POLYNOMIAL-COMPLEXITY MAXIMUM-LIKELIHOOD BLOCK NONCOHERENT MPSK DETECTION

Dimitris S. Papailiopoulos and George N. Karystinos

Department of Electronic and Computer Engineering
Technical University of Crete
73100 Kounoupidiana, Chania, Greece
E-Mail: dpapailiopoulos@isc.tuc.gr, karystinos@telecom.tuc.gr

ABSTRACT

In wireless channels, maximum-likelihood (ML) block noncoherent detection offers significant gains over conventional symbol-by-symbol detection when the fading channel coefficients are not available and cannot be estimated at the receiver. Certainly, in general the complexity of the block detector grows exponentially with the symbol sequence length. However, it has been recently shown that for M-ary phase-shift keying (MPSK) modulation block noncoherent detection can be performed with polynomial complexity. In this work, we develop a new ML block noncoherent detector for MPSK transmission of arbitrary order and multiple-antenna reception. The proposed algorithm introduces auxiliary spherical variables and constructs with polynomial complexity a polynomial-size set which includes the ML data sequence. It is shown that the complexity of the proposed algorithm is polynomial in the sequence length and at least one order of magnitude lower than the complexity of computational-geometry based noncoherent detection algorithms that have been developed recently.

Index Terms — Maximum-likelihood sequence detection, noncoherent detection, single-input multiple-output channels.

1. INTRODUCTION

Multiple-antenna wireless systems are well known to attain increased orders of diversity resulting in substantially higher system capacity compared to single-antenna systems. When perfect channel state information (CSI) is available or can be retrieved through adequate channel estimation at the receiver, several coherent detection schemes can be followed. However, the very nature of wireless channels suggests rapidly changing channel conditions, thus making channel estimation complex and cost inefficient. Even when channel fades occur slowly, phase distortion is introduced and must be accounted for at the receiver end to avoid performance loss.

Alternatively, noncoherent detection has been studied extensively [1]-[5] and implemented in modern digital communication standards. Since noncoherent detection does not need any channel knowledge or estimation, it is applicable even in most degraded and fast fading channels, making it much more attractive than coherent detection under unfavorable channel conditions. Due to the memory in the received data sequence induced by fading channel memory, noncoherent maximum likelihood sequence detection (MLSD) has recently been a subject of extensive research [1]-[4]. Optimal receivers that suffer from exponential complexity with respect to the data sequence length as well as approximate and sub-optimal detection algorithms were developed in [1], [5]. However, very recent studies [3], [4] proved the existence of efficient noncoherent MLSD receiver schemes that attain optimality with polynomial complexity by utilizing computational-geometry (CG) based optimization algorithms.

The present work shows that noncoherent MLSD of MPSK symbols in SIMO systems can be expressed as a rank-deficient quadratic form maximization problem and computed efficiently in polynomial time. We follow a completely different approach than [3],[4] and, inspired by the work in [8],[1] construct a polynomial-complexity noncoherent MLSD method that is at least one order of magnitude faster than the method in [4]. The proposed method that is developed in this present work is also applicable to any arbitrary-order MPSK modulation. Further analysis shows that the computational complexity depends only on the data sequence length and receive-diversity order and does not depend on SNR.

2. SYSTEM MODEL

We consider the transmission of a sequence of N uncoded M-ary phase-shift keying (MPSK) data symbols $\mathbf{s} = \sqrt{P}[s_1, s_2, ..., s_N]^T$ where P is the constant transmitted power per symbol and s_i is selected from an M-ary alphabet $\mathcal{A}_M \triangleq \{e^{j\frac{\pi}{M}(2m+1)} | m = 0, 1, \ldots, M-1\}$, $i = 1, 2, \ldots, N$. The

[1] The work in [6]-[8] considers the efficient computation of the binary vector that maximizes a rank-deficient quadratic form. The authors prove the existence of the optimal solution and develop a method that computes it in polynomial time. Although rank-deficient quadratic form maximization was also treated in [10] based on CG principles, the method in [6]-[8] requires at least one order of magnitude less complexity compared to the method in [10].

data sequence is shaped and transmitted over D independent and identically distributed (i.i.d.) frequency flat Rayleigh fading wireless channels. The downconverted and pulse-matched equivalent received signal at the ith antenna is

$$\mathbf{y}_i = h_i\mathbf{s} + \mathbf{n}_i \tag{1}$$

where h_i denotes the coefficient of the channel between the transmit antenna and the ith receive antenna and is modeled as zero-mean complex Gaussian with variance σ_h^2. Furthermore, $\mathbf{n}_i$ represents additive white complex Gaussian noise (AWGN) and is modeled as a zero-mean complex Gaussian vector with co-variance matrix $\sigma_n^2\mathbf{I}$. We collect all received data from the D receive antennas and form the $N \times D$ "received matrix" $\mathbf{Y} \stackrel{\triangle}{=} [\mathbf{y}_1\mathbf{y}_2 \ldots \mathbf{y}_D]$.

The D channel coefficients h_i, $i = 1, 2, \ldots, D$, are assumed unknown to both the transmitter and the receiver, implying that noncoherent detection has to be performed. The MLSD decision for the transmitted sequence $\mathbf{s}$ given the $N \times D$ observation matrix $\mathbf{Y}$ maximizes the conditional probability density function (pdf) of $\mathbf{Y}$ given $\mathbf{s}$. Thus, the maximization problem becomes

$$\mathbf{s}_{opt} \stackrel{\triangle}{=} \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} f(\mathbf{Y}|\mathbf{s}) = \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} f(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_D|\mathbf{s}). \tag{2}$$

Due to independence among the D channels, the columns of the received matrix $\mathbf{Y}$ are i.i.d. given the transmitted sequence $\mathbf{s}$. Therefore,

$$\mathbf{s}_{opt} = \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} \prod_{i=1}^{D} f(\mathbf{y}_i|\mathbf{s}) = \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} \sum_{i=1}^{D} \ln f(\mathbf{y}_i|\mathbf{s}). \tag{3}$$

The conditional received vector at the ith antenna given the transmitted sequence is $\mathbf{y}_i|\mathbf{s} = h_i\mathbf{s} + \mathbf{n}_i$ where $h_i\mathbf{s}$ is a singular complex Gaussian vector independent from $\mathbf{n}_i$, $i = 1, 2, \ldots, D$. The following proposition identifies the pdf of $\mathbf{y}_i|\mathbf{s}$. The proof is omitted due to lack of space.

Proposition 1 *The sum of a singular complex Gaussian vector and an independent complex Gaussian vector results in a complex Gaussian vector.* □

According to Proposition 1, since h_i and $\mathbf{n}_i$ are both zero-mean, $\mathbf{y}_i|\mathbf{s}$ is a zero-mean complex Gaussian vector with covariance matrix $\mathbf{R} = \sigma_n^2\mathbf{I} + P\sigma_h^2\mathbf{s}\mathbf{s}^H$. As a result, the MLSD receiver of (3) becomes

$$\begin{aligned}\mathbf{s}_{opt} &= \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} \sum_{i=1}^{D} \ln \frac{1}{\pi^N|\mathbf{R}|} \exp\left\{-\mathbf{y}_i^H\mathbf{R}^{-1}\mathbf{y}_i\right\} \\ &= \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} \sum_{i=1}^{D} \left(-\mathbf{y}_i^H\mathbf{R}^{-1}\mathbf{y}_i + \ln\frac{1}{\pi^N|\mathbf{R}|}\right).\end{aligned} \tag{4}$$

Using $|\mathbf{A} + \mathbf{c}\mathbf{d}^H| = |\mathbf{A}|(1 + \mathbf{d}^H\mathbf{A}^{-1}\mathbf{c})$ [11], we compute $|\mathbf{R}| = \sigma_n^{2N}(1 + NP\frac{\sigma_h^2}{\sigma_n^2})$. Therefore, $|\mathbf{R}|$ is not a function of $\mathbf{s}$ and, hence, can be dropped from the detector in (4). Moreover, using the matrix inversion lemma, the inverse of $\mathbf{R}$ becomes $\mathbf{R}^{-1} = \frac{1}{\sigma_n^2}\left(\mathbf{I} - \frac{\sigma_h^2}{\sigma_n^2 + NP\sigma_h^2}\mathbf{s}\mathbf{s}^H\right)$, implying that the decision rule in (4) is simplified to

$$\begin{aligned}\mathbf{s}_{opt} &= \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} \sum_{i=1}^{D} \frac{1}{\sigma_n^2}\left(-||\mathbf{y}_i||^2 + \frac{\sigma_h^2}{\sigma_n^2 + NP\sigma_h^2}\mathbf{y}_i^H\mathbf{s}\mathbf{s}^H\mathbf{y}_i\right) \\ &= \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} \sum_{i=1}^{D} |\mathbf{y}_i^H\mathbf{s}|^2 \\ &= \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} ||\mathbf{Y}^H\mathbf{s}||.\end{aligned} \tag{5}$$

If the above optimization is performed through exhaustive search, then it costs $\mathcal{O}(M^N)$ computations which is an intractable complexity even for moderate vales of N. In the next section, we follow an approach similar to the one of [6]-[8] but tailored to our detection problem in (5). Specifically, we introduce $2D-1$ spherical coordinates and develop an efficient algorithm to build a set $\mathcal{S}(\mathbf{Y}_{N\times D}) \subset \mathcal{A}_M^N$ that consists of $|\mathcal{S}(\mathbf{Y}_{N\times D})| = \mathcal{O}((NM)^{2D-1})$ signal vectors, is constructed with $\mathcal{O}((MN)^{2D})$ computations, and contains the optimal vector $\mathbf{s}_{opt}$ in (5).

3. EFFICIENT ML BLOCK NONCOHERENT MPSK DETECTION

We introduce $2D-1$ auxiliary hyperspherical coordinates $\phi_1 \in (-\pi, \pi]$, $\phi_2, \ldots, \phi_{2D-1} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and define the $2D \times 1$ hyperspherical vector $\tilde{\mathbf{c}}(\phi_1, \ldots, \phi_{2D-1}) \stackrel{\triangle}{=} [\sin\phi_1$ $\cos\phi_1 \sin\phi_2$ $\ldots$ $\cos\phi_1 \ldots \cos\phi_{2D-2} \sin\phi_{2D-1}$ $\cos\phi_1 \ldots \cos\phi_{2D-2} \cos\phi_{2D-1}]^T$ as well as the $D\times 1$ hyperspherical complex vector $\mathbf{c}(\phi_1, \ldots, \phi_{2D-1}) \stackrel{\triangle}{=} \tilde{\mathbf{c}}_{1:D,1}(\phi_1, \ldots, \phi_{2D-1}) + j\tilde{\mathbf{c}}_{D+1:2D,1}(\phi_1, \ldots, \phi_{2D-1})$. Then, the problem in (5) is rewritten as

$$\begin{aligned}&\mathbf{s}_{opt} = \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} ||\mathbf{Y}^H\mathbf{s}|| = \\ &\arg\max_{\mathbf{s}\in\mathcal{A}_M^N} \max_{\phi_1\in(-\pi,\pi]} \max_{\phi_2,\ldots,\phi_{2D-1}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right]} \left|\mathbf{s}^H\mathbf{Y}\mathbf{c}(\phi_1, \ldots, \phi_{2D-1})\right|\end{aligned} \tag{6}$$

due to Cauchy-Schwartz Inequality which states that for any $\mathbf{v} \in \mathbb{C}^D$ $|\mathbf{v}^H\mathbf{c}(\phi_1, \ldots, \phi_{2D-1})| \leq ||\mathbf{v}||||\mathbf{c}(\phi_1, \ldots, \phi_{2D-1})||$ with equality if and only if $\phi_1, \ldots, \phi_{2D-1}$ are the hyperspherical coordinates of $\mathbf{v}$. Furthermore $\forall\mathbf{v} \in \mathbb{C}^D$, $\Re\{\mathbf{v}^H \mathbf{c}(\phi_1, \ldots, \phi_{2D-1})\} \leq |\mathbf{v}^H\mathbf{c}(\phi_1, \ldots, \phi_{2D-1})|$, with equality if and only if $\phi_1, \ldots, \phi_{2D-1}$ are the hyperspherical coordinates of $\mathbf{v}$. Hence, the optimization problem in (6) becomes

$$\begin{aligned}\mathbf{s}_{opt} = \arg\max_{\mathbf{s}\in\mathcal{A}_M^N} \max_{\phi_1\in(-\pi,\pi]} \max_{\phi_2,\ldots,\phi_{2D-1}\in(-\frac{\pi}{2},\frac{\pi}{2}]} \Re\{ \\ \mathbf{s}^H\mathbf{Y}\mathbf{c}(\phi_1, \ldots, \phi_{2D-1})\}.\end{aligned} \tag{7}$$

We interchange the maximizations in (7) and obtain the equivalent problem

$$\max_{\phi_1\in(-\pi,\pi]}\max_{\phi_2,\ldots,\phi_{2D-1}\in(-\frac{\pi}{2},\frac{\pi}{2}]}\sum_{n=1}^{N}\max_{s_n\in\mathcal{A}_M}\Re\{ s_n^*\mathbf{Y}_{n,1:D}\mathbf{c}(\phi_1,\phi_2,\ldots,\phi_{2D-1})\}. \quad (8)$$

For a given set of angles $(\phi_1,\ldots,\phi_{2D-1})\in(-\pi,\pi]\times\left(-\frac{\pi}{2},\frac{\pi}{2}\right]^{2D-2}$, the maximizing argument of each term of the sum in (8) depends only on the corresponding row of $\mathbf{Y}$. As $\phi_1,\phi_2,\ldots,\phi_{2D-1}$ vary, the decision in favor of s_n is maintained as long as a decision boundary is not crossed. Due to the structure of $\mathcal{A}_M$, the $\frac{M}{2}$ decision boundaries that affect the maximization in (8) are given by

$$\mathbf{Y}_{n,1:D}\mathbf{c}(\phi_1,\ldots,\phi_{2D-1})=Ae^{j2\pi\frac{k}{M}}, \quad k=0,1,\ldots,\frac{M}{2}-1,\quad n=1,2,\ldots,N. \quad (9)$$

The decision boundaries in (9) can be rewritten as $\Im\{e^{-j2\pi\frac{k}{M}}\mathbf{Y}_{n,1:D}\mathbf{c}(\phi_1,\ldots,\phi_{2D-1})\}=0, k=0,1,\ldots,\frac{M}{2}-1,\quad n=1,2,\ldots,N$, which is equivalent to

$$\tilde{\mathbf{Y}}_{l,1:2D}\tilde{\mathbf{c}}(\phi_1,\ldots,\phi_{2D-1})=0,\quad l=1,\ldots,\frac{MN}{2}, \quad (10)$$

where $\tilde{\mathbf{Y}}\triangleq\left[\Im(\hat{\mathbf{Y}})\quad\Re(\hat{\mathbf{Y}})\right]$, $\hat{\mathbf{Y}}\triangleq\mathbf{Y}\otimes[1\ e^{j\frac{2\pi}{M}}\ e^{j\frac{4\pi}{M}}\ldots e^{j\frac{2\pi}{M}(\frac{M}{2}-1)}]^T$, and $\otimes$ denotes Kronecker product.

The inner maximization rule in (8) motivates us to define a *decision function s* that maps a set of angles $(\phi_1,\phi_2,\ldots,\phi_{2D-1})$ to a certain value of set $\mathcal{A}_M$ according to

$$\mathbf{s}(\mathbf{y}^T;\phi_1,\phi_2,\ldots,\phi_{2D-1})\triangleq \arg\max_{\mathbf{s}\in\mathcal{A}_M}\Re\left\{s^*\mathbf{y}^T\mathbf{c}(\phi_1\ldots,\phi_{2D-1})\right\} \quad (11)$$

for any $\mathbf{y}\in\mathbb{C}^D$. Then, for the given $N\times D$ matrix $\mathbf{Y}$, each set of angles in $(-\pi,\pi]\times\left(-\frac{\pi}{2},\frac{\pi}{2}\right]^{2D-2}$ is mapped to a candidate binary vector

$$\mathbf{s}(\mathbf{Y}_{N\times D};\phi_1,\ldots,\phi_{2D-1})\triangleq\begin{bmatrix} s(\mathbf{Y}_{1,1:D};\phi_1,\ldots,\phi_{2D-1})\\ s(\mathbf{Y}_{2,1:D};\phi_1,\ldots,\phi_{2D-1})\\ \vdots\\ s(\mathbf{Y}_{N,1:D};\phi_1,\ldots,\phi_{2D-1})\end{bmatrix} \quad (12)$$

and the optimal vector $\mathbf{s}_{opt}$ in (7) belongs to the reduced set $\bigcup_{\phi_1\in(-\pi,\pi]}\bigcup_{\phi_2,\ldots,\phi_{2D-1}\in(-\frac{\pi}{2},\frac{\pi}{2}]}\mathbf{s}(\mathbf{Y}_{N\times D};\phi_1,\ldots,\phi_{2D-1})$. Furthermore, since opposite M-ary vectors result in the same metric in (5), we can ignore the values of ϕ_1 in $\left(-\pi,-\frac{\pi}{2}\right]\cup\left(\frac{\pi}{2},\pi\right]$ and consider $\phi_1,\ldots,\phi_{2D-1}\in\Phi\triangleq\left(-\frac{\pi}{2},\frac{\pi}{2}\right]$. Finally, we collect all candidate M-ary vectors to set

$$\mathcal{S}(\mathbf{Y}_{N\times D})\triangleq\bigcup_{\phi_1,\ldots,\phi_{2D-1}\in\Phi}\{\mathbf{s}(\mathbf{Y}_{N\times D};\phi_1,\ldots,\phi_{2D-1})\}\subseteq\mathcal{A}_M^N, \quad (13)$$

hence,

$$\mathbf{s}_{opt}=\arg\max_{s\in\mathcal{S}(\mathbf{Y})}||\mathbf{Y}^H\mathbf{s}||. \quad (14)$$

Therefore, $\mathbf{s}_{opt}$ belongs to a set $\mathcal{S}(\mathbf{Y}_{N\times D})$ whose cardinality is later proved to be $|\mathcal{S}(\mathbf{Y}_{N\times D})|=\mathcal{O}((NM)^{2D-1})$ and construction is achieved with complexity $\mathcal{O}((NM)^{2D})$.

From (11), we observe that the rows of the $\frac{MN}{2}\times 2D$ matrix $\tilde{\mathbf{Y}}$ determine $\frac{MN}{2}$ hypersurfaces $\mathcal{F}(\tilde{\mathbf{Y}}_{1,1:2D})$ $,\mathcal{F}(\tilde{\mathbf{Y}}_{2,1:2D}),\ldots,\mathcal{F}(\tilde{\mathbf{Y}}_{\frac{MN}{2},2:D})$ that partition the hypercube Φ^{2D-1} into K cells $C_1,C_2,\ldots,C_K$ such that $\bigcup_{k=1}^K C_k=\Phi^{2D-1}$, $C_k\cap C_j\neq 0\ \forall\ k\neq j$, with each cell C_k corresponding to a unique $\mathbf{s}_k\in\mathcal{A}_M^N$. Let $\{i_1,i_2,\ldots,i_{2D-1}\}\subset\{1,2,\ldots,\frac{MN}{2}\}$ be a subset of $2D-1$ indices (that correspond to the $\frac{MN}{2}$ hypersurfaces) and $\phi(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times D};i_1,\ldots,i_{2D-1})\in\Phi^{2D-1}$ equal the vector of coordinates of the intersection of hypersurfaces $\mathcal{F}(\tilde{\mathbf{Y}}_{i_1,1:2D}),\ldots,\mathcal{F}(\tilde{\mathbf{Y}}_{i_{2D-1},1:2D})$. It can be shown that a "collection" of $2D-1$ hypersurfaces, say $\mathcal{F}(\tilde{\mathbf{Y}}_{i_1,1:2D}),\mathcal{F}(\tilde{\mathbf{Y}}_{i_2,1:2D}),\ldots,\mathcal{F}(\tilde{\mathbf{Y}}_{i_{2D-1},1:2D})$, has a unique intersection (which is a vertex of a cell) if and only if no more than two hypersurfaces originate from the same row of the observation matrix $\mathbf{Y}$. Such a cell, say $C\left(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2D};i_1,\ldots,i_{2D-1}\right)$, is associated with a unique vector $\mathbf{s}\left(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2D};i_1,\ldots,i_{2D-1}\right)$. We collect all such vectors to set

$$J(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2D})\triangleq\bigcup_{\{i_1,\ldots,i_{2D-1}\}\subset\{1,\ldots,\frac{MN}{2}\}}\left\{\mathbf{s}\left(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2D};i_1,\ldots,i_{2D-1}\right)\right\}\subseteq\mathcal{A}_M^N \quad (15)$$

with cardinality $|J(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2D})|=\sum_{n=0}^{D-1}\binom{N}{n}\binom{N-n}{2D-(1+2n)}\left(\frac{M}{2}\right)^{2D-1-n}=\mathcal{O}((NM)^{2D-1})$. Thus, $J(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2D})$ contains $\mathcal{O}((NM)^{2D-1})$ M-ary vectors. Then, it can be shown [8] that all candidate vectors form the set

$$\mathcal{S}(\mathbf{Y}_{N\times D})=J(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2D})\cup\ldots\cup J(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2}) =\bigcup_{d=0}^{D-1}J(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2(D-d)}). \quad (16)$$

To summarize, we have utilized $2D-1$ auxiliary spherical coordinates, and partitioned the hypercube Φ^{2D-1} into $\mathcal{O}((NM)^{2D-1})$ cells associated with unique M-ary vectors that constitute the set $\mathcal{S}(\mathbf{Y}_{N\times D})\subseteq\mathcal{A}_M^N$ which includes $\mathbf{s}_{opt}$ in (5). Therefore, the initial detection problem in (5) has been converted into a maximization among $\mathcal{O}((NM)^{2D-1})$ candidate vectors.

The construction of $\mathcal{S}(\mathbf{Y}_{N\times D})$ is of special interest since it determines the overall performance of the proposed method. According to (16), it reduces to the parallel construction of $J(\tilde{\mathbf{Y}}_{\frac{MN}{2}\times 2d})$, for $d=2D,2D-2,\ldots,2$, which can be also fully parallelized since cells in the hypersurface arrangement are examined independently from each other. It can be shown that the decision function in (11) determines

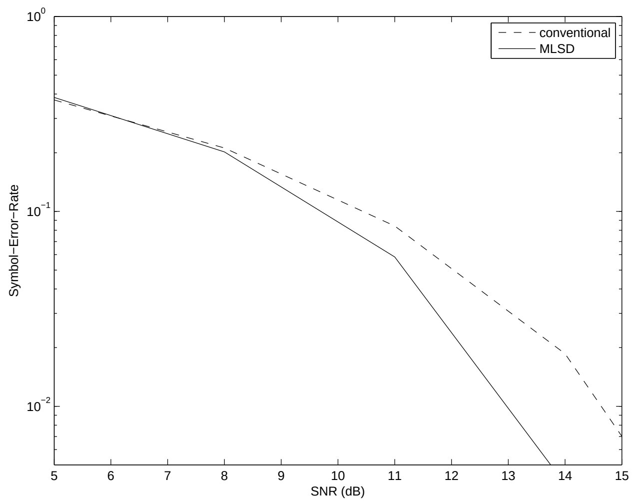

Figure 1: SER versus SNR for conventional and (proposed) MLSD noncoherent receivers.

definitely the corresponding symbol s_n if and only if no hypersurface originates from $\mathbf{Y}_{n,1:d}$. For the hypersurfaces that pass through the cell intersection, the rule in (11) becomes ambiguous. In such a case, definite determination of s_n is attained if ϕ_{2D-1} is set to $\frac{\pi}{2}$ and (11) is examined at the intersection of the same hypersurfaces except from the hypersurface of interest.

The algorithm for the construction of $\mathcal{S}(\mathbf{Y}_{N\times D})$ is available at http://www.telecom.tuc.gr/∼karystinos. The algorithm visits independently the $|\mathcal{S}(\mathbf{Y}_{N\times D})| = \mathcal{O}((NM)^{2D-1})$ intersections and computes the candidate vector in $\mathcal{A}_M^N$ for each intersection. The cost of the algorithm for each candidate vector is $\mathcal{O}(MN)$. Therefore, the overall complexity for the construction of $\mathcal{S}(\mathbf{Y}_{N\times D})$ becomes $\mathcal{O}((NM)^{2D-1})$ $\mathcal{O}(MN) = \mathcal{O}((NM)^{2D})$.

We recall that the corresponding complexity of [4] is $\mathcal{O}((NM)^{2D}\text{LP}(NM, 2D))$ where LP$(NM, 2D)$ is the complexity of a linear programming (LP) optimization problem with MN inequalities and 2D variables. Provided that the worst-case complexity of LP$(NM, 2D)$ in linear in NM [12], it turns out that the method in [4] costs $\mathcal{O}((NM)^{2D+1})$ calculations, i.e., one order of magnitude more calculations than the proposed algorithm. In addition, [4] treats only the case $M = 2$ (BPSK) and $M = 4$ (QPSK).

As an illustration, we consider differentially encoded 8-PSK ($M = 8$) transmission of a sequence of length $N = 10$ and reception by $D = 2$ antennas. In Fig. 1, we present the symbol error rate (SER) of the conventional (1-lag) differential detector and the maximum-likelihood sequence detector (MLSD) implemented using our proposed algorithm of complexity $\mathcal{O}((NM)^{2D})$. Our algorithm appears as an efficient noncoherent MLSD method that is applicable to any order of MPSK constellation.

4. REFERENCES

[1] D. Divsalar and M. K. Simon, “Multiple-symbol differential detection of MPSK,” *IEEE Trans. Commun.*, vol. 38, pp. 300-308, Mar. 1990.

[2] K. M. Mackenthun, Jr., “A fast algorithm for multiple-symbol differential detection of MPSK,” *IEEE Trans. Commun.*, vol. 42, pp. 1471-1474, Feb./Mar./Apr. 1994.

[3] I. Motedayen, A. Krishnamoorthy, and A. Anastasopoulos, “Optimal joint detection/estimation in fading channels with polynomial complexity,” *IEEE Trans. Inform. Theory*, vol. 53, no. 1, pp. 209 - 223, Jan. 2007.

[4] V. Pauli, L. Lampe, R. Schober, and K. Fukuda, “Multiple-symbol differential detection based on combinatorial geometry,” *IEEE International Conference on Communications (ICC 2007)*, Glasgow, Scotland, Jun. 2007, pp. 827 - 832.

[5] H. Leib, “Data-aided noncoherent demodulation of DPSK,” *IEEE Trans. Commun.*, vol. 43, pp. 722-725 Feb/Mar/Apr 1995.

[6] G. N. Karystinos and A. P. Liavas, “Efficient computation of the binary vector that maximizes a rank-3 quadratic form,” in *Proc. 2006 Allerton Conf. Commun., Control, and Computing*, Allerton House, Monticello, IL, Sept. 2006, pp. 1286-1291.

[7] G. N. Karystinos and D. A. Pados, “Rank-2-optimal adaptive design of binary spreading codes,” *IEEE Trans. Inform. Theory*, vol. 53, pp. 3075-3080, Sept. 2007.

[8] G. N. Karystinos and A. P. Liavas, “Efficient computation of the binary vector that maximizes a rank-deficient quadratic form,” in preparation.

[9] H. Edelsbrunner, J. O’Rouke, and R. Seiel, “Constructing arrangements of lines and hyperplanes with applications,” *SIAM J. Computing*, vol. 15, pp. 341-363, 1986.

[10] D. Avis and K. Fukuda, “Reverse search for enumeration,” *Discrete Applied Mathematics*, vol. 65, pp. 21-46, 1996.

[11] C. D. Meyer, *Matrix Analysis and Applied Linear Algebra*, siam, 2004.

[12] Y. Ye, *Interior Point Algorithms: Theory & Analysis*, Wiley, 1997.