PHYSICS X0323: Scientific Analysis & Modeling - Fall 2024

PHYS 20323/60323: Fall 2024

December 7, 2024

1

An electron is to be found in the spin state (in the z-basis): $\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$

(a) (5 points) Determine if the possible values of A such that the state is normalized.

(b) (5 points) Find the expectation values of the operators S_x, S_y, S_z and $\vec{S}^2$.

The matrix representations in the z-basis for the components of electron spin operators are given by:

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

2. The average electrostatic field in the earth's atmosphere in the fair weather is approximately given:

$$\vec{E} = E_0(Ae^{-\alpha}z + Be^{-\beta z}, \tag{1}$$

where A,B,α, β are positive constants and z is the height above the (locally flat) earth surface.

(a) (5 points) Find the average charge density in the atmosphere as a function of height.

(b) (5 points) Find the electric potential as a function height above the earth.

3. **The following questions refer to stars in the Table below.**

Note: There may be multiple answers.

Name	Mass	Luminosity	Lifetime	Temperature	Radius
β Cyg.	$1.3\,M_\odot$	$3.5\,L_\odot$			$1\,R_\odot$
α Cen.	$1.0\,M_\odot$				
η Car.	$60.\,M_\odot$	$10^6\,L_\odot$	8.0×10^5 years		
ε Eri.	$6.0\,M_\odot$	$10^3\,L_\odot$		20,000 K	
δ Scu.	$2.0\,M_\odot$		5.0×10^8 years		$2\,R_\odot$
γ Del.	$0.7\,M_\odot$		4.5×10^{10} years	5000 K	

(a) (4 points) Which one of these stars will produce a planetary nebula.

(b) (4 points) Elements heavier than *Carbon* will be produced in which stars.