Problem #1

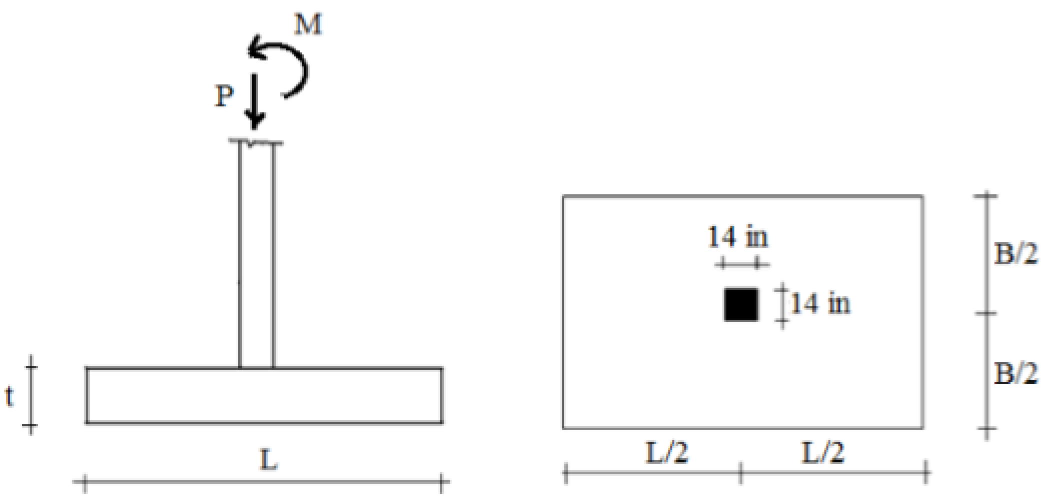
A square/rectangular footing supporting a square column is shown below.

(a) Determine the footing dimensions L and B. Use 3 inch increments
(b) Determine the required t. Use 2 inch increments.
(c) Determine the required reinforcing. Use #8 bars. Show the reinforcing arrangements.

Assume:

$f'_c = 4000$ psi $f_y = 60{,}000$ psi , $q_e = 4.5$ kip/ft^2 .

$P_D = 140$ kip , $P_L = 160$ kip , $M_D = 60$ kip ft , $M_L = 80$ kip ft



Concrete Properties:

fcp := 4000 psi   fy := 60000 psi   yc := .15 kcf

$\alpha := 40$

$$\beta 1 := \begin{cases} .85 & \text{if } fcp \le 4000 \\ .65 & \text{if } fcp \ge 8000 \\ \left[.85 - (fcp - 4000)\cdot\frac{0.05}{1000}\right] & \text{otherwise} \end{cases}$$

$\beta 1 = 0.85$

Soil Properties:

qe := 4.5 ksf   ys := .12 kcf

Assumptions:

Footing Thickness: t := 20 in

Soil depth: Df := 3 ft

Loading Conditions:

Pd := 140 kip   Pl := 160 kip   Md := 60 kip·ft   Ml := 80 kip·ft

P := Pd + Pl = 300 kip   M := Md + Ml = 140 kip·ft   ft

Factored Loads:

Pu1 := 1.4·Pd kip   Pu2 := 1.2·Pd + 1.6·Pl = 42 kip   Pu := max(Pu1, Pu2) = 424 kip

Mu1 := 1.4·Md = 84 kip·ft   Mu2 := 1.2·Md + 1.6·Ml = 20(kip·ft   Mu := max(Mu1, Mu2) = 200 kip·ft

Footing Dimensions:

a1 := 14 in   a2 := 14 in

Factors:

ϕt := .9 ϕv := .75

FOOTING DESIGN:

1. Determining L, B, t, As and Id:

Trial t: $t = 20 \quad in$

$hs := Df \cdot 12 - t = 16 \quad in$

Assuming: $B := 10 \quad ft$ $\quad L := 10$ $\quad A := B \cdot L = 100$

$qe = 4.5 \quad ksf$

$$q1 := \frac{P}{B \cdot L} + 6 \cdot \frac{M}{B \cdot L^2} = 3.84 \qquad q2 := \frac{P}{B \cdot L} + -6 \cdot \frac{M}{B \cdot L^2} = 2.16$$

Satisfy Equation

$$qu := \frac{Pu}{A} = 4.24 \qquad qum := \frac{Mu}{A \cdot B^2} = 0.02$$

Area satisfy for area and moment;

Critical Eccentricity: if e < ec, OK. $e := \frac{M}{P} = 0.467 \quad < \quad ec := \frac{L}{6} = 1.667 \quad$ ok

Footing Thickness:

Reinforcement:

Using #8 bars: $dbar8 := 1 \quad As8 := 0.79 \qquad cover := 3 + 1.5 \cdot dbar8 \qquad d := t - cover = 15.5$

$$b0 := 2 \cdot (a1 + d) + 2 \cdot (a2 + d) = 118$$

$$\beta c := \begin{cases} \frac{a1}{a2} & \text{if } a2 \le a1 \\ \left(\frac{a2}{a1}\right) & \text{otherwise} \end{cases} \qquad \beta c = 1$$

$$Vu1 := qu \cdot B \cdot \left[\left(\frac{L}{2}\right) - \left(\frac{a1}{2 \cdot 12}\right) - \left(\frac{d}{12}\right)\right] = 132.5 \qquad Vu2 := qu \cdot \left[B \cdot L - (a1 + d) \cdot \frac{(a2 + d)}{12 \cdot 12}\right] = 398.376$$

$$d1req := Vu1 \cdot \frac{1000}{\phi v \cdot 2 \cdot \sqrt{fcp} \cdot B \cdot 12} = 11.639 \qquad d2a := Vu2 \cdot \frac{1000}{\phi v \cdot 4 \cdot \sqrt{fcp} \cdot b0} = 17.793$$

$$d2b := Vu2 \cdot \frac{1000}{\phi v \cdot \left(2 + \frac{4}{\beta c}\right) \cdot \sqrt{fcp} \cdot b0} = 11.862 \qquad d2c := Vu2 \cdot \frac{1000}{\phi v \cdot \left(2 + \frac{\alpha \cdot d}{b0}\right) \cdot \sqrt{fcp} \cdot b0} = 9.811$$

$$d2reqd := \max(d2a, d2b, d2c) = 17.793 \qquad dreqd := \max(d2reqd, d1req) = 17.793$$

Compare Required d1 and d2 with assumed d: $d = 15.5$ **Value Assumed is OK**

Reinforcing in long direction

$$MuL := qu\cdot\left[\left(\frac{L}{2}\right) - a1\cdot\frac{1}{24}\right]^2\cdot\frac{B}{2} = 413.547$$

$$RnreqL := MuL\cdot\frac{12000}{\phi t\cdot B\cdot 12\cdot d^2} = 191.258$$

$$\rho reqL := .85\cdot\frac{fcp\cdot\left(1 - \sqrt{1 - \frac{2\cdot RnreqL}{.85\cdot fcp}}\right)}{fy} = 3.283\times 10^{-3}$$

$$\rho max := .85\cdot 0.428\cdot\beta 1\cdot\frac{fcp}{fy} = 0.021$$

$$\rho min1 := 3\cdot\frac{\sqrt{fcp}}{fy} = 3.162\times 10^{-3} \qquad \rho min2 := \frac{200}{fy} = 3.333\times 10^{-3} \qquad \rho min := \max(\rho min1, \rho min2) = 3.333\times 10^{-3}$$

Area of Steel Required:

$$AsreqL := \begin{cases} \rho reqL\cdot B\cdot 12\cdot d & \text{if } \rho reqL \ge \rho min \\ (\rho min\cdot B\cdot 12\cdot d) & \text{otherwise} \end{cases}$$

Using #8 bars: $AsreqL = 6.2$

$$\frac{AsreqL}{As8} = 7.848 \qquad Nbars := 8 \qquad As := Nbars\cdot As8 = 6.32 \qquad \rho := \left(\frac{As}{B\cdot 12\cdot d}\right) = 3.398\times 10^{-3}$$

$$\rho L := \begin{cases} \rho & \text{if } \rho max \ge \rho \\ \text{"Re-design"} & \text{otherwise} \end{cases} \qquad \rho L = 3.398\times 10^{-3}$$

$$a := As\cdot\frac{fy}{0.85\cdot fcp\cdot B\cdot 12} = 0.929 \qquad c := \frac{a}{\beta 1} = 1.093 \qquad \varepsilon t := (d - c)\cdot\frac{0.003}{c} = 0.04$$

$$\phi t := \begin{cases} .65 & \text{if } \varepsilon t \le 0.002 \\ .9 & \text{if } \varepsilon t \ge 0.005 \\ \left[.65 + (\varepsilon t - 0.002)\cdot\frac{250}{3}\right] & \text{otherwise} \end{cases} \qquad \phi t = 0.9$$

$$\phi MnL := \phi t\cdot As\cdot fy\cdot\frac{\left(d - \frac{a}{2}\right)}{12000} = 427.604 \qquad MuL = 413.547$$

$$BL := \begin{cases} \text{"OK"} & \text{if } \phi MnL \ge MuL \\ \text{"Re-design"} & \text{otherwise} \end{cases} \qquad BL = \text{"OK"}$$

Spacing Between Bars:

$$\text{spacing} := \frac{\left[B\cdot 12 - 2\cdot\left(3 + \frac{\text{dbar8}}{2}\right)\right]}{\text{Nbars} - 1} = 16.143\ \text{in}$$

Spacing to be used:

$$\text{spacingL} := 16\ \ \text{in}$$

Development Length

$$\text{Cb1} := \frac{\text{spacing}}{2} = 8.071 \qquad \text{Cb2} := \left(3 + \frac{\text{dbar8}}{2}\right) = 3.5 \qquad \text{Cb} := \min(\text{Cb1}, \text{Cb2}) = 3.5\ \text{in}$$

ACI Code factors, for bottom reinforcing, non coated bars, for no. 7 and larger bars and normal weight concrete, respectively:

$$\psi t := 1 \qquad \psi e := 1 \qquad \psi s := 1 \qquad \lambda := 1 \qquad \text{Ktr} := 0$$

$$\text{coef} := \begin{cases} 2.5 & \text{if } \frac{(\text{Ktr} + \text{Cb})}{\text{dbar8}} \ge 2.5 \\ \frac{(\text{Ktr} + \text{Cb})}{\text{dbar8}} & \text{otherwise} \end{cases} \qquad \text{coef} = 2.5$$

$$\text{ld} := \frac{3\cdot\left(\frac{\text{fy}}{\sqrt{\text{fcp}}}\right)\cdot\left(\psi t\cdot\psi e\cdot\psi s\cdot\frac{\lambda}{\text{coef}}\right)\cdot\text{dbar8}}{40} = 28.46 \qquad \frac{\text{ld}}{12} = 2.372\ \ \text{ft}$$

Reinforcing in short direction

$$\text{Mus} := \text{qu}\cdot\left[\left(\frac{B}{2}\right) - \text{a2}\cdot\frac{1}{24}\right]^2\cdot\frac{L}{2} = 413.547\ \ \text{kip}\cdot\text{ft}$$

$$\text{Rnreqs} := \text{Mus}\cdot\frac{12000}{\phi t\cdot L\cdot 12\cdot d^2} = 191.258$$

$$\rho\text{reqs} := .85\cdot\frac{\text{fcp}\cdot\left(1 - \sqrt{1 - \frac{2\cdot\text{Rnreqs}}{.85\cdot\text{fcp}}}\right)}{\text{fy}} = 3.283\times 10^{-3}$$

Area of Steel Required:

$$\text{Asreqs} := \begin{cases} \rho\text{reqs}\cdot B\cdot 12\cdot d & \text{if } \rho\text{reqs} \ge \rho\text{min} \\ (\rho\text{min}\cdot B\cdot 12\cdot d) & \text{otherwise} \end{cases} \qquad \text{Asreqs} = 6.2$$

Using #8 bars:

$$\frac{\text{Asreqs}}{\text{As8}} = 7.848 \qquad \text{Nsbars} := 8 \qquad \text{AsUs} := \text{Nsbars}\cdot\text{As8} = 6.32 \qquad \rho\text{B1} := \left(\frac{\text{AsUs}}{B\cdot 12\cdot d}\right) = 3.398\times 10^{-3}$$

$$\rho B := \begin{cases} \rho B1 & \text{if } \rho B1 \ge \rho reqs \\ \text{"Re-design"} & \text{otherwise} \end{cases} \qquad \rho B = 3.398 \times 10^{-3}$$

$$\beta := \frac{L}{B} = 1 \qquad \gamma s := \frac{2}{(\beta + 1)} = 1 \qquad Ff := \gamma s \cdot Asreqs = 6.2$$

Number of Bars placed within the Bandwidth area: $NBb := \frac{Ff}{As8} = 7.848$ $\quad Nbb := 8$

Bars Spacing

Spacing to be used with amount of Reinf:

$$B \cdot \frac{12}{(Nsbars - 1)} = 17.143 \qquad L \cdot \frac{12}{(Nsbars - 1)} = 17.143 \qquad B \cdot \frac{12}{(NBb - 1)} = 17.523 \qquad spS := 17.5 \quad in$$

Additional Reinforcement configuration dependent on Bandwidth Area:

Additional Bars needed: $AB := \frac{(Asreqs - \gamma s \cdot Asreqs)}{2} = 0$

Outer Length to be reinforced: $OL1 := (L - B) \cdot \frac{12}{2} = 0$ $\quad OL2 := 0.018 \cdot t \cdot (L - B) \cdot \frac{12}{2} = 0$

$$OL3 := \left[(L - B) \cdot \frac{12}{2}\right] - 3.5 = -3.5 \qquad OL := \min(OL1, OL2, OL3) = -3.5$$

spacing: $sS := \frac{OL}{AB} = \blacksquare$

Strength Check:

$$a := As \cdot \frac{fy}{0.85 \cdot fcp \cdot B \cdot 12} = 0.929 \qquad c := \frac{a}{\beta 1} = 1.093 \qquad \varepsilon t := (d - c) \cdot \frac{0.003}{c} = 0.04$$

$$\phi t := \begin{cases} .65 & \text{if } \varepsilon t \le 0.002 \\ .9 & \text{if } \varepsilon t \ge 0.005 \\ \left[.65 + (\varepsilon t - 0.002) \cdot \frac{250}{3}\right] & \text{otherwise} \end{cases} \qquad \phi t = 0.9$$

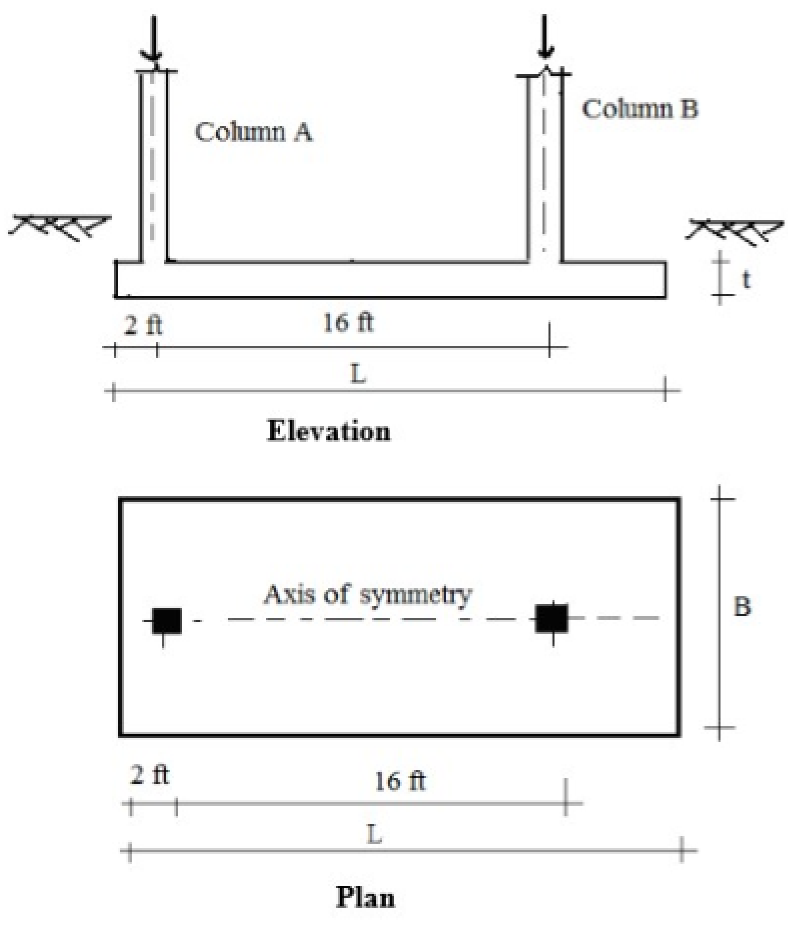
$$\phi MnS := \phi t \cdot As \cdot fy \cdot \frac{\left(d - \frac{a}{2}\right)}{12000} = 427.604 \qquad MuS := 291.176$$

$$BS := \begin{cases} \text{"OK"} & \text{if } \phi MnS \ge MuS \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

$$BS = \text{"OK"}$$

Problem #2

A combined footing supports two square columns. Column A is 14 inches x 14 inches and carries a dead load of 140 kip and a live load of 220 kip. Column B is 16 inches x 16 inches and carries a dead load of 260 kip and a live load of 300 kip. The effective soil pressure is $q_e = 4.0 \text{ kip/ft}^2$. Assume the soil pressure distribution is uniform.



(a) Determine the footing dimensions L and B. Use 3 inch increments.

(b) Establish the shear and moment diagrams corresponding to the factored loading, $P_u = 1.2\ P_D + 1.6\ P_L$.

(c) Determine the required t. Use 2 inch increments.

(d) Determine the required reinforcing. Use #9 bars. Show the reinforcing arrangements.

Assume: $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Concrete Properties:

$fcp := 4000$ psi $\quad fy := 60000$ psi $\quad \gamma c := .15$ kcf

$\alpha := 40$

$$\beta 1 := \begin{cases} .85 & \text{if } fcp \le 4000 \\ .65 & \text{if } fcp \ge 8000 \\ \left[.85 - (fcp - 4000)\cdot\dfrac{0.05}{1000}\right] & \text{otherwise} \end{cases}$$

$\beta 1 = 0.85$

Soil Properties:

$qe := 4.0$ ksf $\quad \gamma s := .12$ kcf

Assumptions:

Footing Thickness: $t := 20$ in

Soil depth: $Df := 3$ ft

Factors:

$\phi t := .9 \quad \phi v := .75$

Footing Dimensions:

$a1 := 14$ in $\quad a2 := 14$ in

$b1 := 16$ in $\quad b2 := 16$ in

edge distance: $e1 := 2$ ft

distance between columns: $d := 16$ ft

Loading Conditions:

<u>Column A</u>

$Pd1 := 140$ kip $\quad Pl1 := 220$ kip

$P1 := Pd1 + Pl1 = 360$ kip

$Md1 := 0$ kip·ft $\quad Ml1 := 0$ kip·ft

$M1 := Md1 + Ml1 = 0$ kip·ft

<u>Column B</u>

$Pd2 := 260$ kip $\quad Pl2 := 300$ kip

$P2 := Pd2 + Pl2 = 560$ kip

$Md2 := 0$ kip·ft $\quad Ml2 := 0$ kip·ft

$M2 := Md2 + Ml2 = 0$ kip·ft

Factored Loads:

$Pu1 := 1.2\cdot Pd1 + 1.6\cdot Pl1 = 520$ kip

$Mu1 := 1.2\cdot Md1 + 1.6\cdot Ml1 = 0$ kip·ft

$Pu2 := 1.2\cdot Pd2 + 1.6\cdot Pl2 = 792$ kip

$Mu2 := 1.2\cdot Md2 + 1.6\cdot M2 = 0$ kip·ft

Combined Footing Resultant:

$Rc := P1 + P2 = 920$ kip

$Mc := M1 + M2 = 0$

$Rcu := Pu1 + Pu2 = 1.312 \times 10^3$ kip

Distance force application to resultant:

$x1 := \dfrac{P1\cdot d}{Rc} = 6.261$ ft $\quad x2 := \dfrac{P2\cdot d}{Rc} = 9.739$ ft

$x1 + x2 = 16$

$Mcu := Mu1 + Mu2 = 0$

For the design of this footing, lets set Combined Area centroid to the location where resultant is applied, i.e, x1.

FOOTING DESIGN:

Trial Area:

$TA := \frac{Rc}{qe} = 230 \quad ft^2$

Centroid of the area should be in x, at cx:

$cx := x1 + e1 = 8.261$ $\quad L := L1 + L2$

L is larger than 18, thus use trial L $\quad Lt := 22\ ft \quad L := Lt \quad e2 := L - e1 - d = 4\ \ ft$

$Bt := \frac{TA}{Lt} = 10.455$ Use $Bt := 16 \quad ft \quad B := Bt \quad A := B \cdot L = 352 \quad ft^2$ area·checks

$qu := \frac{Rcu}{A} = 3.727 \ \ ksf \quad wu := \frac{Rcu}{L} = 59.636 \quad klf \quad V := A \cdot t = 7.04 \times 10^3$

1. Determining L, B, t, As and ld:

$qe = 4 \quad ksf$

$q1 := \frac{Rc}{B \cdot L} + 6 \cdot \frac{Mc}{B \cdot L^2} = 2.614$ Satisfy Equation

Critical Eccentricity: if e < ec, OK. $\quad e := \frac{Mc}{Rc} = 0 \quad < \quad ec := \frac{L}{6} = 3.667 \quad$ ok

$Eccentricity := \begin{cases} \text{"OK"} & \text{if } ec \geq e \\ \text{"Re-design"} & \text{otherwise} \end{cases}$

Eccentricity = "OK"

3. Determining the size of each individual length L1 and L2: $\quad B2 := B = 16\ \ ft \quad B1 := 14 \quad ft$

$d2 := d - x1 = 9.739 \quad d1 := x1$

$L1 := \frac{(B2 \cdot L \cdot d2)}{(B1 \cdot d1 + d2 \cdot B2)} = 14.08 \quad L2 := L - L1 = 7.92$

$A1 := L1 \cdot B1 = 197.12 \quad ft^2 \quad A2 := L2 \cdot B2 = 126.72 \quad ft^2 \quad AT := A1 + A2 = 323.84\ ft^2$

$AreaCheck := \begin{cases} \text{"OK"} & \text{if } AT \geq TA \\ \text{"Re-design"} & \text{otherwise} \end{cases}$ $\quad quT := \frac{Rcu}{AT} = 4.051 \quad$ ultimate·bearingcheck

Eccentricity = "OK"

Footings Dimensions: $L1 = 14.08 \quad L2 = 7.92$

$B1 = 14 \quad B2 = 16$

Moment of Inertia with Respect to Individual Area Centroid:

$$Iy1 := B1 \cdot \frac{L1^3}{12} = 3.257 \times 10^3 \quad ft^4 \qquad Iy2 := B2 \cdot \frac{L2^3}{12} = 662.391 \ ft^4$$

Parallel Axis Theorem: $Iyp1 := Iy1 + A1 \cdot d1^2 = 1.098 \times 10^4 \quad ft^4 \qquad Iyp2 := Iy2 + A2 \cdot d2^2 = 1.268 \times 10^4 \quad ft^4$

Combined Footing Information: $IT := Iyp1 + Iyp2 = 2.367 \times 10^4 \quad ft^4$

Create different vectors for each position x, dx:

$$x := 1$$

$$qx := \frac{Rc}{AT} + Mc \cdot \frac{x}{IT} = 2.841 \qquad qux := \frac{Rcu}{AT} + Mcu \cdot \frac{x}{IT} = 4.051$$

2. Shear and Moment Diagrams for Factored Load Ru: For values up to x, xc:

$$Vax := \int_0^x qux \cdot B \, dx = 64.822 \qquad Vux1 := qux \cdot B \cdot x = 64.822 \quad kip$$

$$Va1 := qux \cdot B \cdot e1 = 129.644 \quad kip \qquad Vux2 := qux \cdot B \cdot x - Pu1 = -455.178 \quad kip$$

$$Mux1 := Vux1 \cdot x = 64.822 \quad kip \cdot ft \qquad Mux2 := \left[qux \cdot B \cdot \left(\frac{x}{2} \right)^2 \right] - Pu1 \cdot x + Mux1 = -438.972 \ kip \cdot ft$$

Exactly where the load A is located: $Va2 := qux \cdot B \cdot \left(e1 + \frac{a1}{12} \right) - Pu1 = -314.73$

V @ xv = 0

$$Vb2 := qux \cdot B \cdot e2 = 259.289 \ kip$$

$$Vb1 := -qux \cdot B \cdot \left(e2 + \frac{b1}{12} \right) + Pu2 = 446.282 \quad kip$$

$$x := e2$$

$$Vbx := \int_x^0 qux \cdot B \, dx = -259.289 \qquad Vb1 := qux \cdot B \cdot e2 = 259.289 \quad kip$$

$$Vbux1 := -qux \cdot B \cdot x = -259.289 \ kip \qquad Vbux2 := -qux \cdot B \cdot x + Pu2 = 532.711 \ kip$$

$$Mubx1 := Vbux1 \cdot x = -1.037 \times 10^3 \ kip \cdot ft \qquad Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2} \right)^2 + Pu2 \cdot x + Mubx1 = 1.872 \times 10^3 \ kip \cdot ft$$

at Shear V=0, Mumax, in xv:

$$Vu := B \cdot qux \cdot xv - Pu1 = 0 \qquad xv := \frac{Pu1}{B \cdot qux} = 8.02\ ft$$

Using Equation Vux1 and Mux1 @ points of shear inflection: $x := xv = 8.022 \quad ft$

$$Vux1 := qux \cdot B \cdot x = 520 \qquad Mux1 := Vux1 \cdot x = 4.171 \times 10^{3}\ kip \cdot ft$$

$$Vux2 := qux \cdot B \cdot x - Pu1 \qquad Mux2 := \left[qu \cdot B \cdot \left(\frac{x}{2}\right)^2\right] - Pu1 \cdot x + Mux1 = 959.425 \quad kip \cdot ft$$

$$x := e1$$

$$Vux1 := qux \cdot B \cdot x = 129.644 \qquad Mux1 := Vux1 \cdot x = 259.289 \quad kip \cdot ft$$

$$Vux2 := qux \cdot B \cdot x - Pu1 = -390.356 \qquad Mux2 := \left[qu \cdot B \cdot \left(\frac{x}{2}\right)^2\right] - Pu1 \cdot x + Mux1 = -721.075 \quad kip \cdot ft$$

$$x := e1 + \frac{a1}{12}$$

$$Vux1 := qux \cdot B \cdot x = 205.27 \qquad Mux1 := Vux1 \cdot x = 650.022 \quad kip \cdot ft$$

$$Vux2 := qux \cdot B \cdot x - Pu1 = -314.73 \qquad Mux2 := \left[qu \cdot B \cdot \left(\frac{x}{2}\right)^2\right] - Pu1 \cdot x + Mux1 = -847.14 \quad kip \cdot ft$$

$$x := cx$$

$$Vux1 := qux \cdot B \cdot x = 535.487 \qquad Mux1 := Vux1 \cdot x = 4.424 \times 10^{3}\ kip \cdot ft$$

$$Vux2 := qux \cdot B \cdot x - Pu1 = 15.487 \qquad Mux2 := \left[qu \cdot B \cdot \left(\frac{x}{2}\right)^2\right] - Pu1 \cdot x + Mux1 = 1.145 \times 10^{3} \quad kip \cdot ft$$

$$x := xv = 8.022 \quad ft$$

$$Vbx := \int_{x}^{0} qux \cdot B\, dx = -520 \qquad Vb1 := qux \cdot B \cdot e2 = 259.289 \quad kip$$

$$Vbux1 := -qux \cdot B \cdot x = -520 \quad kip \qquad Vbux2 := -qux \cdot B \cdot x + Pu2 = 272 \quad kip$$

$$Mubx1 := Vbux1 \cdot x = -4.171 \times 10^{3}\ kip \cdot ft \qquad Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = 1.139 \times 1\ kip \cdot ft$$

$$x := cx$$

$$Vbx := \int_{x}^{0} qux \cdot B\, dx = -535.487$$

$$Vbux1 := -qux \cdot B \cdot x = -535.487\ kip \qquad Vbux2 := -qux \cdot B \cdot x + Pu2 = 256.51\ kip$$

$Mubx1 := Vbux1 \cdot x = -4.424 \times 10^{3}\ kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = 1.013 \times 1\ kip \cdot ft$

$xb := cx + d2 = 18$

$x := xb = 18 \quad ft$

$Vbx := \int_x^0 qux \cdot B\, dx = -1.167 \times 10^3$

$Vb1 := qux \cdot B \cdot e2 = 259.289 \quad kip$

$Vbux1 := -qux \cdot B \cdot x = -1.167 \times\ kip$

$Vbux2 := -qux \cdot B \cdot x + Pu2 = -374.79\ kip$

$Mubx1 := Vbux1 \cdot x = -2.1 \times 10^4 \quad kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = -1.2 \times 10\ kip \cdot ft$

$x := xb - \frac{b1}{12 \cdot 2} = 17.333 \quad ft$

$Vbx := \int_x^0 qux \cdot B\, dx = -1.124 \times 10^3$

$Vb1 := qux \cdot B \cdot e2 = 259.289 \quad kip$

$Vbux1 := -qux \cdot B \cdot x = -1.124 \times\ kip$

$Vbux2 := -qux \cdot B \cdot x + Pu2 = -331.58\ kip$

$Mubx1 := Vbux1 \cdot x = -1.948 \times 10^4\ kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = -1.062 \times 10^4 \quad kip \cdot ft$

$x := xb + \frac{b1}{2 \cdot 12} = 18.667 \quad ft$

$Vbx := \int_x^0 qux \cdot B\, dx = -1.21 \times 10^3$

$Vbux1 := -qux \cdot B \cdot x = -1.21 \times 1\ kip$

$Vbux2 := -qux \cdot B \cdot x + Pu2 = -418.01\ kip$

$Mubx1 := Vbux1 \cdot x = -2.259 \times 10^4\ kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = -1.345 \times 10^4 \quad kip \cdot ft$

$x := xb + e2 = 22 \quad ft \qquad L = 22 \quad ft$

$Vbx := \int_x^0 qux \cdot B\, dx = -1.426 \times 10^3$

$Vb1 := qux \cdot B \cdot e2 = 259.289 \quad kip$

$Vbux1 := -qux \cdot B \cdot x = -1.426 \times\ kip$

$Vbux2 := -qux \cdot B \cdot x + Pu2 = -634.08\ kip$

$Mubx1 := Vbux1 \cdot x = -3.137 \times 10^4\ kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = -2.179 \times 10^4 \quad kip \cdot ft$

Reinforcement:

Footing Thickness: $t := 42$ in

$hs := Df \cdot 12 - t = -6$ in

Using #9 bars: $dbar9 := 1.128$ $As9 := 1$

$cover := 3 + 1.5 \cdot dbar9$

$d := t - cover = 37.308$ in

$b0 := 2 \cdot (a1 + d) + 2 \cdot (a2 + d) = 205.232$ in

$$\beta c := \begin{cases} \dfrac{a1}{a2} & \text{if } a2 \le a1 \\ \left(\dfrac{a2}{a1}\right) & \text{otherwise} \end{cases}$$

$\beta c = 1$

$$Vu1 := qu \cdot B \cdot \left[\left(\frac{L}{2}\right) - \left(\frac{a1}{2 \cdot 12}\right) - \left(\frac{d}{12}\right)\right] = 435.803$$

$$Vu2 := qu \cdot \left[B \cdot L - (a1 + d) \cdot \frac{(a2 + d)}{12 \cdot 12}\right] = 1.244 \times 10^{3} \text{ kip}$$

$Vu1 := Va1 = 129.644$ kip

$$d1req := Vu1 \cdot \frac{1000}{\phi v \cdot 2 \cdot \sqrt{fcp} \cdot B \cdot 12} = 7.118 \text{ in}$$

$$d2a := Vu2 \cdot \frac{1000}{\phi v \cdot 4 \cdot \sqrt{fcp} \cdot b0} = 31.943 \text{ in}$$

$d2b := Vu2 \cdot \frac{1000}{\phi v \cdot \left(2 + \frac{4}{\beta c}\right) \cdot \sqrt{fcp} \cdot b0} = 21.295 \text{ in}$

$d2c := Vu2 \cdot \frac{1000}{\phi v \cdot \left(2 + \frac{\alpha \cdot d}{b0}\right) \cdot \sqrt{fcp} \cdot b0} = 13.781 \text{ in}$

$d2reqd := \max(d2a, d2b, d2c) = 31.943$

$dreqd := \max(d2reqd, d1req) = 31.943$

Compare Required d1 and d2 with assumed d: $d = 37.308$ **Value Assumed is OK**

$$Thickness := \begin{cases} \text{"t trial is OK"} & \text{if } d \ge dreqd \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

$Thickness = \text{"t trial is OK"}$

Use Thickness $tu := \frac{t}{12} = 3.5$ ft

Reinforcing in long direction

$$MuL := qu \cdot \left[\left(\frac{L}{2}\right) - a1 \cdot \frac{1}{24}\right]^2 \cdot \frac{B}{2} = 3.235 \times 10^3$$

$$RnreqL := MuL \cdot \frac{12000}{\phi t \cdot B \cdot 12 \cdot d^2} = 161.426$$

$$\rho reqL := .85 \cdot \frac{fcp \cdot \left(1 - \sqrt{1 - \frac{2 \cdot RnreqL}{.85 \cdot fcp}}\right)}{fy} = 2.758 \times 10^{-3}$$

$$\rho max := .85 \cdot 0.428 \cdot \beta 1 \cdot \frac{fcp}{fy} = 0.021$$

$\rho min1 := 3 \cdot \frac{\sqrt{fcp}}{fy} = 3.162 \times 10^{-3}$ $\rho min2 := \frac{200}{fy} = 3.333 \times 10^{-3}$ $\rho min := \max(\rho min1, \rho min2) = 3.333 \times 10^{-3}$

Area of Steel Required:

$$AsreqL := \begin{cases} \rho reqL \cdot B \cdot 12 \cdot d & \text{if } \rho reqL \ge \rho min \\ (\rho min \cdot B \cdot 12 \cdot d) & \text{otherwise} \end{cases}$$

Using #9 bars: $AsreqL = 23.877$

$rA := \frac{AsreqL}{As9}$ $Nbars := \text{ceil}(rA) = 24$ $As := Nbars \cdot As9 = 24$ $\rho := \left(\frac{As}{B \cdot 12 \cdot d}\right) = 3.35 \times 10^{-3}$

$$\rho L := \begin{cases} \rho & \text{if } \rho max \ge \rho \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

$\rho L = 3.35 \times 10^{-3}$

$a := As \cdot \frac{fy}{0.85 \cdot fcp \cdot B \cdot 12} = 2.206$ $c := \frac{a}{\beta 1} = 2.595$ $\varepsilon t := (d - c) \cdot \frac{0.003}{c} = 0.04$

$$\phi t := \begin{cases} .65 & \text{if } \varepsilon t \le 0.002 \\ .9 & \text{if } \varepsilon t \ge 0.005 \\ \left[.65 + (\varepsilon t - 0.002)\cdot\dfrac{250}{3}\right] & \text{otherwise} \end{cases} \qquad \phi t = 0.9$$

$$\phi MnL := \phi t\cdot As\cdot fy\cdot\frac{\left(d - \frac{a}{2}\right)}{12000} = 3.91\times 10^{3} \qquad MuL = 3.235\times 10^{3}$$

$$BL := \begin{cases} \text{"OK"} & \text{if } \phi MnL \ge MuL \\ \text{"Re-design"} & \text{otherwise} \end{cases} \qquad BL = \text{"OK"}$$

Spacing Between Bars:

Spacing to be used:

$$spacing := \frac{\left[B\cdot 12 - 2\cdot\left(3 + \frac{dbar8}{2}\right)\right]}{Nbars - 1} = 8.043 \quad in \qquad spacingL := floor(spacing) = 8 \quad in$$

Development Length

$$Cb1 := \frac{spacing}{2} = 4.022 \qquad Cb2 := \left(3 + \frac{dbar8}{2}\right) = 3.5 \qquad Cb := \min(Cb1, Cb2) = 3.5 \;\; in$$

ACI Code factors, for bottom reinforcing, non coated bars, for no. 7 and larger bars and normal weight concrete, respectively:

$$\psi t := 1 \qquad \psi e := 1 \qquad \psi s := 1 \qquad \lambda := 1 \qquad Ktr := 0$$

$$coef := \begin{cases} 2.5 & \text{if } \dfrac{(Ktr + Cb)}{dbar8} \ge 2.5 \\ \dfrac{(Ktr + Cb)}{dbar8} & \text{otherwise} \end{cases} \qquad coef = 2.5$$

$$ld := \frac{3\cdot\left(\frac{fy}{\sqrt{fcp}}\right)\cdot\left(\psi t\cdot\psi e\cdot\psi s\cdot\frac{\lambda}{coef}\right)\cdot dbar8}{40} = 28.46 \qquad \frac{ld}{12} = 2.372 \quad ft$$

Reinforcing in short direction

$$Mus := qu\cdot\left[\left(\frac{B}{2}\right) - a2\cdot\frac{1}{24}\right]^{2}\cdot\frac{L}{2} = 2.255\times 10^{3}\, kip\cdot ft$$

$$Rnreqs := Mus \cdot \frac{12000}{\phi t \cdot L \cdot 12 \cdot d^2} = 81.834$$

$$\rho reqs := .85 \cdot \frac{fcp \cdot \left(1 - \sqrt{1 - \frac{2 \cdot Rnreqs}{.85 \cdot fcp}}\right)}{fy} = 1.381 \times 10^{-3}$$

$$\rho L := \begin{array}{|l} \rho \ \text{ if } \ \rho max \geq \rho \\ \text{"Re-design"} \ \text{ otherwise} \end{array}$$

Area of Steel Required:

$$Asreqs := \begin{array}{|l} \rho reqs \cdot B \cdot 12 \cdot d \ \text{ if } \ \rho reqs \geq \rho min \\ (\rho min \cdot B \cdot 12 \cdot d) \ \text{ otherwise} \end{array} \qquad Asreqs = 23.877$$

Using #9 bars:

$$rAs := \frac{Asreqs}{As9} \qquad Nsbars := ceil(rAs) = 24 \qquad AsUs := Nsbars \cdot As9 = 24 \qquad \rho B1 := \left(\frac{AsUs}{B \cdot 12 \cdot d}\right) = 3.35 \times 10^{-3}$$

$$\rho B := \begin{array}{|l} \rho B1 \ \text{ if } \ \rho B1 \geq \rho reqs \\ \text{"Re-design"} \ \text{ otherwise} \end{array} \qquad \rho B = 3.35 \times 10^{-3}$$

$$\beta := \frac{L}{B} = 1.375 \qquad \gamma s := \frac{2}{(\beta + 1)} = 0.842 \qquad Ff := \gamma s \cdot Asreqs = 20.107$$

Number of Bars placed within the Bandwidth area: $NBb := \frac{Ff}{As9} = 20.107$ $\quad Nbb := ceil(NBb) = 21$

Bars Spacing

$$B \cdot \frac{12}{(Nsbars - 1)} = 8.348 \qquad L \cdot \frac{12}{(Nsbars - 1)} = 11.478$$

Spacing to be used with amount of Reinf:

$$B \cdot \frac{12}{(Nbb - 1)} = 9.6 \qquad spS := 9.5 \ \text{ in}$$

Additional Reinforcement configuration dependent on Bandwidth Area:

Amount of Additional Bars: $Ab := Nsbars - Nbb = 3$

Lets place each half a half of Ab:

Additional Bars needed: $AB := \frac{(Asreqs - \gamma s \cdot Asreqs)}{2} = 1.885$

Outer Length to be reinforced: $OL1 := (L - B) \cdot \frac{12}{2} = 36$ $\quad OL2 := 0.018 \cdot t \cdot (L - B) \cdot \frac{12}{2} = 27.216$

$$OL3 := \left[(L - B) \cdot \frac{12}{2}\right] - 3.5 = 32.5 \qquad OL := min(OL1, OL2, OL3) = 27.216 \ \text{ in}$$

spacing: $sS := \frac{OL}{AB} = 14.438$ in

Strength Check:

$a := As \cdot \frac{fy}{0.85 \cdot fcp \cdot B \cdot 12} = 2.206$

$c := \frac{a}{\beta 1} = 2.595$

$\varepsilon t := (d - c) \cdot \frac{0.003}{c} = 0.04$

$$\phi t := \begin{cases} .65 & \text{if } \varepsilon t \le 0.002 \\ .9 & \text{if } \varepsilon t \ge 0.005 \\ \left[.65 + (\varepsilon t - 0.002) \cdot \frac{250}{3}\right] & \text{otherwise} \end{cases}$$

$\phi t = 0.9$

$\phi MnS := \phi t \cdot As \cdot fy \cdot \frac{\left(d - \frac{a}{2}\right)}{12000} = 3.91 \times 10^{3}$

$MuS := 291.176$

$$BS := \begin{cases} \text{"OK"} & \text{if } \phi MnS \ge MuS \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

$BS = \text{"OK"}$

Problem #3

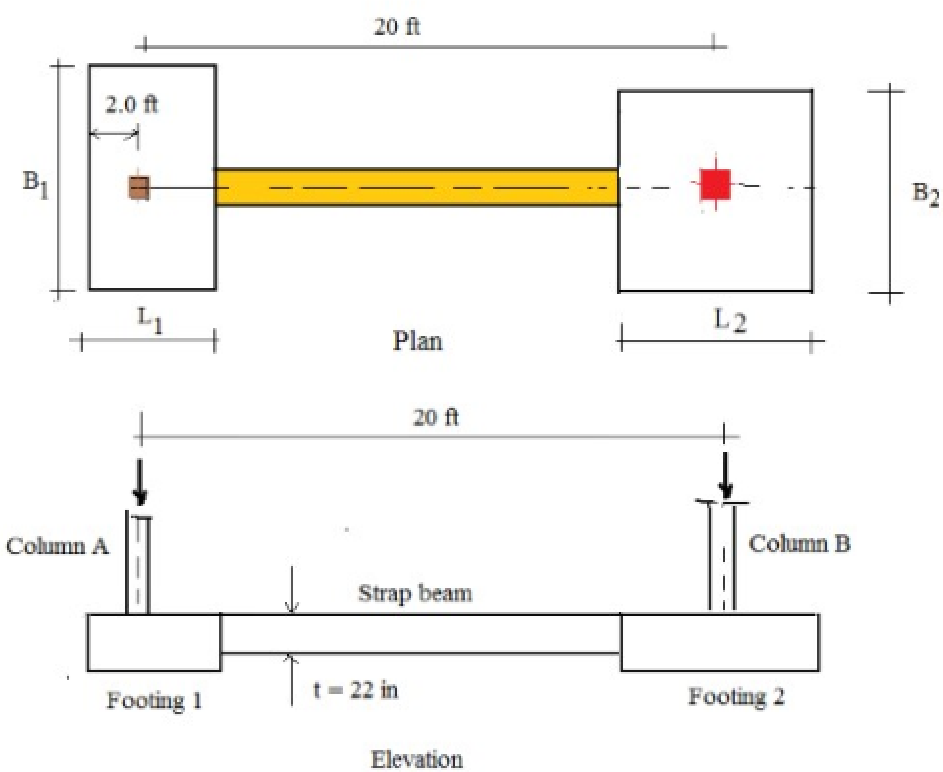
Column A is 14 inches x 14 inches and carries a dead load of 160 kip and a live load of 140 kip. The interior column B is 18 inches x 18 inches and carries a dead load of 230 kip and a live load of 200 kip. The distance between the center lines of the columns is 18 ft. A strap footing is used to support the columns. The center line of column A is 2.ft. from the property line. Assume the strap is placed such that it does not bear directly on the soil. Assume $f_c' = 4000$ psi and $f_y = 60{,}000$ psi.

(a) Determine the dimensions L_1, B_1, B_2 and L_2 for the pad footings that will result in a uniform effective soil pressure not exceeding 4.5 kip/ft^2 under each pad footing. Use ¼ ft. increments.

(b) Establish the shear and moment diagrams corresponding to the factored loading, $P_u = 1.2\, P_D + 1.6\, P_L$.

(c) Design the strap beam (i.e., determine the required width and reinforcing). Use #8 bars. Show the reinforcing arrangements.

(d) Determine the soil pressure profile under the footings determined in part (a) when an additional loading, consisting of an uplift force of 90 kip at the exterior column and an uplift force of 40 kip at the interior column, is applied to the factored loads.



Concrete Properties:

$fcp := 4000 \ \text{psi}$ $fy := 60000 \ \text{psi}$ $\gamma c := .15 \ \text{kcf}$

$\alpha exa := 40$

$\alpha inb := 20$

$$\beta 1 := \begin{cases} .85 & \text{if } fcp \le 4000 \\ .65 & \text{if } fcp \ge 8000 \\ \left[.85 - (fcp - 4000)\cdot\frac{0.05}{1000}\right] & \text{otherwise} \end{cases}$$

$\beta 1 = 0.85$

Soil Properties:

$qe := 4.5 \ \text{ksf}$ $\gamma s := .12 \ \text{kcf}$

Assumptions:

Footing Thickness: $t := 20 \ \text{in}$

Soil depth: $Df := 3 \ \text{ft}$

Factors:

$\phi t := .9$ $\phi v := .75$

Footing Dimensions:

$a1 := 14 \ \text{in}$ $a2 := 14 \ \text{in}$ $b1 := 18 \ \text{in}$ $b2 := 18 \ \text{in}$

edge distance: $e1 := 2 \ \text{ft}$ $ts := 22 \ \text{in}$

distance between columns: $d := 18 \ \text{ft}$

Loading Conditions:

<u>Column A</u>

$Pd1 := 160 \ \text{kip}$ $Pl1 := 140 \ \text{kip}$

$P1 := Pd1 + Pl1 = 300 \ \text{kip}$

$Md1 := 0 \ \text{kip}\cdot\text{ft}$ $Ml1 := 0 \ \text{kip}\cdot\text{ft}$

$M1 := Md1 + Ml1 = 0 \ \text{kip}\cdot\text{ft}$

<u>Column B</u>

$Pd2 := 230 \ \text{kip}$ $Pl2 := 200 \ \text{kip}$

$P2 := Pd2 + Pl2 = 430 \ \text{kip}$

$Md2 := 0 \ \text{kip}\cdot\text{ft}$ $Ml2 := 0 \ \text{kip}\cdot\text{ft}$

$M2 := Md2 + Ml2 = 0 \ \text{kip}\cdot\text{ft}$

Factored Loads:

$Pu1 := 1.2\cdot Pd1 + 1.6\cdot Pl1 = 416 \ \text{kip}$

$Mu1 := 1.2\cdot Md1 + 1.6\cdot Ml1 = 0 \ \text{kip}\cdot\text{ft}$

$Pu2 := 1.2\cdot Pd2 + 1.6\cdot Pl2 = 596 \ \text{kip}$

$Mu2 := 1.2\cdot Md2 + 1.6\cdot M2 = 0 \ \text{kip}\cdot\text{ft}$

Combined Footing Resultant:

$Rc := P1 + P2 = 730 \ \text{kip}$

$Mc := M1 + M2 = 0$

$Rcu := Pu1 + Pu2 = 1.012 \times 10^3 \ \text{kip}$

$Mcu := Mu1 + Mu2 = 0$

Distance force application to resultant:

$x1 := \frac{P1\cdot d}{Rc} = 7.397 \ \text{ft}$ $x2 := \frac{P2\cdot d}{Rc} = 10.60\text{:} \ \text{ft}$ $x1 + x2 = 18$

$x1u := \frac{Pu1\cdot d}{Rcu} = 7.39\text{:} \ \text{ft}$ $x2u := \frac{Pu2\cdot d}{Rcu} = 10.601 \ \text{ft}$ $x1 + x2 = 18$

<u>For the design of this footing, lets set Combined Area centroid to the location where resultant is applied, i.e, x1.</u>

FOOTING DESIGN:

Trial Area:

$TA := \frac{Rc}{qe} = 162.2\text{:} \ \text{ft}^2$

Centroid of the area should be in x, at cx:

$cx := x1 + e1 = 9.397$ $L := L1 + L2$

L is larger than 18, thus use trial L $Lt := 20 \ \text{ft}$ $L := Lt$ $e2 := L - e1 - d = 0 \ \text{ft}$

$Bt := \frac{TA}{Lt} = 8.111$ Use $Bt := 12$ ft $B := Bt$ $A := B \cdot L = 240$ ft^2 area·checks

$qu := \frac{Rcu}{A} = 4.217$ ksf $wu := \frac{Rcu}{L} = 50.6$ klf $V := A \cdot t = 4.8 \times 10^3$ ft^3

1. Determining L, B, t, As and ld:

$qe = 4.5$ ksf

$q1 := \frac{Rc}{B \cdot L} + 6 \cdot \frac{Mc}{B \cdot L^2} = 3.042$ Satisfy Equation

Critical Eccentricity: if e < ec, OK. $e := \frac{Mc}{Rc} = 0$ < $ec := \frac{L}{6} = 3.333$ ok

$$Eccentricity := \begin{cases} \text{"OK"} & \text{if } ec \geq e \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

Eccentricity = "OK"

Determining the size of the strap based on the following assumptions: $t := \frac{ts}{12} = 1.833$

Assumptions: $L1 := 10$ ft $s := 2.5$ $B1 := 14$ ft
$B2 := B = 12$ ft

Results: $L2 := L - s - L1 = 7.5$

$A1 := L1 \cdot B1 = 140$ ft^2

$A2 := L2 \cdot B2 = 90$ ft^2

$AT := A1 + A2 = 230$ ft^2

$quT := \frac{Rcu}{AT} = 4.4$ ultimate·bearingcheck

3. Determining the size of each individual length L1 and L2:

$d2 := d - x1 = 10.603$ $x2 := d2 = 10.603$ $d1 := x1 = 7.397$

$$AreaCheck := \begin{cases} \text{"OK"} & \text{if } AT \geq TA \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

Eccentricity = "OK"

Footings Dimensions: $L1 = 10$ $L2 = 7.5$

$B1 = 14$ $B2 = 12$

Moment of Inertia with Respect to Individual Area Centroid:

$Iy1 := B1 \cdot \frac{L1^3}{12} = 1.167 \times 10^3 \ ft^4$ $\quad$ $Iy2 := B2 \cdot \frac{L2^3}{12} = 421.875 \ ft^4$

Parallel Axis Theorem: $Iyp1 := Iy1 + A1 \cdot d1^2 = 8.827 \times 10^3 \ ft^4$ $\quad$ $Iyp2 := Iy2 + A2 \cdot d2^2 = 1.054 \times 10^4 \ ft^4$

Combined Footing Information: $IT := Iyp1 + Iyp2 = 1.937 \times 10^4 \ ft^4$

Create different vectors for each position x, dx:

$x := 1$

$qx := \frac{Rc}{AT} + Mc \cdot \frac{x}{IT} = 3.174$ $\quad$ $qux := \frac{Rcu}{AT} + Mcu \cdot \frac{x}{IT} = 4.4$

2. Shear and Moment Diagrams for Factored Load Ru: For values up to x, xc:

$Vax := \int_0^x qux \cdot B \, dx = 52.8$ $\quad$ $Vux1 := qux \cdot B \cdot x = 52.8 \ kip$

$Va1 := qux \cdot B \cdot e1 = 105.6 \ kip$ $\quad$ $Vux2 := qux \cdot B \cdot x - Pu1 = -363.2 \ kip$

$Mux1 := Vux1 \cdot x = 52.8 \ kip \cdot ft$ $\quad$ $Mux2 := \left[qux \cdot B \cdot \left(\frac{x}{2}\right)^2\right] - Pu1 \cdot x + Mux1 = -350 \ kip \cdot ft$

Exactly where the load A is located: $Va2 := qux \cdot B \cdot \left(e1 + \frac{a1}{12}\right) - Pu1 = -248.8$

V @ xv = 0

$Vb2 := qux \cdot B \cdot e2 = 0 \ kip$

$Vb1 := -qux \cdot B \cdot \left(e2 + \frac{b1}{12}\right) + Pu2 = 516.8 \ kip$

$x := e2$

$Vbx := \int_x^0 qux \cdot B \, dx = 0$ $\quad$ $Vb1 := qux \cdot B \cdot e2 = 0 \ kip$

$Vbux1 := -qux \cdot B \cdot x = 0 \ kip$ $\quad$ $Vbux2 := -qux \cdot B \cdot x + Pu2 = 596 \ kip$

$Mubx1 := Vbux1 \cdot x = 0 \ kip \cdot ft$ $\quad$ $Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = 0 \ kip \cdot ft$

at Shear V=0, Mumax, in xv: $Vu := B \cdot qux \cdot xv - Pu1 = 0$ $\quad$ $xv := \frac{Pu1}{B \cdot qux} = 7.87! \ ft$

Using Equation Vux1 and Mux1 @ points of shear inflection: $x := xv = 7.879$ ft

$Vux1 := qux \cdot B \cdot x = 416$ $\qquad Mux1 := Vux1 \cdot x = 3.278 \times 10^{3}$ kip·ft

$Vux2 := qux \cdot B \cdot x - Pu1$ $\qquad Mux2 := \left[qu \cdot B \cdot \left(\frac{x}{2}\right)^{2}\right] - Pu1 \cdot x + Mux1 = 785.253$ kip·ft

$x := e1$

$Vux1 := qux \cdot B \cdot x = 105.6$ $\qquad Mux1 := Vux1 \cdot x = 211.2$ kip·ft

$Vux2 := qux \cdot B \cdot x - Pu1 = -310.4$ $\qquad Mux2 := \left[qu \cdot B \cdot \left(\frac{x}{2}\right)^{2}\right] - Pu1 \cdot x + Mux1 = -570.2$ kip·ft

$x := e1 + \frac{a1}{12}$

$Vux1 := qux \cdot B \cdot x = 167.2$ $\qquad Mux1 := Vux1 \cdot x = 529.467$ kip·ft

$Vux2 := qux \cdot B \cdot x - Pu1 = -248.8$ $\qquad Mux2 := \left[qu \cdot B \cdot \left(\frac{x}{2}\right)^{2}\right] - Pu1 \cdot x + Mux1 = -661.015$ kip·ft

$x := cx$

$Vux1 := qux \cdot B \cdot x = 496.175$ $\qquad Mux1 := Vux1 \cdot x = 4.663 \times 10^{3}$ kip·ft

$Vux2 := qux \cdot B \cdot x - Pu1 = 80.175$ $\qquad Mux2 := \left[qu \cdot B \cdot \left(\frac{x}{2}\right)^{2}\right] - Pu1 \cdot x + Mux1 = 1.871 \times 10^{3}$ kip·ft

$x := xv = 7.879$ ft

$Vbx := \int_{x}^{0} qux \cdot B \, dx = -416$ $\qquad Vb1 := qux \cdot B \cdot e2 = 0$ kip

$Vbux1 := -qux \cdot B \cdot x = -416$ kip $\qquad Vbux2 := -qux \cdot B \cdot x + Pu2 = 180$ kip

$Mubx1 := Vbux1 \cdot x = -3.278 \times 10^{3}$ kip·ft $\qquad Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^{2} + Pu2 \cdot x + Mubx1 = 598.788$ kip·ft

$x := cx$

$Vbx := \int_{x}^{0} qux \cdot B \, dx = -496.175$

$Vbux1 := -qux \cdot B \cdot x = -496.175$ kip $\qquad Vbux2 := -qux \cdot B \cdot x + Pu2 = 99.825$ kip

$Mubx1 := Vbux1 \cdot x = -4.663 \times 10^{3}\ kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = -227.594\ kip \cdot ft$

$xb := cx + d2 = 20$

$x := xb = 20 \quad ft$

$Vbx := \int_x^0 qux \cdot B\, dx = -1.056 \times 10^3$

$Vb1 := qux \cdot B \cdot e2 = 0 \quad kip$

$Vbux1 := -qux \cdot B \cdot x = -1.056 \times\ kip$

$Vbux2 := -qux \cdot B \cdot x + Pu2 = -460 \quad kip$

$Mubx1 := Vbux1 \cdot x = -2.112 \times 10^4\ kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = -1.448 \times\ kip \cdot ft$

$x := xb - \frac{b1}{12 \cdot 2} = 19.25 \quad ft$

$Vbx := \int_x^0 qux \cdot B\, dx = -1.016 \times 10^3$

$Vb1 := qux \cdot B \cdot e2 = 0 \quad kip$

$Vbux1 := -qux \cdot B \cdot x = -1.016 \times\ kip$

$Vbux2 := -qux \cdot B \cdot x + Pu2 = -420.4\ kip$

$Mubx1 := Vbux1 \cdot x = -1.957 \times 10^4\ kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = -1.298 \times 10^4\ kip \cdot ft$

$x := xb + \frac{b1}{2 \cdot 12} = 20.75 \quad ft$

$Vbx := \int_x^0 qux \cdot B\, dx = -1.096 \times 10^3$

$Vbux1 := -qux \cdot B \cdot x = -1.096 \times\ kip$

$Vbux2 := -qux \cdot B \cdot x + Pu2 = -499.6\ kip$

$Mubx1 := Vbux1 \cdot x = -2.273 \times 10^4\ kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = -1.605 \times 10^4\ kip \cdot ft$

$x := xb + e2 = 20 \quad ft$ $L = 20 \quad ft$

$Vbx := \int_x^0 qux \cdot B\, dx = -1.056 \times 10^3$

$Vb1 := qux \cdot B \cdot e2 = 0 \quad kip$

$Vbux1 := -qux \cdot B \cdot x = -1.056 \times\ kip$

$Vbux2 := -qux \cdot B \cdot x + Pu2 = -460 \quad kip$

$Mubx1 := Vbux1 \cdot x = -2.112 \times 10^4\ kip \cdot ft$

$Mubx2 := -qux \cdot B \cdot \left(\frac{x}{2}\right)^2 + Pu2 \cdot x + Mubx1 = -1.448 \times 10^4\ kip \cdot ft$

Part c) Determine the required width and reinforcing

Reinforcement:

Footing Thickness: $t := 42$ in $hs := Df \cdot 12 - t = -6$ in

Using #9 bars: $dbar9 := 1.128$ $As9 := 1$ $cover := 3 + 1.5 \cdot dbar9$ $d := t - cover = 37.308$ in

$b0 := 2 \cdot (a1 + d) + 2 \cdot (a2 + d) = 205.232$ in

$$\beta c := \begin{cases} \dfrac{a1}{a2} & \text{if } a2 \le a1 \\ \left(\dfrac{a2}{a1}\right) & \text{otherwise} \end{cases} \qquad \beta c = 1$$

$$Vu1 := qu \cdot B \cdot \left[\left(\frac{L}{2}\right) - \left(\frac{a1}{2 \cdot 12}\right) - \left(\frac{d}{12}\right)\right] = 319.168$$

$$Vu2 := qu \cdot \left[B \cdot L - (a1 + d) \cdot \frac{(a2 + d)}{12 \cdot 12}\right] = 934.914 \text{ kip}$$

$Vu1 := Va1 = 105.6$ kip

$$d1req := Vu1 \cdot \frac{1000}{\phi v \cdot 2 \cdot \sqrt{fcp} \cdot B \cdot 12} = 7.73 \text{ in}$$

$$d2a := Vu2 \cdot \frac{1000}{\phi v \cdot 4 \cdot \sqrt{fcp} \cdot b0} = 24.009 \text{ in}$$

$$d2b := Vu2 \cdot \frac{1000}{\phi v \cdot \left(2 + \dfrac{4}{\beta c}\right) \cdot \sqrt{fcp} \cdot b0} = 16.006 \text{ in}$$

$$d2c := Vu2 \cdot \frac{1000}{\phi v \cdot \left(2 + \dfrac{\alpha \cdot d}{b0}\right) \cdot \sqrt{fcp} \cdot b0} = 10.358 \text{ in}$$

$d2reqd := \max(d2a, d2b, d2c) = 24.009$ $dreqd := \max(d2reqd, d1req) = 24.009$

Compare Required d1 and d2 with assumed d: $d = 37.308$ **Value Assumed is OK**

$$Thickness := \begin{cases} \text{"t trial is OK"} & \text{if } d \ge dreqd \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

Thickness = "t trial is OK"

Use Thickness $tu := \frac{t}{12} = 3.5$ ft

Reinforcing in long direction

$$MuL := qu\cdot\left[\left(\frac{L}{2}\right) - a1\cdot\frac{1}{24}\right]^2\cdot\frac{B}{2} = 2.243\times10^{3}$$

$$RnreqL := MuL\cdot\frac{12000}{\phi t\cdot B\cdot 12\cdot d^2} = 149.241$$

$$\rho reqL := .85\cdot\frac{fcp\cdot\left(1-\sqrt{1-\frac{2\cdot RnreqL}{.85\cdot fcp}}\right)}{fy} = 2.544\times10^{-3}$$

$$\rho max := .85\cdot 0.428\cdot\beta1\cdot\frac{fcp}{fy} = 0.021$$

$$\rho min1 := 3\cdot\frac{\sqrt{fcp}}{fy} = 3.162\times10^{-3} \qquad \rho min2 := \frac{200}{fy} = 3.333\times10^{-3} \qquad \rho min := \max(\rho min1, \rho min2) = 3.333\times10^{-3}$$

Area of Steel Required:

$$AsreqL := \begin{cases} \rho reqL\cdot B\cdot 12\cdot d & \text{if } \rho reqL \ge \rho min \\ (\rho min\cdot B\cdot 12\cdot d) & \text{otherwise}\end{cases}$$

Using #9 bars:

$AsreqL = 17.908$

$$rA := \frac{AsreqL}{As9} \qquad Nbars := \text{ceil}(rA) = 18 \qquad As := Nbars\cdot As9 = 18 \qquad \rho := \left(\frac{As}{B\cdot 12\cdot d}\right) = 3.35\times10^{-3}$$

$$\rho L := \begin{cases} \rho & \text{if } \rho max \ge \rho \\ \text{"Re-design"} & \text{otherwise}\end{cases}$$

$\rho L = 3.35\times10^{-3}$

$$a := As\cdot\frac{fy}{0.85\cdot fcp\cdot B\cdot 12} = 2.206 \qquad c := \frac{a}{\beta1} = 2.595 \qquad \varepsilon t := (d-c)\cdot\frac{0.003}{c} = 0.04$$

$$\phi t := \begin{cases} .65 & \text{if } \varepsilon t \le 0.002 \\ .9 & \text{if } \varepsilon t \ge 0.005 \\ \left[.65 + (\varepsilon t - 0.002)\cdot\frac{250}{3}\right] & \text{otherwise}\end{cases}$$

$\phi t = 0.9$

$$\phi MnL := \phi t\cdot As\cdot fy\cdot\frac{\left(d-\frac{a}{2}\right)}{12000} = 2.933\times10^{3}$$

$MuL = 2.243\times10^{3}$

$$BL := \begin{cases} \text{"OK"} & \text{if } \phi MnL \ge MuL \\ \text{"Re-design"} & \text{otherwise}\end{cases}$$

BL = "OK"

Spacing Between Bars:

$$\text{spacing} := \frac{\left[B\cdot 12 - 2\cdot\left(3 + \frac{\text{dbar8}}{2}\right)\right]}{\text{Nbars} - 1} = 8.059 \quad \text{in}$$

Spacing to be used:

$$\text{spacingL} := \text{floor(spacing)} = 8 \quad \text{in}$$

Development Length

$$\text{Cb1} := \frac{\text{spacing}}{2} = 4.029 \qquad \text{Cb2} := \left(3 + \frac{\text{dbar8}}{2}\right) = 3.5 \qquad \text{Cb} := \min(\text{Cb1}, \text{Cb2}) = 3.5 \ \text{in}$$

ACI Code factors, for bottom reinforcing, non coated bars, for no. 7 and larger bars and normal weight concrete, respectively:

$$\psi t := 1 \qquad \psi e := 1 \qquad \psi s := 1 \qquad \lambda := 1 \qquad \text{Ktr} := 0$$

$$\text{coef} := \begin{cases} 2.5 & \text{if } \frac{(\text{Ktr} + \text{Cb})}{\text{dbar8}} \geq 2.5 \\ \frac{(\text{Ktr} + \text{Cb})}{\text{dbar8}} & \text{otherwise} \end{cases} \qquad \text{coef} = 2.5$$

$$\text{ld} := \frac{3\cdot\left(\frac{\text{fy}}{\sqrt{\text{fcp}}}\right)\cdot\left(\psi t\cdot\psi e\cdot\psi s\cdot\frac{\lambda}{\text{coef}}\right)\cdot\text{dbar8}}{40} = 28.46 \qquad \frac{\text{ld}}{12} = 2.372 \quad \text{ft}$$

Reinforcing in short direction

$$\text{Mus} := \text{qu}\cdot\left[\left(\frac{B}{2}\right) - \text{a2}\cdot\frac{1}{24}\right]^2\cdot\frac{L}{2} = 1.237 \times 10^3 \ \text{kip}\cdot\text{ft}$$

$$\text{Rnreqs} := \text{Mus}\cdot\frac{12000}{\phi t\cdot L\cdot 12\cdot d^2} = 49.381$$

$$\rho\text{reqs} := .85\cdot\frac{\text{fcp}\cdot\left(1 - \sqrt{1 - \frac{2\cdot\text{Rnreqs}}{.85\cdot\text{fcp}}}\right)}{\text{fy}} = 8.291 \times 10^{-4}$$

$$\rho L := \begin{cases} \rho & \text{if } \rho\max \geq \rho \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

Area of Steel Required:

$$\text{Asreqs} := \begin{cases} \rho\text{reqs}\cdot B\cdot 12\cdot d & \text{if } \rho\text{reqs} \geq \rho\min \\ (\rho\min\cdot B\cdot 12\cdot d) & \text{otherwise} \end{cases} \qquad \text{Asreqs} = 17.908$$

Using #9 bars:

$rAs := \frac{Asreqs}{As9}$ $\quad Nsbars := ceil(rAs) = 18$ $\quad AsUs := Nsbars \cdot As9 = 18$ $\quad \rho B1 := \left(\frac{AsUs}{B \cdot 12 \cdot d}\right) = 3.35 \times 10^{-3}$

$$\rho B := \begin{vmatrix} \rho B1 \text{ if } \rho B1 \geq \rho reqs \\ \text{"Re-design" otherwise} \end{vmatrix} \qquad \rho B = 3.35 \times 10^{-3}$$

$\beta := \frac{L}{B} = 1.667$ $\quad \gamma s := \frac{2}{(\beta + 1)} = 0.75$ $\quad Ff := \gamma s \cdot Asreqs = 13.431$

Number of Bars placed within the Bandwidth area: $NBb := \frac{Ff}{As9} = 13.431$ $\quad Nbb := ceil(NBb) = 14$

Bars Spacing

Spacing to be used with amount of Reinf:

$B \cdot \frac{12}{(Nsbars - 1)} = 8.471$ $\quad L \cdot \frac{12}{(Nsbars - 1)} = 14.118$ $\quad B \cdot \frac{12}{(Nbb - 1)} = 11.077$ $\quad spS := 9.5 \text{ in}$

Additional Reinforcement configuration dependent on Bandwidth Area:

Amount of Additional Bars: $Ab := Nsbars - Nbb = 4$

Lets place each half a half of Ab:

Additional Bars needed: $AB := \frac{(Asreqs - \gamma s \cdot Asreqs)}{2} = 2.238$

Outer Length to be reinforced: $OL1 := (L - B) \cdot \frac{12}{2} = 48$ $\quad OL2 := 0.018 \cdot t \cdot (L - B) \cdot \frac{12}{2} = 36.288$

$OL3 := \left[(L - B) \cdot \frac{12}{2}\right] - 3.5 = 44.5$ $\quad OL := \min(OL1, OL2, OL3) = 36.288 \text{ in}$

spacing: $sS := \frac{OL}{AB} = 16.211 \text{ in}$

Strength Check:

$a := As \cdot \frac{fy}{0.85 \cdot fcp \cdot B \cdot 12} = 2.206$ $\quad c := \frac{a}{\beta 1} = 2.595$ $\quad \varepsilon t := (d - c) \cdot \frac{0.003}{c} = 0.04$

$$\phi t := \begin{vmatrix} .65 \text{ if } \varepsilon t \leq 0.002 \\ .9 \text{ if } \varepsilon t \geq 0.005 \\ \left[.65 + (\varepsilon t - 0.002) \cdot \frac{250}{3}\right] \text{ otherwise} \end{vmatrix} \qquad \phi t = 0.9$$

$\phi MnS := \phi t \cdot As \cdot fy \cdot \dfrac{\left(d - \dfrac{a}{2}\right)}{12000} = 2.933 \times 10^{3}$ $\quad MuS := 291.176$ $\quad BS := \begin{cases} \text{"OK"} & \text{if } \phi MnS \ge MuS \\ \text{"Re-design"} & \text{otherwise} \end{cases}$

$BS = \text{"OK"}$

d) Determining the Soil Pressure if Uplift forces occur:

$\alpha exa := 40 \quad Pap := -90$

$\alpha inb := 20 \quad Pbp := -40$

Loading Conditions:

Column A

$Pd1 := 160 \text{ kip} \quad Pl1 := 140 \text{ kip}$

$P1 := Pd1 + Pl1 + Pap = 210 \text{ kip}$

$Md1 := 0 \text{ kip}\cdot\text{ft} \quad Ml1 := 0 \text{ kip}\cdot\text{ft}$

$M1 := Md1 + Ml1 = 0 \text{ kip}\cdot\text{ft}$

Column B

$Pd2 := 230 \text{ kip} \quad Pl2 := 200 \text{ kip}$

$P2 := Pd2 + Pl2 + Pbp = 390 \text{ kip}$

$Md2 := 0 \text{ kip}\cdot\text{ft} \quad Ml2 := 0 \text{ kip}\cdot\text{ft}$

$M2 := Md2 + Ml2 = 0 \text{ kip}\cdot\text{ft}$

Combined Footing Resultant:

$Rc := P1 + P2 = 600 \text{ kip}$

$Mc := M1 + M2 = 0$

Distance force application to resultant:

$x1 := \dfrac{P1 \cdot d}{Rc} = 13.058 \text{ ft} \quad x2 := \dfrac{P2 \cdot d}{Rc} = 24.25 \text{ ft} \quad x1 + x2 = 37.308$

Factored Loads:

$Pu1 := 1.2 \cdot Pd1 + 1.6 \cdot Pl1 = 416 \text{ kip}$

$Mu1 := 1.2 \cdot Md1 + 1.6 \cdot Ml1 = 0 \text{ kip}\cdot\text{ft}$

$Pu2 := 1.2 \cdot Pd2 + 1.6 \cdot Pl2 = 596 \text{ kip}$

$Mu2 := 1.2 \cdot Md2 + 1.6 \cdot M2 = 0 \text{ kip}\cdot\text{ft}$

$Rcu := Pu1 + Pu2 = 1.012 \times 10^{3} \text{ kip}$

$Mcu := Mu1 + Mu2 = 0$

$x1u := \dfrac{Pu1 \cdot d}{Rcu} = 15.33 \text{ ft} \quad x2u := \dfrac{Pu2 \cdot d}{Rcu} = 21.972 \text{ ft} \quad x1 + x2 = 37.308$

For the design of this footing, lets set Combined Area centroid to the location where resultant is applied, i.e, x1.

FOOTING DESIGN:

Trial Area:

$TA := \dfrac{Rc}{qe} = 133.333 \text{ ft}^{2}$

Centroid of the area should be in x, at cx:

$cx := x1 + e1 = 15.058$

$d := 18$

L is larger than 18, thus use trial L $\quad Lt := 22 \text{ ft} \quad L := Lt \quad e2 := L - e1 - d = 2 \text{ ft}$

$Bt := \dfrac{TA}{Lt} = 6.061$ Use $Bt := 10 \text{ ft} \quad B := Bt \quad A := B \cdot L = 220 \text{ ft}^{2}$ area·checks

$qu := \dfrac{Rcu}{A} = 4.6 \text{ ksf} \quad wu := \dfrac{Rcu}{L} = 46 \text{ klf} \quad V := A \cdot t = 9.24 \times 10^{3} \text{ ft}^{3}$

1. Determining L, B, t, As and Id:

$qe = 4.5$ ksf

$q1 := \frac{Rc}{B \cdot L} + 6 \cdot \frac{Mc}{B \cdot L^2} = 2.727$

$q2 := \frac{Rc}{B \cdot L} + -6 \cdot \frac{Mc}{B \cdot L^2} = 2.727$

Satisfy Equation

Critical Eccentricity: if e < ec, OK.

$e := \frac{Mc}{Rc} = 0$ < $ec := \frac{L}{6} = 3.667$ ok

Eccentricity := | "OK" if ec ≥ e
| "Re-design" otherwise

Eccentricity = "OK"

Problem # 4

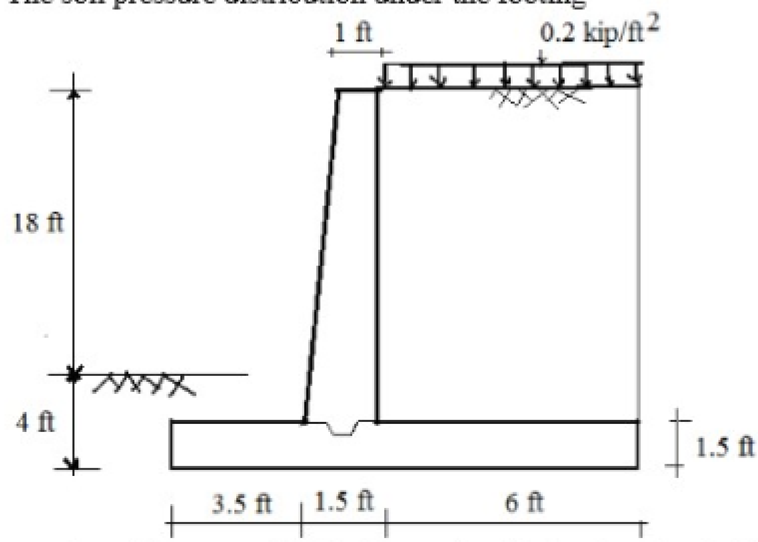
(a) For the cantilever retaining wall shown below, determine the following:

(i) The soil pressure acting on the wall

(ii) The factor of safety for overturning

(iii) The factor of safety for sliding

(iv) The soil pressure distribution under the footing



Concrete Properties:

$fcp := 4000$ psi

$fy := 60000$ psi

$\gamma c := .15$ kcf

Soil Properties:

$qe := 4.5$ ksf

$\gamma s := .12$ kcf

Factors:

$$\beta 1 := \begin{cases} .85 & \text{if } fcp \le 4000 \\ .65 & \text{if } fcp \ge 8000 \\ \left[.85 - (fcp - 4000)\cdot\frac{0.05}{1000}\right] & \text{otherwise} \end{cases}$$

$\alpha exa := 40$ $\quad \beta 1 = 0.85$ $\quad \phi t := .9$

$\alpha inb := 20$ $\quad \mu := .58$ $\quad \phi v := .75$

Soil depth: $Df := 4\ ft$

Angle of Repose: $\varphi := 30\cdot deg$

Surcharge Properies:

$qs := 0.2\quad ksf$

Surcharge Angle, αs: $\alpha s := 0\cdot deg$

Known:

$H := 18\ ft$

Footing Information:

Footing Thickness: $tf := 1.5\cdot 12 = 18\quad in$

toe length: $tl := 3.5\quad ft$

heel length: $hl := 6\quad ft$

Footing length: $L := tl + wb + hl = 11\quad ft$

$$e1 := tl + \frac{wb}{2} = 4.25 \qquad e2 := hl + \frac{wb}{2} = 6.75$$

Retaining Wall Information:

$wt := 1\ ft \qquad wb := 1.5\ ft$

$$Hw := \left(H + Df - \frac{tf}{12}\right) = 20.5$$

$$ang := atan\left[\frac{(wb - wt)}{Hw}\right] = 0.024$$

distance between columns: $d := 0\quad ft \qquad ts := 22\quad in$

Into Plane Info:

$a2 := 12$ in

$B := 1\quad ft$

Rankine Theory:

Active Case: $Ka := \frac{1}{3}$

Passive Case: $Kp := \frac{1}{Ka} = 3$

$$Ka := \cos(\alpha s)\cdot\left[\frac{\left[\cos(\alpha s) - \sqrt{(\cos(\alpha s))^2 - (\cos(\varphi))^2}\right]}{\left[\cos(\alpha s) + \sqrt{(\cos(\alpha s))^2 - (\cos(\varphi))^2}\right]}\right] = 0.333$$

$$Ka := \frac{(1 - \sin(\varphi))}{(1 + \sin(\varphi))} = 0.333$$

Angle on back of the wall: $\theta a := 90deg$

Angle on front of the wall: $\theta p := 90deg + (ang) = 1.595$

Amount of Soil: $z := Hw = 20.5$ $\qquad zp := Df = 4$

$$W1 := \frac{-B\cdot 1\cdot ys\cdot z^2}{2\cdot\sin(\theta a)} = -25.215\ kip \qquad W1P := \frac{B\cdot 1\cdot ys\cdot zp^2}{2\cdot\sin(\theta p)} = 0.96\quad kip$$

Active Force: $Pa := Ka\cdot W1 = -8.405\ kip$

Force Components:

$Pah := Pa\cdot\sin(\theta a) = -8.405$

$Pav := Pa\cdot\cos(\theta a) = 0$

Active Force: $Pp := Kp\cdot W1P = 2.881\quad kip$

Force Components:

$Pph := Pp\cdot\sin(\theta p) = 2.88$

$Ppv := Pp\cdot\cos(\theta p) = -0.07$

Located at: $ya := \frac{Hw}{3} = 6.833$ ft

$xa := wb + tl = 5$

Located at: $yp := \frac{zp}{3} = 1.333$ ft

$xp := tl = 3.5$

Stability Analysis on Wall:

Amount of Concrete: $Af := L \cdot \frac{tf}{12} = 16.5\ ft^2$ $Aw := (wb + wt) \cdot \frac{Hw}{2} = 25.625\ ft^2$ $AT := Af + Aw = 42.125\ ft^2$

$Wfc := B \cdot Af \cdot yc = 2.475$ kip $Wwc := B \cdot Aw \cdot yc = 3.844$ kip $WTc := B \cdot AT \cdot yc = 6.319$ kip

Structure Centroids distance to toe: $xtf := \frac{L}{2} = 5.5$ ft $xw := \left[tl + \frac{wt}{2} + (wb - wt) \cdot \frac{2}{3}\right] = 4.333$ ft

Factor Against Sliding: $N := WTc + W1 + W1P = -17.936$ kip

$F := Pah + Pph = -5.525$ kip $Fmax := N \cdot \mu = -10.403$ kip

$FSs := 1.5$ $Fs := \frac{Fmax}{F} = 1.883$

$$SlidingDesign := \begin{vmatrix} \text{"OK"} & \text{if } Fs \geq FSs \\ \text{"Re-design"} & \text{otherwise} \end{vmatrix}$$

SlidingDesign = "OK"

Net Moment: $Mnet := Pah \cdot ya + Pav \cdot (xa) + [Pph \cdot yp + Ppv \cdot (xp)] + Wfc \cdot xtf + Wwc \cdot xw$

$xcbar := \frac{Mnet}{N}$

Overturning Moments: $Mot := -Pah \cdot ya = 57.434$ kip

Resisting Moments: $Mres := Pph \cdot yp + -Ppv \cdot (xp) + (Wfc \cdot xtf + Wwc \cdot xw) + Pav \cdot xa = 34.355$ kip

Factor Against Overturning: $FO := \frac{Mres}{Mot} = 0.598$

$FSo := 2$

$$OverturningDesign := \begin{vmatrix} \text{"OK"} & \text{if } FO \geq FSo \\ \text{"Re-design"} & \text{otherwise} \end{vmatrix}$$

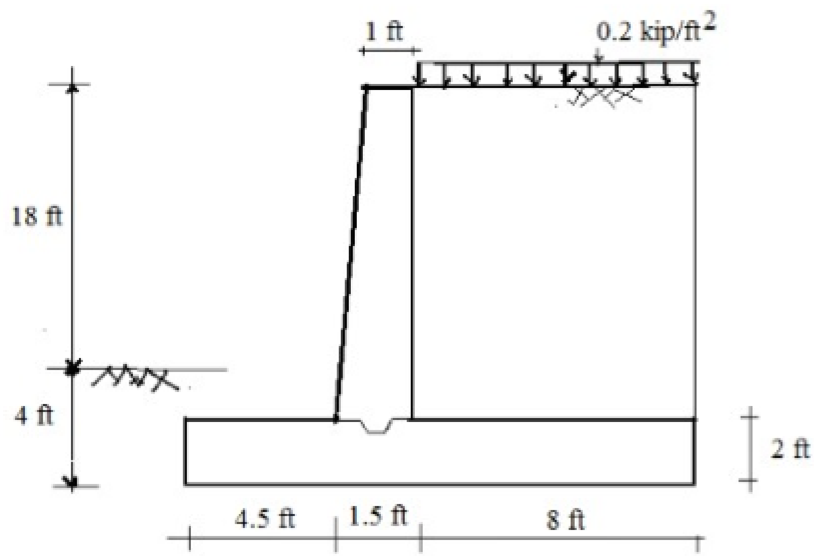
OverturningDesign = "Re-design"

Soil Pressure Distribution Under the Footing: $Iy := L \cdot \frac{B^3}{12} = 0.917$ $x := 1$

$qx := \frac{N}{L \cdot B} + 6 \cdot Mnet \cdot \frac{x}{Iy} = -155.915$

(b) To improve the soil pressure distribution under the footing, the footing size is increased as shown below. Select the reinforcing and show the reinforcing details. Use #8 or #4 bars.

Assume: Allowable soil pressure = 4.5 kip/ft^2, $f_c' = 4000$ psi, $f_y = 60{,}000$ psi, $\gamma_{soil} = 0.12$ kip/ft^3, $\gamma_{concrete} = 0.15$ kip/ft^3, $k_a = 1/3$, and $\mu = .58$



(c) Are the dimensions of the stem and base appropriate for optimal design? What is your suggestion?

Concrete Properties:

$fcp := 4000$ psi $\quad fy := 60000$ psi $\quad \gamma c := .15$ kcf

Factors:

$$\beta 1 := \begin{cases} .85 & \text{if } fcp \le 4000 \\ .65 & \text{if } fcp \ge 8000 \\ \left[.85 - (fcp - 4000)\cdot\frac{0.05}{1000}\right] & \text{otherwise} \end{cases}$$

$\alpha exa := 40 \quad \beta 1 = 0.85 \quad \phi t := .9$

$\alpha inb := 20 \quad \mu := .58 \quad \phi v := .75$

Soil Properties:

$qe := 4.5$ ksf $\quad \gamma s := .12$ kcf

Soil depth: $Df := 4$ ft

Angle of Repose: $\varphi := 30\cdot deg$

Surcharge Properies:

$qs := 0.2$ ksf

Surcharge Angle, αs: $\alpha s := 0\cdot deg$

Known:

$H := 18$ ft

Footing Information:

Footing Thickness: $tf := 2\cdot 12 = 24$ in

toe length: $tl := 4.5$ ft

heel length: $hl := 8$ ft

Footing length: $L := tl + wb + hl = 14$ ft

Retaining Wall Information:

$wt := 1$ ft $\quad wb := 1.5$ ft

$$Hw := \left(H + Df - \frac{tf}{12}\right) = 20$$

$$ang := atan\left[\frac{(wb - wt)}{Hw}\right] = 0.025$$

Into Plane Info:

$a2 := 12$ in

$B := 1$ ft

$e1 := tl + \frac{wb}{2} = 5.25$ $e2 := hl + \frac{wb}{2} = 8.75$ distance between columns: $d := 0$ ft $ts := 22$ in

Rankine Theory:

Active Case: $Ka := \frac{1}{3}$

Passive Case: $Kp := \frac{1}{Ka} = 3$

$$Ka := \cos(\alpha s)\cdot\left[\frac{\left[\cos(\alpha s) - \sqrt{(\cos(\alpha s))^2 - (\cos(\varphi))^2}\right]}{\left[\cos(\alpha s) + \sqrt{(\cos(\alpha s))^2 - (\cos(\varphi))^2}\right]}\right] = 0.333$$

$$Ka := \frac{(1 - \sin(\varphi))}{(1 + \sin(\varphi))} = 0.333$$

Angle on back of the wall: $\theta a := 90deg$ Angle on front of the wall: $\theta p := 90deg + (ang) = 1.596$

Amount of Soil: $z := Hw = 20$ $zp := Df = 4$

$W1 := \frac{-B\cdot 1\cdot ys\cdot z^2}{2\cdot\sin(\theta a)} = -24$ kip $W1P := \frac{B\cdot 1\cdot ys\cdot zp^2}{2\cdot\sin(\theta p)} = 0.96$ kip

Active Force: $Pa := Ka\cdot W1 = -8$ kip

Force Components: $Pah := Pa\cdot\sin(\theta a) = -8$

$Pav := Pa\cdot\cos(\theta a) = 0$

Located at: $ya := \frac{Hw}{3} = 6.667$ ft

$xa := wb + tl = 6$

Active Force: $Pp := Kp\cdot W1P = 2.881$ kip

Force Components: $Pph := Pp\cdot\sin(\theta p) = 2.88$

$Ppv := Pp\cdot\cos(\theta p) = -0.072$

Located at: $yp := \frac{zp}{3} = 1.333$ ft

$xp := tl = 4.5$

Stability Analysis on Wall:

Amount of Concrete: $Af := L\cdot\frac{tf}{12} = 28$ ft^2 $Aw := (wb + wt)\cdot\frac{Hw}{2} = 25$ ft^2 $AT := Af + Aw = 53$ ft^2

$Wfc := B\cdot Af\cdot yc = 4.2$ kip $Wwc := B\cdot Aw\cdot yc = 3.75$ kip $WTc := B\cdot AT\cdot yc = 7.95$ kip

Structure Centroids distance to toe: $xtf := \frac{L}{2} = 7$ ft $xw := \left[tl + \frac{wt}{2} + (wb - wt)\cdot\frac{2}{3}\right] = 5.333$ ft

Factor Against Sliding: $N := WTc + W1 + W1P = -15.09$ kip

$F := Pah + Pph = -5.12$ kip $Fmax := N\cdot\mu = -8.752$ kip

$FSs := 1.5$ $Fs := \frac{Fmax}{F} = 1.709$

$$SlidingDesign := \begin{cases} \text{"OK"} & \text{if } Fs \ge FSs \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

SlidingDesign = "OK"

Net Moment:

$$Mnet := Pah \cdot ya + Pav \cdot (xa) + [Pph \cdot yp + Ppv \cdot (xp)] + Wfc \cdot xtf + Wwc \cdot xw$$

$$xcbar := \frac{Mnet}{N}$$

Overturning Moments: $Mot := -Pah \cdot ya = 53.333$ kip

Resisting Moments: $Mres := Pph \cdot yp + -Ppv \cdot (xp) + (Wfc \cdot xtf + Wwc \cdot xw) + Pav \cdot xa = 53.564$ kip

Factor Against Overturning: $FO := \frac{Mres}{Mot} = 1.004$

$FSo := 2$

$$OverturningDesign := \begin{cases} \text{"OK"} & \text{if } FO \geq FSo \\ \text{"Re-design"} & \text{otherwise} \end{cases}$$

OverturningDesign = "Re-design"

Soil Pressure Distribution Under the Footing: $Iy := L \cdot \frac{B^3}{12} = 1.167$ $x := 1$

$$qx := \frac{N}{L \cdot B} + 6 \cdot Mnet \cdot \frac{x}{Iy} = -3.224$$

Part c)