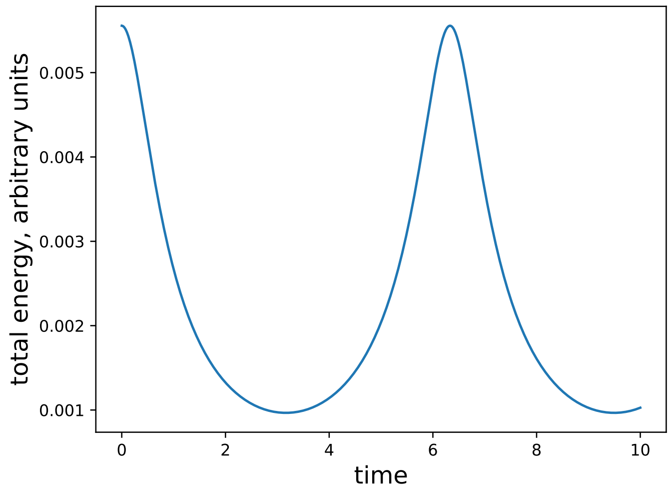


**Fig. 5.** Energy oscillations for Proxima around $\alpha$ Cen

**Table 2.** Adimensional initial conditions in the reference frame of the center of mass for the $\alpha Cen$ A+B.

| Star | Position | Velocity |
|---|---|---|
| $\alpha Cen$ A+B | [-0.060,-0.002,-0.029] | $[0.031, -0.055, -0.060]$ |
| Proxima Cen | [1, 0.035 ,0.495] | $[-0.526, 0.920, 1]$ |
| Test body: (1) | [4,0,0] | $[-1, 1, 0]$ |

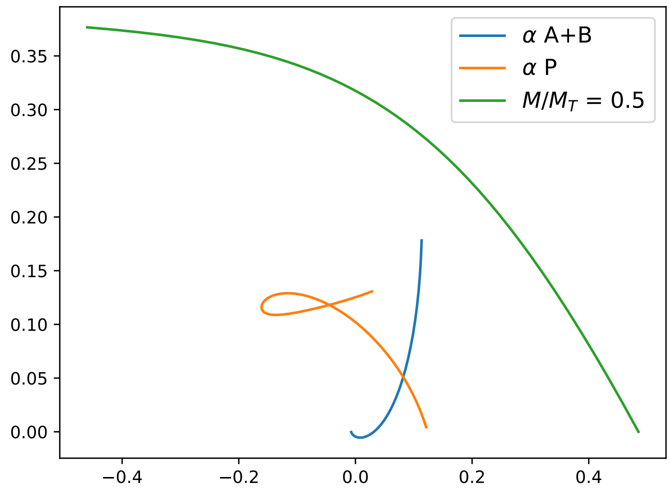


**Fig. 6.** Example of a trajectory on the initial conditions chosen, for $M = 0.5M_T$, where $M_T$ refers to the total mass of the initial configuration. The extra body is shown in green.

so aiming the particle towards the center of mass of the system would give rise to results that do not reflect a real system. A mass range of $M \in [0.0001, 25]M_T$ was studied. [1].

### 4.3. Results

For each order of magnitude of mass in the range studied ($[0.001, 0.01, 0.1, 1, 10, 25]M_T$), several tests were performed, in order to understand the system's behaviour and to look for patterns in trajectories and energy profiles. Resulting in this study, in general, for the initial conditions chosen, approximately five regimens were found:

[1] As a reference, Proxima Centauri is around 0.05 $M_T$

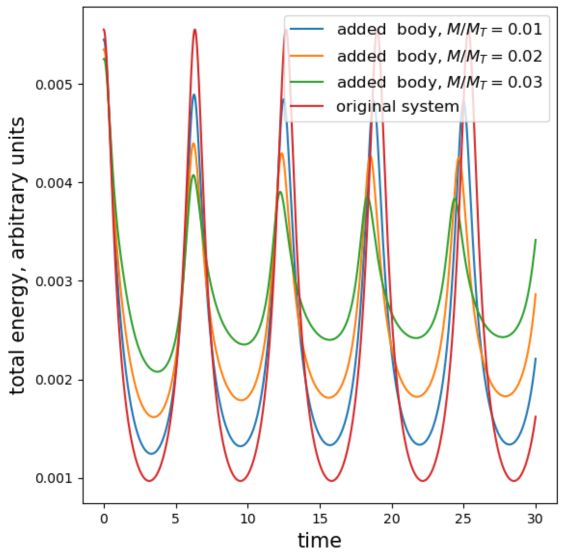


**Fig. 7.** Energy: regimen (1). For small masses, the system's energy decreases as it transfers energy to the projectile.

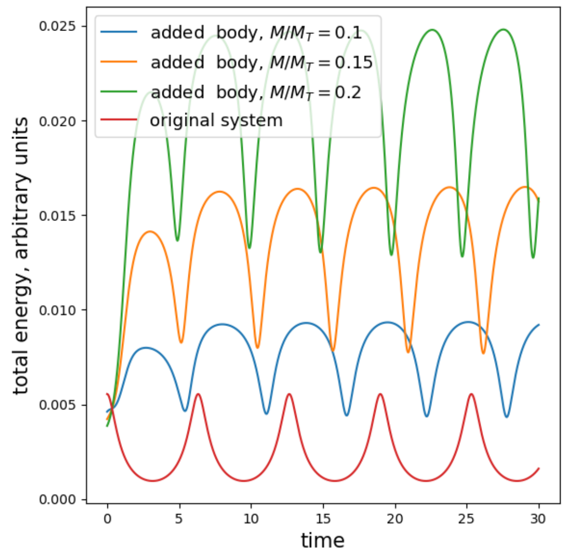


**Fig. 9.** Energy: regimen (2). For intermediate mass, the projectile transfers energy to the system.

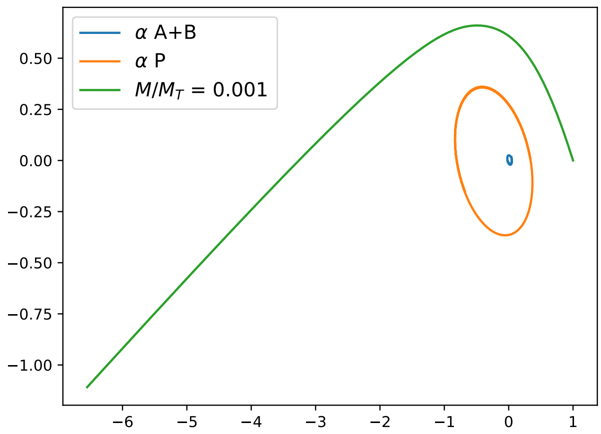


**Fig. 8.** Trajectory: regimen (1). The projectile is accelerated, after receiving energy from the system.

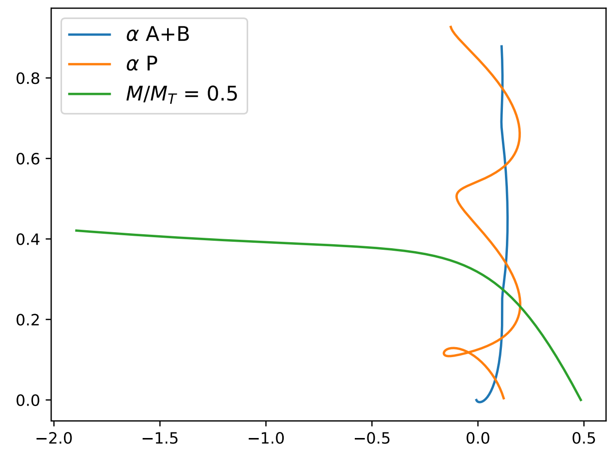


**Fig. 10.** Trajectory: regimen (2). After receiving energy from the projectile, the system's center of mass has an added velocity. Note how the lines shown are trajectories after a certain time. The initial conditions and algorithm are set for the extra body not to collide with the stars in the system, it only has a close encounter.

- (1) $M \leq 0.035 M_T$: The system transfers energy to the extra body, accelerating it - figures (7) and (8);
- (2) $0.035 M_T \leq M \leq 1 M_T$ The extra body transfers some energy to the initial system, but its initial configuration of the system remains - figures (9) and (10);
- (3) $1.5 \leq M \leq 14 M_T$ For masses close to 1.5 $M_T$, proxima initially remains gravitationally bound to the triple system, but is eventually ejected. For $M \geq 4 M_T$ it is ejected instantaneously - figures (11) and (12).
- (4) $15 M_T \leq M \leq 24 M_T$ Quadruple bound system - here a hierarchical mass system is created and found to be stable - figures (13) and (14).
- (5) $M \geq 25 M_T$ Proxima is again ejected. In a way, it is analogous to (3), however more energetic. Here, Proxima is spontaneously ejected. Not many tests were performed on this mass range, due to code limitations - it runs very slowly for masses larger than 20 $M_T$ for a small step. Running it on a larger step would compromise the fidelity of results.

## 5. Discussion

### 5.1. Results

For the initial conditions chosen, it takes a mass around the total mass of the system to disturb it's stability, and of around $1.5 M_T$ to eject Proxima of it's orbit around $\alpha Cen$ A+B. For a mass of $\sim 15 M_T$, a hierarchical mass system is formed and the projectile becomes bound to the initial configuration, forming a triple system. After around $\sim 24 M_T$, Proxima is again ejected, and the projectile becomes bound to the binary star.

A curious result is that regimen (3), in which Proxima is ejected and the projectile forms a binary with $\alpha$ A+B, seems to be more energetic than regimen (4), where a triple system is formed. It is also notable that the latter's orbits are of a higher frequency.

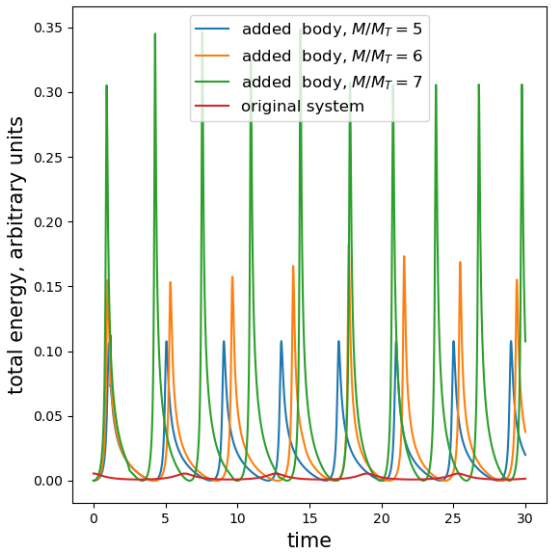


**Fig. 11.** Energy: regimen (3), for masses in which Proxima is is instantaneously ejected. $\alpha CenA + B$ is now bound to the test body, in a significantly more energetic orbit. Note that, for growing masses, the orbit's period decreases. The energy peak differences are probably due to numerical errors and not to orbital instability.

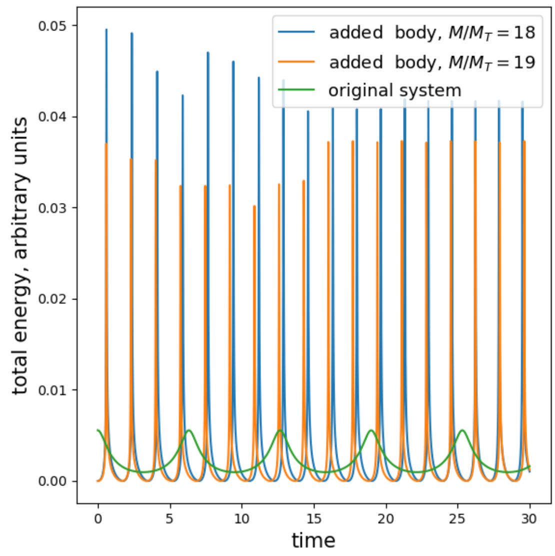


**Fig. 13.** Energy: regimen (4), a more energetic regimen is found, although less energetic than regimen (3). Note how the oscillations are more frequent. Here, once again, the energy peak differences are probably due to numerical errors and not to orbital instability.

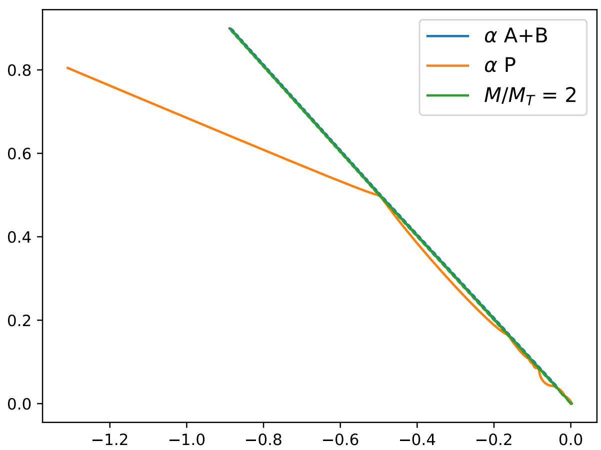


**Fig. 12.** Trajectory: Regimen (3) for $M = 2M_T$. Since the mass of the extra object is close to the total system's, Proxima initially remains bound, in a very unstable orbit. After enough time, Proxima is ejected, and the projectile forms a triple system with $\alpha CenA + B$ (Blue and green lines coincide, the stars are orbiting each other in a triple system).

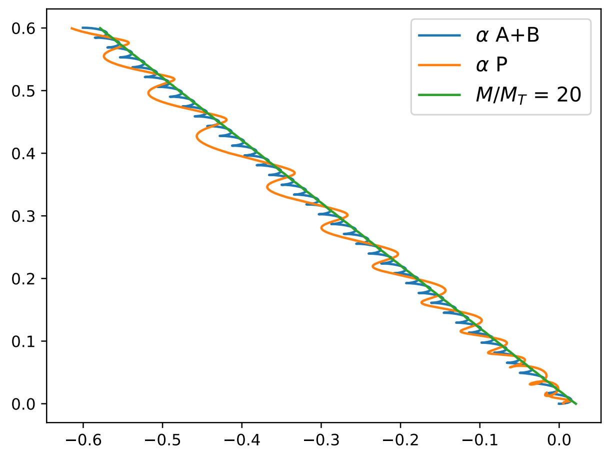


**Fig. 14.** Trajectory: Regimen (4) for $M = 20M_T$. The masses are now in a hierarchical system, where the existence of a stable quadruple system is possible.

### 5.2. Limitations

The main limitation of this algorithm is the absence of collisions, ie, transfer of momentum. We also studied this issue for a fixed velocity, aiming for the study of a particular close encounter and only varying the object's mass. We found the existence of a few different energetic regimens, however, a more general study could be performed, varying for example the angle of incidence and the velocity's magnitude, or even starting position.

For each range of masses, several tests were performed in order to estimate each regimen's limits. This means that another limitation is that, since this code was written based on cycles, it is slow, making it impractical to perform many tests for larger masses. The range of masses studied is, nevertheless, large ($[0.01, 25]M_T$), but could be maximized even further, simulating for example the encounter with a black hole. A way of improving this algorithm would be a broader use of numpy, for example, with methods such as broadcasting, which would allow this study to be performed without the use of cycles.

## 6. Conclusion

- An algorithm for the study of N-body systems was developed, aiming for the analysis of a close encounter between a projected object and a gravitationally bound system, $\alpha$ Centauri.
- It was found that, for fixed initial velocity and position, different masses of the projectile lead to different energetic reg-

imens, some leading to stellar ejection and an altered configuration.
- One can conclude that Proxima's orbit is stable, since it is only disturbed for masses of around $1M_T$, ie, $\sim 2.15M_{\odot}$, and it is only ejected for masses on the range of around $1.5M_T \leq M \leq 14M_T$ and $M \geq 25M_T$.
- One can also conclude that hierarchical mass configurations lead to stable multiple stellar systems.

## 7. Appendix

**Table 3.** Data used for the coordinates and velocities of $\alpha$ Cen A+B and Proxima, in heliocentric coordinates.

| Parameter | $\alpha$CenA + B | Proxima |
|---|---|---|
| $X$(pc) | $+0.95845 \pm 0.00078$ | $+0.90223 \pm 0.00043$ |
| $Y$(pc) | $-0.93402 \pm 0.00076$ | $-0.93599 \pm 0.00045$ |
| $Z$(pc) | $-0.01601 \pm 0.00001$ | $-0.04386 \pm 0.00002$ |
| $V_X\left(\text{kms}^{-1}\right)$ | $-29.291 \pm 0.026$ | $-29.390 \pm 0.027$ |
| $V_Y\left(\text{km}^{-1}\right)$ | $+1.710 \pm 0.020$ | $+1.883 \pm 0.018$ |
| $V_Z\left(\text{kms}^{-1}\right)$ | $+13.589 \pm 0.013$ | $+13.777 \pm 0.009$ |

## References

Aarseth, S. J. 2003, Gravitational N-Body Simulations: Tools and Algorithms (Cambridge University Press), google-Books-ID: WYQmYyy3f98C
Kiseleva, L. G., Eggleton, P. P., & Anosova, J. P. 1994, Monthly Notices of the Royal Astronomical Society, 267, 161
Meng, T., Ji, J., & Dong, Y. 2019, Monthly Notices of the Royal Astronomical Society, 482, 372, arXiv: 1809.08210
Monteiro, M. J. P. 2019, Astronomia Computacional