INTRODUCTION TO AI AND ML

INDEPENDENT COMPONENT ANALYSIS

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INTRODUCTION

In signal processing,independent component analysis(ICA) is a computational method for separating multivariate signals into additive subcomponents.This is done by assuming the subcomponents are non-gaussian signals and are statistically independent from eachother.

A common example application is the cocktail party problem. Thus independent component analysis attempts to decompose a multivariate signal into independent nongaussian signals.

ICA separation of mixed signals

ICA separation of mixed signals gives very good results which is based on two assumptions.

Two assumptions:

(1)The source signals are independent of eachother.
(2)The values of each source signal have non-gaussian distribution.

Properties of mixed signals

There are three properties that the mixed signal statisfies:
Independence:
The source signals are independent but the mixture signals are not independent
Gaussianity:
By the Central limit theorem, the mixed signals are Gaussian.This property can be used for searching for the Non-Gaussian signals within the mixture signals to extract source or independent signals.
Complexity:
The mixed signals re more complex then source signals.

Definitions

Central Limit Theorem

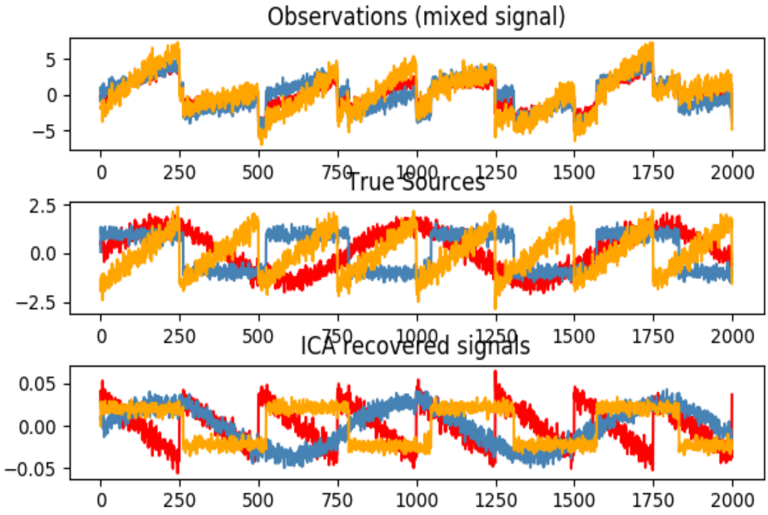
(1) Distribution of a sum of independent random variables tends toward a Gaussian distribution.
(2) Thus, a sum of two independent random variables usually has a distribution that is closer to gaussian than any of the two original random variables.

White signal

White signals are defned as any z which satisfies conditions of

E[z] = 0, E[zz^T] $= I$

Signal plot


Observations (mixed signal)
5
0
−5
0
250
500
750
1000
1250
1500
1750
2000
True Sources
2.5
0.0
−2.5
ICA recovered signals
0.05
0.00
−0.05

ICA separation of mixed signals

We can express the mixed signal as the linear combination of independent source signal.

Assume that we have n mixtures $x_1, x_2, x_3......, x_n$ *of n independent components*.

$x_j = a_{1j}s_1 + a_{2j}s_2 + a_{3j}s_3....... + a_{nj}s_n$ *for all j*

We assume that the mixed signals and the individual components are random variables instead of proper time signal.

Without loss of generality, we assume that the mean of mixture and the individual components is zero.

ICA separation of mixed signals

$$\mathbf{X = AS}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix}, S = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ . \\ . \\ . \\ s_n \end{bmatrix}, A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & . & a_{2n} \\ . & & & & & & \\ . & & & & & & \\ . & & & & & & \\ a_{n1} & a_{n2} & a_{n3} & . & . & . & a_{nn} \end{bmatrix}$$

Where A is the mixing matrix of elements a_{ij}.

ICA separation of mixed signals

The main task is to estimate the independent components of S. Let the approximation of S be Y.To estimate the independent components we consider the linear combination of x_i.

$$y = w^T x \tag{1}$$

where w is the vector to be determined.w is one of the rows of inverse of A.W is the column matrix containing w vectors. In practice, we cannot determine such a W which gives the original independent signal because, we have no knowledge of matrix A.

ICA Estimation

Let's assume z to be a transformation of w,defining

$$z = A^T w. \tag{2}$$

From equation (1),we have

$$y = w^T x = w^T As = z^T s \tag{3}$$

Thus, y is the linear combination of s_i.
From the central limit theorem, $Z^T s$ is more gaussian than any of the s_i.

Maximization of Kurtosis

Kurtosis is the measure of non-gaussianity.Kurtosis is defined by

$$kurt(y) = E[y^4] - 3E[y^2]^2 \tag{4}$$

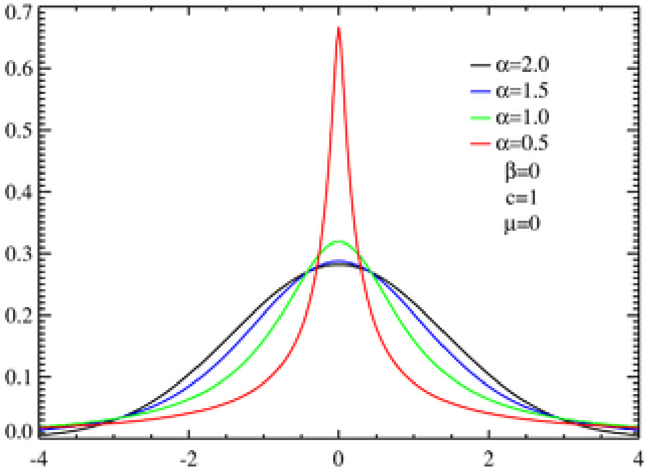
Substituting the value of y from the above equation..

$|\text{kurt}(z^T s)| = |E(z^T s)^4 - 3E[(z^T s)^2]^2|$

For the maximum value of kurtosis, We can differentiate the above equation with z and set it to zero.

We know that, s is white signal $(i, e., E[s] = 0, E[s^2] = 1)$
Now, we can get the independent signals from the mixed signals.

Kurtosis graph



Example of ICA
Cocktail party problem

The cocktail party effect is the phenomenon of the brains ability to focus one's auditory attention or the sound of interest while filtering out the other sound disturbances.

We can record the mixed signals and then separate the desired independent sound signal using ICA.