Efficient Maximum-likelihood Noncoherent Orthogonal STBC Detection

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Overview

- Noncoherent detection of orthogonal space-time block coded signals.
- Unknown channel at receiver ⇒ ML sequence detection.
- Past work:
  - exponential-complexity exhaustive search.
  - suboptimal approaches [Hughes2000], [LarsonStoikaLi2002], [MaVoDavidson2006].
- We prove that the ML sequence detection problem is polynomially solvable.
- We develop an algorithm that performs ML sequence detection with polynomial complexity.
- The order of the polynomial complexity is determined by the number of transmit and receive antennas.

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System Model

- $M_t \times M_r$ MIMO system.
- $\mathbf{s} \in \{\pm 1\}^N$: binary data sequence
- N: information sequence length
- $\mathbf{X}_n \in \mathbb{C}^{M_t \times T}$: space-time encoding matrix, $n = 1, 2, \ldots, N$.
- T: coded sequence length.

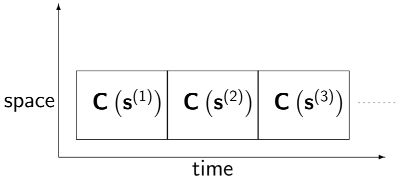
Space-time encoding

$$\mathbf{C}(\mathbf{s}) = \sum_{n=1}^{N} \mathbf{X}_n s_n$$

$$\mathbf{C}(\mathbf{s})\mathbf{C}^H(\mathbf{s}) = \|\mathbf{s}\|^2 \mathbf{I}_{M_t} = T\mathbf{I}_{M_t}$$

- Information rate $= \frac{N}{T}$.

System Model



- $p = 1, 2, 3, \ldots$: transmitted space-time block index.
- pth received block of size $M_r \times T$: $\mathbf{Y}^{(p)} = \mathbf{HC}\left(\mathbf{s}^{(p)}\right) + \mathbf{V}^{(p)}$.
- $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$: channel matrix (assumption: zero-mean complex gaussian i.i.d. coefficients with variance σ_h^2).
- $\mathbf{V}^{(p)} \in \mathbb{C}^{M_r \times T}$: noise matrix (assumption: zero-mean complex gaussian i.i.d. coefficients with variance σ_v^2).

Maximum-likelihood Coherent OSTBC Detection

- ML coherent detection (**H** available at the receiver):

$$\hat{\mathbf{s}}^{(p)} = \arg \min_{\mathbf{s}^{(p)} \in \{\pm 1\}^N} \left\| \mathbf{Y}^{(p)} - \mathbf{HC}\left(\mathbf{s}^{(p)}\right) \right\|_F^2$$

$$\Rightarrow \hat{s}_n^{(p)} = \operatorname{sign}\left(\Re\left\{\operatorname{tr}\left\{\mathbf{Y}^{(p)}\mathbf{X}_n^H\mathbf{H}^H\right\}\right\}\right), \ n = 1, 2, \ldots, N, \ p = 1, 2, 3, \ldots.$$

due to STBC orthogonality.

- Complexity of one-shot ML detector: $\mathcal{O}(N)$.

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$$\Rightarrow \hat{s}_n^{(p)} = \text{sign}\left(\Re\left\{ \text{tr}\left\{ \mathbf{Y}^{(p)} \mathbf{X}_n^H \mathbf{H}^H \right\} \right\} \right),\ n = 1, 2, \ldots, N,\ p = 1, 2, 3, \ldots.$$

  due to STBC orthogonality.

- Complexity of one-shot ML detector: $\mathcal{O}(N)$.

- ML noncoherent detection ($\mathbf{H}$ not available at the receiver).
- Channel uncertainty induces memory.
- ML noncoherent detector operates on a sequence of P consecutive space-time blocks:

$$\mathbf{Y}_{M_r \times TP} \triangleq \left[\mathbf{Y}^{(1)} \, \mathbf{Y}^{(2)} \, \ldots \, \mathbf{Y}^{(P)} \right].$$

- Assumption: Channel matrix $\mathbf{H}$ remains constant during P consecutive space-time block transmissions.

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Maximum-likelihood Noncoherent OSTBC Detection

- Transmitted data sequence:

$$\mathbf{s} = \left[\left(\mathbf{s}^{(1)}\right)^T \left(\mathbf{s}^{(2)}\right)^T \dots \left(\mathbf{s}^{(P)}\right)^T \right]^T \in \{\pm 1\}^{NP}.$$

- ML sequence detection:

$$\begin{aligned} \hat{\mathbf{s}}_{\text{opt}} &= \arg \max_{\mathbf{s} \in \{\pm 1\}^{NP}} f(\mathbf{Y}|\mathbf{s}) = \arg \max_{\mathbf{s} \in \{\pm 1\}^{NP}} f\left([\mathbf{Y}]_{1,:}, [\mathbf{Y}]_{2,:}, \dots, [\mathbf{Y}]_{M_r,:} | \mathbf{s}\right) \\ &= \arg \max_{\mathbf{s} \in \{\pm 1\}^{NP}} \prod_{m=1}^{M_r} f\left([\mathbf{Y}]_{m,:} | \mathbf{s}\right) \end{aligned}$$

due to i.i.d. channel coefficients.

Maximum-likelihood Noncoherent OSTBC Detection

Definitions

$$\mathbf{G}(\mathbf{s}) \triangleq \left[\mathbf{C}\left(\mathbf{s}^{(1)}\right) \mathbf{C}\left(\mathbf{s}^{(2)}\right) \ldots \mathbf{C}\left(\mathbf{s}^{(P)}\right)\right] \in \mathbb{C}^{M_{\mathrm{t}} \times TP}$$

$$\mathbf{V} \triangleq \left[\mathbf{V}^{(1)} \mathbf{V}^{(2)} \ldots \mathbf{V}^{(P)}\right] \in \mathbb{C}^{M_{\mathrm{r}} \times TP}$$

$$\tilde{\mathbf{G}}(\mathbf{s}) \triangleq \mathbf{G}^{*}(\mathbf{s}).$$

Proposition

$[\mathbf{Y}]_{m,:} = [\mathbf{H}\mathbf{G}(\mathbf{s})]_{m,:} + [\mathbf{V}]_{m,:}$ *given* $\mathbf{s}$ *is a complex Gaussian row vector.*

- Mean:

$$\begin{aligned} E\{[\mathbf{Y}]_{m,:}|\mathbf{s}\} &= \mathrm{E}\left\{[\mathbf{H}\mathbf{G}(\mathbf{s})]_{m,:} + [\mathbf{V}]_{m,:} \middle| \mathbf{s}\right\} \\ &= E\{[\mathbf{H}]_{m,:}\}\mathbf{G}(\mathbf{s}) + \mathrm{E}\{[\mathbf{V}]_{m,:}\} = \mathbf{0}. \end{aligned}$$

- Covariance matrix:

$$\begin{aligned} \mathbf{R}_m(\mathbf{s}) &= E\left\{ [\mathbf{Y}]_{m,:}^{T}[\mathbf{Y}]_{m,:}^{*} \middle| \mathbf{s} \right\} = E^{*}\left\{ [\mathbf{Y}]_{m,:}^{H}[\mathbf{Y}]_{m,:} \middle| \mathbf{s} \right\} \\ &= E^{*}\left\{ \mathbf{G}^{H}(\mathbf{s})[\mathbf{H}]_{m,:}^{H}[\mathbf{H}]_{m,:}\mathbf{G}(\mathbf{s}) \middle| \mathbf{s} \right\} + E^{*}\left\{ [\mathbf{V}]_{m,:}^{H}[\mathbf{V}]_{m,:} \right\} \\ &= \sigma_h^2 \tilde{\mathbf{G}}^{H}(\mathbf{s})\tilde{\mathbf{G}}(\mathbf{s}) + \sigma_v^2 \mathbf{I}_{TP}. \end{aligned}$$

Maximum-likelihood Noncoherent OSTBC Detection

$$\hat{\mathbf{s}}_{\text{opt}} = \arg \max_{\mathbf{s} \in \{\pm 1\}^{NP}} \prod_{m=1}^{M_r} \frac{1}{\pi^{TP} \left| \mathbf{R}_m(\mathbf{s}) \right|} \exp\left\{ -[\mathbf{Y}]_{m,:}^{*} \mathbf{R}_m^{-1}(\mathbf{s}) [\mathbf{Y}]_{m,:}^{T} \right\}.$$

- Using linear algebra identities, we obtain

$$|\mathbf{R}_m(\mathbf{s})| = \sigma_v^{2TP} \left(1 + \frac{TP\sigma_h^2}{\sigma_v^2} \right)^{M_t}$$

and

$$\mathbf{R}_m^{-1}(\mathbf{s}) = \frac{1}{\sigma_v^2} \mathbf{I}_{TP} - \left(1 + \frac{TP\sigma_h^2}{\sigma_v^2} \right)^{-1} \frac{\sigma_h^2}{\sigma_v^4} \tilde{\mathbf{G}}^H(\mathbf{s}) \tilde{\mathbf{G}}(\mathbf{s}).$$

Maximum-likelihood Noncoherent OSTBC Detection

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$$= \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \sum_{m=1}^{M_r}[\mathbf{Y}^*]_{m,:}\tilde{\mathbf{G}}^H(\mathbf{s})\tilde{\mathbf{G}}(\mathbf{s})[\mathbf{Y}^T]_{:,m} = \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \sum_{m=1}^{M_r}[\mathbf{Y}]_{m,:}\mathbf{G}^H(\mathbf{s})\mathbf{G}(\mathbf{s})[\mathbf{Y}^H]_{:,m}$$

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$$\hat{\mathbf{s}}_{\text{opt}} = \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \operatorname{tr}\left\{\mathbf{Y}\mathbf{G}^H(\mathbf{s})\mathbf{G}(\mathbf{s})\mathbf{Y}^H\right\}.$$

- Optimal solution: Exhaustive search among all 2^{NP} binary sequences $\mathbf{s} \in \{\pm 1\}^{NP} \Rightarrow$ Exponential complexity, impractical even for small NP.
- Suboptimal solutions:
  - 1-lag differential space-time decoding [Hughes00]
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Polynomial-complexity ML Noncoherent Detection

- The detection problem can be re-expressed as

$$\begin{aligned}\hat{\mathbf{s}}_{\text{opt}} &= \arg\max_{\mathbf{s}\in\{\pm 1\}^{NP}} \operatorname{tr}\left\{\mathbf{G}(\mathbf{s})\mathbf{Y}^H\mathbf{Y}\mathbf{G}^H(\mathbf{s})\right\} \\ &= \arg\max_{\mathbf{s}\in\{\pm 1\}^{NP}} \sum_{m=1}^{M_t}\left[\mathbf{G}(\mathbf{s})\mathbf{Y}^H\right]_{m,:}\left[\mathbf{Y}\mathbf{G}^H(\mathbf{s})\right]_{:,m} \\ &= \arg\max_{\mathbf{s}\in\{\pm 1\}^{NP}} \sum_{m=1}^{M_t}[\mathbf{G}(\mathbf{s})]_{m,:}\mathbf{Y}^H\mathbf{Y}[\mathbf{G}^H(\mathbf{s})]_{:,m}\end{aligned}$$

where

$$[\mathbf{G}(\mathbf{s})]_{m,:} = \left[\left[\mathbf{C}\left(\mathbf{s}^{(1)}\right)\right]_{m,:}\ \left[\mathbf{C}\left(\mathbf{s}^{(2)}\right)\right]_{m,:}\ \dots\ \left[\mathbf{C}\left(\mathbf{s}^{(P)}\right)\right]_{m,:}\right]$$

and, for $p = 1, 2, \dots, P$,

$$\left[\mathbf{C}\left(\mathbf{s}^{(p)}\right)\right]_{m,:} = \sum_{n=1}^{N}[\mathbf{X}_n]_{m,:}s_n^{(p)} = \left(\mathbf{s}^{(p)}\right)^T\begin{bmatrix}[\mathbf{X}_1]_{m,:}\\ [\mathbf{X}_2]_{m,:}\\ \vdots\\ [\mathbf{X}_N]_{m,:}\end{bmatrix}.$$

Polynomial-complexity ML Noncoherent Detection

- Therefore,

$$[\mathbf{G}(\mathbf{s})]_{m,:} = \left[\left(\mathbf{s}^{(1)}\right)^T \begin{bmatrix} [\mathbf{X}_1]_{m,:} \\ [\mathbf{X}_2]_{m,:} \\ \vdots \\ [\mathbf{X}_N]_{m,:} \end{bmatrix} \; \ldots \; \left(\mathbf{s}^{(P)}\right)^T \begin{bmatrix} [\mathbf{X}_1]_{m,:} \\ [\mathbf{X}_2]_{m,:} \\ \vdots \\ [\mathbf{X}_N]_{m,:} \end{bmatrix} \right] = \mathbf{s}^T \mathbf{Z}_m$$

where

$$\mathbf{Z}_m = \mathbf{I}_P \otimes \begin{bmatrix} [\mathbf{X}_1]_{m,:} \\ [\mathbf{X}_2]_{m,:} \\ \vdots \\ [\mathbf{X}_N]_{m,:} \end{bmatrix}.$$

Polynomial-complexity ML Noncoherent Detection

- ML sequence detection becomes

$$\begin{aligned}
\hat{\mathbf{s}}_{\text{opt}} &= \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \sum_{m=1}^{M_t} [\mathbf{G}(\mathbf{s})]_{m,:} \mathbf{Y}^H \mathbf{Y} [\mathbf{G}^H(\mathbf{s})]_{:,m} \\
&= \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \sum_{m=1}^{M_t} \mathbf{s}^T \mathbf{Z}_m \mathbf{Y}^H \mathbf{Y} \mathbf{Z}_m^H \mathbf{s} \\
&= \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \mathbf{s}^T \left(\sum_{m=1}^{M_t} \mathbf{Z}_m \mathbf{Y}^H \mathbf{Y} \mathbf{Z}_m^H \right) \mathbf{s}.
\end{aligned}$$

- Observation:

$$\sum_{m=1}^{M_t} \mathbf{Z}_m \mathbf{Y}^H \mathbf{Y} \mathbf{Z}_m^H \text{ is a rank-deficient matrix.}$$

Polynomial-complexity ML Noncoherent Detection

- $$\mathbf{Y} \in \mathbb{C}^{M_\mathrm{r} \times TP} \Rightarrow \operatorname{rank}(\mathbf{Y}) \le \min(M_\mathrm{r}, TP)$$
  $$\mathbf{Z}_m \in \mathbb{C}^{NP \times TP} \Rightarrow \operatorname{rank}(\mathbf{Z}_m) \le \min(NP, TP) = NP$$

- $$\begin{aligned}
\operatorname{rank}\left(\sum_{m=1}^{M_\mathrm{t}} \mathbf{Z}_m \mathbf{Y}^H \mathbf{Y} \mathbf{Z}_m^H\right) &\le \min\left(\sum_{m=1}^{M_\mathrm{t}} \operatorname{rank}\left(\mathbf{Z}_m \mathbf{Y}^H \mathbf{Y} \mathbf{Z}_m^H\right), NP\right) \\
&\le \min\left(\sum_{m=1}^{M_\mathrm{t}} \min\left(\operatorname{rank}(\mathbf{Y}), \operatorname{rank}\left(\mathbf{Z}_m\right)\right), NP\right) \\
&\le \min\left(\sum_{m=1}^{M_\mathrm{t}} \min(M_\mathrm{r}, TP, NP), NP\right) \\
&= \min\left(M_\mathrm{t} \min(M_\mathrm{r}, TP, NP), NP\right) = \min(M_\mathrm{t} M_\mathrm{r}, NP).
\end{aligned}$$

- Eigendecomposition:

$$\sum_{m=1}^{M_t} \mathbf{Z}_m \mathbf{Y}^H \mathbf{Y} \mathbf{Z}_m^H = \mathbf{Q}\mathbf{Q}^H, \ \mathbf{Q} \in \mathbb{C}^{NP \times \min(M_t M_r, NP)}.$$

- The ML detector becomes

$$\begin{aligned}
\hat{\mathbf{s}}_{\text{opt}} &= \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \mathbf{s}^T \mathbf{Q}\mathbf{Q}^H \mathbf{s} = \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \Re\left\{\mathbf{s}^T \mathbf{Q}\mathbf{Q}^H \mathbf{s}\right\} \\
&= \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \mathbf{s}^T \Re\left\{\mathbf{Q}\mathbf{Q}^H\right\} \mathbf{s} \\
&= \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \mathbf{s}^T \left[\Re(\mathbf{Q})\ \Im(\mathbf{Q})\right] \left[\Re(\mathbf{Q})\ \Im(\mathbf{Q})\right]^T \mathbf{s} \\
&= \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \mathbf{s}^T \mathbf{V}\mathbf{V}^T \mathbf{s} = \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \left\|\mathbf{V}^T \mathbf{s}\right\|
\end{aligned}$$

where $\mathbf{V} = [\Re(\mathbf{Q})\ \Im(\mathbf{Q})] \in \mathbb{R}^{NP \times 2\min(M_t M_r, NP)}$.

- If $NP > 2M_t M_r$, then $\mathbf{V}$ is a "tall" matrix of size $NP \times 2M_t M_r$.

Polynomial-complexity ML Noncoherent Detection

- Special case: 2×1 MISO system, sequence length $NP > 4$.

$$\mathbf{V}_{NP\times 4} = [\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3\ \mathbf{v}_4] \text{ and } \hat{\mathbf{s}}_{\text{opt}} = \arg \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \left\| \mathbf{V}^T \mathbf{s} \right\|.$$

- We introduce three auxiliary angles $\phi \in (-\pi, \pi]$ and $\theta, \omega \in (-\frac{\pi}{2}, \frac{\pi}{2}]$, and the spherical vector

$$\mathbf{c}(\phi, \theta, \omega) \triangleq \begin{bmatrix} \sin\phi \\ \cos\phi \sin\theta \\ \cos\phi\cos\theta\sin\omega \\ \cos\phi\cos\theta\cos\omega \end{bmatrix}.$$

- Cauchy-Schwartz Inequality:

$$\mathbf{c}(\phi,\theta,\omega)^T \mathbf{a} \le \|\mathbf{c}(\phi,\theta,\omega)\| \, \|\mathbf{a}\| = \|\mathbf{a}\|$$
$$\Rightarrow \max_{\phi,\theta,\omega} \{\mathbf{c}(\phi,\theta,\omega)^T \mathbf{a}\} = \|\mathbf{a}\|$$

- Then, our optimization problem becomes

$$\max_{\mathbf{s}\in\{\pm 1\}^{NP}} \left\| \mathbf{V}^T \mathbf{s} \right\| = \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \max_{\phi,\theta,\omega\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right]} \left\{ \mathbf{c}^T(\phi,\theta,\omega)\mathbf{V}^T\mathbf{s} \right\}$$

$$= \max_{\phi,\theta,\omega\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right]} \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \left\{ \mathbf{c}^T(\phi,\theta,\omega)\mathbf{V}^T\mathbf{s} \right\}$$

$$= \max_{\phi,\theta,\omega\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right]} \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \left\{ \sum_{n=1}^{NP} s_n \mathbf{V}_{n,1:4}\mathbf{c}(\phi,\theta,\omega) \right\}$$

- Maximization problem:

$$\max_{\phi,\theta,\omega\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]} \left\{ \sum_{n=1}^{NP} \max_{s_n=\pm 1} s_n(V_{n,1}\sin\phi + V_{n,2}\cos\phi\sin\theta + V_{n,3}\cos\phi\cos\theta\sin\omega + V_{n,4}\cos\phi\cos\theta\cos\omega) \right\}$$

- $\forall \;\; n = 1, 2, \ldots, NP,$ the maximizing argument

$$s_n(\phi,\theta,\omega) \stackrel{\triangle}{=} \arg\max_{s_n=\pm 1} s_n(V_{n,1}\sin\phi+V_{n,2}\cos\phi\sin\theta+V_{n,3}\cos\phi\cos\theta\sin\omega+V_{n,4}\cos\phi\cos\theta\cos\omega)$$

is determined by

$$V_{n,1}\sin\phi+V_{n,2}\cos\phi\sin\theta+V_{n,3}\cos\phi\cos\theta\sin\omega+V_{n,4}\cos\phi\cos\theta\cos\omega \underset{s_n(\phi,\theta,\omega)=-1}{\overset{s_n(\phi,\theta,\omega)=1}{\gtrless}} 0.$$

Polynomial-complexity ML Noncoherent Detection

- Then, our optimization problem becomes

$$\max_{\mathbf{s}\in\{\pm1\}^{NP}} \left\|\mathbf{V}^T\mathbf{s}\right\| = \max_{\mathbf{s}\in\{\pm1\}^{NP}} \max_{\phi,\theta,\omega\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right]} \left\{\mathbf{c}^T(\phi,\theta,\omega)\mathbf{V}^T\mathbf{s}\right\}$$

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- Maximization problem:

$$\max_{\phi,\theta,\omega\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]} \left\{\sum_{n=1}^{NP} \max_{s_n=\pm1} s_n(V_{n,1}\sin\phi + V_{n,2}\cos\phi\sin\theta + V_{n,3}\cos\phi\cos\theta\sin\omega + V_{n,4}\cos\phi\cos\theta\cos\omega)\right\}$$

- $\forall \;\; n = 1, 2, \dots, NP,$ the maximizing argument

$$s_n(\phi,\theta,\omega) \stackrel{\triangle}{=} \arg\max_{s_n=\pm1} s_n(V_{n,1}\sin\phi + V_{n,2}\cos\phi\sin\theta + V_{n,3}\cos\phi\cos\theta\sin\omega + V_{n,4}\cos\phi\cos\theta\cos\omega)$$

is determined by

$$V_{n,1}\sin\phi + V_{n,2}\cos\phi\sin\theta + V_{n,3}\cos\phi\cos\theta\sin\omega + V_{n,4}\cos\phi\cos\theta\cos\omega \underset{s_n(\phi,\theta,\omega)=-1}{\overset{s_n(\phi,\theta,\omega)=1}{\gtrless}} 0.$$

Polynomial-complexity ML Noncoherent Detection

- Element-by-element decision rule (for a certain set of ϕ, θ, ω):

$$s_n(\phi, \theta, \omega) = \begin{cases} -\mathrm{sgn}(V_{n,1}), & \phi \in \left[-\frac{\pi}{2}, \tan^{-1}\left(-\frac{V_{n,2}\sin\theta + V_{n,3}\cos\theta\sin\omega + V_{n,4}\cos\theta\cos\omega}{V_{n,1}}\right)\right) \\ \mathrm{sgn}(V_{n,1}), & \phi \in \left[\tan^{-1}\left(-\frac{V_{n,2}\sin\theta + V_{n,3}\cos\theta\sin\omega + V_{n,4}\cos\theta\cos\omega}{V_{n,1}}\right), \frac{\pi}{2}\right]. \end{cases}$$

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Polynomial-complexity ML Noncoherent Detection

Notes:

- Each triple of surfaces (n, m, l) has a unique intersection which corresponds to the spherical coordinates of the *zero* left singular vector of

$$\left[\mathbf{V}_{n,1:4}^T\,\mathbf{V}_{m,1:4}^T\,\mathbf{V}_{l,1:4}^T\right]_{4\times 3}$$

- Every intersection is a vertex of a cell.
- Each cell associated with a distinct binary vector $\mathbf{s}(n, m, l)$.
- $\#$ intersections $= \binom{NP}{3} \Rightarrow \#$ binary vectors $= \binom{NP}{3}$
- We collect all such binary vectors to set

$$\mathcal{J}(\mathbf{V}) \triangleq \bigcup_{\{n,m,l\}\subset\{1,\ldots,NP\}} \{\mathbf{s}(n, m, l)\}$$

  with cardinality

$$|\mathcal{J}(\mathbf{V})| = \binom{NP}{3}.$$

Polynomial-complexity ML Noncoherent Detection

$$\mathbf{c}\left(\phi, \frac{\pi}{2}, \frac{\pi}{2}\right) = \begin{bmatrix} \sin\phi \\ \cos\phi \\ 0 \\ 0 \end{bmatrix}$$

Polynomial-complexity ML Noncoherent Detection

- All candidate vectors form the set

$$\mathcal{S}(\mathbf{V}_{NP\times 4}) = \mathcal{J}(\mathbf{V}_{NP\times 4}) \cup \mathcal{J}(\mathbf{V}_{NP\times 2})$$

  of size

$$|\mathcal{S}(\mathbf{V}_{NP\times 4})| = |\mathcal{J}(\mathbf{V}_{NP\times 4})| + |\mathcal{J}(\mathbf{V}_{NP\times 2})| = \binom{NP}{3} + NP = \mathcal{O}((NP)^3).$$

- Each candidate vector is computed with complexity $\mathcal{O}(NP)$.
- $\hat{\mathbf{s}}_{\mathrm{opt}} \in \mathcal{S}(\mathbf{V}_{NP\times 4})$ and is obtained by exhaustive search in polynomial time $\mathcal{O}((NP)^4)$.

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General case: $M_\mathrm{t} \times M_\mathrm{r}$ MIMO system, sequence length $NP \geq 2M_\mathrm{t}M_\mathrm{r}$.

$$\mathbf{V}_{NP \times 2M_\mathrm{t}M_\mathrm{r}} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \dots \ \mathbf{v}_{2M_\mathrm{t}M_\mathrm{r}}] \text{ and } \hat{\mathbf{s}}_{\mathrm{opt}} = \arg \max_{\mathbf{s} \in \{\pm 1\}^{NP}} \left\| \mathbf{V}^T \mathbf{s} \right\|.$$

- We introduce $2M_\mathrm{t}M_\mathrm{r} - 1$ auxiliary angles $\phi_1 \in (-\pi, \pi]$, $\phi_2, \dots, \phi_{2M_\mathrm{t}M_\mathrm{r}-1} \in (-\frac{\pi}{2}, \frac{\pi}{2}]$, and the spherical vector

$$\mathbf{c}(\phi_1, \phi_2, \dots, \phi_{2M_\mathrm{t}M_\mathrm{r}-1}) \triangleq \begin{bmatrix} \sin\phi_1 \\ \cos\phi_1 \sin\phi_2 \\ \vdots \\ \cos\phi_1 \dots \cos\phi_{2M_\mathrm{t}M_\mathrm{r}-2} \sin\phi_{2M_\mathrm{t}M_\mathrm{r}-1} \\ \cos\phi_1 \dots \cos\phi_{2M_\mathrm{t}M_\mathrm{r}-2} \cos\phi_{2M_\mathrm{t}M_\mathrm{r}-1} \end{bmatrix}.$$

- Cauchy-Schwartz Inequality:

$$\mathbf{c}(\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1})^T \mathbf{a} \leq \|\mathbf{c}(\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1})\| \, \|\mathbf{a}\| = \|\mathbf{a}\|$$

$$\Rightarrow \max_{\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1}} \left\{\mathbf{c}(\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1})^T \mathbf{a}\right\} = \|\mathbf{a}\|$$

- Then, our optimization problem becomes

$$\begin{aligned}
\max_{\mathbf{s}\in\{\pm 1\}^{NP}} \left\|\mathbf{V}^T \mathbf{s}\right\| &= \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \max_{\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]} \left\{\mathbf{c}^T(\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1}) \mathbf{V}^T \mathbf{s}\right\} \\
&= \max_{\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]} \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \left\{\mathbf{c}^T(\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1}) \mathbf{V}^T \mathbf{s}\right\} \\
&= \max_{\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]} \max_{\mathbf{s}\in\{\pm 1\}^{NP}} \left\{\sum_{n=1}^{NP} s_n \mathbf{V}_{n,1:2M_t M_r} \mathbf{c}(\phi_1, \phi_2, \ldots, \phi_{2M_t M_r-1})\right\}
\end{aligned}$$

Polynomial-complexity ML Noncoherent Detection

- All candidate vectors form the set

$$\mathcal{S}(\mathbf{V}_{NP\times 2M_tM_r}) = \mathcal{J}(\mathbf{V}_{NP\times 2M_tM_r}) \cup \mathcal{J}(\mathbf{V}_{NP\times(2M_tM_r-2)}) \cup \ldots \cup \mathcal{J}(\mathbf{V}_{NP\times 2})$$

- The size of the set that includes all candidate vectors is

$$\begin{aligned}|\mathcal{S}(\mathbf{V}_{NP\times 2M_tM_r})| &= |\mathcal{J}(\mathbf{V}_{NP\times 2M_tM_r})| + |\mathcal{J}(\mathbf{V}_{NP\times(2M_tM_r-2)})| + \ldots + |\mathcal{J}(\mathbf{V}_{NP\times 2})| \\ &= \binom{NP}{2M_tM_r-1} + \binom{NP}{2M_tM_r-3} + \ldots + \binom{NP}{1} \\ &= \sum_{d=0}^{\left\lfloor\frac{2M_tM_r-1}{2}\right\rfloor}\binom{NP}{2M_tM_r-1-2d} = \sum_{d=0}^{2M_tM_r-1}\binom{NP-1}{d} = \mathcal{O}((NP)^{2M_tM_r-1}).\end{aligned}$$

- Each candidate vector is computed with complexity $\mathcal{O}(NP)$.
- $\hat{\mathbf{s}}_{\text{opt}} \in \mathcal{S}(\mathbf{V}_{NP\times 2M_tM_r})$ and is obtained by exhaustive seach in polynomial time $\mathcal{O}((NP)^{2M_tM_r})$.

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&= \binom{NP}{2M_tM_r-1} + \binom{NP}{2M_tM_r-3} + \ldots + \binom{NP}{1} \\
&= \sum_{d=0}^{\left\lfloor \frac{2M_tM_r-1}{2} \right\rfloor} \binom{NP}{2M_tM_r-1-2d} = \sum_{d=0}^{2M_tM_r-1} \binom{NP-1}{d} = \mathcal{O}((NP)^{2M_tM_r-1}).
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MATLAB Code

```
function X=compute_candidates(V)
[N D]=size(V);
if D>2
    combinations=nchoosek(1:N,D-1);
    X=zeros(N,size(combinations,1));
    for i=1:length(combinations)
        I=combinations(i,:); VI=V(I,:);
        c=find_intersection(VI);
        c=c*determine_sign(c);
        X(:,i)=sign(V*c);
        for d=1:D-1
            c=find_intersection([VI([1:d-1 d+1:D-1],1:D-1)]);
            c=c*determine_sign(c);
            X(I(d),i)=sign(VI(d,1:end-1)*c);
        end
    end
    X=[X compute_candidates(V(:,1:D-2))];
else
    phi_crosses=atan(-V(:,2)./V(:,1));
    [phi_sort,phi_ind]=sort(phi_crosses);
    X(phi_ind,1:N+1)=(repmat(-sign(V(phi_ind,1)),[1 N+1])).*(2*tril(ones(N,N+1))-1);
end
```

Table: MATLAB code of the proposed algorithm.

BER of ML noncoherent and ML coherent detector versus SNR. $M_{\mathrm{t}} = 2$, $M_{\mathrm{r}} = 1$, Alamouti STBC.

Complexity Results

(a) BER of ML noncoherent and ML coherent detector vs. number of space-time blocks P. $M_{\mathrm{t}} = 2$, $M_{\mathrm{r}} = 1$, Alamouti STBC.

(b) Complexity of exhaustive search and proposed polynomial-complexity search vs. sequence length N. $M_{\mathrm{t}} = 2$, $M_{\mathrm{r}} = 1$, Alamouti STBC.

Conclusion

- ML noncoherent OSTBC detection is polynomially-solvable in the sequence length.
- Efficient algorithm constructs polynomial-size set of binary vectors.
- ML binary vector is identified with $\mathcal{O}((NP)^{2M_t M_r})$ calculations.
- Complexity exponent is only a function of transmit and receiver antennas.