Optimal OSTBC Sequence Detection over Unknown Correlated Fading Channels*

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***Abstract*—We prove that maximum-likelihood (ML) noncoherent sequence detection of orthogonal space-time block coded (OSTBC) signals can be performed in polynomial time with respect to the sequence length for time-varying Rayleigh or Ricean distributed, correlated (in general) channel coefficients, if the mean channel vector belongs to the range of the channel covariance matrix. We develop a novel algorithm that, provided the above condition, performs ML noncoherent sequence detection with polynomial complexity the order of which equals twice the rank of the channel covariance matrix. Therefore, for Rayleigh fading, polynomial ML noncoherent detection complexity is always guaranteed; the lower the channel covariance rank the lower the receiver complexity. Instead, for Ricean fading, full-rank channel correlation is desired to guarantee polynomial ML noncoherent detection complexity.**

I. Introduction

ML noncoherent detection of OSTBC signals takes the form of sequence detection on the entire coherence interval for best performance, when the receiver has no channel state information (CSI) [1], [2]-[4]. However, if sequence detection is performed through exhaustive search among all possible data sequences [1], [2]-[4], then exponential computational complexity is required. To avoid the exponential complexity of the optimal receiver many suboptimal schemes have been proposed in the literature [4]-[6].

In this work, we prove that ML noncoherent OSTBC detection upon Rayleigh fading "channel processing" can always be performed in polynomial time whose order is completely determined by the rank of the covariance matrix of the vectorized channel matrix. Furthermore, motivated by the works in [8]-[12] which treat the problem of rank-deficient quadratic form maximization, we provide an algorithm that solves the ML noncoherent OSTBC detection problem in polynomial time. In particular, the optimal data sequence is proven to belong to a polynomial in size set of binary vectors that is built in polynomial time, altogether resulting in an efficient, fixed-complexity algorithm.

II. System Model and Problem Statement

We consider a multiple-input multiple-output (MIMO) system with $M_{\rm t}$ transmit and $M_{\rm r}$ receive antennas that employs orthogonal space-time coded transmission of size $M_{\rm t} \times T$ and rate $R = \frac{N}{T}$, $N \leq T$. We assume transmission of binary data that are split into vectors of N bits. Each bit vector forms a corresponding space-time block (matrix) of size $M_{\rm t} \times T$. The $M_{\rm t} \times T$ space-time block $\mathbf{C}(\mathbf{s}) \in \mathbb{C}^{M_{\rm t} \times T}$ that corresponds to the $N \times 1$ data vector $\mathbf{s} \in \{\pm 1\}^N$ is given by $\mathbf{C}(\mathbf{s}) = \sum_{n=1}^{N} \mathbf{X}_n s_n$ where $s_n = \pm 1$ denotes the nth element (bit) of $\mathbf{s}$, $n = 1, 2, \ldots, N$, and $\mathbf{X}_n \in \mathbb{C}^{M_{\rm t} \times T}$, $n = 1, 2, \ldots, N$, are orthogonal space-time codes that satisfy the property

$$\mathbf{C}(\mathbf{s})\mathbf{C}^H(\mathbf{s}) = \|\mathbf{s}\|^2 \mathbf{I}_{M_{\rm t}} = T\mathbf{I}_{M_{\rm t}}, \tag{1}$$

for any $\mathbf{s} \in \{\pm 1\}^N$.

Let $\mathbf{s}^{(p)} = \left[s_1^{(p)} \; s_2^{(p)} \; \ldots \; s_N^{(p)}\right]^T$ denote the data vector contained in the pth transmitted code block, $p = 1, 2, 3, \ldots$. The downconverted and pulse-matched equivalent pth received block of size $M_{\rm r} \times T$ is $\mathbf{Y}^{(p)} = \mathbf{H}^{(p)}\mathbf{C}\left(\mathbf{s}^{(p)}\right) + \mathbf{V}^{(p)}$ where $\mathbf{H}^{(p)} \in \mathbb{C}^{M_{\rm r} \times M_{\rm t}}$ refers to the pth transmission and represents the channel matrix between the $M_{\rm t}$ transmit and $M_{\rm r}$ receive antennas. In general, $\mathbf{H}^{(p)}$ consists of correlated coefficients that are modeled as circular complex Gaussian random variables and account for flat fading. We assume that all collected energy is absorbed by the channel matrix $\mathbf{H}^{(p)}$. In addition, $\mathbf{V}^{(p)} \in \mathbb{C}^{M_{\rm r} \times T}$ denotes zero-mean additive spatially and temporally white circular complex Gaussian noise with variance σ_v^2. The channel and noise matrices $\mathbf{H}^{(p)}$ and $\mathbf{V}^{(p)}$, respectively, $p = 1, 2, 3, \ldots$, are independent of each other.

In this work, we assume that the channel matrices $\mathbf{H}^{(p)}$, $p = 1, 2, 3, \ldots$, are not available to the receiver, hence, the optimum receiver takes the form of a sequence detector. We consider a sequence of P space-time blocks consecutively transmitted by the source and collected by the receiver, say $\mathbf{Y}^{(1)}, \ldots, \mathbf{Y}^{(P)}$, and form the $M_{\rm r} \times TP$ observation matrix

$$\begin{aligned}\mathbf{Y} &\triangleq \left[\mathbf{Y}^{(1)} \ldots \mathbf{Y}^{(P)}\right] \qquad (2)\\ &= \left[\mathbf{H}^{(1)}\mathbf{C}\left(\mathbf{s}^{(1)}\right) \ldots \mathbf{H}^{(P)}\mathbf{C}\left(\mathbf{s}^{(P)}\right)\right] + \left[\mathbf{V}^{(1)} \ldots \mathbf{V}^{(P)}\right].\end{aligned}$$

In the sequel, based on the observation of P blocks at the receiver we present ML noncoherent detection developments.

III. Maximum-likelihood Noncoherent Detection

We consider a time-varying Ricean fading MIMO channel, derive an efficient algorithm for the implementation of the

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ML noncoherent receiver, and prove that the complexity of the proposed ML receiver implementation is polynomial in the sequence length P if the mean channel vector belongs to the range of the channel covariance matrix whose rank is not a function of the sequence length. Interestingly, the order of the polynomial complexity depends strictly on the rank of the channel covariance matrix.

We assume that the channel matrix $\mathbf{H}^{(p)}$ changes during different transmissions and define the concatenated channel matrix $\mathbf{H} \triangleq \left[\mathbf{H}^{(1)} \dots \mathbf{H}^{(P)}\right] \in \mathbb{C}^{M_\text{r} \times P M_\text{t}}$. Due to Ricean fading, the vectorized channel matrix $\mathbf{h} \triangleq \text{vec}(\mathbf{H})$ is a circular complex Gaussian vector of length $M_\text{t} M_\text{r} P$ with mean vector $\boldsymbol{\mu} \in \mathbb{C}^{M_\text{t} M_\text{r} P}$ and covariance matrix $\mathbf{C}_h = \text{E}\left\{(\mathbf{h}-\boldsymbol{\mu})(\mathbf{h}-\boldsymbol{\mu})^H\right\} = \mathbf{Q}\,\mathbf{Q}^H \in \mathbb{C}^{M_\text{t} M_\text{r} P \times M_\text{t} M_\text{r} P}$ where $\mathbf{Q} \in \mathbb{C}^{M_\text{t} M_\text{r} P \times D}$ consists of orthogonal columns and $D \leq M_\text{t} M_\text{r} P$. Given the $M_\text{r} \times TP$ observation matrix $\mathbf{Y}$, the ML detector for the transmitted data sequence $\mathbf{s} = \left[\left(\mathbf{s}^{(1)}\right)^T \dots \left(\mathbf{s}^{(P)}\right)^T\right]^T \in \{\pm 1\}^{NP}$ maximizes the conditional probability density function (pdf) of $\mathbf{Y}$ given $\mathbf{s}$. Thus, the optimal decision is given by

$$\hat{\mathbf{s}}_\text{opt} = \underset{\mathbf{s} \in \{\pm 1\}^{NP}}{\arg\max}\, f(\mathbf{y}|\mathbf{s}) \tag{3}$$

where $\mathbf{y} \triangleq \text{vec}(\mathbf{Y}) \in \mathbb{C}^{M_\text{r} TP}$ and $f(\cdot|\cdot)$ represents the pertinent matrix/vector probability density function of the channel output conditioned on the transmitted bit sequence.

We define the block-diagonal matrix $\mathbf{D}(\mathbf{s}) \triangleq \text{diag}\left(\left[\mathbf{C}\left(\mathbf{s}^{(1)}\right), \dots, \mathbf{C}\left(\mathbf{s}^{(P)}\right)\right]\right) \in \mathbb{C}^{M_\text{t} P \times TP}$. Note that the orthogonality property also holds for $\mathbf{D}(\mathbf{s})$, since $\mathbf{D}(\mathbf{s})\mathbf{D}^H(\mathbf{s}) = T\mathbf{I}_{M_\text{t} P}$. Then, the received matrix $\mathbf{Y}$ in (2) becomes $\mathbf{Y} = \mathbf{H}\mathbf{D}(\mathbf{s}) + \mathbf{V}$ where $\mathbf{V} \triangleq \left[\mathbf{V}^{(1)}\, \mathbf{V}^{(2)} \dots \mathbf{V}^{(P)}\right] \in \mathbb{C}^{M_\text{r} \times TP}$. Due to [13]

$$\text{vec}(\mathbf{ABC}) = \left(\mathbf{C}^T \otimes \mathbf{A}\right)\text{vec}(\mathbf{B}), \tag{4}$$

we obtain $\mathbf{y} = \left(\mathbf{D}^T(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}}\right)\mathbf{h} + \mathbf{v}$ where $\mathbf{v} = \text{vec}(\mathbf{V}) \in \mathbb{C}^{M_\text{r} TP}$ and operator $\otimes$ denotes Kronecker tensor product. Then, it can be proven that $\mathbf{y}$ given $\mathbf{s}$ is a complex Gaussian vector with mean $E\{\mathbf{y}|\mathbf{s}\} = \left(\mathbf{D}^T(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}}\right)\boldsymbol{\mu}$ and covariance matrix $\mathbf{C}_y(\mathbf{s}) = \left(\mathbf{D}^T(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}}\right)\mathbf{Q}\,\mathbf{Q}^H\left(\mathbf{D}^*(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}}\right) + \sigma_v^2 \mathbf{I}_{M_\text{r} TP}$. Therefore, the optimization problem in (3) becomes

$$\begin{aligned}\hat{\mathbf{s}}_\text{opt} = \underset{\mathbf{s} \in \{\pm 1\}^{NP}}{\arg\max} \frac{1}{\pi^{M_\text{r} TP}\,|\mathbf{C}_y(\mathbf{s})|} \exp\{-(\mathbf{y} - (\mathbf{D}^T(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}})\boldsymbol{\mu})^H \\ \mathbf{C}_y^{-1}(\mathbf{s})(\mathbf{y} - (\mathbf{D}^T(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}})\boldsymbol{\mu})\}.\end{aligned} \tag{5}$$

A natural approach to (5) would be an exhaustive search among all 2^{NP} data sequences $\mathbf{s} \in \{\pm 1\}^{NP}$, but such a receiver is impractical even for moderate values of P, since its complexity grows exponentially with P. In the sequel, we present an efficient algorithm that performs the maximization in (5) with $\mathcal{O}(P^{2D})$ calculations.

Using identities for the determinant and inverse of a rank-deficient update [14], we compute $|\mathbf{C}_y(\mathbf{s})| = \sigma_v^{2M_\text{r} TP}\left|\mathbf{I}_D + \frac{T}{\sigma_v^2}\boldsymbol{\Sigma}\right|$ and

$$\begin{aligned}\mathbf{C}_y^{-1}(\mathbf{s}) &= \frac{1}{\sigma_v^2}\mathbf{I}_{M_\text{r} TP} \\ &- \frac{1}{\sigma_v^4}\left(\mathbf{D}^T(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}}\right)\mathbf{Q}\left(\mathbf{I}_D + \frac{T}{\sigma_v^2}\boldsymbol{\Sigma}\right)^{-1}\mathbf{Q}^H\left(\mathbf{D}^*(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}}\right),\end{aligned} \tag{6}$$

where $\boldsymbol{\Sigma} \triangleq \mathbf{Q}^H\mathbf{Q}$ is a $D \times D$ diagonal matrix with the D positive eigenvalues of $\mathbf{C}_h$ on its diagonal. For notation simplicity, we define $\mathbf{U} \triangleq \mathbf{Q}\left(\mathbf{I}_D + \frac{T}{\sigma_v^2}\boldsymbol{\Sigma}\right)^{-\frac{1}{2}}$ and observe that $\mathbf{U}^H\mathbf{U} = \left(\boldsymbol{\Sigma}^{-1} + \frac{T}{\sigma_v^2}\mathbf{I}_D\right)^{-1}$. We note that $|\mathbf{C}_y(\mathbf{s})|$ is independent of the transmitted sequence $\mathbf{s}$, drop it from the maximization in (5), and substitute (6) in (5) to obtain (7).

We continue our algorithmic developments by defining the matrices $\mathbf{X} \triangleq [\mathbf{X}_1 \dots \mathbf{X}_N] \in \mathbb{C}^{M_\text{t} \times TN}$, $\mathbf{S} \triangleq \left[\mathbf{s}^{(1)} \dots \mathbf{s}^{(P)}\right] \in \{\pm 1\}^{N \times P}$, and $\mathbf{G}(\mathbf{s}) \triangleq \left[\mathbf{C}\left(\mathbf{s}^{(1)}\right) \dots \mathbf{C}\left(\mathbf{s}^{(P)}\right)\right]$. We observe that $\mathbf{s} = \text{vec}(\mathbf{S})$, $\mathbf{G}(\mathbf{s})\mathbf{G}^H(\mathbf{s}) = TP\mathbf{I}_{M_\text{t}}$ due to (1), and

$$\mathbf{G}(\mathbf{s}) = \left[\sum_{n=1}^{N}\mathbf{X}_n s_n^{(1)} \dots \sum_{n=1}^{N}\mathbf{X}_n s_n^{(P)}\right] = \mathbf{X}\left(\mathbf{S} \otimes \mathbf{I}_T\right). \tag{8}$$

Moreover, we denote by $\mathbf{e}_p$ the pth column of $\mathbf{I}_P$, $p = 1, 2, \dots, P$, and rewrite $\mathbf{D}(\mathbf{s})$ as

$$\mathbf{D}(\mathbf{s}) = \left(\mathbf{I}_P \otimes \mathbf{G}(\mathbf{s})\right)\left(\tilde{\mathbf{I}}_P \otimes \mathbf{I}_T\right) \overset{(8)}{=} \left(\mathbf{I}_P \otimes \mathbf{X}(\mathbf{S} \otimes \mathbf{I}_T)\right)\left(\tilde{\mathbf{I}}_P \otimes \mathbf{I}_T\right) \tag{9}$$

where $\tilde{\mathbf{I}}_P \triangleq \left[\mathbf{e}_1\mathbf{e}_1^T \cdots \mathbf{e}_P\mathbf{e}_P^T\right]^T \in \{0,1\}^{P^2 \times P}$. Then, the vector $\left(\mathbf{D}^*(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}}\right)\mathbf{y}$ that appears in the maximization problem in (7) can be written as

$$\begin{aligned}\left(\mathbf{D}^*(\mathbf{s}) \otimes \mathbf{I}_{M_\text{r}}\right)\mathbf{y} &\underset{(9)}{\overset{(4)}{=}} \left(\mathbf{I}_{M_\text{t} P} \otimes \mathbf{Y}\left(\tilde{\mathbf{I}}_P^T \otimes \mathbf{I}_T\right)\right) \\ &\times \text{vec}\left(\mathbf{I}_P \otimes \left(\mathbf{S}^T \otimes \mathbf{I}_T\right)\mathbf{X}^H\right).\end{aligned} \tag{10}$$

We observe that

$$\text{vec}\left(\mathbf{I}_P \otimes \left(\mathbf{S}^T \otimes \mathbf{I}_T\right)\mathbf{X}^H\right) = \begin{bmatrix} \text{vec}\left(\mathbf{e}_1 \otimes \left(\mathbf{S}^T \otimes \mathbf{I}_T\right)\mathbf{X}^H\right) \\ \vdots \\ \text{vec}\left(\mathbf{e}_P \otimes \left(\mathbf{S}^T \otimes \mathbf{I}_T\right)\mathbf{X}^H\right) \end{bmatrix} \tag{11}$$

where, for any $p = 1, 2, \dots, P$,

$$\begin{aligned}\text{vec}\left(\mathbf{e}_p \otimes \left(\mathbf{S}^T \otimes \mathbf{I}_T\right)\mathbf{X}^H\right) &\overset{(4)}{=} \left(\mathbf{I}_{M_\text{t}} \otimes \mathbf{e}_p \otimes \mathbf{I}_{PT}\right) \\ &\times \text{vec}\left(\left(\mathbf{S}^T \otimes \mathbf{I}_T\right)\mathbf{X}^H\right).\end{aligned} \tag{12}$$

We also denote by $\tilde{\mathbf{X}}_i$ the matrix that contains the ith rows of all N space-time matrices, that is

$$\tilde{\mathbf{X}}_i \triangleq \begin{bmatrix} [\mathbf{X}_1]_{i,:} \\ \vdots \\ [\mathbf{X}_N]_{i,:} \end{bmatrix} \in \mathbb{C}^{N \times T},\ i = 1, \dots, M_\text{t}, \tag{13}$$

and observe that $\mathbf{X}^H = \left[\text{vec}\left(\tilde{\mathbf{X}}_1^H\right) \dots \text{vec}\left(\tilde{\mathbf{X}}_{M_\text{t}}^H\right)\right]$. Then,

$$\begin{aligned}&\text{vec}\left(\left(\mathbf{S}^T \otimes \mathbf{I}_T\right)\mathbf{X}^H\right) \\ &= \text{vec}\left(\left(\mathbf{S}^T \otimes \mathbf{I}_T\right)\left[\text{vec}\left(\tilde{\mathbf{X}}_1^H\right) \dots \text{vec}\left(\tilde{\mathbf{X}}_{M_\text{t}}^H\right)\right]\right) \\ &= \text{vec}\left(\left[\mathbf{Z}_1^H\mathbf{s} \dots \mathbf{Z}_{M_\text{t}}^H\mathbf{s}\right]\right) = \mathbf{Z}^H\mathbf{s}.\end{aligned} \tag{14}$$

$$\hat{\mathbf{s}}_{\text{opt}} = \underset{\mathbf{s}\in\{\pm1\}^{NP}}{\arg\max}\left\{-\left(\mathbf{y}-\left(\mathbf{D}^T(\mathbf{s})\otimes\mathbf{I}_{M_\text{r}}\right)\boldsymbol{\mu}\right)^H\left(\frac{1}{\sigma_v^2}\mathbf{I}_{M_\text{t}TP}-\frac{1}{\sigma_v^4}\left(\mathbf{D}^T(\mathbf{s})\otimes\mathbf{I}_{M_\text{r}}\right)\mathbf{U}\,\mathbf{U}^H\left(\mathbf{D}^*(\mathbf{s})\otimes\mathbf{I}_{M_\text{r}}\right)\right)\left(\mathbf{y}-\left(\mathbf{D}^T(\mathbf{s})\otimes\mathbf{I}_{M_\text{r}}\right)\boldsymbol{\mu}\right)\right\}$$

$$= \underset{\mathbf{s}\in\{\pm1\}^{NP}}{\arg\max}\left(\begin{array}{c}\frac{1}{\sigma_v^2}\mathbf{y}^H\left(\mathbf{D}^T(\mathbf{s})\otimes\mathbf{I}_{M_\text{r}}\right)\mathbf{U}\,\mathbf{U}^H\left(\mathbf{D}^*(\mathbf{s})\otimes\mathbf{I}_{M_\text{r}}\right)\mathbf{y}\\ +\mathbf{y}^H\left(\mathbf{D}^T(\mathbf{s})\otimes\mathbf{I}_{M_\text{r}}\right)\left(\mathbf{I}_{M_\text{t}M_\text{r}P}-\frac{T}{\sigma_v^2}\mathbf{U}\,\mathbf{U}^H\right)\boldsymbol{\mu}+\boldsymbol{\mu}^H\left(\mathbf{I}_{M_\text{t}M_\text{r}P}-\frac{T}{\sigma_v^2}\mathbf{U}\,\mathbf{U}^H\right)\left(\mathbf{D}^*(\mathbf{s})\otimes\mathbf{I}_{M_\text{r}}\right)\mathbf{y}\end{array}\right) \tag{7}$$

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where $\mathbf{Z} \triangleq [\mathbf{Z}_1 \;\ldots\; \mathbf{Z}_{M_\text{t}}] \in \mathbb{C}^{NP\times M_\text{t}TP}$ and $\mathbf{Z}_m \triangleq \left(\mathbf{I}_P\otimes\tilde{\mathbf{X}}_m\right)\in\mathbb{C}^{NP\times TP},\ m=1,\ldots,M_\text{t}$. Substituting (14) in (12) and then back in (11), we obtain

$$\text{vec}(\mathbf{I}_P\otimes\left(\mathbf{S}^T\otimes\mathbf{I}_T\right)\mathbf{X}^H) = \left[\begin{array}{c}(\mathbf{I}_{M_\text{t}}\otimes\mathbf{e}_1\otimes\mathbf{I}_{PT})\mathbf{Z}^H\mathbf{s}\\ \vdots\\ (\mathbf{I}_{M_\text{t}}\otimes\mathbf{e}_P\otimes\mathbf{I}_{PT})\mathbf{Z}^H\mathbf{s}\end{array}\right] = \left(\left[\begin{array}{c}\mathbf{I}_{M_\text{t}}\otimes\mathbf{e}_1\otimes\mathbf{I}_P\\ \vdots\\ \mathbf{I}_{M_\text{t}}\otimes\mathbf{e}_P\otimes\mathbf{I}_P\end{array}\right]\otimes\mathbf{I}_T\right)\mathbf{Z}^H\mathbf{s}. \tag{15}$$

Using (15), eq. (10) becomes

$$\left(\mathbf{D}^*(\mathbf{s})\otimes\mathbf{I}_{M_\text{r}}\right)\mathbf{y} = \left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}\right)\left(\left[\begin{array}{c}\mathbf{I}_{M_\text{t}}\otimes\mathbf{e}_1\mathbf{e}_1^T\\ \vdots\\ \mathbf{I}_{M_\text{t}}\otimes\mathbf{e}_P\mathbf{e}_P^T\end{array}\right]\otimes\mathbf{I}_T\right)\mathbf{Z}^H\mathbf{s} = \left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}\right)\mathbf{E}\mathbf{Z}^H\mathbf{s} \tag{16}$$

where $\mathbf{E}\triangleq\left[\begin{array}{c}\mathbf{I}_{M_\text{t}}\otimes\mathbf{e}_1\mathbf{e}_1^T\\ \vdots\\ \mathbf{I}_{M_\text{t}}\otimes\mathbf{e}_P\mathbf{e}_P^T\end{array}\right]\otimes\mathbf{I}_T\in\{0,1\}^{M_\text{t}P^2T\times M_\text{t}PT}$. Due to (16), the first, second, and third part of the maximization argument in (7) become

$$\frac{1}{\sigma_v^2}\mathbf{s}^T\mathbf{Z}\mathbf{E}^T\left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}^H\right)\mathbf{U}\,\mathbf{U}^H\left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}\right)\mathbf{E}\mathbf{Z}^H\mathbf{s},$$

$$\mathbf{s}^T\mathbf{Z}\mathbf{E}^T\left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}^H\right)\left(\mathbf{I}_{M_\text{t}M_\text{r}}-\frac{T}{\sigma_v^2}\mathbf{U}\,\mathbf{U}^H\right)\boldsymbol{\mu},$$

$$\text{and}\ \ \boldsymbol{\mu}^H\left(\mathbf{I}_{M_\text{t}M_\text{r}}-\frac{T}{\sigma_v^2}\mathbf{U}\,\mathbf{U}^H\right)\left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}\right)\mathbf{E}\mathbf{Z}^H\mathbf{s}, \tag{17}$$

respectively. We append a 1 to the end of the data vector $\mathbf{s}$, define $\tilde{\mathbf{s}}\triangleq\left[\mathbf{s}^T\ 1\right]^T$, and obtain $\hat{\mathbf{s}}_{\text{opt}}=\left[\hat{\tilde{\mathbf{s}}}_{\text{opt}}\right]_{1:NP,1}$ where, using (17) in (7), we obtain (18).

In the sequel, we show that (18) is polynomially solvable when the matrix of interest up to diagonal manipulations has at most D nonzero eigenvalues that are also positive. For this purpose, we present the following proposition and lemma.

Proposition 1: Every matrix $\mathbf{A}\in\mathbb{C}^{N\times N}$ is binary-quadratic-form-optimization-equivalent (BQFO-equivalent) to $\dot{\mathbf{A}}=\mathbf{A}+\text{diag}(\mathbf{x}),\ \mathbf{x}\in\mathbb{R}^{N\times1}$, i.e. $\mathbf{s}^T\mathbf{A}\mathbf{s}$ and $\mathbf{s}^T\dot{\mathbf{A}}\mathbf{s}$ are maximized (minimized) by the same binary vector $\mathbf{s}\in\{\pm1\}^N$.

Proof: Omitted due to lack of space. □

Lemma 1: Let $\mathbf{B}\in\mathbb{C}^{(N-1)\times M}$, $\mathbf{C}\in\mathbb{C}^{M\times D}$, $\mathbf{x}\in\mathbb{C}^{M\times1}$, and

$$\mathbf{A}\triangleq\left[\begin{array}{cc}\mathbf{B}\mathbf{C}\mathbf{C}^H\mathbf{B}^H & \mathbf{B}\left(\mathbf{I}_M-\mathbf{C}\mathbf{C}^H\right)\mathbf{x}\\ \mathbf{x}^H\left(\mathbf{I}_M-\mathbf{C}\mathbf{C}^H\right)\mathbf{B}^H & 0\end{array}\right]\in\mathbb{C}^{N\times N}. \tag{19}$$

If $\mathbf{x}$ belongs to the range of $\mathbf{C}$, that is $\mathbf{x}=\mathbf{C}\mathbf{a},\ \mathbf{a}\in\mathbb{C}^{D\times1}$, then $\mathbf{A}$ is BQFO-equivalent to the positive (semi)definite matrix

$$\dot{\mathbf{A}}\triangleq\left[\begin{array}{c}\mathbf{B}\mathbf{C}\\ \mathbf{a}^H\left(\mathbf{I}_D-\mathbf{C}^H\mathbf{C}\right)\end{array}\right]\left[\mathbf{C}^H\mathbf{B}^H\ \ \left(\mathbf{I}_D-\mathbf{C}^H\mathbf{C}\right)\mathbf{a}\right]. \tag{20}$$

Proof: Omitted due to lack of space. □

If $\boldsymbol{\mu}$ belongs to the range of $\mathbf{C}_h=\mathbf{Q}\,\mathbf{Q}^H$ (hence, the range of $\mathbf{U}$, that is $\boldsymbol{\mu}=\frac{1}{\sigma_v}\mathbf{U}\mathbf{a}$, $\mathbf{a}=\sigma_v\left(\mathbf{U}^H\mathbf{U}\right)^{-1}\mathbf{U}^H\boldsymbol{\mu}$), then we set $\mathbf{B}\triangleq\frac{1}{\sqrt{T}}\mathbf{Z}\mathbf{E}^T\left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}^H\right)$, $\mathbf{C}\triangleq\frac{\sqrt{T}}{\sigma_v}\mathbf{U}$, and $\mathbf{x}\triangleq\sqrt{T}\boldsymbol{\mu}$ in Proposition 2 and rewrite (18) as where $\mathbf{V}\triangleq[\Re\{\mathbf{A}\}\ \ \Im\{\mathbf{A}\}]\in\mathbb{R}^{(NP+1)\times2D}$ and

$$\mathbf{A}\triangleq\left[\begin{array}{c}\frac{1}{\sigma_v}\mathbf{Z}\mathbf{E}^T\left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}^H\right)\mathbf{Q}\\ \sigma_v\boldsymbol{\mu}^H\mathbf{Q}\boldsymbol{\Sigma}^{-1}\end{array}\right]\left(\mathbf{I}_D+\frac{T}{\sigma_v^2}\boldsymbol{\Sigma}\right)^{-\frac{1}{2}}. \tag{22}$$

The computation of $\hat{\mathbf{s}}_{\text{opt}}$ in (21) can be implemented with complexity $\mathcal{O}\left(P^{2D}\right)$ if we follow the multiple-auxiliary-angle methodology that has been introduced in [10]-[12] for the problem of rank-deficient quadratic form maximization and resulted in the algorithm that is available at http://www.telecom.tuc.gr/∼karystinos.

In our developments in this section, $\mathbf{V}$ (which is a function of $\mathbf{A}$) has to be computed by the receiver and subsequently fed to the algorithm of [12]. In (21), it is seen that $\mathbf{A}$ is a function of the received data matrix $\mathbf{Y}$, matrices $\mathbf{Z}$, $\mathbf{E}$, $\mathbf{Q}$, and $\boldsymbol{\Sigma}$, vector $\boldsymbol{\mu}$, and scalar σ_v. We note that

$$\mathbf{Z}\mathbf{E}^T\left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}^H\right)\mathbf{Q}=\sum_{p=1}^{P}[\mathbf{Z}_1\ \ldots\ \mathbf{Z}_{M_\text{t}}] \times\left(\mathbf{I}_{M_\text{t}}\otimes\left(\mathbf{e}_p\mathbf{e}_p^T\otimes\mathbf{I}_T\right)\mathbf{Y}^H\right)[\,\mathbf{Q}]_{(p-1)M_\text{t}M_\text{r}+1:pM_\text{t}M_\text{r},:}. \tag{23}$$

For convenience, we define $\mathbf{Q}_p\triangleq[\,\mathbf{Q}]_{(p-1)M_\text{t}M_\text{r}+1:pM_\text{t}M_\text{r},:}$ and observe that, for any $p=1,2,\ldots,P$,

$$[\mathbf{Z}_1\ \ldots\ \mathbf{Z}_{M_\text{t}}]\left(\mathbf{I}_{M_\text{t}}\otimes\left(\mathbf{e}_p\mathbf{e}_p^T\otimes\mathbf{I}_T\right)\mathbf{Y}^H\right)\mathbf{Q}_p = \left[\begin{array}{c}\mathbf{0}_{(p-1)N\times D}\\ \sum_{m=1}^{M_\text{t}}\tilde{\mathbf{X}}_m\left[\mathbf{Y}^H\right]_{(p-1)T+1:pT,:}[\mathbf{Q}_p]_{(m-1)M_\text{r}+1:mM_\text{r},:}\\ \mathbf{0}_{(P-p)N\times D}\end{array}\right]. \tag{24}$$

Computation of the product $\tilde{\mathbf{X}}_m[\mathbf{Y}^H]_{(p-1)T+1:pT,:}[\mathbf{Q}_p]_{(m-1)M_\text{r}+1:mM_\text{r},:}$, $m=1,\ldots,M_\text{t}$, requires $\mathcal{O}(NT{\cdot}M_\text{r}{\cdot}D)$ calculations and the sum in (24) consists of M_t such products resulting in a total of $\mathcal{O}(M_\text{t}\ M_\text{r}NTD)$ calculations. In addition, (23) contains P such sums, hence the computational complexity of $\mathbf{Z}\mathbf{E}^T\left(\mathbf{I}_{M_\text{t}P}\otimes\mathbf{Y}^H\right)\mathbf{Q}$ is $\mathcal{O}(M_\text{t}M_\text{r}NPTD)$. Computation of the row vector $\sigma_v\boldsymbol{\mu}^H\mathbf{Q}\boldsymbol{\Sigma}^{-1}$ that appears in the bottom row of $\mathbf{A}$ requires $\mathcal{O}\left(M_\text{t}M_\text{r}PD+D\right)$ calculations. Finally, the multiplication of the leftmost matrix in (22) with $\left(\mathbf{I}_D+\frac{T}{\sigma_v^2}\boldsymbol{\Sigma}\right)^{-\frac{1}{2}}$ costs $\mathcal{O}\left((NP+1)D\right)$. Therefore, the overall complexity overhead for the computation of $\mathbf{V}$ becomes

$$\mathcal{O}(M_\text{t}M_\text{r}NPTD+M_\text{t}M_\text{r}PD+D+(NP+1)D) \tag{25}$$

$$\hat{\tilde{\mathbf{s}}}_{\text{opt}} = \underset{\substack{\tilde{\mathbf{s}}\in\{\pm1\}^{NP+1}\\ \tilde{s}_{NP+1}=1}}{\arg\max}\ \tilde{\mathbf{s}}^T \begin{bmatrix} \frac{1}{\sigma_v^2}\mathbf{Z}\mathbf{E}^T\left(\mathbf{I}_{M_tP}\otimes\mathbf{Y}^H\right)\mathbf{U}\,\mathbf{U}^H\left(\mathbf{I}_{M_tP}\otimes\mathbf{Y}\right)\mathbf{E}\mathbf{Z}^H & \mathbf{Z}\mathbf{E}^T\left(\mathbf{I}_{M_tP}\otimes\mathbf{Y}^H\right)\left(\mathbf{I}_{M_tM_r}-\frac{T}{\sigma_v^2}\mathbf{U}\,\mathbf{U}^H\right)\boldsymbol{\mu} \\ \boldsymbol{\mu}^H\left(\mathbf{I}_{M_tM_r}-\frac{T}{\sigma_v^2}\mathbf{U}\,\mathbf{U}^H\right)\left(\mathbf{I}_{M_tP}\otimes\mathbf{Y}\right)\mathbf{E}\mathbf{Z}^H & 0\end{bmatrix}\tilde{\mathbf{s}} \tag{18}$$

$$\begin{aligned}\hat{\tilde{\mathbf{s}}}_{\text{opt}} &= \underset{\substack{\tilde{\mathbf{s}}\in\{\pm1\}^{NP+1}\\ \tilde{s}_{NP+1}=1}}{\arg\max}\left\|\underbrace{\left[\frac{1}{\sigma_v}\left(\mathbf{I}_D+\frac{T}{\sigma_v^2}\boldsymbol{\Sigma}\right)^{-\frac{1}{2}}\mathbf{Q}^H\left(\mathbf{I}_{M_tP}\otimes\mathbf{Y}\right)\mathbf{E}\mathbf{Z}^H \quad \sigma_v\boldsymbol{\Sigma}^{-1}\left(\mathbf{I}_D+\frac{T}{\sigma_v^2}\boldsymbol{\Sigma}\right)^{-\frac{1}{2}}\mathbf{Q}^H\boldsymbol{\mu}\right]}_{\mathbf{A}^H}\tilde{\mathbf{s}}\right\| \\ &= \underset{\substack{\tilde{\mathbf{s}}\in\{\pm1\}^{NP+1}\\ \tilde{s}_{NP+1}=1}}{\arg\max}\ \tilde{\mathbf{s}}^T\mathbf{A}\mathbf{A}^H\tilde{\mathbf{s}} = \underset{\substack{\tilde{\mathbf{s}}\in\{\pm1\}^{NP+1}\\ \tilde{s}_{NP+1}=1}}{\arg\max}\ \Re\left\{\tilde{\mathbf{s}}^T\mathbf{A}\mathbf{A}^H\tilde{\mathbf{s}}\right\} = \underset{\substack{\tilde{\mathbf{s}}\in\{\pm1\}^{NP+1}\\ \tilde{s}_{NP+1}=1}}{\arg\max}\left\|\mathbf{V}^T\tilde{\mathbf{s}}\right\|\end{aligned} \tag{21}$$

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which is linear in the sequence length P for constant values of M_t, M_r, N, T, and D (that is, fixed number of antennas, space-time coding rate, and channel covariance rank). We conclude that the overall complexity of the proposed receiver is $\mathcal{O}\left(P^{2D}\right)$.

IV. Special Cases

The time-invariant Ricean, time-varying Rayleigh, and time-invariant Rayleigh channel models are special cases of the general model that we considered, thus polynomial ML noncoherent detection complexity can also be attained for such scenaria. In this section, we examine separately these special cases.

Case I: Time-varying Rayleigh fading

Due to Rayleigh fading, we have $\boldsymbol{\mu}=\mathbf{0}$, hence the bottom row of $\mathbf{A}$ in (22) becomes zero and the ML detector of (21) simplifies to $\hat{\mathbf{s}}_{\text{opt}} = \underset{s\in\{\pm1\}^{NP}}{\arg\max}\left\|\mathbf{V}_1^T\mathbf{s}\right\|$ where $\mathbf{V}_1 \triangleq [\Re\{\mathbf{A}_1\}\ \Im\{\mathbf{A}_1\}]$ and $\mathbf{A}_1 \triangleq \mathbf{Z}\mathbf{E}^T\left(\mathbf{I}_{M_tP}\otimes\mathbf{Y}^H\right)\mathbf{Q}\left(\mathbf{I}_D+\frac{T}{\sigma_v^2}\boldsymbol{\Sigma}\right)^{-\frac{1}{2}}$.

Case II: Time-invariant Ricean fading

We have $\mathbf{h} = \mathbf{1}_P\otimes\underline{\mathbf{h}}$ where $\underline{\mathbf{h}} = \text{vec}(\underline{\mathbf{H}})$ is a circular complex Gaussian vector with mean $\underline{\boldsymbol{\mu}}$ and covariance matrix $\mathbf{C}_{\underline{h}} = \underline{\mathbf{Q}}\underline{\mathbf{Q}}^H$ and $\underline{\mathbf{H}}$ is the channel matrix that remains constant for P consecutive space-time blocks. Then, the covariance matrix of the channel vector $\mathbf{h}$ equals $\mathbf{C}_h = (\mathbf{1}_P\otimes\underline{\mathbf{Q}})(\mathbf{1}_P\otimes\underline{\mathbf{Q}})^H$, therefore $\mathbf{Q} = \mathbf{1}_P\otimes\underline{\mathbf{Q}}$ and $\mathbf{Q}^H\left(\mathbf{I}_{M_tP}\otimes\mathbf{Y}\right)\mathbf{E} = \underline{\mathbf{Q}}^H\left(\mathbf{I}_{M_t}\otimes\mathbf{Y}\right)$. In addition, $\boldsymbol{\Sigma} = P\underline{\mathbf{Q}}^H\underline{\mathbf{Q}} = P\underline{\boldsymbol{\Sigma}}$ and $\underline{\boldsymbol{\mu}} = \frac{1}{\sigma_v}\underline{\mathbf{Q}}\left(\mathbf{I}_D+\frac{TP}{\sigma_v^2}\underline{\boldsymbol{\Sigma}}\right)^{-\frac{1}{2}}\mathbf{a}$. Then, the ML detector of (21) simplifies to $\hat{\tilde{\mathbf{s}}}_{\text{opt}} = \underset{\substack{\tilde{s}\in\{\pm1\}^{NP+1}\\ \tilde{s}_{NP+1}=1}}{\arg\max}\left\|\mathbf{V}_2^T\tilde{\mathbf{s}}\right\|$ where $\mathbf{V}_2 \triangleq [\Re\{\mathbf{A}_2\}\ \Im\{\mathbf{A}_2\}]$ and $\mathbf{A}_2 \triangleq \begin{bmatrix}\frac{1}{\sigma_v}\mathbf{Z}\left(\mathbf{I}_{M_t}\otimes\mathbf{Y}^H\right)\underline{\mathbf{Q}} \\ \sigma_v\underline{\boldsymbol{\mu}}^H\underline{\mathbf{Q}}\underline{\boldsymbol{\Sigma}}^{-1}\end{bmatrix}\left(\mathbf{I}_D+\frac{TP}{\sigma_v^2}\underline{\boldsymbol{\Sigma}}\right)^{-\frac{1}{2}}$. Of course, such a simplification is possible when $\boldsymbol{\mu}$ belongs to the range of $\mathbf{C}_h$ (or, equivalently, the range of $\underline{\mathbf{U}} = \underline{\mathbf{Q}}\left(\mathbf{I}_D+\frac{TP}{\sigma_v^2}\underline{\boldsymbol{\Sigma}}\right)^{-\frac{1}{2}}$).

Case III: Time-invariant Rayleigh fading

We have $\underline{\boldsymbol{\mu}} = \mathbf{0}$, hence the ML detector becomes $\hat{\mathbf{s}}_{\text{opt}} = \underset{s\in\{\pm1\}^{NP}}{\arg\max}\left\|\mathbf{V}_3^T\mathbf{s}\right\|$ where $\mathbf{V}_3 \triangleq [\Re\{\mathbf{A}_3\}\ \Im\{\mathbf{A}_3\}]$ and $\mathbf{A}_3 \triangleq \mathbf{Z}\left(\mathbf{I}_{M_t}\otimes\mathbf{Y}^H\right)\underline{\mathbf{Q}}\left(\mathbf{I}_D+\frac{TP}{\sigma_v^2}\underline{\boldsymbol{\Sigma}}\right)^{-\frac{1}{2}}$. Note that the latter is identical to the detector derived in Section III and is always feasible since $\underline{\boldsymbol{\mu}} = \mathbf{0}$ always belongs to the range of $\mathbf{C}_h$.

It is interesting to mention that the ML noncoherent detector in the case of time-invariant Rayleigh fading (Case III) simplifies to the popular trace detector [1] in the special cases of independent and identically distributed (i.i.d.) channel coefficients or joint channel estimation and data detection due to channel statistics uncertainty at the receiver. Indeed, if the channel coefficients are i.i.d., then $\mathbf{R}_h$, $\mathbf{Q}$, and $\boldsymbol{\Sigma}$ are scaled versions of $\mathbf{I}_{M_tM_r}$, $D = M_tM_r$, and the ML detector becomes

$$\hat{\mathbf{s}}_{\text{opt}} = \underset{\mathbf{s}\in\{\pm1\}^{NP}}{\arg\max}\left\|\left(\mathbf{G}^*(\mathbf{s})\otimes\mathbf{I}_{M_r}\right)\mathbf{y}\right\|^2 = \underset{\mathbf{s}\in\{\pm1\}^{NP}}{\arg\max}\left\|\mathbf{Y}\mathbf{G}^H(\mathbf{s})\right\|_{\text{F}}^2. \tag{26}$$

In the second special case, the receiver does not have knowledge of the channel statistics, hence joint ML estimation of $\mathbf{H}$ and detection of $\mathbf{s}$ is performed according to $\widehat{\{\mathbf{H},\mathbf{s}\}} = \underset{\substack{\mathbf{H}\in\mathbb{C}^{M_r\times M_t}\\ \mathbf{s}\in\{\pm1\}^{NP}}}{\arg\min}\|\mathbf{Y}-\mathbf{H}\mathbf{G}(\mathbf{s})\|_{\text{F}}^2$. Then,

$$\hat{\mathbf{s}}_{\text{GLRT}} = \underset{\mathbf{s}\in\{\pm1\}^{NP}}{\arg\min}\left\{\min_{\mathbf{H}\in\mathbb{C}^{M_r\times M_t}}\|\mathbf{Y}-\mathbf{H}\mathbf{G}(\mathbf{s})\|_{\text{F}}^2\right\} \tag{27}$$

is the generalized-likelihood ratio test (GLRT) detection of $\mathbf{s}$ [5]. For any $\mathbf{s}\in\{\pm1\}^{NP}$, the inner minimization in (27) results in $\hat{\mathbf{H}}(\mathbf{s}) = \frac{1}{TP}\mathbf{Y}\mathbf{G}^H(\mathbf{s})$. Using $\hat{\mathbf{H}}(\mathbf{s})$, the GLRT decision in (27) becomes

$$\begin{aligned}\hat{\mathbf{s}}_{\text{GLRT}} &= \underset{\mathbf{s}\in\{\pm1\}^{NP}}{\arg\min}\left\|\left(\mathbf{Y}-\frac{1}{TP}\mathbf{Y}\mathbf{G}^H(\mathbf{s})\mathbf{G}(\mathbf{s})\right)\right\|_{\text{F}}^2 \\ &= \underset{\mathbf{s}\in\{\pm1\}^{NP}}{\arg\max}\ \text{tr}\left\{\mathbf{Y}\mathbf{G}^H(\mathbf{s})\mathbf{G}(\mathbf{s})\mathbf{Y}^H\right\}.\end{aligned} \tag{28}$$

Apparently, (26) and (28) are identical problems. Both constitute special cases of Case III and can be solved in polynomial time $\mathcal{O}(P^{2M_tM_r})$ if we follow the proposed approach.

We conclude that for time-invariant Rayleigh fading the ML noncoherent detector can *always* operate with polynomial complexity in the sequence length P, the order of which is determined strictly by the rank D of the channel covariance matrix. That is, the lower the Rayleigh channel covariance rank the lower the receiver complexity. Therefore, the worst-case complexity is $\mathcal{O}\left(P^{2M_tM_r}\right)$.

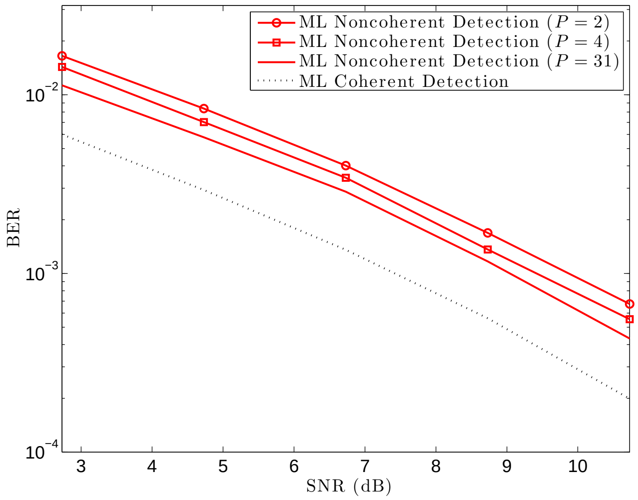

Fig. 1. BER versus SNR for ML coherent OSTBC receiver and ML noncoherent OSTBC receivers with sequence length $P = 2$ (conventional receiver), $P = 4$ (8 bits), and $P = 31$ (62 bits) upon Ricean fading.

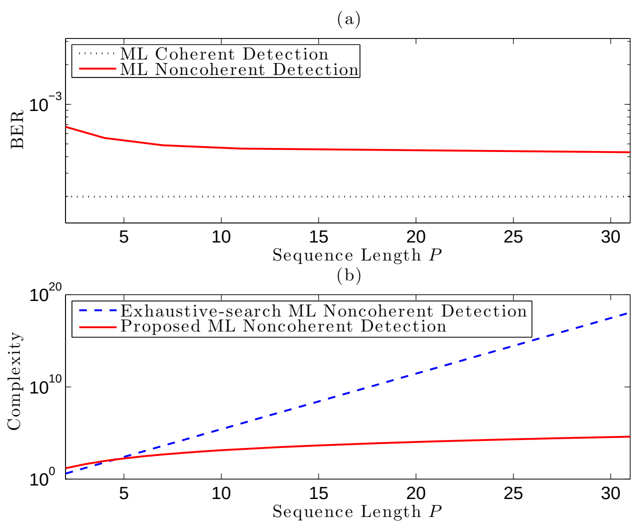

Fig. 2. (a) BER and (b) complexity versus sequence length P for SNR $= 10.7$dB and Ricean fading.

V. Simulation Studies

As an illustration, we consider a 2×2 MIMO system employing Alamouti coding (with rate $R = \frac{N}{T} = 1$, since $N = T = 2$) in an unknown Ricean fading channel environment. Space-time ambiguity induced by the rotatability of the Alamouti code [15] is resolved by employing differential space-time modulation [7]. For the covariance matrix of the vectorized channel matrix we adopt a model presented in [16], according to which $\mathbf{C}_h = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$. We consider binary transmission and present results averaged over 250 channel realizations. In Fig. 1, we present the bit error rate (BER) of the one-shot coherent MRC receiver and the proposed ML block noncoherent receiver of complexity $\mathcal{O}\left(P^4\right)$ as a function of the transmitted signal-to-noise ratio (SNR). In Fig. 2, we set the SNR to 10.7dB and present BER and computational complexity curves of the proposed ML noncoherent receiver versus block length P. In Fig. 2(a), the BER of the MRC receiver is also presented as a performance lower bound. As the block size P increases, the performance of the ML noncoherent detector approaches that of the coherent one. We emphasize that for $P > 15$ the exponential complexity ML noncoherent receiver cannot be implemented in reasonably small time while the proposed receiver maintains polynomial computational complexity. Fig. 2(b) demonstrates the significant complexity gain offered by the proposed receiver. For example, if $P = 31\,(NP+1 = 63)$, then the conventional ML receiver requires an exhaustive search among $2^{61} \approx 3 \cdot 10^{18}$ binary vectors of length 62 while the proposed ML receiver performs a search among $\binom{63}{1} + \binom{63}{3} \approx 4 \cdot 10^4$ binary vectors of length 62.

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