

**Theorem 1.** Let  $X \sim (v_i, f_i)_{0 \leq i \leq n-1}$ , where  $v_i < v_{i+1}$  for all  $0 \leq i < n-1$  are possible reservations and  $f_i = \mathbb{P}(v_{i-1} < X \leq v_i)$  is the probability of an application to finish during reservation  $v_i$  (with  $\sum_{i=0}^{n-1} f_i = 1$ ).

Denote by  $F_j = \sum_{\ell=j}^{n-1} f_\ell$  and  $\mathbb{E}[X] = \sum_{\ell=0}^{n-1} f_\ell v_\ell$ . Then the minimal expected makespan is returned by  $E(0)$  where:

$$E(n) = \beta \cdot \mathbb{E}[X]$$

$$E(i) = \min_{i \leq i' < n} \left( (\alpha \cdot v_{i'} + \gamma) \cdot F_i + \beta \cdot v_{i'} \cdot F_{i'+1} + E(i' + 1) \right)$$

The sequence that minimizes  $E(0)$  can be computed in  $O(n^2)$ .

For a job of actual length  $t$ , the cost of a reservation  $t_1$  is defined as:  $\alpha t_1 + \beta \min(t, t_1) + \gamma$ , where the  $\alpha$  part of this cost is the reservation cost, and the  $\beta$  part is the utilization cost.