Term Test 1

Graded

Student

Patrick Hu

Total Points

72 / 80 pts

Question 1

Q1a **6** / 6 pts

part 1

✔ **+ 3 pts** Correct

+ 2 pts Mostly Correct

+ 1 pt Mostly Incorrect

+ 0 pts Missing/Incorrect

---

part 2

✔ **+ 3 pts** Correct

+ 2 pts Mostly correct

+ 1 pt Mostly incorrect

+ 0 pts Missing/Incorrect

Question 2

Q1b

4 / 4 pts

0.4 for each option correctly identified

✔ **+ 4 pts** Click here to replace this description.

+ 3.6 pts Click here to replace this description.

+ 3.2 pts Click here to replace this description.

+ 2.8 pts Click here to replace this description.

+ 2.4 pts Click here to replace this description.

+ 2 pts Click here to replace this description.

+ 1.6 pts Click here to replace this description.

+ 1.2 pts Click here to replace this description.

+ 0.8 pts Click here to replace this description.

+ 0.4 pts Click here to replace this description.

+ 0 pts Empty

Question 3

Q1c **5** / 5 pts

✔ + 5 pts Correct

+ 4 pts Small error

+ 3 pts Partially correct

+ 2 pts Correctly determines non-equivalence, but missing/incorrect counterexample

+ 0 pts Missing/Tries to prove equivalence

Question 4

Q1d **5** / 5 pts

✔ **+ 5 pts** Correct

+ 4 pts Small error

+ 3 pts Partially correct / laws missing

+ 2 pts Correctly determines equivalence, but missing/incorrect proof

+ 0 pts Missing/Tries to show non-equivalence

Question 5

Q1e **5** / 5 pts

✔ **+ 5 pts** Correct

+ 4 pts Small error

+ 3 pts Partially correct

+ 2 pts Mostly Incorrect / Incorrect but some work shown

+ 1 pt Correct but no work shown

+ 0 pts Missing / Incorrect

Question 6

Q2 **5** / 5 pts

✔ **+ 5 pts** Fully correct

+ 4 pts Small error or correct but one law missing

+ 3 pts Partially correct OR correct but more than one law missing

+ 2 pts Mostly Incorrect OR Incorrect but some relevant work shown

+ 0 pts Missing / Incorrect

Question 7

Q3abc

Resolved **14** / 15 pts

Part (a)

+ 5 pts Correct

✔ **+ 4 pts** Small error

+ 3 pts Partially correct

+ 2 pts Mostly incorrect but some work shown

+ 1 pt Quantifiers correct or barely correct

+ 0 pts Missing / Incorrect

Part (b)

✔ **+ 5 pts** Correct

+ 4 pts Small error

+ 3 pts Partially correct

+ 2 pts Mostly incorrect but some work shown / not a valid predicate (eg. put a predicate inside another / put commas instead of logical

operators) / did not attempt to simplify/ just add negation everywhere / only know to flip ∃ and ∀

+ 0 pts Missing / Incorrect

Part (c)

✔ **+ 5 pts** Correct

+ 4 pts Small error

+ 3 pts Partially correct

+ 2 pts Mostly incorrect, but some work shown

+ 0 pts Missing / Incorrect

Regrade Request **Submitted on: Nov 09**

Hi! I was just wondering what was the justification for a mark being taken off here for Question 3 a). If it was because I didn't use the defined predicate Q(x, y) wouldn't

my answer still be correct because I wrote y | x, and the question did not specify for us to use the predicate? If my answer could be reconsidered I would greatly appreciate it!

I'm sorry I can not grant you another mark since the question says use predicate logic and the predicates are defined for you

Reviewed on: Nov 10

Question 8

Q4ab **6** / 6 pts

part (a)

✔ **+ 3 pts** Correct

+ 2 pts Mostly Correct

+ 1 pt Mostly Incorrect

+ 0 pts Missing/Incorrect

+ 2.5 pts Minor Mistake

---

part (b)

✔ **+ 3 pts** Correct

+ 2 pts Mostly correct

+ 1 pt Mostly incorrect

+ 0 pts Missing/Incorrect

+ 2.5 pts Minor Mistake

Question 9

Q4cd

11 / 11 pts

part (c)

- ✔ **+ 5 pts** Correct
- **+ 4 pts** Mostly correct or minor error
- **+ 3 pts** Partially correct
- **+ 2 pts** Mostly Incorrect
- **+ 1 pt** Incorrect but some work shown
- **+ 0 pts** Missing / Incorrect

---

part (d)

- ✔ **+ 4 pts** Correct
- **+ 3 pts** Mostly correct
- **+ 2 pts** Partially correct
- **+ 1 pt** Mostly incorrect, but some work shown
- **+ 0 pts** Missing / Incorrect

✔ **+ 2 pts** English statement correct

+ 1 pt English statement partially correct

+ 0 pts English statement missing or entirely incorrect

Question 10

Q5ab

7 / 12 pts

part (a)

+ 6 pts Correct

+ 5 pts Correct quantifiers, mostly correct explanation

✔ **+ 4 pts** Correct quantifiers, Partially correct explanation or prove by example(never do this again)

+ 3 pts Correct quantifiers, explanation incorrect(for example re-write the statement in English without any further explanation).

+ 2 pts Correct quantifiers, explanation missing

+ 1 pt Partially Correct quantifiers, explanation missing.

+ 0 pts Missing / Incorrect

---

part (b)

+ 6 pts Correct

+ 5 pts Small error

+ 4 pts Counterexample partially correct

✔ + 3 pts Counterexample missing/incorrect, but work shown

+ 2 pts Explanation missing

+ 0 pts Missing / tries to prove equivalence

Question 11

Q6 4 / 6 pts

+ 0 pts Missing / Incorrect

+ 2 pts 1/3 Correct

✔ + 4 pts 2/3 Correct

+ 6 pts Correct

CSC/MAT A67H3 F 2023 Term Test 1
Duration — 90 minutes
Aids allowed: none

Family (Last) Name: Hu **First Name:** Patrick

UTORID: hupatri4 **Student #:** 1 1 0 1 1 0 1 2 4 3 1 6 1 3 1 9

Instructor: Bretscher

---

*Do **not** turn this page until you have received the signal to start.*
Fill out the identification section above.
Good Luck!

This term test is double sided and consists of 6 questions on 14 pages (including this one).

Read each question carefully. If you are unsure how to answer, try writing down all the information you have first.

If you use any space for rough work, indicate clearly what you want marked.

Good Luck!!

1: _____/25
2: _____/15
3: _____/15
4: _____/17
5: _____/12
6: _____/ 6

TOTAL: _____/90

Logical Equivalences

Commutative	$p \wedge q \iff q \wedge p$	$p \vee q \iff q \vee p$
Associative	$(p \wedge q) \wedge r \iff p \wedge (q \wedge r)$	$(p \vee q) \vee r \iff p \vee (q \vee r)$
Distributive	$p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$
Identity	$p \wedge T \iff p$	$p \vee F \iff p$
Negation	$p \vee \neg p \iff T$	$p \wedge \neg p \iff F$
Double Negative	$\neg(\neg p) \iff p$	
Idempotent	$p \wedge p \iff p$	$p \vee p \iff p$
Universal Bound	$p \vee T \iff T$	$p \wedge F \iff F$
De Morgan's	$\neg(p \wedge q) \iff (\neg p) \vee (\neg q)$	$\neg(p \vee q) \iff (\neg p) \wedge (\neg q)$
Absorption	$p \vee (p \wedge q) \iff p$	$p \wedge (p \vee q) \iff p$
Conditional or ($\rightarrow$) Law	$(p \rightarrow q) \iff (\neg p \vee q)$	$\neg(p \rightarrow q) \iff (p \wedge \neg q)$
Biconditional	$(p \leftrightarrow q) \iff (p \rightarrow q) \wedge (q \rightarrow p)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$

Inference Laws

Modus Ponens

$p \rightarrow q$
p
$\therefore q$

Modus Tollens

$p \rightarrow q$
$\neg q$
$\therefore \neg p$

Generalization

a. p
$\therefore p \vee q$

b. q
$\therefore p \vee q$

Specialization

a. $p \wedge q$
$\therefore p$

b. $p \wedge q$
$\therefore q$

Conjunction

p
q
$\therefore p \wedge q$

Elimination

a. $p \vee q$
$\neg q$
$\therefore p$

b. $p \vee q$
$\neg p$
$\therefore q$

Transitivity

$p \rightarrow q$
$q \rightarrow r$
$\therefore p \rightarrow r$

Proof by Division into Cases

$p \vee q$
$p \rightarrow r$
$q \rightarrow r$
$\therefore r$

Contradiction Rule

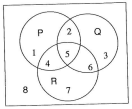
$\neg p \rightarrow c$
$\therefore p$

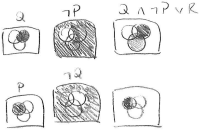
Question 1. [25 MARKS]

Answer the following short answer questions.

Part (a) [6 MARKS]

For each of the following statements, list the numbered region(s) of the Venn diagram that represent(s) when the statement is true.





1. $(Q \to P) \to R$

⇔ $(\neg Q \vee P) \to R$ ② 4,5,6,7,3

⇔ $(Q \wedge \neg P) \vee P$

2. $\neg((P \wedge \neg Q) \to \neg R)$ ③

① ⇔ $\neg(\neg P \vee Q \vee \neg R)$ 4

⇔ $P \wedge \neg Q \wedge R$

Part (b) [4 MARKS]

Circle the statements below that are equivalent to $c \to (a \to b)$.

1. $(a \vee \neg b) \to \neg c$ ⇝ $\neg a \wedge b \vee \neg c$ ⇔ $\neg c \vee (a \wedge \neg b)$
2. $\neg c \to \neg(a \to b)$ ⇔ $\neg c \vee (\neg a \vee b)$
3. c is necessary for $\neg a$ or b. ⇔ $\neg c \vee \neg a \vee b$
4. c is sufficient for $\neg a$ or b. ⇔ $\neg(a \wedge c) \vee b$
5. a and c are sufficient for b. . ⇔ $a \wedge c \to b$
6. a is necessary for b both of which are necessary for c. $c \to b \to a$
7. $(c \wedge a) \to b$ $\neg a \vee \neg c \vee b$
8. $(a \to b) \to \neg c$
9. All of the above.
10. None of the above.

(¬p ∨ q) → r
p ∧ ¬q ∨ r

Question 1. (CONTINUED)

Part (c) [5 MARKS]

Determine whether the two statements are equivalent or not. If they are equivalent provide a proof using equivalence laws and if they are not, give a counter example.

$$\overset{F}{p} \to \overset{T}{(q \to r)} \text{ and } \overset{F}{(p \to q)} \to \overset{F}{r}$$

①
LHS: $p \to (q \to r)$
$\Leftrightarrow \neg p \vee (q \to r)$ Implication
$\Leftrightarrow \neg p \vee (\neg q \vee r)$ Implication

RHS: $(p \to q) \to r$
$\neg(p \to q) \vee r$ Implication
$p \wedge \neg q \vee r$ Implication

② Appears not equivalent,
Test $p = F$, $q = T$, and $r = F$,
Then LHS: $F \to (T \to F)$
$\Leftrightarrow T$

RHS: $(F \to T) \to F$
$\Leftrightarrow T \to F$
$\Leftrightarrow F$

③ ∴ not equivalent

Part (d) [5 MARKS]

Determine whether the two statements are equivalent or not. If they are equivalent provide a proof using equivalence laws and if they are not, give a counter example.

$$(p \vee q) \to r \text{ and } (p \to r) \wedge (q \to r)$$

LHS: $(p \vee q) \to r$
$\Leftrightarrow (\neg(p \vee q)) \vee r$ Implication
$\Leftrightarrow (\neg p \wedge \neg q) \vee r$ DeMorgan's
$\Leftrightarrow (p \vee r) \wedge (\neg q \vee r)$ Distributive
$\Leftrightarrow (p \to r) \wedge (q \to r)$ Implication ×2
$\Leftrightarrow$ RHS
∴ are equivalent

Question 1. (CONTINUED)

Part (e) [5 MARKS]

You are stranded again on an island. You encounter three people A, B and C. People on the island are either truth-tellers or liars. Determine from A, B and C's statements what they are.

A: If B is lying then so is C.
B: If C is lying then so is A.
C: If A is lying then so is B.

A B C	A says: B is lying → C is lying	B says: C is lying → A is lying	C says: A is lying → B is lying
T T T	T	T	T ✓
T T F	T	F	T
T F T	F	T	T
T F F	T	F	T
F T T	T	T	F
F T F	T	T	F
F F T	F	T	T
F F F	T	T	T

∴ A, B, and C are truth tellers.

Question 2. [15 MARKS]

Part (a) [5 MARKS]

Write a proof sequence for the following assertions. Include your justification for each step. The equivalence and inference laws are listed on page 2 for your reference.

1. $\neg(a \wedge \neg b)$ *given*
2. $\neg b$ *given*

$\neg a$ *conclusion*

1. $\neg(a \wedge \neg b)$ Premise
2. $\neg b$ Premise
3. $\neg a \vee b$ DeMorgan's (1)
4. $\neg a$ Elimination (2, 3)

Question 2. (CONTINUED)

Part (b). [10 MARKS]

Write a proof sequence for the following assertion. Include your justification. The equivalence and inference laws are listed on page 2 for your reference.

1.	$r \vee q$	*given*
2.	$q \to r$	*given*
3.	$p \wedge s \to t$	*given*
4.	$\neg r$	*given*
5.	$\neg q \to u \wedge s$	*given*
	q	*conclusion*

Question 3. [15 MARKS]

Short answer questions on predicates. For the following questions we define $P(x)$ and $Q(x, y)$.

Let $P(x)$ be the predicate *x is prime.*
Let $Q(x, y)$ be the predicate for *x divides y.*
Consider the statement:

Domain is set of naturals

For every x that is not prime, there is some prime y that divides it.

Part (a) [5 MARKS]

Write the statement in predicate logic.

$\forall x \in \mathbb{N}\ ,\ \neg P(x) \rightarrow \exists y \in \mathbb{N}\ ,\quad P(y) \wedge y \mid x$

Part (b) [5 MARKS]

Negate your statement from (a).

$\exists x \in \mathbb{N}\ ,\ \neg P(x) \wedge (\forall y \in \mathbb{N}\ ,\ \neg P(y) \vee y \nmid x)$

Part (c) [5 MARKS]

Write the English translation of your statement from (b).

There exists a natural number x that is not prime and every natural number y is either composite or does not divide x.

Question 4. [17 MARKS]

For the following questions we will use the following domain and predicate definitions to translate the statements into precise symbolic notation.

Simplify your answers such that *only predicates* are negated (not entire statements). For example, do not leave your statement as $\neg(a \to b)$ but rather leave it as $(a \wedge \neg b)$.

$\neg(a \to b)$

$\boldsymbol{X}$: The set of all problems

$\boldsymbol{p(x)}$: Problem x is polynomial time solvable

$\boldsymbol{np(x)}$: Problem x is non-deterministically polynomial time solvable

Part (a) [3 MARKS]

$p(x)$

Every polynomial time solvable problem is non-deterministically polynomial time solvable.

$np(x)$

$\forall x \in X,\ p(x) \to np(x)$

Part (b) [3 MARKS]

Some non-deterministically polynomial time solvable problem is not polynomial time solvable.

$\exists x \in X,\ np(x) \wedge \neg p(x)$

Question 4. (CONTINUED)

Part (c) [5 MARKS]

every problem is poly solvable ⇔

Consider the statement S:

> *"If every problem is polynomial time solvable if and only if it is non-deterministically polynomial time solvable, then the problem SAT is polynomial time solvable."*

Translate statement S into precise symbolic notation. Simplify your answer such that *only predicates* are negated (not entire statements).

Let SAT be the problem SAT.

$(\forall x \in X,\ p(x) \Leftrightarrow np(x)) \rightarrow p(SAT)$

$\neg(p \wedge \neg q) \wedge \neg(\neg p \wedge \neg q)$

$(\neg p \vee q) \wedge (p \vee \neg q)$

$\neg(p \Leftrightarrow q)$

$(p \wedge q) \vee (\neg p \wedge \neg q)$

Part (d) [6 MARKS]

Give the contrapositive of S first in English, then in precise symbolic notation. Simplify your answer such that *only predicates* are negated.

① If the problem SAT is not polynomial time solvable, then there exists a problem that is both not polynomial time solvable or not non-deterministically polynomial time solvable and is either polynomial polynomial time solvable or non-deterministically polynomial time solvable.

② $\neg p(SAT) \rightarrow \exists x \in X,\ (\neg p(x) \vee \neg q(x)) \wedge (p(x) \vee q(x))$

Question 5. [12 MARKS]

Part (a) [6 MARKS]

Fill in the blanks with either $\exists x$ or $\forall x$. Marks will also be given for explaining your thought process. A formal proof is not required.

$$\underset{①}{\exists x} \in X, (a(x) \to b(x)) \iff \underset{②}{(\forall x} \in X, a(x)) \to (\exists x \in X, b(x)).$$

Statement ① says there exists an x such that if $a(x)$ holds, then $b(x)$ also holds.

Statement ② says if every x has the property $a(x)$, then there exists an x with the property $b(x)$.
This guarantees that there exists an x with both properties $a(x)$ and $b(x)$, if each x already has the property $a(x)$.
Thus it is the same as saying Statement ①.

Part (b) [6 MARKS]

Prove or disprove

$$\underset{①}{\forall x} \in X, (a(x) \vee b(x)) \iff \underset{②}{(\forall x} \in X, a(x)) \vee (\forall x \in X, b(x)).$$

Explain your reasoning. Marks will only be given for your *explanation*.

The two statements are not equivalent.

① says every x has either properties $a(x)$ or $b(x)$.

② says every x has property $a(x)$ OR every x has property $b(x)$.
This means all x's only have $a(x)$, or all x's only have $b(x)$, or all x's have both $a(x)$ and $b(x)$.
This is not the same as ① where some x's could have $a(x)$ and some other x's could have $b(x)$.

They are not equivalent.

Question 6. [6 MARKS]

You are very excited to start your journey at UTSC, and make new friends at the student residence. Your dorm floor has 5 rooms in total, numbered 201-205. You're in room 205. Due to flight delays, you missed the move-in day yesterday, and couldn't meet your dormmates. The semester has already started, so you want to meet them as soon as possible before things get busy. Eager to meet them, you quickly unlock your dorm room to unpack, and find a piece of paper that was slipped under the door. This is what it reads:

> Hey dormmate! It's unfortunate you weren't able to make it to move-in day. We got to know a lot about each other, especially the fact that each of us are very different. Here's a little puzzle for you to get to know us better:
>
> | | |
> |---|---|
> | **Names:** | Alex, Ivy, Kai, Zenith |
> | **Favorite Snacks:** | Cheetos, Strawberry Milkshake, Choco-chip cookies, Vanilla Ice-cream |
> | **Programs:** | CS, Env. Science, Psychology, Management |
> | **Hobbies:** | Basketball, Knitting, Crime documentaries, Hiking |
>
> They are not in order, but maybe these facts will help you figure it out:
>
> 1. The person in dorm 204 loves to play basketball.
> 2. There are two dorms between the person who likes Cheetos and the Psychology student, who also has the higher dorm number of the two.
> 3. Kai lives directly to the right of the person that loves to watch Crime documentaries.
> 4. The Management student lives directly to the left of the CS student.
> 5. Alex loves to eat Choco-chip cookies.
> 6. The person that likes Crime documentaries also loves Choco-chip cookies.
> 7. There is one dorm between the Env. Science student and the Management student.
> 8. Ivy loves Strawberry milkshakes.
> 9. There are two dorms between Kai and the person that enjoys Hiking, who also has the lower dorm number of the two.

You try to knock on their doors, but no one seems to be in their room.

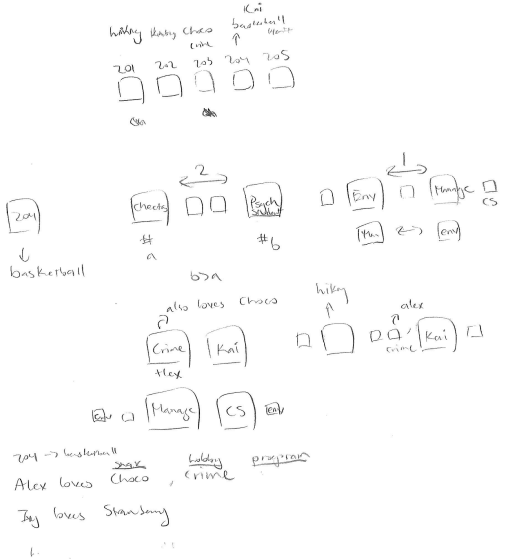
(a) There's Walk-in Basketball this afternoon at PanAm, what name do you have to ask around to find your dormmate? **Name:** Kai

(b) There's a new crime series that came out this week called 'The Devil on Trial'. What dorm room do you slip a message into with a request to watch this show together? **Dorm Room:** 203

(c) For the CMS orientation tomorrow, what snack should you bring for your dormmate? **Snack:** Vanilla Ice-cream

Use this page for rough work.

Use this page for rough work.



Use this page for rough work.