Automatic covariates selection in dynamic regression models with application to COVID-19 evolution

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Introduction

The **linear dynamic regression model** defines the linear dependence between a stochastic process Y_t and a set of processes $\mathcal{X} = \{X_t^{(1)}, ..., X_t^{(m)}\}$:

$$Y_t = \beta_0 + \beta_1 X_{t-r_1}^{(1)} + \cdots \beta_m X_{t-r_m}^{(m)} + \eta_t$$

constrained to $r_i \geq 0$ for $i = 1, ..., m$ and $\eta_t \sim \text{ARMA}(p, q)$[1].

- [Cryer and Chan, 2008] proposed the *prewhitening* as a technique for removing spurious correlation between processes in order to detect linear correlation.
- We propose a forward-selection method that iteratively adds regressor variables (from a set of candidates) Y_t is *significantly* dependent with.
- Widespread application in financial, economic, political and biomedical fields.
- **Procedure**: analyze the mutual impact of several variables in order to define mathematical relationships and use them in forecasting.

[1] here we denote the AutoRegressive Moving Average model as ARMA.

Prewhitening

[Cryer and Chan, 2008] states that if a linear correlation between two processes X_t and Y_t exists for some $k \leq 0$ then

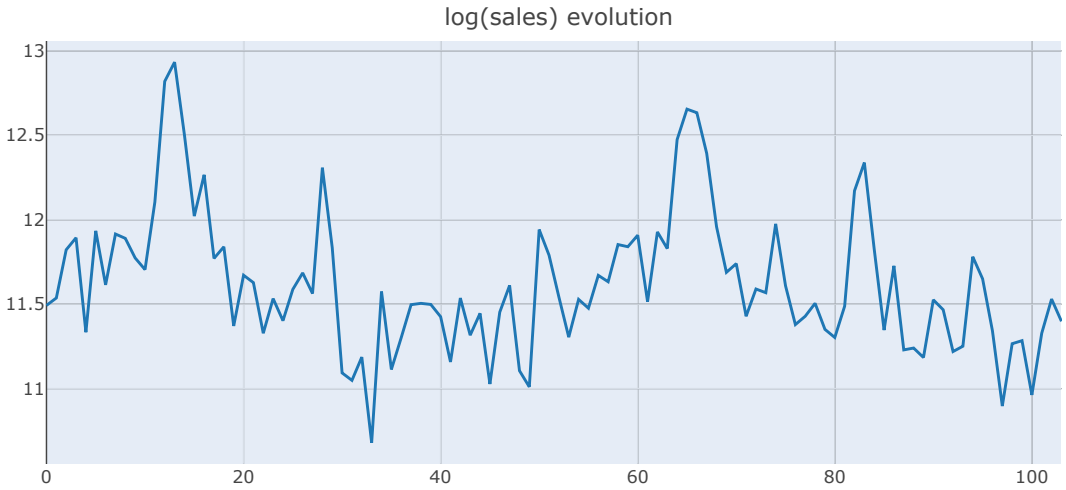
$$\rho_k(\ddot{X}_t, \ddot{Y}_t) \qquad \text{is statistically significant,}$$

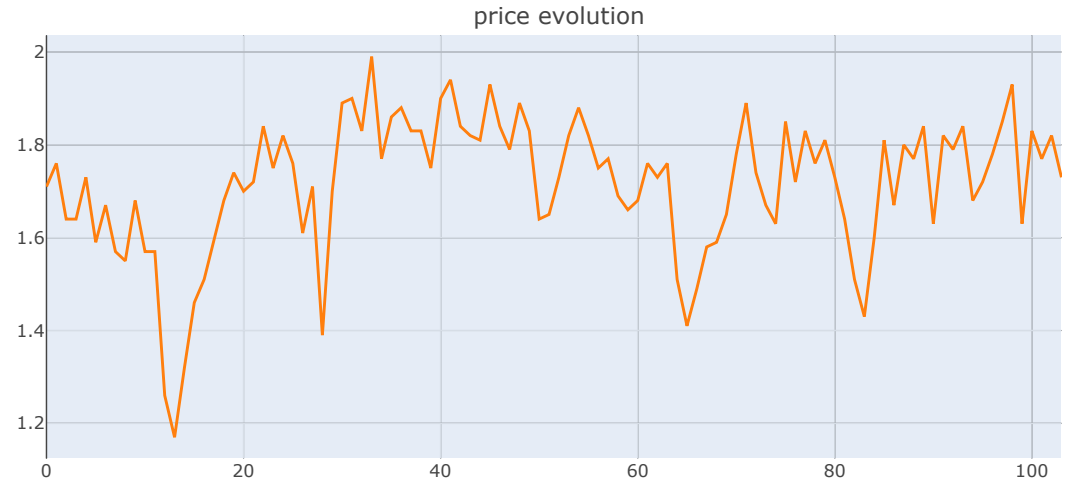
- $\rho_k(X_t^{(1)}, X_t^{(2)})$ is the cross-correlation function between processes $X_t^{(1)}$ and $X_t^{(2)}$ lagged k moments.
- $\ddot{X}_t$ and $\ddot{Y}_t$ are obtained via some linear filter application to X_t and Y_t, respectively, ensuring one of them is white noise and the other is a stationary process (*prewhitening*).

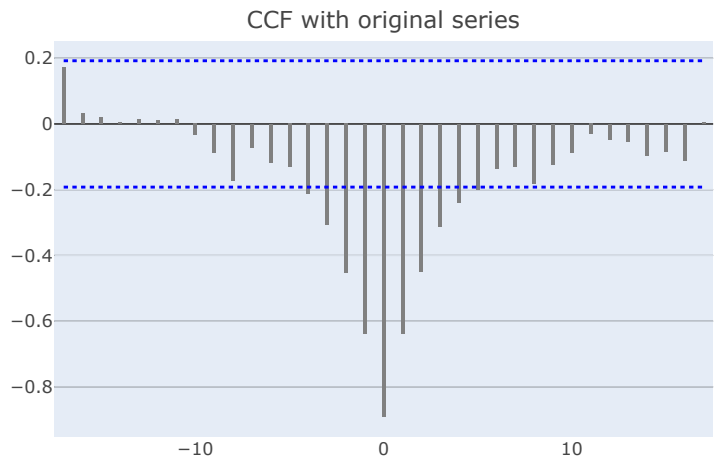
Our proposal

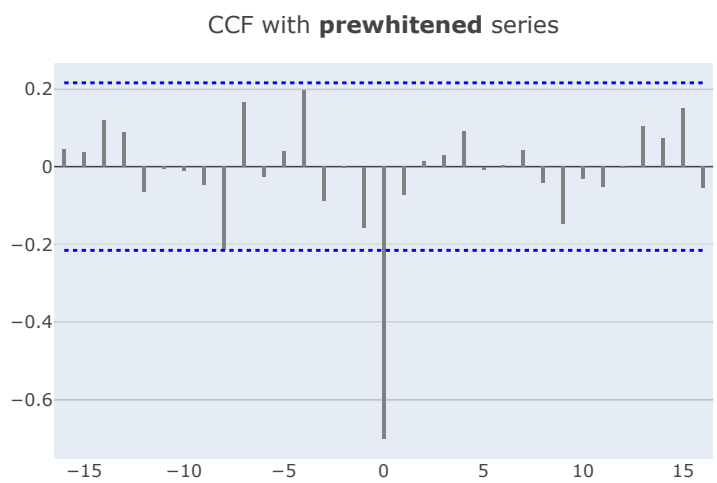
- Iteratively adds in the model significant covariates with their respective lags that optimize some information criterion (IC) of the resultant residuals.
- Uses the **residuals** of the last model created (by adding covariates) to check the existence of correlation between the dependent variable and a **new covariate candidate**.

Example of spurious correlation and prewhitening









Methodology

Let Y_t be the dependent variable and $\mathcal{X}$ the set of covariates candidates. Thus, selection proceeds as follows:

1. Initialization. Consider X_t^{best} as the covariate (lagged r moments) that minimizes the IC of the constructed model with Y_t:

$$X_t^{\text{best}} = \underset{X_t \in \mathcal{X}}{\arg\min} \left\{ \text{IC} \left(Y_t = \beta_0 + \beta_1 X_{t-r}^{\text{best}} + \eta_t \right) \right\}$$

2. Iteration. Use the regression errors (η_t) of the last model created to check if some correlation exists between the rest of the covariates not yet added to the model. Find again the "best" variable and add it to the model to obtain a new IC value. If this value improves the last one achieved, repeat this step. If it does not, stop the iteration.
3. Finalization. The errors of the last fitted model must satisfy the stationary property. In other case, consider the regular differentiation of all data and start again the procedure.

Example of the iterative selection step by step

Table 1: From worldmeters source, model the evolution of *exitus* cases in Spain due to the COVID-19 considering other country data

Covariate	**Lag**	**Coefficient est. (s.e)**	**AICc**
`confirmed_spain`	0	0.0064 (0.0009)	5596.641
`recovered_portugal`	-1	-0.0337 (0.0063)	5550.963
`recovered_france`	0	0.0646 (0.0142)	5540.655
`confirmed_france`	0	-0.0033 (0.0007)	5522.699
`confirmed_portugal`	-13	0.0395 (0.0085)	5504.169
`recovered_spain`	-7	-0.0669 (0.0176)	5500.573

- **Note**: This model was fitted with differentiated data.
- `deaths_england`, `deaths_france`, `confirmed_england`, `recovered_england` and `deaths_portugal` were not included in the model.
- The residuals of the model follow an ARMA$(1,1) \times (1,1)_7$ with parameters:

$$\phi_1 = 0.9724(0.0131),\ \theta_1 = -0.7508(0.0370),$$
$$\Phi_1 = 0.6958(0.1892),\ \Theta_1 = -0.5512(0.2215)$$

Simulation procedure

1. Seven time series (modelable by an ARIMA) were generated: six act as covariate candidates $\mathcal{X} = \{X_t^{(1)}, ..., X_t^{(6)}\}$ with random lags $r \in [0, 6]$, and the remaining as the errors of the model.
2. Formally, each simulation follows this scheme:

$$Y_t = \beta_0 + \beta_1 X_{t-r_1}^{(1)} + \beta_2 X_{t-r_2}^{(2)} + \beta X_{t-r_3}^{(3)} + \eta_t$$

   where $\eta_t \sim \text{ARMA}(p, q)$, $\beta_0, ..., \beta_3$ are randomly generated and $r_i \in [0, 6]$ for $i = 1, .., 3$.
3. Selection method was tested with different configurations:
   - Changing the IC with three different options: AIC, BIC or AICc.
   - Changing the method to check stationary: via the Dickey-Fuller test or via adjusting an ARIMA and checking the differentiation order.

Example of one simulation result

Figure 1: Code output when launching the function `auto.fit.arima.regression()` that implements our approach

```
beta0 <- -0.6; beta1 <- 1.7; beta2 <- -2.2; beta3 <- 1.3; r1 <- 2; r3 <- 3
Y <- beta0 + beta1*lag(X1,-r1) + beta2*X2 + beta3*lag(X3,-r3) + residuals
xregs <- cbind(X1, X2, X3, X4, X5, X6)
ajuste <- drm.select(Y, xregs, ic='aicc', st_method='adf.test', show_info=F)
```

```
print(ajuste$history, row.names=F)
```

```
 var lag                ic
  X2   0 -1156.68486061937
  X1  -2 -2171.66958134745
  X3  -3 -3108.15443209894
```

```
print(ajuste, row.names=F)
```

```
Series: serie
Regression with ARIMA(0,0,4) errors

Coefficients:
         ma1     ma2  ma3     ma4  intercept       X2      X1      X3
      0.2498  0.3360    0  0.1589    -0.5947  -2.1868  1.6949  1.3083
s.e.  0.0304  0.0302    0  0.0300     0.0033   0.0105  0.0089  0.0320

sigma^2 = 0.002377:  log likelihood = 1562.15
AIC=-3108.3   AICc=-3108.15   BIC=-3069.26
```

Results of multiple simulations where $\eta_t \sim$ ARMA(p,q)

Table 2: Percentage of covariates correctly added to the model

	AIC	**BIC**	**AICc**
adf.test			
auto.arima			

Results of multiple simulations where $\eta_t \sim$ ARIMA(p,d,q)

Table 3: Percentage data about the performance of the selection method

	AIC	**BIC**	**AICc**	**AIC**	**BIC**	**AICc**
adf.test	93.33%	93.33%	93.33%	4.33%	0.30%	4.33%
auto.arima	94.33%	94.66%	95.33%	5.00%	1.33%	5.00%
	correctly added (TP)			incorrectly added (FP)		

	AIC	**BIC**	**AICc**	**AIC**	**BIC**	**AICc**
adf.test	95.00%	98.66%	95.00%	6.66%	6.66%	6.66%
auto.arima	94.66%	99.66%	95.66%	4.66%	5.33%	4.66%
	correctly **not** added (TN)			incorrectly **not** added (FN)		

Application to COVID19 evolution

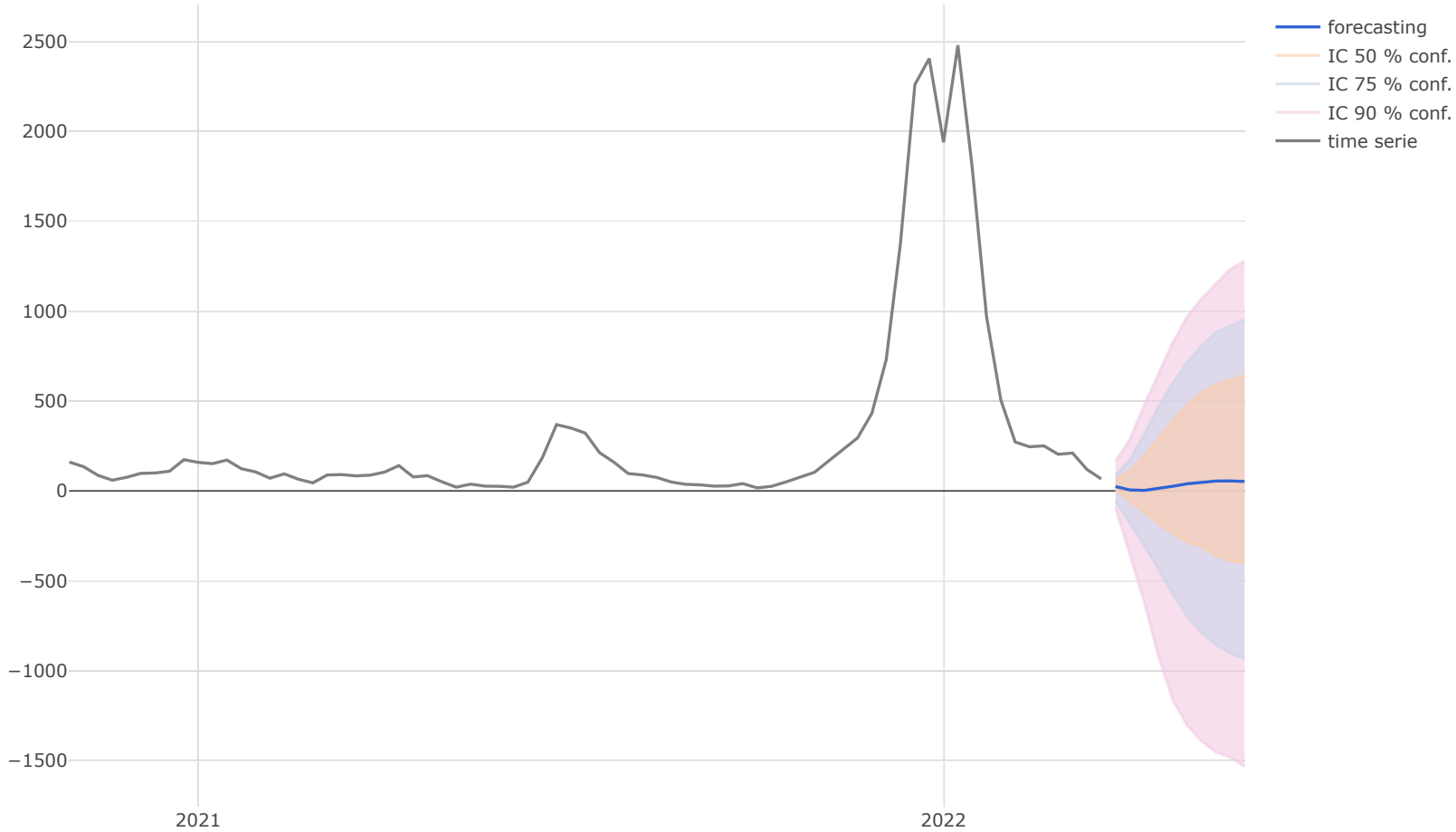
Table 4: Information about the dynamic regression model constructed via selection of multiple vaccination variables to model COVID19 evolution

Covariate	**Lag**	**Coefficient est. (s.e)**
vac4565	-3	-0.0410 (0.0057)
vac6580	-2	-0.0468 (0.0120)
vac1845	-6	-0.0901 (0.0047)
vac1218	Not included in the model	
vac80	Not included in the model	
residuals	ARIMA(4, 0, 0)	$\phi_1 = 2.0816(0.0810)$ $\phi_2 = -1.2837(0.1152)$ $\phi_4 = 0.1919(0.0432)$

- There is a lagged negative correlation between the vaccination data and COVID-19 evolution.
- The vaccination of the young population (from 18 up to 45 years old) has a greater impact in the COVID19 evolution.
- The vaccination of the population older than 80 years has no significative impact in the COVID19 confirmed cases.

Study on vaccination data aggregated by group of ages

Figure 2: Forecasting applied to the dynamic regression model constructed in 4

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2.