# MATRIX PROJECT

EE1390

INTRODUCTION TO AI AND ML

# QUESTION - 20

**(IN GEOMETRIC FORM)**

Let K be an integer such that the triangle with vertices A = (k,-3k) ; B = (5,k) ; C = (-k,2) has area 28.Find the orthocentre of this triangle.

# QUESTION-20

**(IN MATRIX FORM)** Let k be an integer such that the triangle with vertices

$$A = \begin{bmatrix} k \\ -3k \end{bmatrix}, B = \begin{bmatrix} 5 \\ k \end{bmatrix}, C = \begin{bmatrix} -k \\ 2 \end{bmatrix}$$

has area 28.Find the orthocentre of the triangle.

# SOLUTION:

Vertices :

$$A = \begin{bmatrix} k \\ -3k \end{bmatrix}, B = \begin{bmatrix} 5 \\ k \end{bmatrix}, C = \begin{bmatrix} -k \\ 2 \end{bmatrix}$$

area = 28

Area of the triangle in matrix form:

$$\begin{vmatrix} k & 5 & -k \\ -3k & k & 2 \\ 1 & 1 & 1 \end{vmatrix} = 56$$

$$|\ k(k-2) - 5(-3k-2) - k(-3k-k)\ | = 56$$

CASE 1:

$$k(k-2)-5(-3k-2)-k(-3k-k)=56$$

$$k^2-2k+15k+10+3k^2+k^2=56$$

$$5k^2+13k-46=0$$

$$k=2, k=\frac{-23}{5}$$

CASE 2:

$$k(k-2) - 5(-3k-2) - k(-3k-k) = -56$$

$$k^2 - 2k + 15k + 10 + 3k^2 + k^2 = -56$$

$$5k^2 + 13k + 66 = 0$$

$$k\ is\ complex\ in\ this\ case.$$

Since k is given to be an integer,

$$k = 2$$

On substituting the value of k in the given coordinates

$$\text{A} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \text{B} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \text{C} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

# TO FIND THE ORTHOCENTER

The point of intersection of altitudes is the orthocenter of the triangle.

Let D be the foot of perpendicular from A to BC,

The equation of AD is

$$(B-C)^T(X-A)=0$$
$$(B-C)^T(X)=(B-C)^T(A) \tag{1}$$

Let E be the foot of perpendicular from B to AC,

The equation of BE is

$$(C-A)^T(X-B)=0$$
$$(C-A)^T(X)=(C-A)^T(B) \tag{2}$$

The point of intersection of AD and BE(altitudes) is the orthocentre H.

$$\begin{bmatrix} B-C & C-A \end{bmatrix}^T X = \begin{bmatrix} (B-C)^T A \\ (C-A)^T B \end{bmatrix} \tag{3}$$

$$(B-C) = \begin{bmatrix} 5 \\ 2 \end{bmatrix} - \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$$

$$(B-C)^T A = \begin{bmatrix} 7 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \end{bmatrix} = 14$$

$$(C-A) = \begin{bmatrix} -2 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ -6 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \end{bmatrix}$$

$$(C-A)^T B = \begin{bmatrix} -4 & 8 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = -4$$

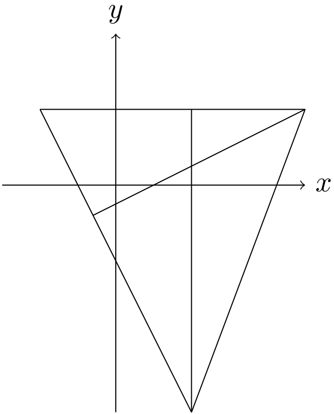
Substituting the above values in equation (3),

$$\begin{bmatrix} 7 & -4 \\ 0 & 8 \end{bmatrix}^T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ -4 \end{bmatrix}$$

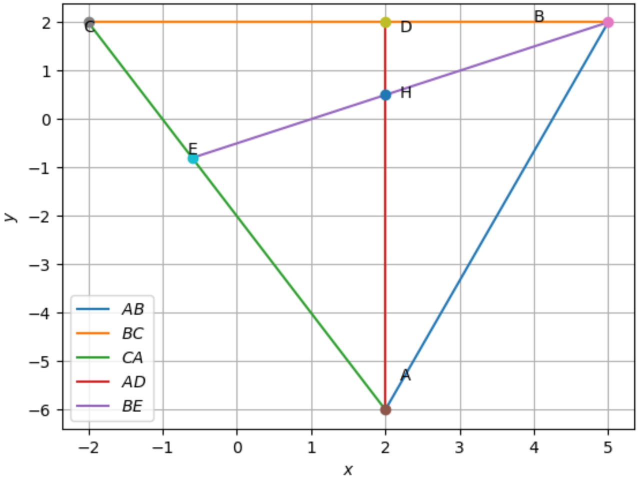
$$\begin{bmatrix} 7 & 0 \\ -4 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ -4 & 8 \end{bmatrix}^{-1} \begin{bmatrix} 14 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{8}{56} & 0 \\ \frac{4}{56} & \frac{7}{56} \end{bmatrix} \begin{bmatrix} 14 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{112}{56} \\ \frac{28}{56} \end{bmatrix} = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$$




Therefore,the orthocenter of the triangle is (2,0.5)



C
D
B
H
E
A
AB
BC
CA
AD
BE
2
1
0
−1
−2
−3
−4
−5
−6
y
−2
−1
0
1
2
3
4
5
x

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