

Overview

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In this notebook we show multiple simulations for our automatic covariates selection method in multiple scenarios with different lags. Diverse examples are explored and our method is tested against a set of covariates where only some of them influence in the target variable with a concrete lag (always lower or equal to 0). In the following sections we propose different scenarios with incremental complexity in order to study the behavior of our proposal in simulated data and infer the performance in real environments.

We used `arima_simulation.R` functions to randomly generate time series from an ARIMA process.

1 Simulation of a dynamic regression model with stationary errors

In this section we show how our automatic selection method works on basic examples where modeling errors are stationary:

$$Y_t = \beta_0 + \beta_1 X_{t-r_1}^{(1)} + \beta_2 X_{t-r_2}^{(2)} + \dots + X_{t-r_p}^{(p)} + \eta_t, \quad \eta_t \sim \text{ARMA}(p,q), \quad r_i \geq 0 \text{ para } i = 1, \dots, p$$

1.1 Model with null lags

Assume a dynamic regression model with three regressor variables with null lags (all lags are equal to zero) following:

$$Y_t = \beta_0 + \beta_1 X_t^{(1)} + \beta_2 X_t^{(2)} + \beta_3 X_t^{(3)} + \eta_t \quad (1)$$

where:

- $\eta_t \sim \text{ARMA}(2,1)$, thus, errors are stationary.
- $X_t^{(1)} \sim \text{ARIMA}(2, 1, 3)$ and its coefficient $\beta_1 = 2.8$.
- $X_t^{(2)} \sim \text{ARIMA}(1, 1, 2)$ and its coefficient $\beta_2 = -1.12$.
- $X_t^{(3)} \sim \text{ARMA}(1, 2)$ and its coefficient $\beta_3 = -2.3$.
- The *intercept* is $\beta_0 = 0.8$.

Assume another set of variables (all following an ARIMA process) which do not influence in the target variable:

- $X_t^{(4)} \sim \text{ARIMA}(1, 0, 3)$.
- $X_t^{(5)} \sim \text{ARIMA}(2, 1, 2)$.
- $X_t^{(6)} \sim \text{ARIMA}(2, 1, 1)$.

```
# ---- Generate all variables of the scenario ----
set.seed(12)
N <- 1000

# residuals ~ ARIMA(2,0,1)
residuals <- sim.arma(model=list(p=2, d=0, q=1), n=N, with.constant=FALSE)

# X1 ~ ARIMA(2,1,3)
X1 <- sim.arma(model=list(p=2, d=1, q=3), n=N, with.constant=FALSE)

# X2 ~ ARIMA(1,1,2)
X2 <- sim.arma(model=list(p=1, d=1, q=2), n=N, with.constant=FALSE)
```

```

# X3 ~ ARIMA(1,0,2)
X3 <- sim.arima(model=list(p=1, d=0, q=2), n=N, with.constant=FALSE)

# X4 ~ ARIMA(1,0,3)
X4 <- sim.arima(model=list(p=1, d=0, q=3), n=N, with.constant=FALSE)

# X5 ~ ARIMA(2,1,2)
X5 <- sim.arima(model=list(p=2, d=1, q=2), n=N, with.constant=FALSE)

# X6 ~ ARIMA(2,1,1)
X6 <- sim.arima(model=list(p=2, d=1, q=1), n=N, with.constant=FALSE)

```

Covariates selection and model fitting: We create the target variable and test the final result of the selection function `drm.select()`:

```

beta0 <- 0.8; beta1 <- 2.8; beta2 <- -1.12; beta3 <- -2.3
Y <- beta0 + beta1 * X1$X + beta2 * X2$X + beta3 * X3$X + residuals$X
regressors <- cbind(X1=X1$X, X2=X2$X, X3=X3$X, X4=X4$X, X5=X5$X, X6=X6$X)
model <- drm.select(Y, regressors, show_info=T)

```

```

Covariate X1 has been tested [ic=-827.077221400029, lag=0]
Covariate X2 has been tested [ic=-337.82881082826, lag=0]
Covariate X3 has been tested [ic=-562.255180843397, lag=0]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-91.2830346467132, lag=-19]
Covariate X6 has been tested [ic=-91.2830346467132, lag=-12]
Covariate X1 has been added [aicc=-827.077221400029, lag=0]
Series: serie
Regression with ARIMA(1,1,3) errors

```

```

Coefficients:
      ar1      ma1      ma2      ma3      xreg
    0.8087  -1.218  0.6331  -0.4012  2.8110
s.e.  0.0272   0.036  0.0428   0.0315  0.0148

```

```

sigma^2 = 0.02491:  log likelihood = 419.58
AIC=-827.16  AICc=-827.08  BIC=-797.85

```

```

Covariate X2 has been tested [ic=-1215.17240040525, lag=0]
Covariate X3 has been tested [ic=-2088.59212838621, lag=0]
Significative correlation with lag<=0 could not be found for X4
Significative correlation with lag<=0 could not be found for X5
Covariate X6 has been tested [ic=-827.077222105698, lag=-12]
Covariate X3 has been added [aicc=-2088.59212838621, lag=0]
Series: serie
Regression with ARIMA(2,1,3) errors

```

```

Coefficients:
      ar1      ar2      ma1      ma2      ma3      X1      X3
    0  0.7373  0.0722  -0.7520  -0.2864  2.8091  -2.2782
s.e.  0  0.0433  0.0314   0.0376   0.0345  0.0123   0.0345

```

```
sigma^2 = 0.006853: log likelihood = 1051.35
AIC=-2088.71 AICc=-2088.59 BIC=-2054.51
```

```
-----
Covariate X2 has been tested [ic=-3150.08602230015, lag=0]
Covariate X4 has been tested [ic=-2088.59212753666, lag=-21]
Significative correlation with lag<=0 could not be found for X5
Significative correlation with lag<=0 could not be found for X6
Covariate X2 has been added [aicc=-3150.08602230015, lag=0]
Series: serie
Regression with ARIMA(2,0,1) errors
```

Coefficients:

	ar1	ar2	ma1	intercept	X1	X3	X2
	0	0.8420	0.6863	0.7896	2.7892	-2.2956	-1.1320
s.e.	0	0.0186	0.0267	0.0238	0.0038	0.0248	0.0233

```
sigma^2 = 0.002321: log likelihood = 1582.1
AIC=-3150.2 AICc=-3150.09 BIC=-3116
```

```
-----
Covariate X4 has been tested [ic=-3150.08624487398, lag=-18]
Significative correlation with lag<=0 could not be found for X5
Significative correlation with lag<=0 could not be found for X6
Covariate X4 has been added [aicc=-3150.08624487398, lag=-18]
Series: serie
Regression with ARIMA(2,0,1) errors
```

Coefficients:

	ar1	ar2	ma1	intercept	X1	X3	X2	X4
	0	0.8421	0.6863	0.7902	2.7892	-2.2956	-1.1317	0
s.e.	0	0.0186	0.0267	0.0238	0.0038	0.0248	0.0233	0

```
sigma^2 = 0.002321: log likelihood = 1582.1
AIC=-3150.2 AICc=-3150.09 BIC=-3116
```

```
-----
Significative correlation with lag<=0 could not be found for X5
Significative correlation with lag<=0 could not be found for X6
No more variables will be added
```

```
-----
| Historical of added covariates to the model (ndiff=0) |
```

```
-----
var lag          ic
X1  0 -827.077221400029
X3  0 -2088.59212838621
X2  0 -3150.08602230015
X4 -18 -3150.08624487398
-----
```

```
Series: serie
Regression with ARIMA(2,0,1) errors
```

Coefficients:

	ar1	ar2	ma1	intercept	X1	X3	X2	X4
	0	0.8421	0.6863	0.7902	2.7892	-2.2956	-1.1317	0
s.e.	0	0.0186	0.0267	0.0238	0.0038	0.0248	0.0233	0

```
sigma^2 = 0.002321: log likelihood = 1582.1
AIC=-3150.2 AICc=-3150.09 BIC=-3116
```

In the output of our method we see that:

1. The selected covariates are $X_t^{(1)}$, $X_t^{(2)}$ and $X_t^{(3)}$.
2. Residuals are stationary and modeled by an ARIMA(2,0,1).
3. Regression coefficients haven been correctly estimated.

Prediction: We can estimate puntual predictions:

```
preds <- forecast_model(Y, regressors, model, h=10, mode='bootstrap')
display(plot_forecast(preds, rang=c(N-50, N+10)), name='example1')
```



1.2 Model where $r_i \geq 0$ para $i = 1, \dots, p$

Let's assume a dynamic regression model similar to our [first example](#) using the same variables but applying a negative lag.

$$Y_t = \beta_0 + \beta_1 X_{t-r_1}^{(1)} + \beta_2 X_{t-r_2}^{(2)} + \beta_3 X_{t-r_3}^{(3)} + \eta_t \quad (2)$$

where:

- $\eta_t \sim \text{ARMA}(2,1)$.
- $X_t^{(1)} \sim \text{ARIMA}(2, 1, 3)$ and its lag $r_1 = 2$.
- $X_t^{(2)} \sim \text{ARMA}(1, 1, 2)$ and its lag $r_2 = 0$.
- $X_t^{(3)} \sim \text{ARMA}(1, 0, 2)$ and its lag $r_3 = 3$.

```

beta0 <- -0.6; beta1 <- 1.7; beta2 <- -2.2; beta3 <- 1.3
r1 <- 2; r3 <- 3
Y <- beta0 + beta1 * lag(X1$X, -r1) + beta2 * X2$X + beta3 * lag(X3$X, -r3) +
  residuals$X

```

Covariates selection and model fitting:

```

regressors <- cbind(X1=X1$X, X2=X2$X, X3=X3$X, X4=X4$X, X5=X5$X, X6=X6$X)
model <- drm.select(Y, regressors, show_info=T, st_method='adf.test')

```

```

Covariate X1 has been tested [ic=-759.91153383683, lag=-2]
Covariate X2 has been tested [ic=-760.646932763231, lag=0]
Covariate X3 has been tested [ic=-607.677287680323, lag=-3]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-373.529141045731, lag=-22]
Covariate X6 has been tested [ic=-373.360141471898, lag=-22]
Covariate X2 has been added [aicc=-760.646932763231, lag=0]
Series: serie
Regression with ARIMA(0,1,0) errors

```

Coefficients:

```

      xreg
      -2.1742
s.e.    0.0868

```

```

sigma^2 = 0.02673: log likelihood = 382.33
AIC=-760.66 AICc=-760.65 BIC=-750.9

```

```

Covariate X1 has been tested [ic=-2039.26823147573, lag=-2]
Covariate X3 has been tested [ic=-1685.34996673435, lag=-3]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-760.646932532334, lag=-17]
Significative correlation with lag<=0 could not be found for X6
Covariate X1 has been added [aicc=-2039.26823147573, lag=-2]
Series: serie
Regression with ARIMA(4,0,0) errors

```

Coefficients:

```

      ar1      ar2      ar3      ar4 intercept      X2      X1
      0.4428 0.6404      0 -0.2322   -0.6284 -2.2504 1.6861
s.e. 0.0274 0.0342      0  0.0280    0.0310  0.0343 0.0044

```

```

sigma^2 = 0.007158: log likelihood = 1026.69
AIC=-2039.38 AICc=-2039.27 BIC=-2005.21

```

```

Covariate X3 has been tested [ic=-3137.63271174962, lag=-3]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-2045.89040493921, lag=-8]
Covariate X6 has been tested [ic=-2045.89039161594, lag=-22]
Covariate X3 has been added [aicc=-3137.63271174962, lag=-3]
Series: serie
Regression with ARIMA(2,0,1) errors

```

```

Coefficients:
      ar1      ar2      ma1  intercept      X2      X1      X3
      0  0.8409  0.6861   -0.6110  -2.2103  1.6896  1.2831
s.e.      0  0.0186  0.0267    0.0238   0.0233  0.0038  0.0249

```

```

sigma^2 = 0.002319: log likelihood = 1575.87
AIC=-3137.75  AICc=-3137.63  BIC=-3103.57

```

```

-----
Covariate X4 has been tested [ic=-3137.63153945379, lag=-18]
Significative correlation with lag<=0 could not be found for X5
Significative correlation with lag<=0 could not be found for X6
No more variables will be added

```

```

-----
|               Historical of added covariates to the model (ndiff=0)               |
-----
var lag          ic
X2  0 -760.646932763231
X1 -2 -2039.26823147573
X3 -3 -3137.63271174962

```

```

Series: serie
Regression with ARIMA(2,0,1) errors

```

```

Coefficients:
      ar1      ar2      ma1  intercept      X2      X1      X3
      0  0.8409  0.6861   -0.6110  -2.2103  1.6896  1.2831
s.e.      0  0.0186  0.0267    0.0238   0.0233  0.0038  0.0249

```

```

sigma^2 = 0.002319: log likelihood = 1575.87
AIC=-3137.75  AICc=-3137.63  BIC=-3103.57

```

In the output of our function we see our method's selection:

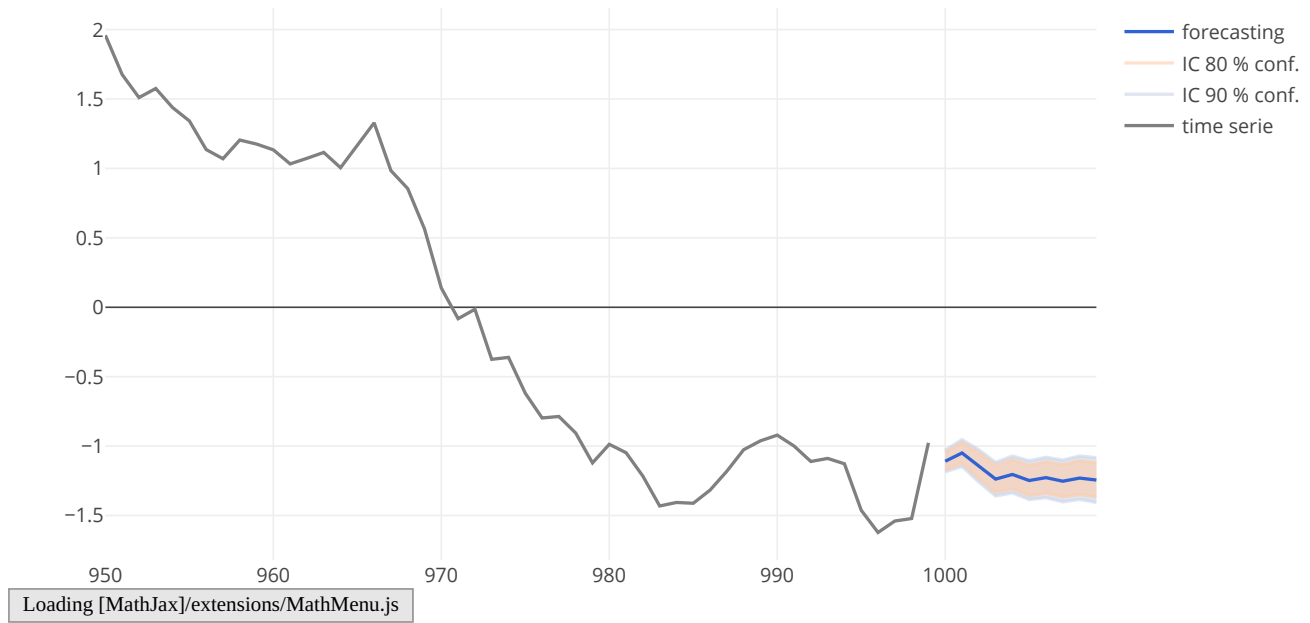
1. The fitted model has stationary residuals generated by an ARIMA(2,0,2).
2. The selected covariates are the same used in the generation of Y .
3. The ARIMA process for the residuals do not match with the original orders (we used an ARIMA(2,0,1))
4. Regressor coefficients have been correctly approximated.

Puntual predictions:

```

preds <- forecast_model(Y, regressors, model, h=10, mode='bootstrap')
display(plot_forecast(preds, rang=c(N-50, N+10)), name='example2')

```



2 Simulation of a dynamic regression model with ARIMA errors ($d \geq 1$)

In this section we consider dynamic regression models with non-stationary errors:

$$Y_t = \beta_0 + \beta_1 X_{t-r_1}^{(1)} + \beta_2 X_{t-r_2}^{(2)} + \dots + X_{t-r_p}^{(p)} + \eta_t, \quad \eta_t \sim \text{ARIMA}(p, d, q)$$

2.1 Model where $r_i = 0$ para $i = 1, \dots, p$

We took the same variables of [our first example](#) but adding non-stationary residuals:

$$Y_t = \beta_0 + \beta_1 X_t^{(1)} + \beta_2 X_t^{(2)} + \beta_3 X_t^{(3)} + \eta_t, \quad \eta_t \sim \text{ARIMA}(1, 2, 2) \quad (3)$$

where:

- $X_t^{(1)} \sim \text{ARIMA}(2, 1, 3)$ and its coefficient $\beta_1 = -1.3$.
- $X_t^{(2)} \sim \text{ARIMA}(1, 1, 2)$ and its coefficient $\beta_2 = 2.12$.
- $X_t^{(3)} \sim \text{ARMA}(1, 2)$ and its coefficient $\beta_3 = 2.3$.
- The *intercept* is $\beta_0 = 0.8$.

Covariates which do not influence in Y are the same as those proposed in the [first scenario](#).

```
set.seed(123)
residuals <- sim.arima(model=list(p=3, d=2, q=2), n=N)

beta0 <- 0.8; beta1 <- -1.3; beta2 <- 1.12; beta3 <- 1.3
Y <- beta0 + beta1 * X1$X + beta2 * X2$X + beta3 * X3$X + 1.1*residuals$X
```


Covariates selection and model fitting: We fit the model with original covariates:

```
regressors <- cbind(X1=X1$X, X2=X2$X, X3=X3$X, X4=X4$X, X5=X5$X, X6=X6$X)
model <- drm.select(Y, regressors, show_info=T)
```

Covariate X1 has been tested [ic=-684.537389465694, lag=0]
Covariate X2 has been tested [ic=-1066.64131742063, lag=0]
Covariate X3 has been tested [ic=-932.382987603382, lag=0]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-528.093854535604, lag=-19]
Significative correlation with lag<=0 could not be found for X6
Covariate X2 has been added [aicc=-1066.64131742063, lag=0]
Series: serie
Regression with ARIMA(5,1,0) errors

Coefficients:

	ar1	ar2	ar3	ar4	ar5	xreg
	0	0.8431	0.8334	-0.1857	-0.4952	1.1556
s.e.	0	0.0269	0.0229	0.0230	0.0271	0.0371

sigma^2 = 0.01934: log likelihood = 539.36
AIC=-1066.73 AICc=-1066.64 BIC=-1037.43

Covariate X1 has been tested [ic=-1419.85693823948, lag=0]
Covariate X3 has been tested [ic=-1655.90896386581, lag=0]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-1066.64131751632, lag=-19]
Significative correlation with lag<=0 could not be found for X6
Covariate X3 has been added [aicc=-1655.90896386581, lag=0]
Series: serie
Regression with ARIMA(5,1,0) errors

Coefficients:

	ar1	ar2	ar3	ar4	ar5	X2	X3
	0.2417	0.7016	0.8344	-0.3732	-0.4079	1.1580	1.2508
s.e.	0.0292	0.0278	0.0237	0.0279	0.0293	0.0291	0.0389

sigma^2 = 0.01054: log likelihood = 836.03
AIC=-1656.06 AICc=-1655.91 BIC=-1616.99

Covariate X1 has been tested [ic=-2538.22194457237, lag=0]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-1655.90896531437, lag=-19]
Significative correlation with lag<=0 could not be found for X6
Covariate X1 has been added [aicc=-2538.22194457237, lag=0]
Series: serie
Regression with ARIMA(4,1,0) errors

Coefficients:

	ar1	ar2	ar3	ar4	X2	X3	X1
	0.6788	0.3937	0.7537	-0.8283	1.1326	1.3125	-1.2702
s.e.	0.0179	0.0148	0.0145	0.0179	0.0212	0.0154	0.0265

sigma^2 = 0.004258: log likelihood = 1277.19
AIC=-2538.37 AICc=-2538.22 BIC=-2499.3

```

-----
Significative correlation with lag<=0 could not be found for X4
Significative correlation with lag<=0 could not be found for X5
Significative correlation with lag<=0 could not be found for X6
No more variables will be added
The global model does not have stationary errors
Trying to adjust a model that do have stationary errors
No valid model with stationary errors could be optimized
Applying regular differentiation (ndiff=1) and calling again the function
-----

```

```

-----
Covariate X1 has been tested [ic=58.8045602196465, lag=0]
Covariate X2 has been tested [ic=104.602030040923, lag=0]
Covariate X3 has been tested [ic=-249.291766899248, lag=0]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=180.614562027151, lag=-19]
Significative correlation with lag<=0 could not be found for X6
Covariate X3 has been added [aicc=-249.291766899248, lag=0]
Series: serie
Regression with ARIMA(0,1,0) errors

```

Coefficients:

```

      xreg
      1.2519
s.e.  0.0537

```

```

sigma^2 = 0.0452:  log likelihood = 126.65
AIC=-249.3   AICc=-249.29   BIC=-239.54

```

```

-----
Covariate X1 has been tested [ic=-500.744606509845, lag=0]
Covariate X2 has been tested [ic=-395.585434188201, lag=0]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-249.291766899249, lag=-13]
Significative correlation with lag<=0 could not be found for X6
Covariate X1 has been added [aicc=-500.744606509845, lag=0]
Series: serie
Regression with ARIMA(0,1,0) errors

```

Coefficients:

```

      X3      X1
      1.3020  -1.2977
s.e.  0.0472   0.0763

```

```

sigma^2 = 0.03489:  log likelihood = 253.38
AIC=-500.77   AICc=-500.74   BIC=-486.12

```

```

-----
Covariate X2 has been tested [ic=-2878.43097732394, lag=0]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-500.744460512912, lag=-13]
Significative correlation with lag<=0 could not be found for X6
Covariate X2 has been added [aicc=-2878.43097732394, lag=0]
Series: serie
Regression with ARIMA(3,1,2) errors

```

```

Coefficients:
      ar1  ar2    ar3    ma1    ma2    X3    X1    X2
-0.2888    0  0.7971 -0.3327  0.6023  1.3111 -1.2893  1.1118
s.e.    0.0144    0  0.0138  0.0272  0.0251  0.0181  0.0195  0.0103

```

```

sigma^2 = 0.003009: log likelihood = 1447.29
AIC=-2878.58  AICc=-2878.43  BIC=-2839.52

```

```

-----
Significative correlation with lag<=0 could not be found for X4
Significative correlation with lag<=0 could not be found for X5
Covariate X6 has been tested [ic=-695.5702280309, lag=-14]
No more variables will be added
The global model does not have stationary errors
Trying to adjust a model that do have stationary errors
-----

```

```

|                Historical of added covariates to the model (ndiff=1)                |
-----
var lag                ic
X3    0 -249.291766899248
X1    0 -500.744606509845
X2    0 -2878.43097732394
-----

```

```

Series: serie
Regression with ARIMA(4,0,0) errors

```

```

Coefficients:
      ar1    ar2    ar3    ar4    X3    X1    X2
0.6786  0.3934  0.7534 -0.8276  1.3126 -1.2702  1.1328
s.e.    0.0179  0.0148  0.0145  0.0179  0.0154  0.0265  0.0212

```

```

sigma^2 = 0.004257: log likelihood = 1277.17
AIC=-2538.34  AICc=-2538.19  BIC=-2499.27

```

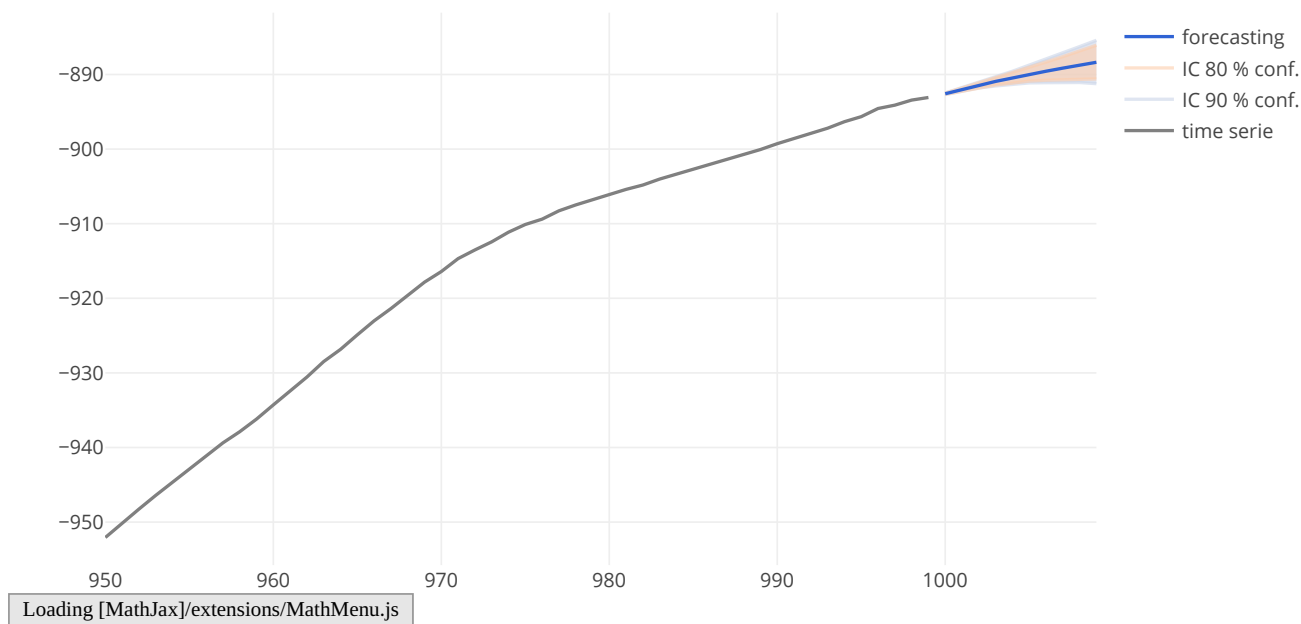
By observing the console output, note that our selection method has applied a regular differentiation to fit an ARIMAX model with stationary errors and has correctly selected the 3 regressor variables which originally influence in the target variable with the correct lags.

Prediction: When using a fitted model with differentiation, the function `forecast_model()` raises a warning about using the original scale of data to compute puntual predictions.

```

preds <- forecast_model(Y, regressors, model, h=10, mode='bootstrap')
Returning predictions in original scale...
display(plot_forecast(preds, rang=c(N-50, N+10)), name='example3')

```



2.2 Model where $r_i \geq 0$ para $i = 1, \dots, p$

We modify the [last example](#) to make regressors to influence in Y with a positive lag.

- $X_t^{(1)}$ is introduced with lag $r_1 = 2$.
- $X_t^{(3)}$ is introduce with lag $r_3 = 1$.

```
beta0 <- 0.8; beta1 <- -1.3; beta2 <- 2.12; beta3 <- 2.3
r1 <- 2; r3 <- 1
Y <- beta0 + beta1 * lag(X1$X, -r1) + beta2 * X2$X + beta3 * lag(X3$X, -r3) + 1.5*residuals$X
```

Covariates selection and model fitting:

```
regressors <- cbind(X1=X1$X, X2=X2$X, X3=X3$X, X4=X4$X, X5=X5$X, X6=X6$X)
model <- drm.select(Y, regressors, show_info=T, st_method='adf.test')
```

```
Covariate X1 has been tested [ic=330.289892350127, lag=-2]
Covariate X2 has been tested [ic=-325.322695806822, lag=0]
Covariate X3 has been tested [ic=-207.18762445319, lag=-1]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=996.205184669821, lag=-13]
Covariate X6 has been tested [ic=1004.29829329033, lag=-17]
Covariate X2 has been added [aicc=-325.322695806822, lag=0]
Series: serie
Regression with ARIMA(5,1,0) errors
```

Coefficients:

	ar1	ar2	ar3	ar4	ar5	xreg
	0	0.9030	0.7509	-0.2139	-0.4444	2.1026
s.e.	0	0.0278	0.0246	0.0247	0.0279	0.0550

```
sigma^2 = 0.04137: log likelihood = 168.7
AIC=-325.41 AICc=-325.32 BIC=-296.1
```

Covariate X1 has been tested [ic=-487.278867401677, lag=-2]
 Covariate X3 has been tested [ic=-1378.23195503316, lag=-1]
 Significant correlation with lag<=0 could not be found for X4
 Covariate X5 has been tested [ic=-325.322695801524, lag=0]
 Covariate X6 has been tested [ic=-325.322695801569, lag=-14]
 Covariate X3 has been added [aicc=-1378.23195503316, lag=-1]
 Series: serie
 Regression with ARIMA(5,1,0) errors

Coefficients:

	ar1	ar2	ar3	ar4	ar5	X2	X3
	0.2661	0.7127	0.8683	-0.4189	-0.4317	2.0864	2.3206
s.e.	0.0289	0.0271	0.0218	0.0271	0.0290	0.0328	0.0433

sigma^2 = 0.01402: log likelihood = 697.19
 AIC=-1378.38 AICc=-1378.23 BIC=-1339.3

Covariate X1 has been tested [ic=-1929.51443203733, lag=-2]
 Significant correlation with lag<=0 could not be found for X4
 Significant correlation with lag<=0 could not be found for X5
 Significant correlation with lag<=0 could not be found for X6
 Covariate X1 has been added [aicc=-1929.51443203733, lag=-2]
 Series: serie
 Regression with ARIMA(4,1,0) errors

Coefficients:

	ar1	ar2	ar3	ar4	X2	X3	X1
	0.6809	0.3918	0.7526	-0.8275	2.1368	2.2828	-1.2912
s.e.	0.0179	0.0148	0.0146	0.0179	0.0290	0.0210	0.0363

sigma^2 = 0.007962: log likelihood = 972.83
 AIC=-1929.66 AICc=-1929.51 BIC=-1890.59

Significant correlation with lag<=0 could not be found for X4
 Significant correlation with lag<=0 could not be found for X5
 Covariate X6 has been tested [ic=-1929.51505675862, lag=-14]
 Covariate X6 has been added [aicc=-1929.51505675862, lag=-14]
 Series: serie
 Regression with ARIMA(4,1,0) errors

Coefficients:

	ar1	ar2	ar3	ar4	X2	X3	X1	X6
	0.6810	0.3918	0.7527	-0.8278	2.1366	2.2828	-1.2912	0
s.e.	0.0179	0.0148	0.0146	0.0180	0.0290	0.0210	0.0363	0

sigma^2 = 0.007961: log likelihood = 972.83
 AIC=-1929.66 AICc=-1929.52 BIC=-1890.59

Significant correlation with lag<=0 could not be found for X4
 Significant correlation with lag<=0 could not be found for X5
 No more variables will be added
 The global model does not have stationary errors
 Trying to adjust a model that do have stationary errors
 No valid model with stationary errors could be optimized

Applying regular differentiation (ndiff=1) and calling again the function

Covariate X1 has been tested [ic=937.298080726452, lag=-2]
Covariate X2 has been tested [ic=846.874907976384, lag=0]
Covariate X3 has been tested [ic=343.397189504434, lag=-1]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=996.205184669483, lag=-13]
Covariate X6 has been tested [ic=1004.29829329002, lag=-17]
Covariate X3 has been added [aicc=343.397189504434, lag=-1]
Series: serie
Regression with ARIMA(0,1,0) errors

Coefficients:

 xreg
 2.2394
s.e. 0.0727

sigma^2 = 0.08298: log likelihood = -169.69
AIC=343.38 AICc=343.4 BIC=353.15

Covariate X1 has been tested [ic=245.087765858082, lag=-2]
Covariate X2 has been tested [ic=56.3688859781321, lag=0]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=343.397189504433, lag=-13]
Covariate X6 has been tested [ic=340.254503404008, lag=-17]
Covariate X2 has been added [aicc=56.3688859781321, lag=0]
Series: serie
Regression with ARIMA(0,1,0) errors

Coefficients:

 X3 X2
 2.1935 2.1423
s.e. 0.0627 0.1168

sigma^2 = 0.06178: log likelihood = -25.17
AIC=56.34 AICc=56.37 BIC=70.99

Covariate X1 has been tested [ic=-2271.62664716549, lag=-2]
Significative correlation with lag<=0 could not be found for X4
Significative correlation with lag<=0 could not be found for X5
Significative correlation with lag<=0 could not be found for X6
Covariate X1 has been added [aicc=-2271.62664716549, lag=-2]
Series: serie
Regression with ARIMA(3,1,2) errors

Coefficients:

 ar1 ar2 ar3 ma1 ma2 X3 X2 X1
 -0.2863 0 0.7984 -0.3346 0.6028 2.2966 2.108 -1.3075
s.e. 0.0144 0 0.0139 0.0273 0.0250 0.0247 0.014 0.0265

sigma^2 = 0.00562: log likelihood = 1143.89
AIC=-2271.78 AICc=-2271.63 BIC=-2232.71

Significative correlation with lag<=0 could not be found for X4
Significative correlation with lag<=0 could not be found for X5
Covariate X6 has been tested [ic=-98.7055858132784, lag=-14]
No more variables will be added
The global model does not have stationary errors
Trying to adjust a model that do have stationary errors

```
-----
|               Historical of added covariates to the model (ndiff=1)               |
-----
var lag              ic
X3  -1  343.397189504434
X2   0  56.3688859781321
X1  -2 -2271.62664716549
-----
```

Series: serie
Regression with ARIMA(4,0,0) errors

```
Coefficients:
      ar1      ar2      ar3      ar4      X3      X2      X1
0.6810  0.3918  0.7526 -0.8277  2.2829  2.1366 -1.2912
s.e.  0.0179  0.0148  0.0146  0.0180  0.0210  0.0290  0.0363
```

```
sigma^2 = 0.00796:  log likelihood = 972.82
AIC=-1929.64  AICc=-1929.49  BIC=-1890.56
```

The output is similar to the result of the [last example](#). Covariate lags have been correctly selected and the method has applied a regular differentiation to data to fit an ARIMAX model with stationary errors.

3 Prewhitening method

3.1 With stationary errors

```
set.seed(123)
residuals <- sim.arima(model=list(p=2, d=0, q=2), n=N)

beta0 <- -0.1; beta1 <- 3.2; beta2 <- -2.5
r1 <- 2; r2 <- 3
Y <- beta0 + beta1 * lag(X1$X, -r1) + beta2 * lag(X2$X, -r2) + residuals$X
regressors <- cbind(X1=X1$X, X2=X2$X, X3=X3$X, X4=X4$X, X5=X5$X, X6=X6$X)
```

Fit a model with the stationary detection method `auto.arima`:

```
model <- drm.select(Y, regressors, show_info=T, st_method='auto.arima')
```

```
Covariate X1 has been tested [ic=-625.961030588487, lag=-2]
Covariate X2 has been tested [ic=-467.58487147781, lag=-3]
Significative correlation with lag<=0 could not be found for X3
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=282.613507688515, lag=-19]
Significative correlation with lag<=0 could not be found for X6
Covariate X1 has been added [aicc=-625.961030588487, lag=-2]
Series: serie
Regression with ARIMA(2,1,3) errors
```

Coefficients:

	ar1	ar2	ma1	ma2	ma3	xreg
	-0.1356	0.7048	0	-0.5217	-0.4048	3.2406
s.e.	0.0265	0.0320	0	0.0325	0.0275	0.0260

```
sigma^2 = 0.03042: log likelihood = 319.02
AIC=-626.05 AICc=-625.96 BIC=-596.78
```

```
Covariate X2 has been tested [ic=-3059.29747191119, lag=-3]
Significative correlation with lag<=0 could not be found for X3
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-625.961030588396, lag=-16]
Covariate X6 has been tested [ic=-625.961030588236, lag=0]
Covariate X2 has been added [aicc=-3059.29747191119, lag=-3]
Series: serie
Regression with ARIMA(2,0,2) errors
```

Coefficients:

	ar1	ar2	ma1	ma2	intercept	X1	X2
	-0.0845	0.7947	0.2370	0.5695	-0.0798	3.1998	-2.4804
s.e.	0.0207	0.0208	0.0279	0.0283	0.0156	0.0024	0.0163

```
sigma^2 = 0.002473: log likelihood = 1537.72
AIC=-3059.45 AICc=-3059.3 BIC=-3020.42
```

```
Significative correlation with lag<=0 could not be found for X3
```


Significative correlation with lag<=0 could not be found for X4
 Covariate X5 has been tested [ic=-3059.29747298372, lag=-18]
 Significative correlation with lag<=0 could not be found for X6
 Covariate X5 has been added [aicc=-3059.29747298372, lag=-18]
 Series: serie
 Regression with ARIMA(2,0,2) errors

Coefficients:

	ar1	ar2	ma1	ma2	intercept	X1	X2	X5
	-0.0844	0.7947	0.2369	0.5695	-0.0798	3.1998	-2.4805	0
s.e.	0.0207	0.0208	0.0279	0.0283	0.0156	0.0024	0.0163	0

sigma^2 = 0.002473: log likelihood = 1537.72
 AIC=-3059.45 AICc=-3059.3 BIC=-3020.42

Significative correlation with lag<=0 could not be found for X3
 Significative correlation with lag<=0 could not be found for X4
 Significative correlation with lag<=0 could not be found for X6
 No more variables will be added

Historical of added covariates to the model (ndiff=0)		
var lag	ic	
X1 -2	-625.961030588487	
X2 -3	-3059.29747191119	
X5 -18	-3059.29747298372	

Series: serie
 Regression with ARIMA(2,0,2) errors

Coefficients:

	ar1	ar2	ma1	ma2	intercept	X1	X2	X5
	-0.0844	0.7947	0.2369	0.5695	-0.0798	3.1998	-2.4805	0
s.e.	0.0207	0.0208	0.0279	0.0283	0.0156	0.0024	0.0163	0

sigma^2 = 0.002473: log likelihood = 1537.72
 AIC=-3059.45 AICc=-3059.3 BIC=-3020.42

Fir a model with the stationary detection method adf.test:

```
model <- drm.select(Y, regressors, show_info=T, st_method='adf.test')
```

Covariate X1 has been tested [ic=-629.798734864938, lag=-2]
 Covariate X2 has been tested [ic=-471.760045300591, lag=-3]
 Significative correlation with lag<=0 could not be found for X3
 Significative correlation with lag<=0 could not be found for X4
 Covariate X5 has been tested [ic=280.1216933898, lag=-19]
 Significative correlation with lag<=0 could not be found for X6
 Covariate X1 has been added [aicc=-629.798734864938, lag=-2]
 Series: serie
 Regression with ARIMA(2,1,3) errors

Coefficients:

	ar1	ar2	ma1	ma2	ma3	xreg
	-0.1359	0.7050	0	-0.5226	-0.4039	3.2404

```
s.e.    0.0265  0.0319    0    0.0325   0.0274  0.0260
```

```
sigma^2 = 0.03038:  log likelihood = 320.94  
AIC=-629.89   AICc=-629.8   BIC=-600.6
```

```
-----  
Covariate X2 has been tested [ic=-3070.66617767731, lag=-3]  
Significative correlation with lag<=0 could not be found for X3  
Significative correlation with lag<=0 could not be found for X4  
Covariate X5 has been tested [ic=-629.798734860246, lag=-16]  
Covariate X6 has been tested [ic=-633.352832663822, lag=-10]  
Covariate X2 has been added [aicc=-3070.66617767731, lag=-3]  
Series: serie  
Regression with ARIMA(2,0,2) errors
```

```
Coefficients:
```

	ar1	ar2	ma1	ma2	intercept	X1	X2
	-0.0825	0.7970	0.2366	0.5694	-0.0809	3.2004	-2.4792
s.e.	0.0207	0.0207	0.0279	0.0282	0.0157	0.0024	0.0163

```
sigma^2 = 0.002476:  log likelihood = 1543.41  
AIC=-3070.82   AICc=-3070.67   BIC=-3031.76
```

```
-----  
Significative correlation with lag<=0 could not be found for X3  
Significative correlation with lag<=0 could not be found for X4  
Covariate X5 has been tested [ic=-3070.6661020866, lag=-18]  
Significative correlation with lag<=0 could not be found for X6  
No more variables will be added  
-----
```

```
|          Historical of added covariates to the model (ndiff=0)          |
```

```
-----  
var lag          ic  
X1  -2 -629.798734864938  
X2  -3 -3070.66617767731  
-----
```

```
Series: serie  
Regression with ARIMA(2,0,2) errors
```

```
Coefficients:
```

	ar1	ar2	ma1	ma2	intercept	X1	X2
	-0.0825	0.7970	0.2366	0.5694	-0.0809	3.2004	-2.4792
s.e.	0.0207	0.0207	0.0279	0.0282	0.0157	0.0024	0.0163

```
sigma^2 = 0.002476:  log likelihood = 1543.41  
AIC=-3070.82   AICc=-3070.67   BIC=-3031.76
```

3.2 With non-stationary errors

```
set.seed(123)  
residuals <- sim.arima(model=list(p=2, d=2, q=1), n=N)  
  
beta0 <- -0.1; beta1 <- 3.2; beta2 <- -2.5
```

```
r1 <- 2; r2 <- 3
Y <- beta0 + beta1 * lag(X1$X, -r1) + beta2 * lag(X2$X, -r2) + residuals$X
regressors <- cbind(X1=X1$X, X2=X2$X, X3=X3$X, X4=X4$X, X5=X5$X, X6=X6$X)
```

Fit a model with the stationary detection method auto.arima:

```
model <- drm.select(Y, regressors, show_info=T, st_method='auto.arima')
```

```
Covariate X1 has been tested [ic=-574.530644513446, lag=-2]
Covariate X2 has been tested [ic=-319.625017869207, lag=-3]
Covariate X3 has been tested [ic=273.546752771765, lag=-24]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=273.546752771765, lag=-19]
Significative correlation with lag<=0 could not be found for X6
Covariate X1 has been added [aicc=-574.530644513446, lag=-2]
Series: serie
Regression with ARIMA(2,2,3) errors
```

Coefficients:

	ar1	ar2	ma1	ma2	ma3	xreg
	1.1202	-0.2548	-1.1101	0	0.2938	3.2284
s.e.	0.0519	0.0435	0.0369	0	0.0267	0.0688

```
sigma^2 = 0.03215: log likelihood = 293.31
AIC=-574.62 AICc=-574.53 BIC=-545.35
```

```
-----
Covariate X2 has been tested [ic=-3061.93131550221, lag=-3]
Covariate X3 has been tested [ic=-574.530646140655, lag=-23]
Significative correlation with lag<=0 could not be found for X4
Significative correlation with lag<=0 could not be found for X5
Covariate X6 has been tested [ic=-574.53065374945, lag=-12]
Covariate X2 has been added [aicc=-3061.93131550221, lag=-3]
Series: serie
Regression with ARIMA(3,1,1) errors
```

Coefficients:

	ar1	ar2	ar3	ma1	X1	X2
	0.8942	0.8695	-0.7658	0.5810	3.1863	-2.5005
s.e.	0.0257	0.0285	0.0209	0.0344	0.0136	0.0163

```
sigma^2 = 0.002465: log likelihood = 1538.02
AIC=-3062.05 AICc=-3061.93 BIC=-3027.89
```

```
-----
Covariate X3 has been tested [ic=-3061.93128181228, lag=-24]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-3065.18302343985, lag=-2]
Covariate X6 has been tested [ic=-3061.93140240731, lag=-16]
Covariate X5 has been added [aicc=-3065.18302343985, lag=-2]
Series: serie
Regression with ARIMA(2,2,1) errors
```

Coefficients:

	ar1	ar2	ma1	X1	X2	X5
	-0.1060	0.7644	0.5824	3.1863	-2.5005	0

s.e. 0.0258 0.0208 0.0343 0.0136 0.0164 0

sigma^2 = 0.002471: log likelihood = 1538.64

AIC=-3065.27 AICc=-3065.18 BIC=-3036

Covariate X3 has been tested [ic=-3065.18302344977, lag=-24]

Significative correlation with lag<=0 could not be found for X4

Covariate X6 has been tested [ic=-3065.18302389084, lag=-16]

Covariate X6 has been added [aicc=-3065.18302389084, lag=-16]

Series: serie

Regression with ARIMA(2,2,1) errors

Coefficients:

	ar1	ar2	ma1	X1	X2	X5	X6
	-0.1060	0.7644	0.5824	3.1863	-2.5005	0	0
s.e.	0.0258	0.0208	0.0343	0.0136	0.0164	0	0

sigma^2 = 0.002471: log likelihood = 1538.64

AIC=-3065.27 AICc=-3065.18 BIC=-3036

Covariate X3 has been tested [ic=-3065.18302389851, lag=-24]

Significative correlation with lag<=0 could not be found for X4

Covariate X3 has been added [aicc=-3065.18302389851, lag=-24]

Series: serie

Regression with ARIMA(2,2,1) errors

Coefficients:

	ar1	ar2	ma1	X1	X2	X5	X6	X3
	-0.1060	0.7644	0.5824	3.1863	-2.5005	0	0	0
s.e.	0.0258	0.0208	0.0343	0.0136	0.0164	0	0	0

sigma^2 = 0.002471: log likelihood = 1538.64

AIC=-3065.27 AICc=-3065.18 BIC=-3036

Significative correlation with lag<=0 could not be found for X4

No more variables will be added

The global model does not have stationary errors

Trying to adjust a model that do have stationary errors

No valid model with stationary errors could be optimized

Applying regular differentiation (ndiff=1) and calling again the function

Covariate X1 has been tested [ic=-574.530471188021, lag=-2]

Covariate X2 has been tested [ic=-372.308688110976, lag=-3]

Covariate X3 has been tested [ic=273.546501338415, lag=-24]

Significative correlation with lag<=0 could not be found for X4

Covariate X5 has been tested [ic=273.546501338415, lag=-19]

Significative correlation with lag<=0 could not be found for X6

Covariate X1 has been added [aicc=-574.530471188021, lag=-2]

Series: serie

Regression with ARIMA(2,1,3) errors

Coefficients:

	ar1	ar2	ma1	ma2	ma3	xreg
--	-----	-----	-----	-----	-----	------

	1.1202	-0.2548	-1.1101	0	0.2938	3.2284
s.e.	0.0519	0.0435	0.0369	0	0.0267	0.0688

sigma^2 = 0.03215: log likelihood = 293.31
AIC=-574.62 AICc=-574.53 BIC=-545.35

Covariate X2 has been tested [ic=-3065.18492727886, lag=-3]
Covariate X3 has been tested [ic=-574.530472814533, lag=-23]
Significative correlation with lag<=0 could not be found for X4
Significative correlation with lag<=0 could not be found for X5
Covariate X6 has been tested [ic=-574.530480425355, lag=-12]
Covariate X2 has been added [aic=-3065.18492727886, lag=-3]
Series: serie
Regression with ARIMA(2,1,1) errors

Coefficients:

	ar1	ar2	ma1	X1	X2
	-0.1060	0.7644	0.5824	3.1863	-2.5005
s.e.	0.0258	0.0208	0.0343	0.0136	0.0164

sigma^2 = 0.00247: log likelihood = 1538.64
AIC=-3065.27 AICc=-3065.18 BIC=-3036

Covariate X3 has been tested [ic=-3065.18492728369, lag=-24]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-3065.18492676853, lag=-2]
Covariate X6 has been tested [ic=-3065.18492719258, lag=-16]
Covariate X3 has been added [aic=-3065.18492728369, lag=-24]
Series: serie
Regression with ARIMA(2,1,1) errors

Coefficients:

	ar1	ar2	ma1	X1	X2	X3
	-0.1060	0.7644	0.5824	3.1863	-2.5005	0
s.e.	0.0258	0.0208	0.0343	0.0136	0.0164	0

sigma^2 = 0.00247: log likelihood = 1538.64
AIC=-3065.27 AICc=-3065.18 BIC=-3036

Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-3065.18492677773, lag=-2]
Covariate X6 has been tested [ic=-3065.18492720179, lag=-16]
No more variables will be added
The global model does not have stationary errors
Trying to adjust a model that do have stationary errors

	Historical of added covariates to the model (ndiff=1)	
var lag	ic	
X1 -2	-574.530471188021	
X2 -3	-3065.18492727886	
X3 -24	-3065.18492728369	

Series: serie

Regression with ARIMA(3,0,1) errors

Coefficients:

	ar1	ar2	ar3	ma1	X1	X2	X3
	0.8941	0.8695	-0.7658	0.5811	3.1863	-2.5005	0
s.e.	0.0257	0.0285	0.0209	0.0343	0.0136	0.0163	0

sigma^2 = 0.002465: log likelihood = 1538.01

AIC=-3062.02 AICc=-3061.91 BIC=-3027.87

Fit a model with the stationary detection method adf.test:

```
model <- drm.select(Y, regressors, show_info=T, st_method='adf.test')
```

Covariate X1 has been tested [ic=-574.530644513446, lag=-2]
Covariate X2 has been tested [ic=-319.625017869207, lag=-3]
Covariate X3 has been tested [ic=273.546752771765, lag=-24]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=273.546752771765, lag=-19]
Significative correlation with lag<=0 could not be found for X6
Covariate X1 has been added [aicc=-574.530644513446, lag=-2]
Series: serie
Regression with ARIMA(2,2,3) errors

Coefficients:

	ar1	ar2	ma1	ma2	ma3	xreg
	1.1202	-0.2548	-1.1101	0	0.2938	3.2284
s.e.	0.0519	0.0435	0.0369	0	0.0267	0.0688

sigma^2 = 0.03215: log likelihood = 293.31

AIC=-574.62 AICc=-574.53 BIC=-545.35

Covariate X2 has been tested [ic=-3061.93131550221, lag=-3]
Covariate X3 has been tested [ic=-574.530646140655, lag=-23]
Significative correlation with lag<=0 could not be found for X4
Significative correlation with lag<=0 could not be found for X5
Covariate X6 has been tested [ic=-574.53065374945, lag=-12]
Covariate X2 has been added [aicc=-3061.93131550221, lag=-3]
Series: serie
Regression with ARIMA(3,1,1) errors

Coefficients:

	ar1	ar2	ar3	ma1	X1	X2
	0.8942	0.8695	-0.7658	0.5810	3.1863	-2.5005
s.e.	0.0257	0.0285	0.0209	0.0344	0.0136	0.0163

sigma^2 = 0.002465: log likelihood = 1538.02

AIC=-3062.05 AICc=-3061.93 BIC=-3027.89

Covariate X3 has been tested [ic=-3061.93128181228, lag=-24]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-3065.18302343985, lag=-2]
Covariate X6 has been tested [ic=-3061.93140240731, lag=-16]
Covariate X5 has been added [aicc=-3065.18302343985, lag=-2]
Series: serie

Regression with ARIMA(2,2,1) errors

Coefficients:

	ar1	ar2	ma1	X1	X2	X5
	-0.1060	0.7644	0.5824	3.1863	-2.5005	0
s.e.	0.0258	0.0208	0.0343	0.0136	0.0164	0

sigma^2 = 0.002471: log likelihood = 1538.64

AIC=-3065.27 AICc=-3065.18 BIC=-3036

Covariate X3 has been tested [ic=-3065.18302344977, lag=-24]
Significative correlation with lag<=0 could not be found for X4
Covariate X6 has been tested [ic=-3065.18302389084, lag=-16]
Covariate X6 has been added [aicc=-3065.18302389084, lag=-16]

Series: serie

Regression with ARIMA(2,2,1) errors

Coefficients:

	ar1	ar2	ma1	X1	X2	X5	X6
	-0.1060	0.7644	0.5824	3.1863	-2.5005	0	0
s.e.	0.0258	0.0208	0.0343	0.0136	0.0164	0	0

sigma^2 = 0.002471: log likelihood = 1538.64

AIC=-3065.27 AICc=-3065.18 BIC=-3036

Covariate X3 has been tested [ic=-3065.18302389851, lag=-24]
Significative correlation with lag<=0 could not be found for X4
Covariate X3 has been added [aicc=-3065.18302389851, lag=-24]

Series: serie

Regression with ARIMA(2,2,1) errors

Coefficients:

	ar1	ar2	ma1	X1	X2	X5	X6	X3
	-0.1060	0.7644	0.5824	3.1863	-2.5005	0	0	0
s.e.	0.0258	0.0208	0.0343	0.0136	0.0164	0	0	0

sigma^2 = 0.002471: log likelihood = 1538.64

AIC=-3065.27 AICc=-3065.18 BIC=-3036

Significative correlation with lag<=0 could not be found for X4
No more variables will be added
The global model does not have stationary errors
Trying to adjust a model that do have stationary errors
No valid model with stationary errors could be optimized
Applying regular differentiation (ndiff=1) and calling again the function

Covariate X1 has been tested [ic=-574.530471188021, lag=-2]
Covariate X2 has been tested [ic=-372.308688110976, lag=-3]
Covariate X3 has been tested [ic=273.546501338415, lag=-24]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=273.546501338415, lag=-19]
Significative correlation with lag<=0 could not be found for X6
Covariate X1 has been added [aicc=-574.530471188021, lag=-2]

Series: serie
Regression with ARIMA(2,1,3) errors

Coefficients:

	ar1	ar2	ma1	ma2	ma3	xreg
	1.1202	-0.2548	-1.1101	0	0.2938	3.2284
s.e.	0.0519	0.0435	0.0369	0	0.0267	0.0688

sigma^2 = 0.03215: log likelihood = 293.31

AIC=-574.62 AICc=-574.53 BIC=-545.35

Covariate X2 has been tested [ic=-3065.18492727886, lag=-3]
Covariate X3 has been tested [ic=-574.530472814533, lag=-23]
Significative correlation with lag<=0 could not be found for X4
Significative correlation with lag<=0 could not be found for X5
Covariate X6 has been tested [ic=-574.530480425355, lag=-12]
Covariate X2 has been added [aicc=-3065.18492727886, lag=-3]

Series: serie

Regression with ARIMA(2,1,1) errors

Coefficients:

	ar1	ar2	ma1	X1	X2
	-0.1060	0.7644	0.5824	3.1863	-2.5005
s.e.	0.0258	0.0208	0.0343	0.0136	0.0164

sigma^2 = 0.00247: log likelihood = 1538.64

AIC=-3065.27 AICc=-3065.18 BIC=-3036

Covariate X3 has been tested [ic=-3065.18492728369, lag=-24]
Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-3065.18492676853, lag=-2]
Covariate X6 has been tested [ic=-3065.18492719258, lag=-16]
Covariate X3 has been added [aicc=-3065.18492728369, lag=-24]

Series: serie

Regression with ARIMA(2,1,1) errors

Coefficients:

	ar1	ar2	ma1	X1	X2	X3
	-0.1060	0.7644	0.5824	3.1863	-2.5005	0
s.e.	0.0258	0.0208	0.0343	0.0136	0.0164	0

sigma^2 = 0.00247: log likelihood = 1538.64

AIC=-3065.27 AICc=-3065.18 BIC=-3036

Significative correlation with lag<=0 could not be found for X4
Covariate X5 has been tested [ic=-3065.18492677773, lag=-2]
Covariate X6 has been tested [ic=-3065.18492720179, lag=-16]
No more variables will be added
The global model does not have stationary errors
Trying to adjust a model that do have stationary errors

	Historical of added covariates to the model (ndiff=1)
--	---

var lag	ic
---------	----


```
X1 -2 -574.530471188021
X2 -3 -3065.18492727886
X3 -24 -3065.18492728369
```

Series: serie

Regression with ARIMA(3,0,1) errors

Coefficients:

	ar1	ar2	ar3	ma1	X1	X2	X3
	0.8941	0.8695	-0.7658	0.5811	3.1863	-2.5005	0
s.e.	0.0257	0.0285	0.0209	0.0343	0.0136	0.0163	0

sigma^2 = 0.002465: log likelihood = 1538.01

AIC=-3062.02 AICc=-3061.91 BIC=-3027.87