

Theorem 1. Let $X \sim (v_i, f_i)_{0 \leq i \leq n-1}$, where $v_i < v_{i+1}$ for all $0 \leq i < n-1$ are possible reservations and $f_i = \mathbb{P}(v_{i-1} < X \leq v_i)$ is the probability of an application to finish during reservation v_i (with $\sum_{i=0}^{n-1} f_i = 1$).

Denote by $F_j = \sum_{\ell=j}^{n-1} f_\ell$ and $\mathbb{E}[X] = \sum_{\ell=0}^{n-1} f_\ell v_\ell$. Then the minimal expected makespan is returned by $E(-1)$ where:

$$E(n-1) = \beta \cdot \mathbb{E}[X]$$

$$E(i) = \min_{i+1 \leq i' < n} \left((\alpha \cdot v_{i'} + \gamma) \cdot F_{i+1} + \beta \cdot v_{i'} \cdot F_{i'+1} + E(i') \right)$$

The sequence that minimizes $E(-1)$ can be computed in $O(n^2)$.

For a job of actual length t , the cost of a reservation t_1 is defined as: $\alpha t_1 + \beta \min(t, t_1) + \gamma$, where the α part of this cost is the reservation cost, and the β part is the utilization cost.