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Near ML Detection of Nonlinearly Distorted OFDM Signals

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) signaling exhibits high peak-to-average power ratio (PAR) which results in large sensitivity to nonlinear distortion created by the use of a high power amplifier (HPA) at the end of a wireless transmitter. In this work, in contrast with the popular approach of amplifier input signal clipping or low-PAR OFDM signal design, we utilize HPA characteristics to develop a low-complexity iterative detector of nonlinearly distorted OFDM signals that follows the principles of steepest descent search. Simulation studies indicate that the proposed method approaches maximum-likelihhod performance and offers a significant gain over plain OFDM or other competing techniques.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a very popular and promising technique for emerging telecommunication systems [1]-[3], mainly due to its robustness against multipath fading and impulsive noise [2]. However, OFDM exhibits Gaussian-like time-domain behavior, resulting in high peak-to-average power ratio (PAR). Consequently, OFDM signals have great sensitivity to nonlinear distortion created by the use of high power amplifiers (HPA) at the transmitter end [4]-[7].

To mitigate the negative effects of high PAR, signal clipping at the transmitter has been introduced in [8], [9]. Although clipping reduces PAR, it does result in large power back offs and additional nonlinear distortion. Other methods introduce the design of low-PAR OFDM signals, as suggested in [10]-[13], nonetheless decreasing the pre-detection signal-to-noise ratio (SNR) of the system and adding to the total complexity of both the transmitter and the receiver. However, all the aforementioned techniques ignore the amplifier nonlinear characteristics by assuming a linear amplifier at the transmitter end. Indeed, only few methods [14], [15] have appeared in the literature that take into account the true power amplifier nonlinear characteristics and their effect on OFDM signals. Certainly, in contrast to the linear complexity of the optimal receiver for linearly distorted OFDM signals, the maximum-likelihood (ML) OFDM receiver requires exponential complexity when nonlinear distortion is induced by the transmitter's HPA [14].

In the present work, we follow a completely different direction than the common approach of pre-amplification signal processing. Instead of modifying the pre-amplification signal at the transmitter, we utilize power amplifier and channel characteristics to develop an efficient iterative detector at the receiver end of nonlinearly distorted OFDM signals. The proposed method follows the principles of steepest descent search [16] to reduce the receiver complexity from $O(M^N)$ to $O(NM)$, where N is the number of independent frequency subcarriers and M is the quadrature amplitude modulation (QAM) order. Extensive simulation studies verify that the proposed method approaches the ML detector in terms of bit-error-rate (BER) or symbol-error-rate (SER). Furthermore, the greater than one order of magnitude gain over plain OFDM clarifies the advantages of trasfer-function-driven receiver optimization as opposed to transmitter modification approaches that have been followed extensively in the past.

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II. System Model

We consider an uncoded cyclic-prefix OFDM scheme with N orthogonal independent frequency subcarriers. The data vector to be modulated and transmitted is denoted by $\mathbf{d}_{N\times 1} = [d_0\, d_1 \ldots d_{N-1}]^T$, where symbol d_i is selected from a set of M constellation points, $d_i \in \mathcal{S} = \{s_0, s_1, ..., s_{M-1}\}$. The discrete-time OFDM symbols are obtained from the inverse discrete Fourier transform (DFT) of $\mathbf{d}$ and denoted in vector form by

$$\mathbf{x}_{N\times 1} = \mathbf{W}^H \mathbf{d}. \tag{1}$$

In (1), $\mathbf{W}$ is the DFT matrix defined as

$$\mathbf{W}_{N\times N} \triangleq \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & \omega^{1\cdot 1} & \ldots & \omega^{1\cdot(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{(N-1)\cdot 1} & \ldots & \omega^{(N-1)\cdot(N-1)} \end{bmatrix} \tag{2}$$

with $\omega = e^{-j\frac{2\pi}{N}}$. Hence, each OFDM symbol is given by

$$x_n = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} d_i e^{j\frac{2\pi n i}{N}} \tag{3}$$

with $E(x_n) = 0$ and $E(|x_n|^2) = p$, $0 \leq n \leq N-1$. To avoid channel induced intersymbol interference, the last G OFDM symbols are added as a cyclic prefix to $\mathbf{x}$, resulting in the extended symbol vector

$$\mathbf{x}^{cp}_{(N+G)\times 1} = [x_{G-N}\, x_{G-N+1} \ldots x_{N-1}\, x_0\, x_1 \ldots x_{N-1}]^T. \tag{4}$$

Before transmission, each element of the CP-OFDM vector $\mathbf{x}^{cp}$ is passed through a power amplifier whose AM/AM and AM/PM characteristics are described by the transfer function $g(x)$ so that the amplifier-output transmited signal in vector form becomes

$$\mathbf{x}^{cp,g}_{(N+G)\times 1} = [x^g_{G-N}\, x^g_{G-N+1} \ldots x^g_{N-1}\, x^g_0\, x^g_1 \ldots x^g_{N-1}]^T \tag{5}$$

with $x^g_i \triangleq g(x_i)$.

The baseband equivalent of the channel is assumed multipath fading with length L and AWGN. The channel coefficients $c_0, c_1, \ldots, c_{L-1}$ are modeled as independent zero-mean complex Gaussian random variables. We denote by $\mathbf{c}_{L\times 1} = [c_0\, c_1 \ldots c_{L-1}]^T$ and $\tilde{\mathbf{c}}_{N\times 1} = \mathbf{W}^H[c_0\, c_1 \ldots c_{L-1} 0 \ldots 0]^T$ the channel vector and its N-point DFT, respectively.

Upon downconversion and pulse-matched filtering or pulse-rate sampling, the receiver observes the convolution of $\mathbf{x}^{cp,g}$ with the channel vector $\mathbf{c}$, distorted by zero-mean additive white Gaussian noise (AWGN) $\mathbf{w}_{(L+G+N-1)\times 1}$ of variance σ^2. Therefore, the received samples form the vector

$$\mathbf{q}^{cp,g}_{(L+G+N-1)\times 1} = \mathbf{c} * \mathbf{x}^{cp,g} + \mathbf{w} \tag{6}$$

The cyclic prefix is removed and the remaining vector $\mathbf{q}^g_{N\times 1} = [q^g_G\, q^g_{G+1} \ldots q^g_{N+G-1}]^T$ is passed through the DFT operation to yield:

$$\mathbf{y}^g_{N\times 1} = \mathbf{W}\mathbf{q}^g = \tilde{\mathbf{c}} \circ (\mathbf{W}\mathbf{x}^g) + \mathbf{n} \tag{7}$$

where $\circ$ denotes element-by-element multiplication of two vectors ($\mathbf{a} \circ \mathbf{b} = [a_0 b_0\; a_1 b_1 \ldots a_{N-1} b_{N-1}]^T$) and $\mathbf{n}_{N\times 1}$ is a zero-mean AWGN vector with covariance matrix σ^2I. Finally, the plain OFDM receiver performs symbol-by-symbol one-shot detection to obtain the detected data vector

$$\hat{\mathbf{d}}_{N\times 1} = \left[\ \left\langle \frac{y^g_0}{\tilde{c}_0} \right\rangle \left\langle \frac{y^g_1}{\tilde{c}_1} \right\rangle \ldots \left\langle \frac{y^g_{N-1}}{\tilde{c}_{N-1}} \right\rangle\ \right] \tag{8}$$

where $\langle\cdot\rangle$ denotes minimum Euclidean distance detection with respect to the transmitter signal constellation. It can be easily shown that (8) represents the ML detection of $\mathbf{d}$ when the amplifier AM/AM and AM/PM curves correspond to a linear function $g(\mathbf{x})$.

III. Near Maximum-likelihood Detection

In wireless applications, the elements of the CP-OFDM block are amplified using a nonlinear HPA, resulting in suboptimality of the plain OFDM receiver in (8). The charateristic funtion considered in most cases is a nonlinear function $g(\mathbf{x})$, where $\mathbf{x}$ is the OFDM block to be transmitted. To examine the nonlinearity efects on the transmitted signal, we express the OFDM symbol x_i using its polar coordinates as

$$x_i = |x_i| e^{j\,\arg(x_i)} = \rho_i e^{j\phi_i}, \quad \rho_i \geq 0, \quad 0 \leq i \leq N-1. \tag{9}$$

Since the nonlinear HPA operation is memoryless, the complex envelope representation of the output signal for an input signal $\rho e^{j\phi}$ is given by

$$g(\rho e^{j\phi}) = F(\rho) e^{j(\phi + \Phi(\rho))} \tag{10}$$

where $F(\cdot)$ and $\Phi(\cdot)$ denote the HPA amplitude and phase, respectively, conversion characteristics. We consider the following three different types of HPA transfer functions.

1) *Soft Limiter (SL)*

The SL conversion characteristics are given by

$$F(\rho) = \begin{cases} G\rho, & \rho \leq A \\ A, & \rho > A \end{cases}, \quad \Phi(\rho) = 0, \tag{11}$$

where G is the small signal gain and A is the saturation amplitude.

2) *Solid-State Power Amplifier (SSPA)*

The SSPA conversion characteristics are usually modeled as

$$F(\rho) = \frac{G\rho}{[1 + (\frac{G\rho}{A})^{2p}]^{\frac{1}{2p}}}, \quad \Phi(\rho) = 0, \tag{12}$$

where G is the small signal gain, A is the saturation amplitude, and p is a “shaping” integer. For large values of p the SL and SSPA AM/AM curves approach each other.

3) *Traveling-Wave Tube (TWT)*

The TWT conversion characteristics are modeled by

$$F(\rho) = \frac{G\rho}{1 + (\frac{G\rho}{2A})^2}, \quad \Phi(\rho) = \frac{\pi}{3}\frac{\rho^2}{\rho^2 + 4A^2}, \tag{13}$$

where G is the small signal gain and A is the saturation amplitude.

Upon signal transmission, “multipath channel processing,” downconversion, and sampling, the ML detector decides in favor of the transmitted data vector $\mathbf{d}$ according to the optimization problem

$$\begin{aligned} \hat{\mathbf{d}}^{\mathrm{ML}} &= \arg \min_{\mathbf{d} \in \mathcal{S}^N} ||\mathbf{y}^g - \tilde{\mathbf{c}} \circ (\mathbf{W} g(\mathbf{W}^H \mathbf{d}))|| \\ &= \arg \min_{j=0,1,\ldots,M^N-1} ||\mathbf{y}^g - \tilde{\mathbf{c}} \circ (\mathbf{W} g(\mathbf{W}^H \mathbf{d}_j))||. \end{aligned} \tag{14}$$

In (18), $\mathbf{d}_j$ denotes the j-th transmitted data combination vector, $j = 0, 1, ..., M^N - 1$.

The ML detector according to (18) requires an exaustive search among all possible input sequences (elements of $\mathcal{S}^N$) resulting in complexity of order $O(M^N)$. To avoid this exponential cost, most receiver schemes ignore the HPA nonlinearity and detect the data vector $\mathbf{d}$ accoring to (8). Regretfully, as it will be seen in the next section, such an approach results in significant performance degradation.

To reduce the performance degradation that is induced by the linear OFDM detection of nonlinearly distorted signals, several works attempt to “linearize” the transmitter HPA by enforcing the pre-amplification signal to the linear amplifier-input region. Although under such a modification the linear OFDM receiver becomes the ML one, simulation results in the next session indicate a significant performance loss in comparison with ML detection of unmodified OFDM transmissions.

In contrast to both above directions, in the present method we do not alter the pre-amplification signal but instead we attempt to efficiently approach the ML detection performance at the receiver. Along these lines, we perform an iterative steepest descent search [16] of Hamming distance one initialized at the linear OFDM receiver output obtained from (8). More specifically, given a detected data vector $\hat{\mathbf{d}}$, we define the set

$$\mathcal{S}^N(\hat{\mathbf{d}}) = \{\mathbf{d} \in \mathcal{S}^N : h(\hat{\mathbf{d}}, \mathbf{d}) = 1\} \tag{15}$$

that consist of the $(M-1)N$ nearest vectors $\mathbf{d}_1(\hat{\mathbf{d}}), ..., \mathbf{d}_{(M-1)N}(\hat{\mathbf{d}})$ that are closest to $\hat{\mathbf{d}}$, i.e the vectors that are at a Hamming distance one far from $\hat{\mathbf{d}}$. In (15) $h(\cdot,\cdot)$ denotes the Hamming distance between two vectors. Then, the elements of $\mathcal{S}^N(\hat{\mathbf{d}})$ as well as $\hat{\mathbf{d}}$ are compared with each other in terms of the likelihood metric and the vector that

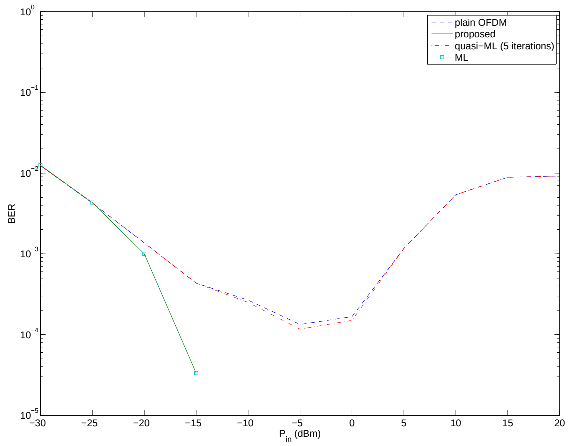

Fig. 1. BER versus P_{in} for the plain OFDM, quasi-ML, proposed, and ML receivers.

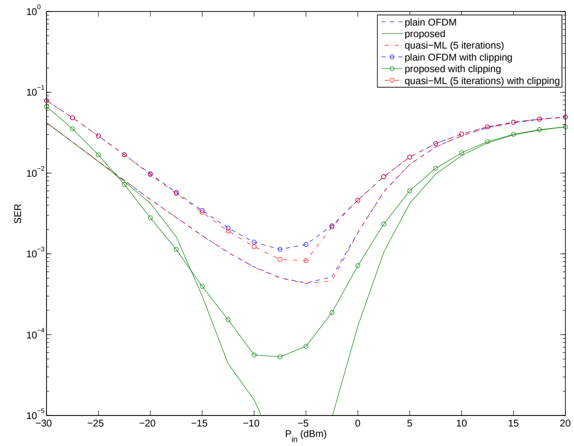

Fig. 2. SER versus P_{in} for the plain OFDM, quasi-ML, and proposed receivers as well as their clipped versions.

minimizes the corresponding Euclidean distance metric is chosen as the locally optimal one. That is,

$$\hat{\mathbf{d}}^{(1)} = \arg \min_{\mathbf{d} \in \mathcal{S}^N(\hat{\mathbf{d}}) \cup \{\hat{\mathbf{d}}\}} ||\mathbf{y}^g - \tilde{\mathbf{c}} \circ (\mathbf{W} g(\mathbf{W}^H \mathbf{d}))||. \quad (16)$$

If we wish to continue the iterative algorithm, then we define the new set

$$\mathcal{S}^N(\hat{\mathbf{d}}^{(1)}) = \{\mathbf{d} \in \mathcal{S}^N : h(\hat{\mathbf{d}}^{(1)}, \mathbf{d}) = 1\} \quad (17)$$

and seek for the locally optimal data vector

$$\hat{\mathbf{d}}^{(2)} = \arg \min_{\mathbf{d} \in \mathcal{S}^N(\hat{\mathbf{d}}^{(1)}) \cup \{\hat{\mathbf{d}}^{(1)}\}} ||\mathbf{y}^g - \tilde{\mathbf{c}} \circ (\mathbf{W} g(\mathbf{W}^H \hat{\mathbf{d}}))||. \quad (18)$$

The iteration may continue until convergence which is guaranteed in a finite number of iterations [16]. Certainly, convergence to the ML detection $\hat{\mathbf{d}}^{ML}$ is not guaranteed. However, one iteration is interestingly enough to approach closely the ML solution, as we will see in the next section. Therefore, with $O((M-1)N)$ cost we attain near ML performance. Supprisingly, this near ML approach offers one to two orders of magnitude gain in terms of BER or SER in comparison with competing PAR reduction techniques. This performance gain is especially identified over the critical signal power range that includes the saturation point of the HPA.

IV. Numerical Results

We consider OFDM transmissions over a multipath fading channel with L independent complex Gaussian coefficients of exponentially decreasing variance. The channel path loss is set to -75dB, a value that holds true in wireless systems and in transmitter-receiver distances of approximately 10-30 meters. The channel also introduces zero-mean AWGN with power σ^2. The OFDM cyclic prefix has length $G = L-1$. The transmitter utilizes a SSPA as described in (12) with gain factor 30dB, saturation input power -5dBm, and smoothing parameter $p = 1$.

In our studies, we examine the error probability of the plain OFDM receiver, the quasi-ML algorithm [14], and the proposed detector upon one iteration, as described in Section III. We also compare the above three schemes when signal clipping is considered at the transmitter amplifier input. In the latter case, the clip level $\mathrm{CL} = 20\log_{10}\frac{A}{\sigma}$dB, where A is the saturation amplitude, is set to 0dB.

In Fig. 1, we set the OFDM block length to $N = 12$, the channel length to $L = 8$, and the channel to noise ratio (CNR) to 43dB. The corresponding noise variance becomes $\sigma^2 = -113$dB. We assume BPSK transmissions and plot the BER as a function of the amplifier input signal power for the three schemes under consideration. As a reference, we also include the BER of the ML detector obtained through exhaustive search. We observe the performance of the plain OFDM and quasi-ML receivers deteriorates significantly for mean input signal power values greater than the saturation input power of -5dBm, which corresponds to an SNR of 35dB. On the contrary, the proposed detector shows robustness to nonlinearity effects and attains near ML performance offering at least one order of magnitude gain in comparison with plain OFDM or quasi-ML receivers for an amplifier input signal power of -15dBm or higher.

In Fig. 2, we consider QPSK modulation, increase the block length to $N = 64$ and the channel length to $L = 17$, and plot the SER as a function of the amplifier input power. The proposed scheme outperforms all other schemes by a gain of one to two orders of magnitude around the saturation point. We also observe that clipping leads to superior performance only for the proposed scheme and for a short range of input power ($-23\text{dBm} < P_{in} < -17$dBm).

In Fig. 3, the SSPA operates at the saturation level ($\mathrm{P}_{in} = -5$dBm). We vary the AWGN variance σ^2 from -80dB to -140dB, resulting in CNR range of 10-70dB and a corresponding SNR range of $0-60$dB. The proposed receiver curve descends faster than the plain OFDM and the quasi-ML receivers, resulting in about 20dB SNR gain at SER=10^{-5}. Once more, clipping leads to inferior performance for all three schemes.

In Fig. 4, the SSPA operates essentially in the linear region ($\mathrm{P}_{in} = -12$dBm) where the proposed iterative receiver still

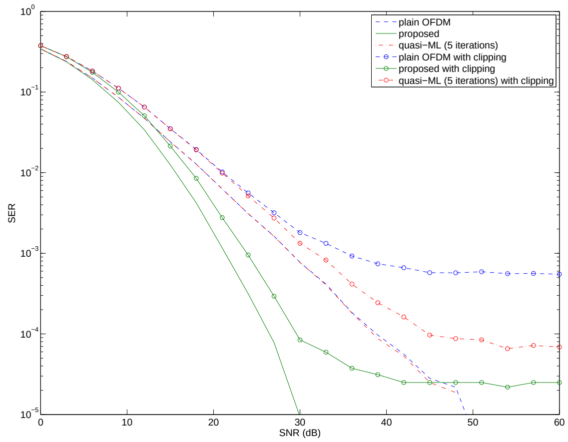

Fig. 3. SER versus SNR for the plain OFDM, quasi-ML, and proposed receivers as well as their clipped versions.

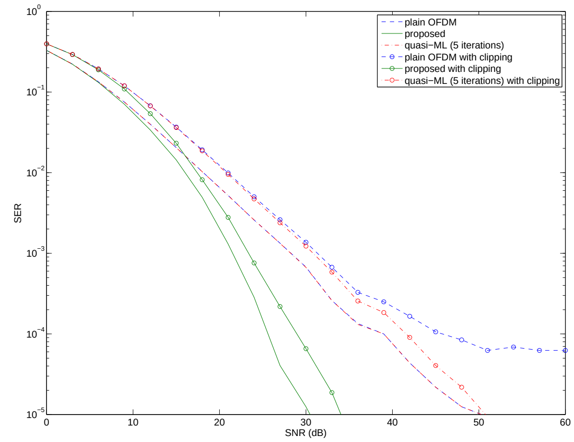

Fig. 4. SER versus SNR for the plain OFDM, quasi-ML, and proposed receivers as well as their clipped versions.

outperforms the other two schemes by 20dB at SER=10^{-5}. The CNR of ranges from 14dB to 74dB and the noise variance σ^2 from -84dB to -144dB.

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