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Subject: Analysis of Spruce-Pine-Fir for a Joist Floor System

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SUMMARY

The objective of this test was to determine whether or not Spruce-Pine-Fir (SPF) beams are suitable for the construction of an exercise balcony in the Costello Athletic Center. Bending testing of 14-samples of 2''x3''x30'' S-P-F beams were conducted. The results of the tests were obtained using an Instron 8511. These results were later used to determine the required dimensions for the Gymnasium's joist floor system. The bending test results showed a consistent value for the modulus of rupture and modulus of rupture of the specimen. The data obtained and calculated shows that the SPF wood is suitable, under the dimensions determined over our calculations, for the construction of the Gymnasium's joist floor system.

EXPERIMENTAL APPROACH

To conduct the experiment the following equipment and materials were used.

Equipment and Materials

A schematic presentation of the experimental setup is shown below in Figure 1. In order to obtain the SPF bending tests data, the following materials were needed as listed below.

- Instron 8511
- Data acquisition software
- Wood saw machine
- 2''x3"x30", stud grade, Spruce-Pine-Fir beam

- Adjustable square
- Caliper
- Measuring tape
- Oven

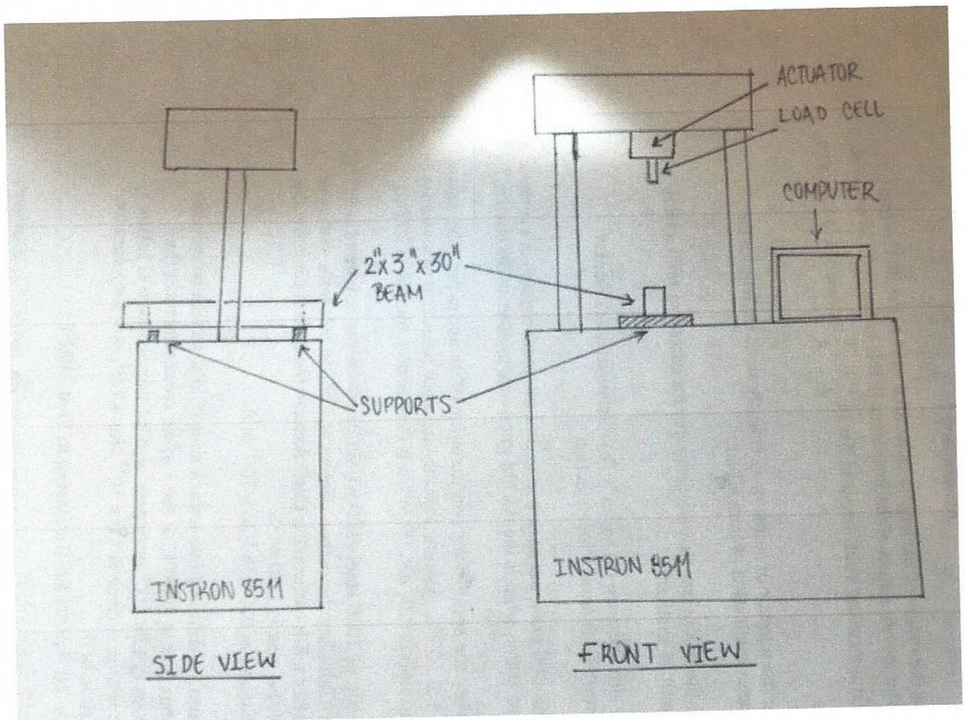

Figure 1 – Instron 8511 experimental setup, showing of the side view of the machine on the left and front view on the right.

Procedure

A clear beam, with cross-sectional dimensions of 2"x3", was chosen and cut into a 30-in. test specimen. Next, the real dimensions of the specimen were determined using the caliper. The center of the beam was then located using the measuring tape. Measurements of 14-in. in length were made from the center of the beam, in both directions to determine the location of the end supports for the specimen. Next, the specimen was taken to the testing machine, Instron 8511. The 14 groups performing the experiments were instructed to position the beams through an X-axis, with the greatest value of I (moment of inertia), by which the largest cross-sectional length (3-in.) would be placed as the height and the smallest (2-in.) as the width (see Figure 2). In placing the specimen, the side with most knots was oriented upwards in The Instron, where the specimen would be subjected to compression rather then tension forces. Given the right placement, the load-displacement of each of the 14 samples were measured. The load-displacements graphs were generated from the data acquisition software (see Appendix A).

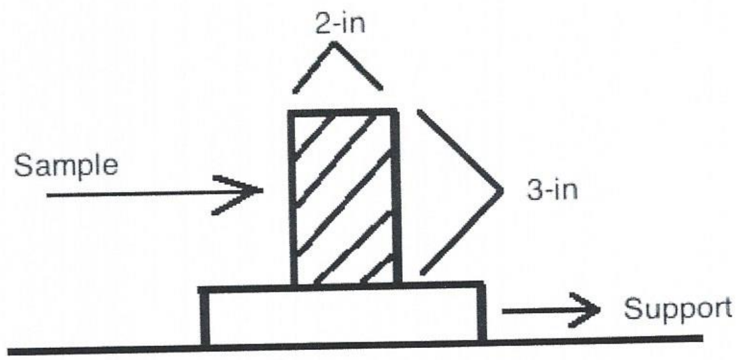

Figure 2 – Side view of the correct positioning of the beams over the supports of the Instron 8511.

DISCUSSION OF RESULTS

To test the strength of the SPF, 14 samples, all with approximately 2"x3"x30" dimensions, were tested for bending strength. The results provided by the software, and calculated from the graphs produced by it, are in Table 1. The modulus of rupture, was obtained using the following equation 1 :

$$f_R = \frac{Mc}{I} = \frac{PL}{4S} = \frac{P\,(28/4)}{1.563} = 4.479P \qquad (1)$$

Where P corresponds to the load input on the beam, the value L, in the formula is input from the span of the beam used, (L= 28-in.), and, the section modulus, S=I/c, was input from standard tabled values. It is important to observe that the moment in the formula is equal to the maximum moment from a simply supported beam with a centered concentrated load (M = PL/4). Therefore, the value of the modulus of rupture for each of the samples were obtained from the maximum load, P, as shown in equation 1. Moreover, an average value of the modulus of rupture was obtained from all the experiments to calculate the dimensions necessary for the joist.

To obtain the Modulus of Elasticity (see Table 1), the secant modulus was applied at a displacement equal to 0.4-in. for each of the samples. As seen in the table, the values of modulus of elasticity are consistent and useful for further calculations. As for the modulus of rupture, an average value of the modulus of elasticity was calculated for the design of the joist.

Table 1 – SPF Bending Results for 14 different specimens:

Test No.	**Load (lb)**	**Slope (lb/in)**	**Modulus of Rupture (lb/in^2)**	**Modulus of Elasticity (lb/in^2)**
1	1,653	2,868	7,404	0.672×10^6
2	1,827	3,350	8,183	0.785×10^6
3	1,300	2,568	5,823	0.601×10^6
4	1,560	2,868	6,987	0.672×10^6
5	1,925	3,395	8,622	0.795×10^6
6	1,740	3,018	7,793	0.707×10^6
7	1,653	3,005	7,404	0.704×10^6
8	2,240	3,750	10,033	0.878×10^6
9	1,416	2,925	6,342	0.662×10^6
10	1,376	2,968	6,163	0.695×10^6
11	1,842	3,095	8,250	0.725×10^6
12	1,571	2,563	7,037	0.600×10^6
13	1,617	2,958	7,243	0.693×10^6
14	1,776	3,043	7,955	0.713×10^6
Average			7,517.1	0.707×10^6

It is noticeable that the values for Modulus of Rupture in the table are on average uniform, however, some of the values show much smaller numbers when compared to the largest Modulus of

Rupture found. For instance, the smallest value obtained, Test No. 3, is 41.9% smaller than the largest value, Test No. 8. It should be observed that, although, the groups were instructed to position the beams along the X-axis, with the majority of knots facing upwards, one group failed to follow the correct positioning for the knots, thus one of the beams ruptured at a smaller value it should have otherwise failed. Considering this technical error during the experiment, we are still able to assume that the results for modulus of rupture can be considered consistent, since most results are close to the average value.

Nevertheless, given that the standard modulus of rupture of SPF is equal to f_U= 9093 lb/in.^{2} (Canada Wood China), and that the average value of the modulus of rupture was found to be f_R = 7517.1 lb/in.^{2} it is observed that the experiments produced an averaged error of e=17.3%. Most importantly, since the value found is smaller than the standard value, the assumption that the safety of the material is preserved, and thus, the experimental value found can be used for further calculations. Moreover, the standard value given by the NDS Supplement of Design Values for Wood Construction, shows that the modulus of elasticity for SPF, is equal to E_S=1.2 x 10^6 lb/in.^{2}, this value when compared to the average value obtained experimentally, E_{AVE}= 0.707 x 10^6 lb/in.^{2} gives an error equal to e=41.1%. As mentioned earlier, the value of modulus of elasticity in the experiment was calculated through a secant modulus, which is probably the reason of the discrepancy between the two values. Once again, since the value obtained is smaller than the standard, the value obtained can be used in the calculations, since in this situation it also preserves the safety of the design.

To determine the necessary dimensions for the joist, the group used the average modulus of rupture, f_R, and a factor of safety, F.S., equal to 6.0 to determine the allowable stress, f_{all}, for the design. For this determination, equation 2 was used:

$$f_{all} = \frac{f_r}{F.S.} \qquad (2)$$

The factor of safety was determined as 6.0. This value was determined given the deviation of the values of the modulus of rupture, the risk of harm to human life in case of material failure, and possible unexpected heavier loadings. For instance, since the area in question is a gymnasium and the load over the floor is considered distributed, and equal to w_l=150 lb/ft^2, it was considered the possibility of much heavier, concentrated loads over different areas on the floor.

Taking these considerations the value of the allowable stress is f_{all}= 1252.8 lb/in^2. In an attempt to use less material and lower the costs of the amount of lumber needed, further calculations demonstrated that a factor of safety equal to 6, is the minimum value to preserve safety and maintain reduced costs in this project.

Moreover, in order to determine the dimensions of the cross-sectional area of the joists, equation 3 was used. The value of S gives the section modulus needed for the joists, and M_{MAX} is equivalent to the maximum bending moment over the beam.

$$f_{all} = \frac{M_{MAX}}{S} \tag{3}$$

In this case, we assume a distributed load over the joist length, thus M_{MAX} in this case is equal to equation 4:

$$M_{MAX} = \frac{wl^2}{8} \tag{4}$$

Combining these two equations, we have equation 5:

$$S = \frac{wl^2}{8\, f_{all}} \tag{5}$$

To use equation 5, it is necessary the linear floor load, w, the span of the joist, l, and the distance d, between joists. (See Figure 3). From the calculations, w=16.66 lb.in., and the section modulus $S=27.66\text{-}in^3$. Given the section modulus, it was possible to determine that the size of the lumber for the joist must be 2"x12". Please see Appendix B for the complete set of calculations.

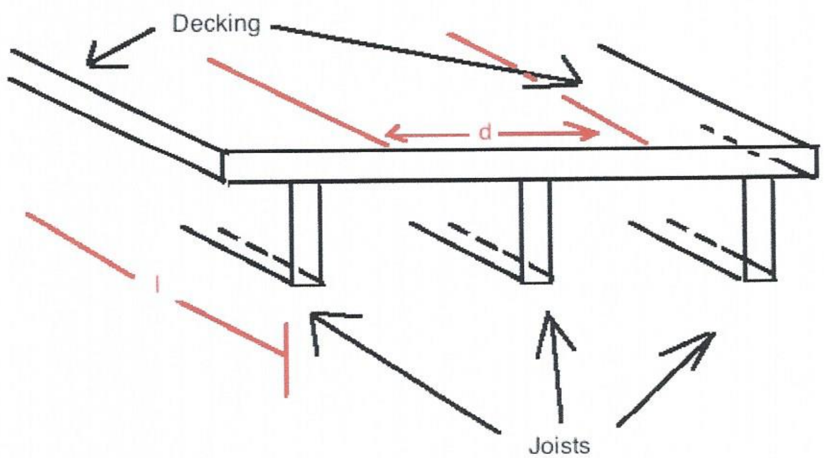

Figure 3 – Schematic representation of the decking and joists at the Athletic Center

Still, it was necessary to check the deflection of the determined beam and its limitations to assure that the lumber size of the SPF chosen met the specifications for the joist floor system design. For the design to be acceptable we must have $\delta < \delta_{MAX}$. Where, for a distributed load over a beam, the deflection is given by equation 6:

$$\delta = \frac{5\,wl^4}{384\,E_{AVE}I} \qquad (6)$$

And the maximum deflection accepted is equal to equation 7:

$$\delta_{Max} = \frac{l}{180} \qquad (7)$$

Given that the floor load is w=16.66 lb.in., span is l=129 in, average modulus of elasticity is E_{AVE}= 0.707 x 10^6 lb/in^2 and I=bh^3/12 (from the cross-sectional area chosen). The resulting deflection obtained from the chosen lumber size is equal to δ=0.478-in. and $\delta_{MAX=}$ 0.717-in., therefore, the lumber size chosen is adequate.

Therefore, the results are acceptably close to our theoretical expectations; they are also adequate and can produce safe calculations of the dimensions of the cross-sectional area of the new joists for the floor system of the Costello Athletic Center

APPENDIX A – Load-Displacement Graphs

Data contained in Figure A-1 were obtained through the Instron 8511 software package, for specimen #14 (Figure A-1 and Figure A-2) and specimens #11 to #14 (Figure A-3).

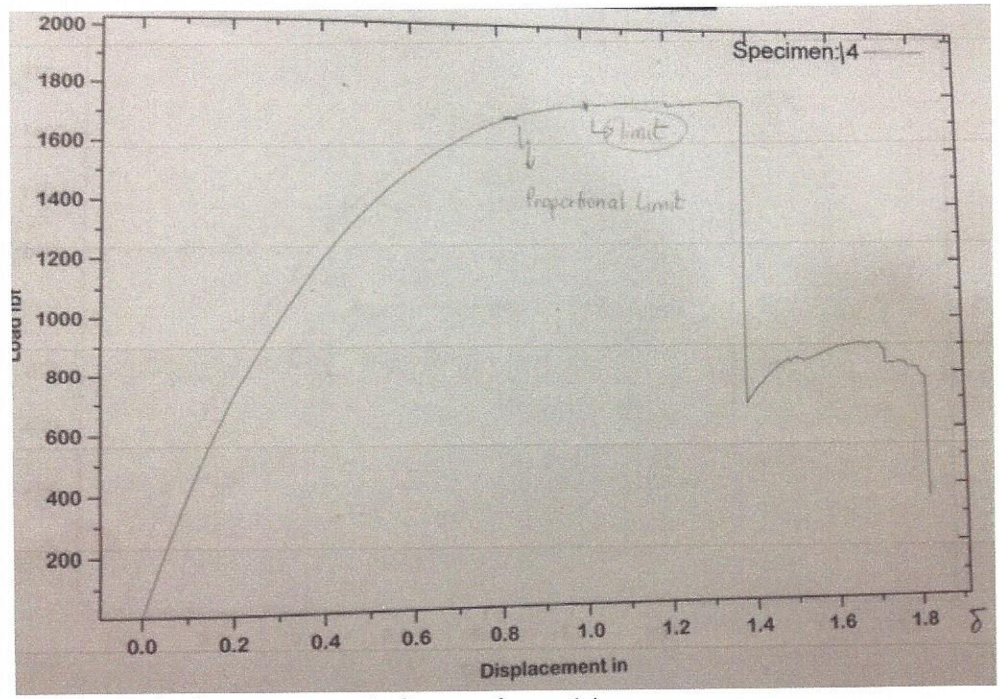

Figure A-1 – Load-Displacement graph for specimen 14

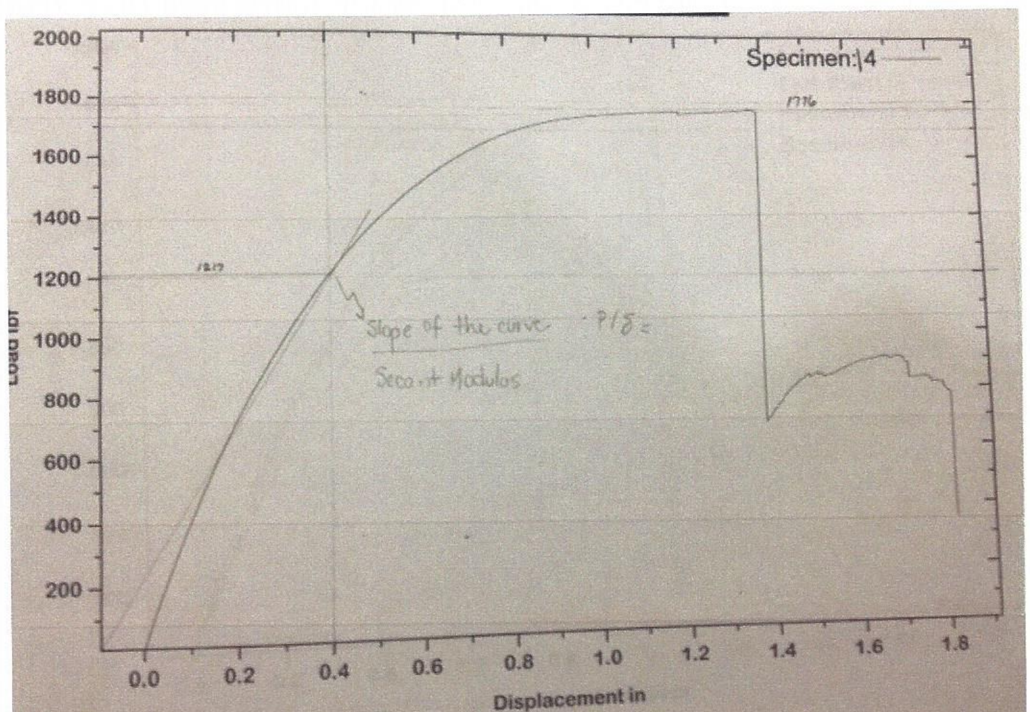

Figure A-2 – Load-Displacement graph with secant modulus lines and maximum load

APPENDIX A – Load-Displacement Graphs

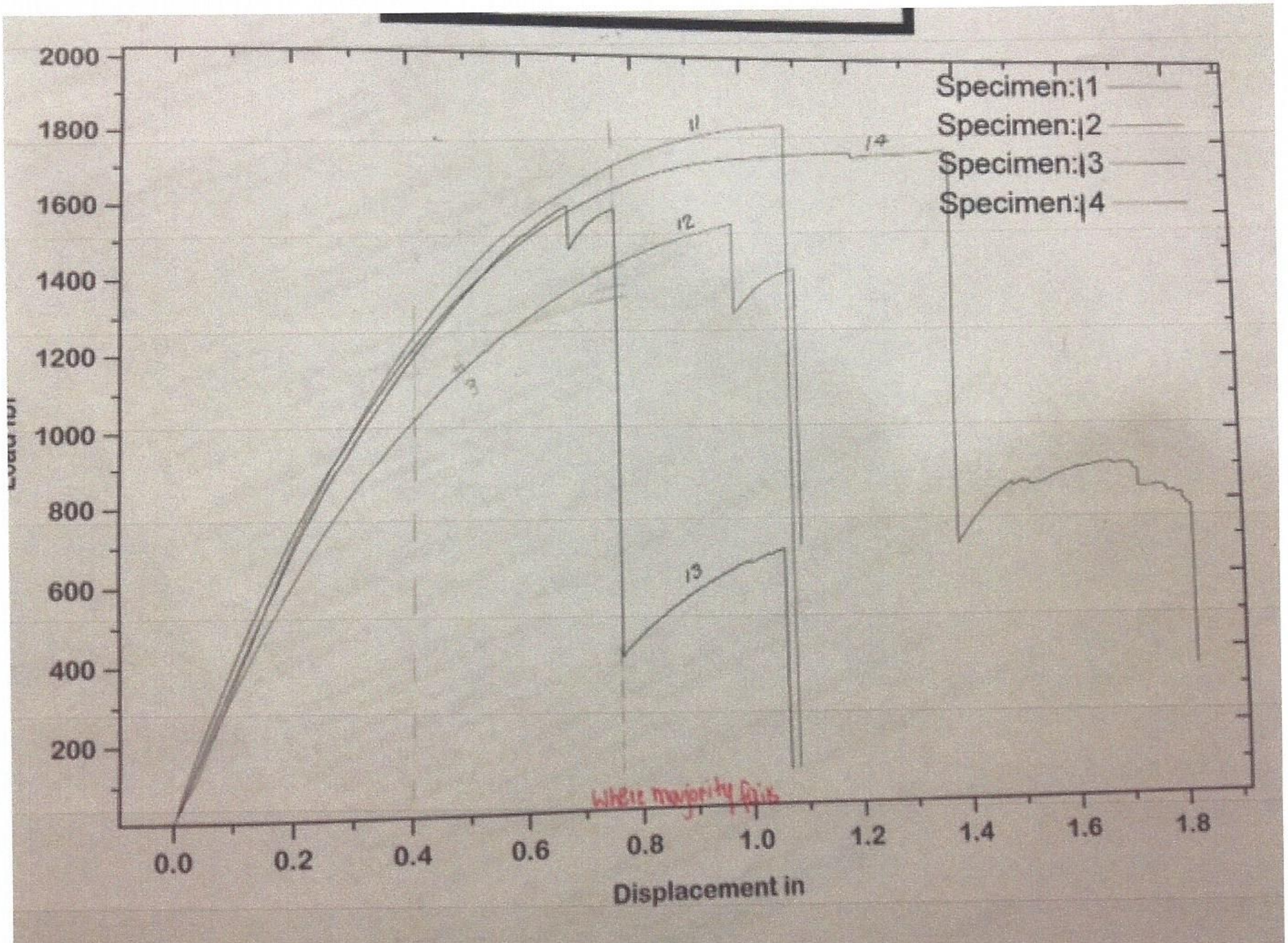

Figure A-3 – Load-Displacement graph for specimens 11 to 14

APPENDIX B – Joist Dimensions Calculations

For the selection of the joist dimensions it was necessary to determine the number of variables that were measured, and the ones obtained from the experiment, its collection and the calculations that resulted in the final resulting dimensions are given on the table 1-B and steps 1 to 6 below:

Table 1-B - Variable Names, Indices and Values

Variable Name	Variable	Variable Value
Average modulus of rupture	f_R	7517.1 lb/in^2
Floor load over an area:	w_1	150 lb/ft^2
Floor load over a length	w	**to be determined**
Width distance of floor	d	16 in.
Joist span	l	129 in.
Average modulus of elasticity	E_{AVE}	0.707 x 10^6 lb/in^2
Factor of Safety	F.S.	6.0
Moment of Inertia	I	Bh^3/12

(1) <u>Allowable Stress Determination:</u> From equation 1:

$$f_{all} = \frac{f_r}{F.S.} = \frac{7517.1\text{ lb/in2}}{6.0} = 1252.8\text{ lb}/in^2 \qquad (1)$$

(2) <u>Floor load over a length:</u>

$$w = \frac{w_1 d}{12^2} = \frac{150\frac{\text{lb}}{\text{ft2}} x\ 16\ in}{12^2\ in2/ft2} = 16.66\ lb/in \qquad (2)$$

(3) <u>Section Modulus Determination:</u> From equation 5:

$$S = \frac{wl^2}{8\ f_{all}} = \frac{16.66\frac{lb}{in} x\ (129in)^2}{8\ x\ 1252.8\frac{\text{lb}}{\text{in2}}} = 27.67\ in^3 \qquad (5)$$

(4) <u>Lumber Size Determination:</u>

From the NDC table, the lumber size to be used is a **2in.x 12in.**

(5) <u>Maximum Allowable Deflection:</u> From equation:

$$\delta_{Max} = \frac{l}{180} = \frac{129in}{180} = 0.717\ in \qquad (6)$$

(6) <u>Deflection From Determined Dimensions:</u> From equation 7:

$$\delta = \frac{5\ wl^4}{384\ E_{AVE} I} = \frac{5\ x\ 16.66 \frac{lb}{in} x\ (129in)^4}{384\ x\ 0.707\ \text{x}\ 106 \frac{\text{lb}}{\text{in2}} x\ I} = 0.478\ in \qquad (7)$$