Kinematic model of 1 DoF elbow joint with a pair of agonist-antagonist muscles

Python code can be found here:

https://github.com/anagamori/1dof_elbow_model.git

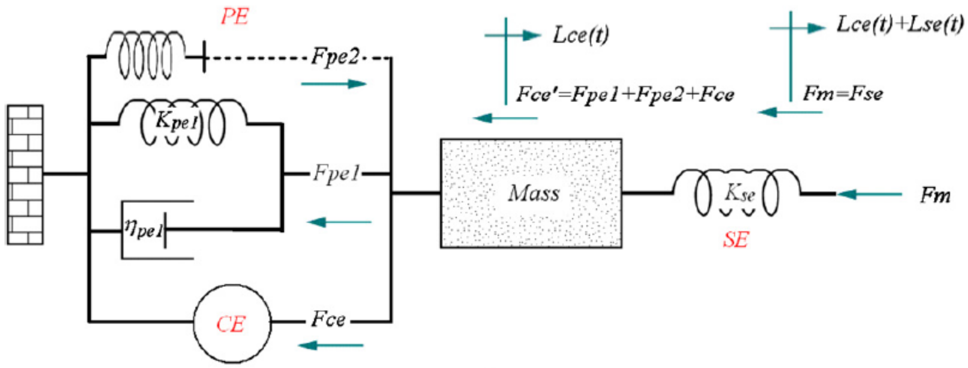

Figure 1: The mechanical structure of a muscle model without pennation angle.

Muscle Model

This model of a musculotendon unit (muscle + tendon + aponeurosis) consists of five elements: a mass, a contractile element, two passive elements and a series elastic element (Fig. 1) (Cheng et al., 2000; Song, Raphael, et al., 2008; Tsianos et al., 2012). The mass, M_m, presents that of a musculotendon unit. The contractile element generates muscle force, F_{ce}, arising from contractions of muscles fibers. This muscle force represents summed forces from individual motor unit forces, each of which contains a single motoneuron and a number of muscle fibers that the motoneuron innervates. The two passive elements, F_{pe1} and F_{pe2}, simulate parallel elasticity of muscle fascicles. The muscle contraction causes a change in the length of a series-elastic element. This length change in the series-elastic element generates force on a skeletal system through its attachment site (i.e. insertion) and cause joint rotation. All model parameters except for those specific to our model are given in Table 1 of Tsianos et al. (2012) (see parameters for slow-twitch units).

Contractile Element

The contractile element represents the force generating mechanisms of muscles (i.e. motoneurons and muscle fibers that they innervate), which converts the neural activation signal, U, into muscle force F_{ce}. The neural activation signal is a simplified representation of overall synaptic input that the entire motor unit population receives and the value is normalized between 0 and 1. For the version of the model presented here, muscle consists of a single fiber-type.

The neural activation signal, U, is converted into muscle activation, A, to account for dynamics due to calcium kinetics using the following two first-order dynamics equations:

$$\dot{A}_{int} = \frac{U - A_{int}}{T_U} \tag{1}$$

$$\dot{A} = \frac{A_{int} - A}{T_U} \tag{2}$$

$$T_U = \begin{cases} 0.0343, U \geq U_{eff} \\ 0.047, U < U_{eff} \end{cases}$$

The length-dependence of muscle force, known as the force-length relationship, arises from the interaction between the muscle length and the number of cross-bridges that can be formed (Tsianos & Loeb, 2017). This relationship is captured by scaling muscle force by FL^i, which can be described as follows:

$$\bar{FL} = exp(-\left|\frac{L_{ce}^{\beta} - 1}{\omega}\right|^{\rho}), \tag{3}$$

where the parameters, ω, β and ρ are fiber-type specific. In our model, we use the parameters for slow-twitch units described in Song, Raphael, et al., 2008.

Active muscle force is known to depend on the speed at which muscle contracts (Tsianos & Loeb, 2017). Compared to an isometric condition (muscle length = constant), muscle force decreases when the muscle is shortening while it increases when the muscle is lengthening. This force-velocity relationship can be described as follows:

$$\bar{FV} = \begin{cases} (V_{max} - V_{ce}/[V_{max} + (c_{v0} + c_{v1} * L_{ce})V_{ce}]), V_{ce} \leq 0 \\ [b_v - (a_{v0} + a_{v1}L_{ce} + a_{v2}L_{ce}^2)V_{ce}]/(b_v + V_{ce}), V_{ce} < 0 \end{cases} . \tag{4}$$

The values of the coefficients in the above equations depend on fiber types as in the force-length relationship (Song, Lan, et al., 2008; Tsianos et al., 2012). In our model, we use the parameters for slow-twitch units described in Song, Raphael, et al., 2008.

The normalized active force from the contractile element is then computed as follows:

$$F_{ce} = A \cdot (\bar{FL} \cdot \bar{FV} + F_{pe2}). \tag{5}$$

F_{pe2} is passive, resistive force against shortening (see its description below).

Passive Elements

The model includes two passive elements in parallel to the contractile element: passive element 1 and 2 generating force against muscle stretch (F_{pe1}) and against shortening (F_{pe2}), respectively. The force generated by passive element 1 can be described as follows:

$$\overline{F}_{pe1}(L_{ce}, V_{ce}) = c_1 k_1 ln \left[exp \left(\frac{L_{ce}/L_{ce}^{max} - L_{r1}}{k_1} \right) + 1 \right] + \eta V_{ce}. \tag{6}$$

The force generated by passive element can be described as follows:

$$\overline{F}_{pe2}(L_{ce}) = c_2 \{ exp[k_2(L_{ce} - L_{r2})] - 1 \}. \tag{7}$$

Series-Elastic Element

The force generated by a series-elastic element is described as follows:

$$\overline{F}_{se}(L_{se}) = c^T k^T ln \left[exp \left(\frac{L_{se} - L_r^T}{k_T} \right) + 1 \right]. \tag{8}$$

Contractile Dynamics

At an equilibrium, force generated by the muscle and tendon become equal (Fig. 1). Any deviation from this equilibrium due to changes in neural activation to the muscle or those inherent in the motor unit force generation mechanisms (i.e. unfused tetanic contractions) causes changes in the length of muscle and a series-elastic element (i.e. tendon + aponeurosis). This contraction dynamics between muscle and the series-elastic element can be described as follows (He et al., 1991):

$$A_{ce}(t) \cdot L_{ce0}(t) = \frac{1}{M_m}[F_0 \cdot \bar{F}_{se}(t) \cdot cos\alpha - (F_{ce}(t) + F_0 \cdot \bar{F}_{pe1}(t)) \cdot cos^2\alpha] + \frac{(V_{ce}(t) \cdot L_{ce0})^2 \cdot tan^2\alpha}{L_{ce}(t) \cdot L_{ce0}}, \tag{9}$$

with an constraint given by the following equation:

$$L_{mt}(t) = L_{ce}(t) \cdot L_{ce0} \cdot cos\alpha + L_{se}(t) \cdot L_{se0}, \tag{10}$$

where α, L_{ce}, V_{ce}, A_{ce}, L_{se} and L_{mt} are pennation angle (angle between muscle's line of action and that of a series-elastic element), muscle length normalized to optimal muscle length, normalized muscle velocity, normalized muscle acceleration, series-elastic element length normalized to its optimal length and musculotendon unit length in cm. F_0 determines maximum tetanic force of muscle as described below. The resulting muscle acceleration is then integrated to update the muscle length, which is used to compute the series-elastic element length from the relationship described in Eq. 10.

Muscle Mass

The relationship between the maximum force that muscle can generate (F_0) and its mass is described as follows (Lieber et al., 1990):

$$F_0 = \frac{M_m \cdot cos(\alpha) \cdot \epsilon}{\rho \cdot L_{ce0}}, \tag{11}$$

where M_m, ρ, ϵ and L_{ce0} are muscle mass, muscle density (1.06 g/cm_3), specific tension (31.8 N/cm_2) and optimal fascicle length, respectively. From this relationship, M_m is derived for a user-specified value of F_0.

One degree-of-freedom elbow joint

A schematic representation of a model of a 1 degree-of-freedom elbow joint is shown in Fig. 2. The model consists of two identical limb segments modeled as a cylindrical rod. The length and

Table 1: **Model parameters for muscle based on architectural parameters of *biceps long head*** (Song, Lan, et al., 2008)

Maximum muscle force, F_0 (N)	434
Optimal muscle length, L_{ce0} (cm)	21.5
Optimal tendon length, L_{se0} (cm)	16.3
Pennation angle, α (deg)	0

diameter of the segment are set to 45 cm and 5 cm, respectively. The mass, M, equals to 100 g. The proximal segment is attached to the ground, permitting no rotation. The distal segment rotates around an axis placed 5 cm inside of its one end. The counter-clockwise rotation (or elbow flexion) is defined as a position angle.

The joint rotation is controlled by two identical muscles defined above. The movement arm, r, of both muscles are also set identical to 3 cm but with opposite signs. The joint kinematics introduced by the activation of those muscles follows the Newton's second law such that

$$\ddot{\theta} = \frac{1}{I}(\tau_1 + \tau_2 - b \cdot \dot{\theta} - k \cdot \theta), \tag{12}$$

where τ_1 and τ_2 are torque generated by muscle 1 and 2, respectively, θ is the joint angle in radian and I is the inertia of the distal limb rotating around the axis defined in Fig. 2. b and k define the external viscosity and stiffness, both of which are set to 0. The resulting joint rotation causes changes in the musculotendon length (i.e. L_{mt}) of muscles, which are computed as follows:

$$\Delta L^i_{mt} = r^i \cdot \Delta\theta, \tag{13}$$

where the superscript, i, denotes muscle. This equation is used to update the musculotendon length in Eq. 10 every time step.

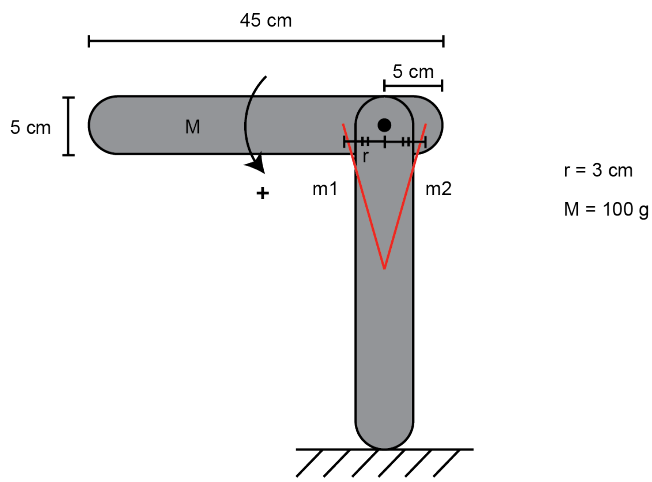

Figure 2: A schematic representation of a 1 DoF elbow joint.

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