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Article in *Annual Reviews in Control* · April 2010

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# Pricing Strategies and Models

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## Abstract

Price is a major parameter that affects company revenue significantly. This is why this paper starts by presenting basic pricing concepts. Strategies, such as market segmentation, discount, revenue management, price skimming, are introduced. A particular attention is paid to the relationship among margin, price and selling level. Then, the impact of prices on selling volume is analyzed, and the notion of selling curve is introduced. Related pricing methods are discussed such as price testing, cost plus method, involvement of experts, market analysis and customer surveying. Included in the last category is the conjoint measurement concerned with finding what parameters of the items are important to customers. The profile method and a simplified version, the two-factor method, are also detailed. They provide a set of part-worths (*i.e.*, numerical values) for each tester. In other words, the opinion of each tester can be represented by a point in a space whose dimension is the number of part-worths. By applying a clustering method, specifically K-mean analysis, a limited number of clusters can be obtained, each of them representing a market segment. A deterministic pricing model with time-dated items is also analyzed. This model provides practical insights into pricing mechanisms.

## Key words

Conjoint measurement, Cost-plus method, Discount strategy, Duopoly market, High price strategy, Low price strategy, Margin, Market segmentation, Part-worth, Penetration pricing, Price skimming, Price testing, Revenue management, Salvage value, Selling curve

## 1. Introduction

Three non exclusive ways exist to increase the competitiveness of a production system:

- decrease production costs;
- increase the market share; and/or
- adjust the prices to the state of the market.

Adjusting the prices is called pricing strategy. A pricing strategy has as goal to establish an optimum price with current profit maximization, maximization of the number of units sold, etc.

In production management, historically, the objectives were the reduction of production costs and augmentation of market shares. Significant (and costly) efforts have been made in companies to reduce the expenses. For instance, in the automotive industry and banking sectors, costs have been reduced by 30% to 50% within the last 10 years. Increasing a market share, which is also costly, depends on the competitiveness of the company. In turn, competitiveness depends also on the ability of the company to meet customer's requirements.

After a long period when in production management the principal objective was the reduction of costs and increase of the market share, now more and more researchers try deal with the non sufficiently studied problem of adjustment of the prices to the state of the market. This area is called pricing or revenue management.

This paper deals with this new domain for production managers which concerns how the prices influence company revenues. It should be noted that changing a price is obviously easier and faster than developing a process to reduce production costs or to increase the market share. Furthermore, the price parameter influences directly and strongly profit margin as well as market share. It has been shown that modifying the price by 1% results in the change of at least 10% in everyday consumption.

Thus, price as an adjustment parameter for profit is the easiest and fastest way to increase competitiveness.

Indeed, setting a price is the first step of any selling process and we will discuss this point, but pricing strategy does more. It tries to take advantage of:

- Time by playing with seasonality of demand, for example.
- Customers' preferences and purchasing behavior.
- Spectrum of available products.

These aspects are the most important when pricing is concerned and not exclusive. As mentioned in (Talluri and Van Ryzin, 2004), pricing strategy is beneficial when:

- Customers are heterogeneous, which means that their purchasing behavior over time varies, their willingness to pay varies from customer to customer, and they are attracted by different benefits offered by the same type of products.
- Demand variability and uncertainty are high, which guarantees a flourishing revenue to those who master pricing.
- Production is rigid, which encourages playing with prices when demand varies.

Note that the pricing problem is novel for production management but has been extensively studied in other fields. Initially, pricing has been used by said airline industry (Belobaba, 1987, 1989) and (Belobaba and Wilson, 1997), followed by retailers (Bernstein and Federgruen, 2003) and (Chen *et al*, 2006), hotel industry (Bitran and Mondschein, 1995; Bitran and Gilbert, 1996) and, more recently, by companies in the energy sector. Note that these sectors are characterized by production (or offer) rigidity, variability of demand and heterogeneity of customers. Interesting research work has been performed in the case of time dated items (Gallego and Van Ryzin, 1994). Unfortunately, very restrictive assumptions were introduced to reach analytical results.

The rest of this paper is organized as follows. In Section 2, we review the most common strategies used to define prices in order to maximize revenue. In Section 3, we discuss the links between selling level, price and costs. The selling curve that connects the price to the sales volume can be obtained as will be shown in Section 4. The conjoint measurement helps finding the characteristics of importance to customers in an item. This will be explained in Section 5. Section 6 provides some insights on pricing strategy for oligopoly markets, that is to say markets dominated by a small number of providers, each of them being aware of the actions of the competitors, having the same access to technologies and resources, and the freedom to enter or exit the system at any time and freely. In Section 7, we propose an analytical dynamic pricing model. Section 8 is the conclusion.

## **2. Common pricing strategies**

### **2.1. High and low price strategies**

High price is accepted if it agrees with the value of the product perceived by the customers, otherwise such a strategy leads to commercial failure.

Note that while price plays a role, it is not always decisive, in particular when the product is new on the market. A recent example is the strategy of Apple to dominate in the MP3 player market: Apple based its marketing strategy on i-Pod quality and aesthetics and won the leadership in the domain despite the fact that the i-Pod was the most expensive among the similar products. So high price did not prevent the i-Pod from becoming the leader in the MP3 player market. Another pertinent example is the Mercedes-Benz class A vehicles. The price of this product has been set at a higher level than the initial cost analysis by the car company. Nevertheless, the production capacity was fully utilized during the first production year. The explanation is the power of the corporate image of Mercedes-Benz. Numerous other examples can be found in the cosmetic industry and, more generally, in the luxury goods industry. In luxury industry, high price is more of a selling point than drawback.

A low price strategy may also lead to a commercial success, especially in the food retailing sector. For instance, low price retailers such as Lidl, Aldi or Leader Price are currently having a great amount of success in Europe. Another example is Dell Computer that allows customers to personalize their PC. Amazon.com gained an significant share of the book market by sometimes reducing the prices by 40 to 50% and providing greater choice. These last two companies base their strategy on the use of Internet to directly distribute their items to customers, which results in a huge reduction of costs which, in turn, allows a large reduction of prices and thus improves their competitiveness.

The success of a low price strategy depends on the number of clients attracted by the product since the low margin should be compensated by a higher number of items sold. In Section 2.2.2, we provide the number of new customers required to compensate a given reduction of price.

Some disadvantages should be outlined in companies applying high or low price strategy. For instance, the image of the items sold by the company is somewhat frozen and a long term price expectation is established, which can reduce the flexibility of the decision-making system.

High and low price strategies could be described as frozen strategies since they try to attract clients by making the most of the corporate image. The drawback is the inability of these strategies to adapt themselves to fundamental disturbances. For instance, an overall impoverishment of a country may sharply penalize companies devoted to luxury and expensive items.

Other strategies are much more adjustable. We provide a short description of these strategies in the next section.

## **2.2. Adjustable strategies**

### **2.2.1. Market segmentation (or price discrimination) strategy**

The development of a plan based upon the fact that different groups of customers attach different levels of importance to diverse benefits offered by a type of product or service is called market segmentation. For instance, the same car model may be proposed in different versions (two-door or four-door, different engine powers, different finishing levels, *etc.*), and each version may attract a particular type of customer.

This strategy is applicable to a type of item in the case of a monopoly market. It consists of segmenting the market and charging a different price for each segment, depending on the willingness of these customers to pay more or less to purchase the item. Indeed, some “rate fences” should be introduced in order to make sure that the customers of a segment will pay the price assigned to the segment. These “rate fences” can be the promotion of some benefits that attract the customers of a specific niche, or by offering some particular services to the customers of a distinct segment. Again, the customers belonging to given segment should be similar or, in other words, characterized by the same parameters, and dissimilar from the customers of others. Similarity and dissimilarity are related to buying habits.

The approach to market segmentation is a four-stage process that can be summarized as follows:

- Identify the parameters that customers are interested in. For instance, in personal computer market, training, software level, memory size, disc and CPU sizes and quality of after sales service are parameters of interest. This identification is usually done by carrying out a survey among customers.
- Identify the part-worths (*i.e.*, the characteristics of the parameters, also called benefits) that are of interest to customers. This can be done using conjoint measurement (see section 5). Using *K*-mean analysis, which is a method of cluster analysis which aims to partition  $n$  observations into  $k$  clusters in which each observation belongs to the cluster with the nearest mean, may be helpful to put part-worth in an obvious.
- Define part-worth (or benefit) subsets that correspond to clusters of customers
- Identify the parameters that characterize the customers of a cluster, for instance, adherence to a socioeconomic class, geographic location, consumption habits, gender, religion, age, *etc.*

The last two steps can be performed using correspondence analysis, see for instance Deville and Saporta (1983), Benzecri (1994), Greenacre (1993) and Tabachnik (1989).

Market segmentation is the relationship between subsets of customers and subsets of benefits. Each subset of benefits is a market segment. To be acceptable, segments should be homogeneous within, and distinct from one segment to another.

*Remark:* It is also possible to use correspondence analysis, a method belonging to data analysis, to establish a relationship between subsets of customers and subsets of benefits. Correspondence analysis is a technique that will provide information about the “proximity” of benefits to characteristics of customers.

### **2.2.2. Discount strategy**

A discount sale consists in selling a given set of items at a reduced price for a limited period. Some examples are proposed in (Bolton, 1989), (Blattberg and Neslin, 1990), (Bemmaor and Mouchoux, 1991). This reduction should generate enough supplementary sales to compensate the reduction in income. However, this is rarely the case. Few companies realize the true discount cost. When a product discount is offered for a given period, it applies to all sales, which often leads to disastrous consequences.

Let us consider a set of items sold at the price  $c$  each. During a period  $T$ , the retailer sold usually  $m$  items and the benefit is  $b$  for each item sold. The retailer decides to apply a  $x\%$  discount, assuming that  $x c/100 \leq b$ , which means that the rebate is less than the benefit. The question is: how many supplementary items should be sold to compensate this reduction in income? We denote by  $z$  the number of supplementary items that should be sold.

It is easy to verify that we must have.

$$z \geq \frac{m x c}{100 b - x c} \quad (1)$$

Assume, for instance, that  $m = 10,000$ ,  $c = 50$  and  $b = 15$ . Figure 1 gives the minimum number of additional items that should be sold to compensate the reduction in income according to the percentage of discount.

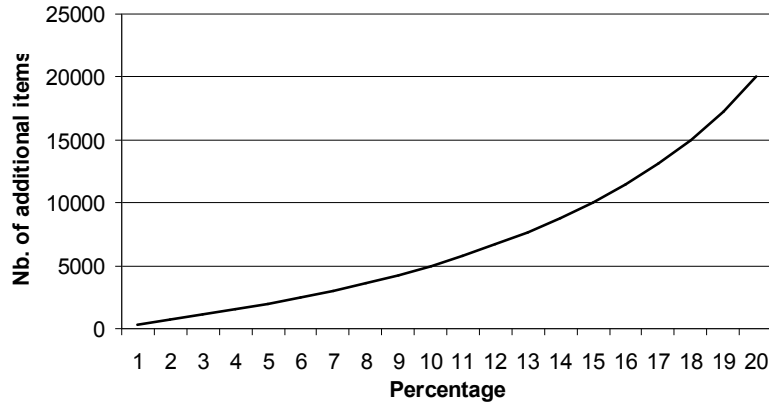


Figure 1: Number of additional items over percentage

### 2.2.3. Price skimming

In this strategy, a relatively high price is set at first, and then lowered over time. This strategy is similar to the price discrimination but with the time factor. Price skimming usually applies when customers are relatively less price sensitive (clients of cosmetic industry, for instance) or when they are attracted by some innovation (in particular electronic items such as computers).

Price skimming is useful to reimburse huge investments made for research and development. Indeed, a high price cannot be maintained for a long time, but only as long as the company is in a monopolistic situation with regard to the item's originality.

### 2.2.4. Penetration pricing

Penetration pricing consists of setting an initial price lower than the one of the market. The expectation is that this price is low enough to break down the purchasing habits of the customers. The objective is to obtain a larger market share. This can be defined as the low-price strategy enriched by the time factor.

Penetration pricing leads to cost reduction pressure and discourage the entry of competitors.

### 2.2.5. Yield management (revenue management)

The goal of yield management is to anticipate customers' and competitors' behavior in order to maximize revenue. Companies that use yield management review periodically earlier situations to analyze the effects of events on past customer and competitor behaviors. They also can take into account future events to adjust their pricing decisions.

Yield management is especially suitable in the case of time-dated items (airplane tickets) or perishable items (fruits, processed food). A mathematical model will be proposed in Section 7.

### 3. Margin, price and selling level

In this section, we explain the mechanism that links costs, price and margin. Note that a strong assumption is made: we consider that any number of available items can be sold.

Price affects margin, but may also affect cost due to the scale effect. The higher the price, the greater the margin per item, but increasing the price may results in a fall-off in sales, which may increase cost. On the contrary, when the price decreases, the number of items sold may increase, and thus the production costs may decrease due to the scale effect.

Let us introduce the following variables and assumptions:

- $c(s)$  is the variable cost per item when the total number of items sold during a given period (a month for instance) is  $s$ . The function  $c(s)$  is a non-increasing function of  $s$ .
- $f(s)$  is the fixed cost, that is to say the cost that applies whatever the number of items sold during the same period as the one chosen for the variable cost. Nevertheless,  $f(s)$  may increase with regard to  $s$  if some additional facilities are necessary to pass a given production threshold. The function  $f(s)$  is a non-decreasing and piecewise continuous function of  $s$ .

In fact,  $c(s)$  (respectively  $f(s)$ ) is either constant, or piecewise constant.

- $R$  is the margin for the period considered.
- $p$  is the price of one item.

We assume that all the items produced are sold, which implies that the market exists and the price remains attractive to customers. Under this assumption, which is strong, relation (2) is straightforward:

$$R = s p - s c(s) - f(s) \quad (2)$$

It is often necessary to compute the number of items to be sold in order to reach a given margin, knowing the price, the variable cost per item and the fixed cost. Let us consider the case when fixed and variable costs are constant.

In this case, Equation (2) becomes:

$$R = s p - s c - f$$

which leads to Relation (3):

$$s = \left\lceil \frac{R + f}{p - c} \right\rceil \quad (3)$$

Remember that  $\lceil a \rceil$  is the smallest integer greater than or equal to  $a$ .

Indeed, the price is always greater than the variable cost per item.

The margin being unchanging, it is easy to see that the number of items to be sold is a decreasing function of the price and an increasing function of the variable cost per item.

Figure 2 presents two examples giving the number of items to be sold to reach a given margin  $R$  with regard to the price. The costs are given ( $f = 5$ ,  $c = 1$ ).

When at least one of the two costs is piecewise constant, the series of positive integer numbers is divided into consecutive intervals for each cost, and the value of this cost is constant on each one of these intervals. To find the number of items to be sold, we consider all the pairs of intervals, a pair being made with an interval associated with the variable cost per item and an interval associated with the fixed cost. We use the costs corresponding to these intervals to apply Relation (3). If the resulting number of items to be sold belongs to both intervals, this number is a solution to the problem; otherwise another pair of intervals is tested.

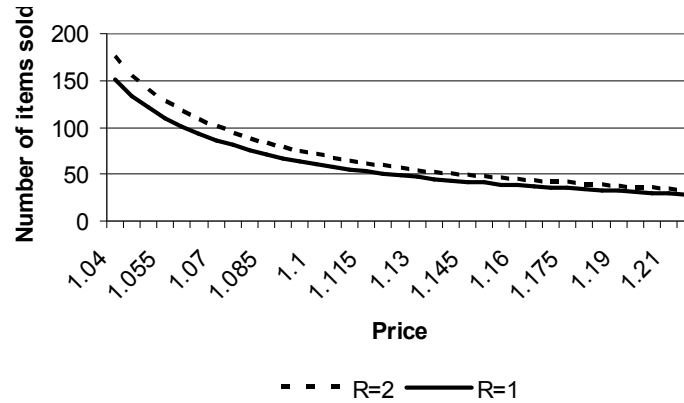


Figure 2: Number of items to be sold versus the price

*Definition:* The equilibrium point is the minimum number of items to sell in order to break-even. We simply have to assign the value 0 to  $R$  and apply Relation (3). We still assume that all the items that are produced are sold.

## 4. The selling curve

The *selling curve* (also called *demand curve*) represents the relationship between the price of an item and the number of items customers are willing to purchase during a given period. Usually, it is assumed that the business environment conditions are steady during this period.

### 4.1 Introduction

Customers intuitively compare the price of an item to the value they associate with it. According to a well-known axiom, customers do not buy items, they buy benefits or, in other words, they buy promise of what the item will deliver. If the evaluation made by the customer is higher than the price of the item, then the customer will buy it; otherwise, he/she will not.

If several types of items are equally attractive to customer, they will buy the item with the biggest difference between their own value evaluation and the price.

Several approaches are available to fix the price of an item:

- Cost-plus method.
- Price testing.
- Estimation made by experts.
- Market analysis.
- Survey.

We will examine these approaches in more detail.

### 4.2 Cost-plus method

Several types of cost-plus methods are available, but the common thread among these methods is to:

1. Calculate the cost per item.
2. Introduce an additional amount that will be the profit.

The profit can be a percentage of the cost or a fixed amount.

The cost per item can be calculated either using a standard accounting method based on arbitrary expense categories for allocating overhead, or deriving it from the resource used (*i.e.*, linking cost to project), or considering incremental cost.

Retailers assume that the purchase price paid to their suppliers is the cost.

The pivotal advantages of the cost-plus method are the following:

- Price is easy to calculate, which is of utmost importance when a huge number of prices must be established every day, for instance in volume retailing.
- Price is easy to manage.
- The method tends to stabilize the market.

The most important disadvantages are:

1. Customers and competitors are ignored.
2. Opportunity costs are ignored.

The *cost-plus* methods should be avoided since they ignore customers' behavior as well as the parameters they use to build their own evaluation. Defining a price requires analyzing the market and the behavior of customers with regard to the price. The objective is to establish a relationship between the number of items sold and the price. The curve that reflects this relationship is called *selling curve*.

#### 4.3 Price testing

This approach consists of modifying the price of the item under consideration and recording the number of items sold or the market share. This can be done playing with a scale shop or a shop simulated on computer with different classes of customers, characterized by their age, gender, the level of incomes and/or the buying habits, *etc.* This helps not only to define a price, but also to select most profitable market niche.

The price testing method is even easier to apply if the item is sold by Internet since changing the price is straightforward. Unfortunately, in this case, clients cannot be sometimes completely identified, and thus marketing cannot take advantage of customers' characteristics.

#### 4.4 Estimation made by experts

This is the only available method when a new type of item is introduced on the market, a fundamental advances in technology modifies an existing item, or when the competition changes drastically.

The first step of the method consists in recording the opinion of several experts (at least ten) that are not in contact with each other. One usually obtains different selling curves that are quite different from each other. The objective of the next step is to organize working sessions with these experts in order to move their estimations/recommendations closer to each other.

Indeed, the drawback, when applying this method, is that customers do not contribute to the process of the selling curve definition, which increases the probability of error.

#### 4.5 Market analysis

This method is based on the history of the item under consideration. This implies that the item (or a similar item) has a history (*i.e.*, is not new in the market) and that the current market environment is the same as (or similar to) the environment when the data were collected.

Assume that the previous conditions hold and that the history provides quantities sold at different prices. Let  $p_i, i = 1, 2, \dots, n$  be the prices and  $s_i, i = 1, 2, \dots, n$  the corresponding quantities sold.

The objective is to define a function  $s = a p + b$  that fits "at the best" with the data. In other words, we compute  $a$  and  $b$  that minimize:

$$\varphi(a, b) = \sum_{i=1}^n (a p_i + b - s_i)^2$$

It is well known that this function is minimal for:



$$\left\{ \begin{array}{l} a = \frac{n \sum_{i=1}^n s_i p_i - \sum_{i=1}^n s_i \sum_{i=1}^n p_i}{n \sum_{i=1}^n p_i^2 - (\sum_{i=1}^n p_i)^2} \\ b = \frac{\sum_{i=1}^n s_i \sum_{i=1}^n p_i^2 - \sum_{i=1}^n s_i p_i \sum_{i=1}^n p_i}{n \sum_{i=1}^n p_i^2 - (\sum_{i=1}^n p_i)^2} \end{array} \right.$$

#### 4.6 Customer surveying

Questioning customers about the price of an item was widely used in the sixties. At that time, direct questions were asked as, for instance:

- What is the maximal amount you are prepared to pay for this item?
- What is, in your opinion, the right price for this item?
- Would you buy this product at the price of  $x$  monetary units?

This approach is rarely in use nowadays since it gives too much importance to the price that is only one of the parameters customers are interested in. Prices cannot be considered in isolation anymore.

This is why methods that try to evaluate characteristics of interest to the customers have been developed. One of them is the well known *Conjoint Measurement*.

### 5. Conjoint measurement and market segmentation

#### 5.1. Introduction

The conjoint measurement is becoming increasingly popular as a tool to find the characteristics of the items (product or services) that are important to customers. As a consequence, it provides information to increase the value added of items and market segments. An example is available in (Cattin and Wittink, 1982).

Conjoint measurement starts by listing the parameters that are supposed to be of some importance to customers in the item under consideration. The characteristics of importance will be extracted from this list. For instance, if the item is a personal computer, we may have:

- A parameter related to the hard disk size. For example, it takes the value 1 if the size belongs to [ 50 GO, 80 GO ], the value 2 if the size belongs to ( 80 GO, 120 GO ] and the value 3 if the size is greater than 120 GO (if we use figures from recent years).
- A parameter related to the memory size. It takes the value 1 if the size belongs to [ 256, 512 KO ] and 2 otherwise.
- A parameter related to the training. It takes the value 2 if the training is free and 1 otherwise.
- A parameter related to after sale service. It takes the values 1, 2 or 3 depending on the type of services.

For this example, we have  $3 \times 2 \times 2 \times 3 = 36$  sequences of five parameter values. Such a sequence is called a *stimulus*. Thus this example gives birth to 36 stimuli.

Note that the value of a parameter does not reflect an assessment, but a choice. This point is important.

Splitting the broad evaluation of an item among the values of its parameters implies that the overall evaluation of an item is the sum of the evaluations assigned to the values of the parameters. In other words, the utility function is assumed to be additive. This assumption is strong since it implies that parameters are disjointed (*i.e.*, independent from each other from the point of view of customers' perception of the item value). The evaluation of a parameter value is called *part-worth*. We obtain a set of part-worth from each tester. It helps answering the following questions:

- How to differentiate customers from each other? Customers that present close part-worths can be considered as similar (*i.e.*, as belonging to the same group in terms of purchasing behavior).
- How to establish the price of an item? It is possible to introduce the price as one of the parameters and ask customer to select a price range among a set of prices (*i.e.*, a set of parameter values).

- How will the market share alter if the price changes?
- Which parameter value should be improved first in order to increase the perceived value of the item?

Conjoint measurement is used not only to evaluate the importance of the values of the parameters of products (*i.e.*, the part-worths). It also applies to service quality, reliability, trademark reputation, *etc.*

Usually, each tester is asked to rank the stimuli in the order of his preference. Another possibility is to give a grade to each stimulus. The objective of the method is to derive from this ranking (or grading) the part-worths or, in other words, the evaluation of each one of the parameter values.

The method that requires ranking (or grading) the whole set of stimuli is called *profile method*. One drawback is the number of stimuli to rank (or grade) since this number increases exponentially with the number of characteristics and their “values”. Assume, for instance, that an item has 5 parameters and 3 values for each, which is a medium-size problem. Then,  $3^5 = 243$  stimuli must be ranked (or graded), which is quite impossible in practice.

The *two-factor method* tries to overcome the previous difficulty. The idea is to consider just two characteristics at a time and then to combine the information to derive the part-worths.

In the following subsections, we present the profile and the two factors methods that lead to the part-worths. Then, we propose a clustering method (*K*-mean analysis) that classifies these part-worths to provide market segments.

## 5.2. Profile method

Assume that  $m$  parameters have been selected for defining a given item and that parameter  $r \in \{1, 2, \dots, m\}$  has  $N_r$  values denoted by  $\{1, 2, \dots, N_r\}$ . The number of stimuli is  $N = \prod_{k=1}^m N_k$ .

Let  $S$  be the set of stimuli. Then  $\text{card}(S) = N$ . A stimuli is the sequence  $\{j_1, j_2, \dots, j_r, \dots, j_m\}$  with  $j_r \in \{1, \dots, N_r\}$  for  $r = 1, \dots, m$ . The rank (or the grade) assigned by a tester to stimulus  $\{j_1, j_2, \dots, j_r, \dots, j_m\}$  is denoted by  $G(j_1, j_2, \dots, j_r, \dots, j_m)$ .

We also denote by  $w_{i,j_i}$ ,  $j_i = 1, \dots, N_i$  and  $i = 1, \dots, m$  the part-worth of the  $j_i$ -th “value” of parameter  $i$  and by  $W = \{w_{i,j_i} \mid i=1, \dots, m; j_i=1, \dots, N_i\}$  the set of part-worths.

Let  $U(W) = \sum_{j_1=1}^{N_1} \dots \sum_{j_m=1}^{N_m} [G(j_1, \dots, j_m) - \sum_{i=1}^m w_{i,j_i} - a]^2$ , where  $a$  is the mean value of the ranks (or grades) assigned to the stimuli of  $S$ :

$$a = \frac{1}{\text{card}(S)} \sum_{(j_1, j_2, \dots, j_r, \dots, j_m) \in S} G(j_1, \dots, j_m)$$

The problem to solve is:

*Minimize*  $U(W)$

Subject to:

$$\sum_{j_i=1}^{N_i} w_{i,j_i} = 0 \text{ for } i = 1, \dots, m$$

## 5.3. Two-factor method

The two factor method consists in considering just two parameters at a time. Assume that  $m$  parameters are under consideration. Then  $C_m^2 = m \times (m-1) / 2$  tests will be conducted to generate the overall part-worths.

Consider the parameter  $r \in \{1, \dots, m\}$  having  $N_r$  “values”. This parameter will be tested with parameters  $1, \dots, r-1, r+1, \dots, m$  to obtain  $m-1$  part-worths for each parameter value  $j_r$  of  $r$  denoted by

$w_{r,j_r}^1, w_{r,j_r}^2, \dots, w_{r,j_r}^{r-1}, w_{r,j_r}^{r+1}, \dots, w_{r,j_r}^m$ . We take into account that the part-worth associated with the parameter value  $j_r$  of  $r$  is the mean value of the above part-worths:

$$w_{r,j_r}^* = \frac{1}{m-1} \sum_{\substack{i=1 \\ i \neq r}}^m w_{r,j_r}^i$$

It is easy to show that, if the grade associated to the stimulus  $(j_r, j_k)$  is the sum of the grades of the stimuli of  $S$  of the profile method where  $j_r$  is the  $r$ -th component and  $j_k$  the  $k$ -th component, then  $w_{r,j_r}^* = w_{r,j_r}$ , where the second member of the equality is the part-worth obtained with the profile method.

#### 5.4. Clustering for market segmentation

Using the profile method or the two factors method leads to a set of part-worths for each tester. Thus, each tester can be represented by a point in the  $m$ -dimensions space where  $m$  is the number of part-worths provided.

Clustering is a technique that classifies the points corresponding to the testers into different groups (*i.e.*, that partitions the sets of part-worths into clusters) so that the sets belonging to the same cluster represent testers (and thus customers) having a similar purchasing behavior. Thus, part-worths can define market segments: the common characteristics of the customers belonging to the same cluster define a segment. In fact, customers belonging to the same cluster are not obliged to have exactly the same characteristics (identical part-worths), they have similar characteristics (close part-worths), not necessary identical, and thus similar purchasing behavior.

Hereafter, we will see how a unique set of part-worths can be assigned to each cluster. Each one of these sets will define the corresponding segment.

To understand clustering and its use to define market segments, we present  $K$ -mean analysis that is a clustering algorithm. Additional aspects are given in (Anderberg, 1973), (Berry, 1994), (Greenleaf, 1995), (Hand, 1981), (Hartigan, 1975).

##### ***K-mean analysis algorithm***

Let  $W_1, \dots, W_r$  be the  $r$  sets of  $m$  part-worths derived from the evaluation of  $r$  testers. Thus,  $W_i = \{w_1^i, \dots, w_m^i\}$  for  $i = 1, \dots, r$ . The  $K$ -mean analysis algorithm can be summarized as follows.

1. Choose  $K$  ( $K \ll r$ ) sets  $V_1, \dots, V_K$  at random among the sets  $W_1, \dots, W_r$ . It is also possible to generate at random  $m$  components for each  $V_k = \{v_1^k, \dots, v_m^k\}$ . In this case,  $v_i^k$  is generated at random in

$$\left[ \min_{k=1, \dots, r} w_i^k, \max_{k=1, \dots, r} w_i^k \right].$$

2. For  $i = 1$  to  $r$  do:

2.1. Find  $u$  such that  $d(W_i, V_u) = \min_{j=1, \dots, K} d(W_i, V_j)$  where  $d$  is a distance or a similarity index. If  $d$  is the

Euclidian distance,  $d(W_i, V_j) = \sqrt{\sum_{q=1}^m (w_q^i - v_q^j)^2}$ .

2.2. Assign  $W_i$  to cluster  $C_u$ .

*Remark: Some clusters may be empty at the end of Step 2. They disappear and  $K$  decreases accordingly but, for the sake of simplicity, we still denote by  $K$  the number of clusters.*

3. For  $j = 1, \dots, K$ , compute the gravity center  $G_j$  of cluster  $C_j$ .

$$G_j = \{g_1^j, \dots, g_m^j\}, \text{ where } g_k^j = \sum_{i|W_i \in C_j} w_k^i / N_j \text{ and } N_j = \text{Card}(C_j) = \text{Number of sets in cluster } C_j.$$

4. If  $(G_1, \dots, G_K) \neq (V_1, \dots, V_K)$ , then set  $V_i = G_i$  for  $i = 1, \dots, K$  and return to step 2. Otherwise, the solution is obtained and  $(G_1, \dots, G_K)$  characterizes the clusters (*i.e.*, the market segments).

### Remarks

- Some testers may be more credible than others. In this case, a weight can be added to each set of part-worths: the more credible the tester, the greater the weight associated to the corresponding set of part-worths. In this case we should replace in Step 3:

$$“G_j = \{g_1^j, \dots, g_m^j\}, \text{ where } g_k^j = \sum_{i|W_i \in C_j} w_k^i / N_j \text{ and } N_j = \text{Card}(C_j) = \text{Number of sets in cluster } C_j.”$$

with:

$$“G_j = \{g_1^j, \dots, g_m^j\}, \text{ where } g_k^j = \sum_{i|W_i \in C_j} p_i \times w_k^i / N_j \text{ and } N_j = \sum_{i|W_i \in C_j} p_i.”$$

Parameters  $p_i$  represent the weights.

- Since the initial set  $V_1, \dots, V_K$  is generated at random, the result may be different from one application to the next. It is why some algorithms proceed as follows:

Launch the algorithm several times (100 times for instance),

Consider that a cluster is composed of  $W_i$  that are in the same cluster at the end of each run. Other  $W_i$  are discarded and considered as belonging to the “grey set”, that is to say the set of elements that will not be considered for further analysis.

The gravity centre of a cluster represents the cluster. In some applications, the  $W_i$  belonging to the “grey set” are redeployed in the cluster represented by the nearest gravity centre.

## 6. Price strategy in oligopoly markets

Let us first provide some definitions required to properly understand this section.

An *oligopoly market* is a market dominated by a small number of providers. Each provider (firm) is aware of the actions of the other providers (competitors) and the actions of one provider influence the others. Providers operate under imperfect competition, that is to say a market where at least one of the following conditions does not hold:

- Numerous providers.
- Perfect information: all providers and customers know the prices set by all providers.
- Freedom of entry and exit: a provider can enter or exit the system at any time and freely.
- Homogeneous output, which means that there is no product differentiation or, in other words, items are perfect substitutes.
- All providers have equal access to technologies and resources.

### 6.1. Forecasting competitors' reactions

The reactions of competitors influence price decisions, in particular in an oligopoly market where each provider is aware of the decisions of the others and react accordingly.

In addition to the demand curve, that describes the purchasing behavior of customers with respect to prices, price definition requires estimating competitors' reactions, which is very difficult due to the unpredictability of their behavior confronting with a decrease or increase in prices. Competitors' reactions also depend on the size of the firm, the production capacity, conjuncture and psychology of the managers.

An effective way to forecast the competitors' reaction is to organize brainstorming sessions with the employees who have some connections with competitors at the technical, commercial, training, scientific, *etc.* levels.

### 6.2. Price war: decreasing prices

To illustrate the notion of price war, let us consider two providers  $A$  and  $B$ . We assume that  $A$  and  $B$  are the only providers for the items under consideration, i.e. duopoly market (Benoit and Krishna, 1987).

If both providers keep their prices constant, then the revenue of both will remain constant too. If  $A$  decreases its prices, some clients will migrate from  $B$  to  $A$ . As a consequence, the revenue of  $B$  will decrease while the

revenue of *A* may increase (see example below). If *B* decreases its prices in reaction, then both will lose a part of their revenues. This example is summarized in Table 1.

Table 1: Prices Decrease

		<i>Provider B</i>	
		<i>Keeps the prices constant</i>	<i>Decreases the prices</i>
<i>Provider A</i>	<i>Keeps the prices constant</i>	The revenues of <i>A</i> and <i>B</i> remain constant	The revenue of <i>B</i> may increase. The revenue of <i>A</i> decreases
	<i>Decreases the prices</i>	The revenue of <i>A</i> may increase. The revenue of <i>B</i> decreases	Both revenues decrease

Obviously, this strategy is inadequate since if one of the providers decreases its prices, the other one cannot do anything but decrease its own prices, and the revenues of both providers will fall. Decreasing the prices can be justified if a provider, *A* for instance, wants to expel *B* from the market, but this strategy may backfire if *B* is able to respond to the price decrease.

To conclude, cutting prices in duopoly or oligopoly market is often a mistake: customers are reluctant to switch to providers who reduce their prices since they know that their provider will follow the move and reduce its prices as well. Thus, the best strategy for all is to maintain prices, except if a company is strong enough to survive a price war and expel the other competitors from the market.

Nevertheless, price wars have often happened in the past 20 years, particularly in the food and computer industries, among Internet access companies or the airlines, to name just a few. A price war originates usually because of:

- Surplus production capacity, which can force companies to reduce their inventory level at “any” price.
- Production of basic products, easy to manufacture, which encourages an aggressive competition.
- Persistence of a low growth market, which incites some providers to try gaining market shares at the expense of competitors.
- Management style.

This list is not exhaustive.

An important question is: how to avoid price wars? Or, in other words, how to avoid competitor aggressiveness when cutting prices? One of the strategies is to differentiate the products (market segmentation) and to reduce the prices in segments that are not important to rival firms. Another strategy is to target products that do not attract competitors.

### 6.3. Price war: increasing prices

A similar progression may be observed if one of the providers increases his / her prices. The results of such a strategy are summarized in Table 2.

Table 2: Prices increase

		<i>Provider B</i>	
		<i>Keeps prices constant</i>	<i>Increases prices</i>
<i>Provider A</i>	<i>Keeps prices constant</i>	The revenues of <i>A</i> and <i>B</i> remain constant	The revenue of <i>A</i> increases. The revenue of <i>B</i> may decrease.
	<i>Increases prices</i>	The revenue of <i>A</i> may decrease. The revenue of <i>B</i> increases.	Both revenues increase

Thus, providers who increase their prices are not sure to increase profits, but the benefit to their competitors will certainly increase. If the competitor does not change its prices, the move may be dangerous, but if the competitor increases its prices, then all providers increase their profit.

To conclude, if a provider increases their prices, the result will be a overall increase of benefits if all competitors increase their prices accordingly. But some competitors may decide to keep prices unchanged, which would results in loss of benefits for the providers who increased their prices. A quite common strategy to avoid this problem is to apply the well-known signal theory. In this approach, one of the competitors gives notice, a long time in advance, of their willingness to increase a price (justified by originality or technical improvement). This strategy also applies when the goal is to reduce a price. Doing so, competitors have time to signal if they are ready to follow the move or not.

## 7. A dynamic pricing model

### 7.1. Introduction

Any dynamic pricing model requires establishing how demand responds to changes in price. This section deals with mathematical models of monopoly systems. The reader will notice that strong assumptions are made to obtain tractable models. Indeed, such mathematical models can hardly represent real-life situations, but they do illustrate the relationship between price and customers' purchasing behavior.

In this section we consider the case of time-dated items, i. e. items that must be sold before a given point in time, say  $T$ . Furthermore, there is no supply option before time  $T$ . This situation is common in food industry, toy business (when toys must be sold before Christmas, for instance), marketing products (associated with special events like movies, football matches, etc.), fashion apparel (because the selling period ends with the season), airplane tickets, etc.

The goal is to find a strategy (dynamic pricing, also called yield or revenue management) that leads to the maximum expected revenue by time  $T$ , assuming that the process starts at time 0. This strategy consists in selecting a set of adequate prices for the items that vary according to the number of unsold items and, in some cases, over the time. At a given point in time, we assume that the price of an item is a non-increasing function of the inventory level. For a given inventory level, prices are going down over time.

Indeed, a huge number of models exist, depending of the situation at hand and the assumptions made to reach a working model. For instance, selling airplane tickets requires a pricing strategy that leads to very cheap tickets as takeoff nears while selling fashion apparel is less constrained since a second market exists, i.e. it is still possible to sell these items at discount after the deadline.

For the models presented in this section we assume that:

- The number of potential customers is infinite. As a consequence, the size of the population does not belong to the set of the parameters of the models.
- A single type of item is concerned and its sales are not affected by other types of items.
- We are in a monopoly situation, which means that there is no competition with other companies selling the same type of item. Note that, due to price discrimination, a company can be monopolistic in one segment of the population while other companies sell the same type of item with slight differences to other segments. This requires a sophisticated fencing strategy that prevents customers for moving to a cheaper segment.
- Customers are myopic, which means that they buy as soon as the price is less than the one they are prepared to pay. Strategic customers who optimize their purchasing behavior in response to the pricing strategy of the company are not considered in this chapter; game theory is used when strategic customers are concerned.

To summarize, this section provides an insight into mathematical pricing models. Note also that few convenient models exist without the assumptions presented above, that is to say monopoly situation, infinite number of potential customers who are myopic and no supply option. The reader will also observe that negative exponential functions are often used to make the model manageable and few persuasive arguments are proposed to justify this choice: it is why we consider that most of these models are more useful to understand dynamic pricing than to treat real-life situations.

More details and example of dynamic pricing are available in (Benait, 1987), (Bodily and Weatherford, 1995), (Cross, 1995, 1998), (Dolan and Jeuland 1981), (Feng and Gallego, 1995), (Gallego and Van Ryzin, 1997), (Kalish, 1983), (Kannan and Kopalle, 2001), (Kimes, 1989), (Leloup and Deveau, 2001), (Levin *et al.*, 2007), (McGill and Van Ryzin 1999), (Pasternack, 1985), (Petruzzi and Dada, 2002), (Zhao and Zheng, 2000) and (Dolgui and Proth, 2009, 2010).

### 7.2. A deterministic model for time-dated items

### 7.2.1 Problem setting

In this model, the initial inventory  $s_0$  is known: it is the maximal quantity that can be sold by time  $T$ . We assume that demands appear at times  $1, 2, \dots, T$ , and  $x_t$  represents the demand at time  $t$ . The demands are real and positive. The price of one item at time  $t$  is denoted by  $p_t$ , and this price is a function of the demand and the time:  $p_t = p(x_t, t)$ .

The following assumptions are made: (i) the number of potential customers is not limited, and as a consequence, the size of the population is not a parameter of the model, (ii) only one type of item is concerned, (iii) a monopoly situation is considered, and (iv) customers buy items as soon as the price is less than or equal to the price they are prepared to pay (myopic customers).

We assume that there exists a one-to-one relationship between demand and price at any time  $t \in \{1, 2, \dots, T\}$ . Thus,  $x_t = x(p_t, t)$  is the relation that provides the demand when the price is fixed.

We also assume that:

- $x(p, t)$  is continuously differentiable with regard to  $p$ .
- $x(p, t)$  is lower and upper bounded and tends to zero as  $p$  tends to its maximal value.

Finally, the problem can be expressed as follows:

$$\text{Maximize } \sum_{t=1}^T p_t x_t \quad (4)$$

subject to:

$$\sum_{t=1}^T x_t \leq s_0 \quad (5)$$

$$x_t \geq 0 \text{ for } t \in \{1, 2, \dots, T\} \quad (6)$$

$$x_t \leq x(p_t^{\min}, t) \text{ for } t \in \{1, 2, \dots, T\} \quad (7)$$

where  $p_t^{\min}$  is the minimal value of  $p_t$ .

Criterion (4) means that the objective is to maximize the total revenue. Constraint (5) guarantees that the total demand at horizon  $T$  does not exceed the initial inventory. Constraints (6) are introduced to make sure that demands are never less than zero. Finally, Constraints (7) provide the upper bound of the demand at any time.

### 7.2.2 Solving the problem: overall approach

To solve this problem, we use the Kuhn and Tucker approach based on Lagrange multipliers. Since  $p_t$  is a function of  $x_t$ , then  $p_t x_t$  is the function of  $x_t$ . Taking into account the constraints of the problem, the Lagrangian is:

$$\begin{aligned} L(x_1, \dots, x_T, \lambda, \mu_1, \dots, \mu_T, l_1, \dots, l_T) = & \sum_{t=1}^T p(x_t, t) x_t - \lambda \left( \sum_{t=1}^T x_t - s_0 \right) \\ & + \sum_{t=1}^T \mu_t x_t - \sum_{t=1}^T l_t (x_t - x(p_t^{\min}, t)) \end{aligned} \quad (8)$$

The goal is to solve the  $T$  equations:

$$\frac{\partial L}{\partial x_t} = 0 \text{ for } t \in \{1, 2, \dots, T\} \quad (9)$$

Together with the  $2T + 1$  complementary slackness conditions:

$$\lambda \left( \sum_{t=1}^T x_t - s_0 \right) = 0 \quad (10)$$

$$\left. \begin{array}{l} \mu_t x_t = 0 \\ l_t (x_t - x(p_t^{\min}, t)) = 0 \end{array} \right\} \text{ for } t \in \{1, 2, \dots, T\} \quad (11)$$

Thus, we have  $3T + 1$  equations for the  $3T + 1$  unknowns that are:

$$x_1, \dots, x_T, \lambda, \mu_1, \dots, \mu_T, l_1, \dots, l_T$$

A solution to the system of Equations (9), (10) and (11) is admissible if  $\lambda \geq 0$ ,  $\mu_t \geq 0$ ,  $l_t \geq 0$  and if Inequalities (5), (6) and (7) hold.

Note that, due to Relations (10) and (11):

$$\lambda = 0 \text{ and/ or } \sum_{t=1}^T x_t = s_0$$

$$\mu_t = 0 \text{ and/ or } x_t = 0 \text{ for } t \in \{1, 2, \dots, T\}$$

$$l_t = 0 \text{ and/or } x_t = x(p_t^{\min}, t) \text{ for } t \in \{1, 2, \dots, T\}$$

### 7.2.3. Remarks

Three remarks can be made concerning this model:

- The main difficulty consists in establishing the deterministic relationship between the demand and the price per item. In fact, establishing such a relationship is a nightmare. Several approaches are usually used to reach this objective. One of them is to carry out a survey among a large population, asking customers the price they are prepared to pay for one item. Let  $n$  be the size of the population and  $s_p$  the number of customers who are prepared to pay  $p$  or more for one item, then  $s_p / n$  is the proportion of customers who will buy at cost  $p$ . Then, evaluating at  $k$  the number of customers who demonstrate some interest for the item, we can consider that the demand is  $k \times s_p / n$  when the price is  $p$ . Another approach is to design a "virtual shop" on Internet and to play with potential customers to extract the same information as before. This is particularly efficient for products sold via Internet. Ebay and other auction sites can often provide this initial function for price and demand surveying.
- If demands are discrete, linear interpolation is usually enough to provide a near-optimal solution.
- In this model, we also assumed that the value of one item equal zero after time  $T$ . We express this situation saying that there is no salvage value.

### 7.2.4. A stochastic model

A stochastic model of dynamic pricing for time-dated products has been developed in (Gallego and Van Ryzin, 1994). It should be noticed that very restrictive conditions apply to this model:

- There is no salvage value, *i.e.* the value of an item equals zero at the horizon of the problem.
- Imperfect competition applies, which means that the vendor has a monopoly over the items. In the case of imperfect competition, customers respond to price.
- The model is a risk-neutral, which means that the objective is only to maximize the expected revenue at time  $T$ , without taking into account the risk of poor performance.
- The distribution function of the price is  $F(p) = 1 - e^{-\alpha p}$  where  $\alpha > 0$

## 8. Conclusion

We concentrated mostly on monopoly systems with myopic customers who do not "play" with providers, *i.e.*, who are not aware of providers' strategies. Note that this kind of market is the one for which most of the efficient models deal with, probably because the management of monopoly systems does not involve psychological components.

Several sections dealt with pricing strategies: high and low price strategies, price discrimination, discount, price skimming, penetration pricing and revenue management. Each one of these strategies targets a particular objective and applies to certain situations.



Significant part of this article took into account the relationship between the margin and the cost and quantity sold. The objective was to explain the mechanism that links these parameters.

Selling curve, that establishes the relationship between price and volume sold, was introduced. Since we were still in a monopoly environment, only provider and customers are concerned. In this section, several methods were listed, from the well-known cost-plus method (which is not recommended since it ignores customers' buying behavior) to market analysis (which, in some sense, assumes steady customers' behavior).

The conjoint measurement and market segmentation has been introduced. It can reveal the characteristics of a product (or service) that are of importance for some testers and which, in turn, may define market segments using *K*-mean analysis; finally, assigning a price per unit in each market segment can be done using one of the strategies mentioned above. Then, oligopoly and duopoly markets were considered, and thus the study was focused on competitors' reactions when a retailer decreases / increases price.

We then provide insight into the domain of pricing models. We limited ourselves to time dated items with no supply option in monopolistic environment with myopic customers. Although these assumptions simplify drastically the problem, many additional restrictive assumptions are required to obtain a mathematical model that is easy to analyze.

In our opinion, the pricing models are more or less tools to help better understand what dynamic pricing is than something used to solve real-life problems.

**Acknowledgements:** The authors thanks Chris Yukna for his help in English

## 9. References

- Anderberg, M.R., (1973). *Cluster Analysis for Applications*, Academic Press, New-York.
- Benoit, J.P., Krishna, V., (1987). Dynamic duopoly: Prices and quantities. *Review of Economic Studies* 54: 23 – 35.
- Belobaba, P.P., (1987). Airline yield management: An overview of seat inventory control. *Transportation Science* 21: 63 – 73.
- Belobaba, P.P., (1989). Application of a probabilistic decision model to airline seat inventory control. *Operations Research* 37: 183 – 197.
- Belobaba, P.P., Wilson, J.L., (1997). Impacts of yield management in competitive airline markets. *Journal of Air Transport Management* 3: 3 – 9.
- Bemmaor, A.C., Mouchoux, D., (1991). Measuring the short-term effect of in-store promotion and retail advertising on brand sales: A factorial experiment. *Journal of Marketing Research* 28: 202 – 214.
- Bernstein, F., Federgruen, A., (2003). Pricing and replenishment strategies in a distribution system with competing retailers. *Operations Research* 51: 409 – 426.
- Benzecri, J.-P., (1994). *Correspondence Analysis Handbook*. New-York: Dekker, P.
- Berry, S., (1994). Estimating discrete-choice models of product differentiation. *RAND Journal of Economics* 25: 242 – 262.
- Bitran, G.R., Mondschein, S.V., (1995). An application of yield management to the hotel industry considering multiple day stays. *Operations Research* 43: 427 – 443.
- Bitran, G.R., Gilbert, S.M., (1996). Managing hotel reservations with uncertain arrivals. *Operations Research* 44: 35 – 49.
- Blattberg, R.C., Neslin, S.A., (1990). *Sales Promotion: Concepts, Methods and Strategies*. Prentice-Hall, Englewood Cliffs, NJ.
- Bodily, S.E., Weatherford, L.R., (1995). Perishable-asset revenue management: Generic and multiple-price yield management with diversion. *Omega - The International Journal of Management Science* 23: 173 – 185.
- Bolton, R.N., (1989). The relationship between market characteristics and promotional price elasticities. *Marketing Science* 8: 153 – 169.

- Cattin, P., Wittink, D.R., (1982). Commercial use of conjoint analysis: A survey. *Journal of Marketing* 46: 44 – 53.
- Chen, F., Federgruen, A., Zheng, Y.S., (2001). Near-optimal pricing and replenishment strategies for a retail / distribution system. *Operations Research* 49(6): 839 – 853.
- Cross, R.G., (1995). An introduction to revenue management. In: Jenkins, D., (ed.), *The Handbook of Airline Economics*, The Aviation Weekly Group of the McGraw-Hill Companies, New York, NY.
- Cross, R.G., (1998). *Revenue Management: Hardcore Tactics for Market Domination*. Broadway Books, Bantam, Doubleday, Dell Publishing Group, New York, NY.
- Deville, J.-C., Saporta, G., (1983). Correspondence analysis with an extension towards nominal time series. *Journal of Econometrics*, 22, 169-189.
- Dolan, R.J., Jeuland, A.P., (1981). Experience curves and dynamic demand models: Implications for optimal pricing strategies. *Journal of Marketing* 45: 52 – 62.
- Dolgui, A., Proth, J.-M., (2009). Stochastic Dynamic Pricing Models of Monopoly Systems, *Proceedings of the 13th IFAC Symposium on Information Control Problems in Manufacturing*, Moscow, Russia, June 3-5, 2009, Dolgui, A., Nof, S.Y., Lototsky, V.A., (Eds.), Elsevier Science, 2009, IFAC-PapersOnline.net
- Dolgui, A., Proth, J.-M., (2010). *Supply Chain Engineering: Useful Methods and Techniques*, Springer, Berlin.
- Feng, Y., Gallego, G., (1995). Optimal starting times for end-of-season sales and optimal stopping times for promotional fares. *Management Science* 41: 1371 – 1391.
- Gallego, G., Van Ryzin, G., (1994). Optimal dynamic pricing of inventories with stochastic demand over finite horizons. *Management Science* 40: 999 – 1020.
- Gallego, G., Van Ryzin, G., (1997). A multi-product dynamic pricing problem and its applications to network yield management. *Operations Research* 45(1): 24 – 41.
- Greenacre, M.J., (1993). *Correspondence Analysis in Practice*. London : Academic Press.
- Greenleaf, E.A., (1995). The impact of reference price effects on the profitability of price promotions. *Marketing Science* 14: 82 – 104.
- Hand, D.J., (1981). *Discrimination and classification*. Wiley, London.
- Hartigan, (1975). *Clustering algorithms*, Wiley, New-York.
- Kalish, C., (1983). Monopolist pricing with dynamic demand and production costs. *Marketing Science* 2: 135 – 159.
- Kannan, P.K., Kopalle, P.K., (2001). Dynamic pricing on the Internet: Importance and implications for consumer behaviour. *International Journal of Electronic Commerce* 5(3): 63 – 83.
- Kimes, S.E., (1989). Yield management: A tool for capacity constrained service firms. *Journal of Operations Management* 8: 348 – 363.
- Leloup, B., Deveaux, L., (2001). Dynamic pricing on the Internet: Theory and simulations. *Journal of Electronic Commerce Research* 1(3): 265 – 276.
- Levin, Y., McGill, J., Nediak, M., (2007). Price guarantees in dynamic pricing and revenue management. *Operations Research* 55(1): 75 – 97.
- McGill, J.I., Van Ryzin, G.J., (1999). Revenue management: Research overview and prospects. *Transportation Science* 33(2): 233 – 256.
- Pasternack, B.A., (1985). Optimal pricing and return policies for perishable commodities. *Marketing Science* 4 (2): 166 – 176.
- Petruzzi, N.C., Dada, M., (2002). Dynamic pricing and inventory control with learning. *Naval Research Logistics* 49: 304 – 325.
- Tabachnik, B.G., (1989). *Using Multivariate Statistics*. New-York : HarperCollins.
- Talluri, K., Van Ryzin, G., (2004). Revenue management under a general discrete dedicated choice model of consumer behaviour. *Management Science* 50(1): 15 – 33

Zhao, W., Zheng, Y.-S., (2000). Optimal dynamic pricing for perishable assets with nonhomogeneous demand. *Management Science* 46(3): 375 – 388.

## Authors' Biographies

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Prof. **Jean-Marie Proth** is currently Consultant, Researcher and an Associate Editor of the *IEEE Transactions on Industrial Informatics*. He has been Research Director at INRIA (National Institute for Computer Science and Automation), leader of the SAGEP (Simulation, Analysis and Management of Production Systems) team and the Lorraine research centre of INRIA, Associate Member in the Laboratory of Mechanical Engineering, University of Maryland, University Professor in France, as well as at the European Institute for Advanced Studies in Management (Brussels). He has carried on close collaboration with several US universities. His main research focuses on operations research techniques, Petri nets and data analysis for production management, especially facility layout, scheduling and supply chains. He has authored or coauthored 15 books (text books and monographs) and more than 150 papers in major peer-reviewed international journals. He is the author or coauthor of about 300 papers for international conferences and 7 book chapters, the editor of 8 proceedings of international conferences, 55 times an invited speaker or invited professor throughout the world. J.-M. Proth was the supervisor of 28 PhD theses in France and the USA. He was also a coeditor of the journal *Applied Stochastic Models and Data Analysis*. He was an Associate Editor of *IEEE Transactions on Robotics and Automation* (1995–1998). He was guest editor of *Engineering Costs and Production Economics* and several other refereed international journals. He has been the Chairman of the Program Committee of various international conferences, an officer of several professional societies, for example: International Society for Productivity Enhancement, Vice-President of Flexible Automation, 1992–1995. For further information, see [proth.jean-marie.neuf.fr/](http://proth.jean-marie.neuf.fr/)