A Causal Effective Field Theory for Dark Energy with Coherence Length L_c and Relaxation Time τ_I

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We propose an effective scalar field $I(x)$ describing an "informational" structure with a characteristic coherence length L_c and relaxation time τ_I. An exponential non-local regulator $\mathcal{F}_{L_c}[\Box] \simeq (1 - L_c^2\Box)^{-1}$ ensures finite response time (causality) without introducing additional ghost-like degrees of freedom. The field exhibits telegraph-like dynamics, induces small anisotropic stresses due to finite L_c, and produces testable, scale-dependent signatures in large-scale structure observables: a knee in $P(k)$, mild scale-dependent growth, and small $\Phi - \Psi$ splitting impacting lensing and ISW. We provide the action, energy-momentum tensor, a causal response kernel, and a mapping to phenomenological functions $\mu(k, a)$ and $\gamma(k, a)$. Pilot parameter values indicate effects at the edge of the sensitivity of Euclid/LSST/DESI.

I. INTRODUCTION

The ΛCDM model provides an excellent fit to current data, yet the microphysics of dark energy and dark matter remains elusive. Here we present a concrete EFT prototype based on a single effective scalar $I(x)$ endowed with two physical scales: a coherence length L_c and a relaxation time τ_I. These parameters control non-instantaneous response and finite-range correlations. **Ghost-free & causal:** we employ the exponential regulator $\mathcal{F}_{L_c}[\Box] = (1 - L_c^2\Box)^{-1}$; in the EFT regime it does not add new propagating poles in the physical sheet, hence preserves unitarity and remains ghost-free while enforcing finite signal speed (no instantaneous propagation).

II. ACTION AND EQUATIONS OF MOTION

We consider the Einstein–Hilbert action coupled to $I(x)$:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{Z}{2}\nabla_\mu I\nabla^\mu I - \frac{\alpha}{2L_c^2}I\,\mathcal{F}_{L_c}[\Box]\,I - V(I)\right], \tag{1}$$

with the non-local operator $\mathcal{F}_{L_c}[\Box] \simeq (1-L_c^2\Box)^{-1}$ acting as an exponential regulator.[1] *Equation of motion.* Variation of the action with respect to I yields a telegraph-type dynamics (schematically):

$$\tau_I\,\nabla^\mu\nabla_\mu(\dot{I}) + \dot{I} \;=\; c_I^2\,\Box I \;-\; V'(I), \tag{2}$$

which ensures finite response time and avoids instantaneous reaction of the medium.

III. ENERGY-MOMENTUM TENSOR AND CONSERVATION

Varying the action with respect to the metric gives

$$T_{\mu\nu}^{(I)} = Z\,\nabla_\mu I\,\nabla_\nu I - g_{\mu\nu}\left[\frac{Z}{2}(\nabla I)^2 + \frac{\alpha}{2L_c^2}I\,\mathcal{F}_{L_c}[\Box]\,I + V(I)\right] + \Pi_{\mu\nu}^{(L_c)}, \tag{3}$$

where $\Pi_{\mu\nu}^{(L_c)}$ encodes small anisotropic stresses induced by finite L_c (the explicit form depends on the chosen regulator representation). Diffeomorphism invariance implies $\nabla^\mu T_{\mu\nu}^{(I)} = 0$, consistent with the Bianchi identity when coupled to GR.

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[1] In the local limit $L_c \to 0$ the theory reduces to a k-essence-like scalar with standard gradient expansion.

IV. BACKGROUND EVOLUTION

In an FLRW background, the effective energy density and pressure read

$$\rho_I = \frac{Z}{2}\dot{I}^2 + V(I) + \rho_{\rm nl}(L_c), \qquad p_I = \frac{Z}{2}\dot{I}^2 - V(I) + p_{\rm nl}(L_c). \tag{4}$$

For a slow drift of I one finds an equation-of-state parameter

$$w_I(a) \simeq -1 + \epsilon(a), \qquad \epsilon \equiv \frac{Z\dot{I}^2}{\rho_I} \ll 1, \tag{5}$$

so the background mimics a near-Λ behavior with controlled departures.

V. PERTURBATIONS AND SCALE-DEPENDENT SOUND SPEED

Linear perturbations feature a scale-dependent sound speed

$$c_s^2(k,a) \approx \frac{c_I^2\, k^2}{k^2 + k_c^2(a)}, \qquad k_c(a) \equiv \frac{a}{L_c}, \tag{6}$$

so long-wavelength modes ($kL_c \ll 1$) are "soft" with small c_s^2, while short modes approach c_I^2. Finite L_c also induces anisotropic stress contributions $\propto (kL_c)^2$, which vanish on very large scales.

VI. CAUSAL RESPONSE KERNEL

In linear response, the field reacts to a source $S(k,a)$ via

$$\delta I(k,a) = \int^a da'\, K\!\left(kL_c, \frac{a-a'}{aH\tau_I}\right) S(k,a'), \tag{7}$$

with a causal kernel

$$K(x,y) = \Theta(y)\, e^{-y}\, \frac{\sin\!\left(\sqrt{x^2 - \beta^2}\, y\right)}{\sqrt{x^2 - \beta^2}}, \tag{8}$$

where $x = kL_c$ and $y = (t - t')/\tau_I$ are dimensionless, $\beta \lesssim 1$ controls damping, and Θ enforces causality ($K = 0$ for $y < 0$).

VII. OBSERVABLE SIGNATURES

At the level of linear cosmological perturbations, the model maps onto effective modifications of the Poisson equation and gravitational slip:

$$\mu(k,a) \simeq 1 + \frac{\beta_I^2(a)\, k^2}{k^2 + k_*^2(a)}, \tag{9}$$

$$\gamma(k,a) \simeq 1 - \frac{\eta_I(a)\, k^2}{k^2 + k_*^2(a)}, \tag{10}$$
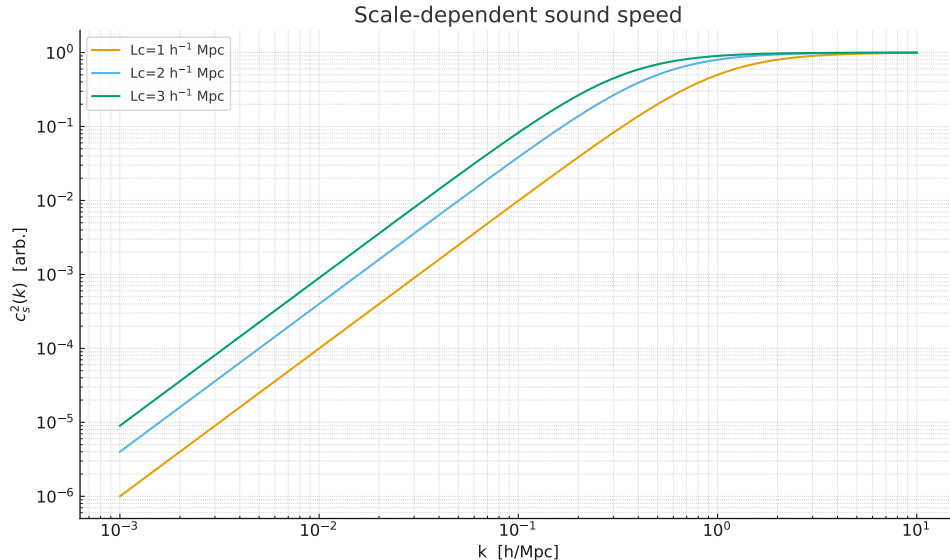
where the transition scale is

$$k_*(a) \sim \frac{a}{\sqrt{L_c^2 + c_I^2\tau_I^2}}. \tag{11}$$

This produces a distinctive "knee" in the matter power spectrum near $k \sim k_*$, a mild k-dependence of the growth rate $f(k,z)$, and a small $\Phi - \Psi$ split affecting lensing and ISW.

Parameter	Value (pilot)	Observable impact
L_c	$1–3\,h^{-1}\mathrm{Mpc}$	$k_* \sim 0.3\text{–}1\,h\,\mathrm{Mpc}^{-1}$ (knee in $P(k)$)
τ_I	$0.1–1\,\mathrm{Gyr}$	ISW lag $\Delta C_\ell^{T\phi}/C_\ell^{T\phi} \sim 2\text{–}5\%$ at $\ell \sim 10\text{–}50$
β_I	~ 0.1	$\Delta(f\sigma_8) \sim 3\text{–}7\%$; lensing shift $\sim 2\text{–}4\%$

TABLE I. Pilot parameter values and indicative observational signatures.

FIG. 1. Scale-dependent sound speed $c_s^2(k)$ for different coherence lengths L_c. Long-wavelength modes $(kL_c \ll 1)$ have suppressed sound speed, while short modes approach c_I^2.

A. Pilot values and expected effects

B. Predicted signatures

- Knee in $P(k)$ at $k \sim k_*$ with weak redshift evolution (via a/τ_I);

- Mild scale dependence of the growth rate $f(k,z)$, especially for $k \lesssim k_*$;

- Small but non-zero gravitational slip $\Phi - \Psi$ impacting lensing and ISW cross-correlations;

- Real-space smoothing of correlation functions on $r \sim L_c$.

VIII.   CONCLUSION AND OUTLOOK

We presented a minimal, causal and ghost-free EFT prototype for dark energy, specified by a scalar field $I(x)$ with scales (L_c, τ_I). The theory preserves covariant conservation, yields scale-dependent perturbations with small anisotropic stress, and predicts testable signatures in $P(k)$, lensing, RSD, and ISW. **Outlook:** the next step is numerical implementation in Boltzmann solvers (CLASS/HiCLASS) to generate parameter forecasts and direct data comparisons with Euclid, LSST and DESI.

[1] Planck Collaboration, *Planck 2018 results. VI. Cosmological parameters*, A&A **641**, A6 (2020).
[2] C. Armendariz-Picon, V. Mukhanov, and P. J. Steinhardt, *Essentials of k-essence*, Phys. Rev. D **63**, 103510 (2001).
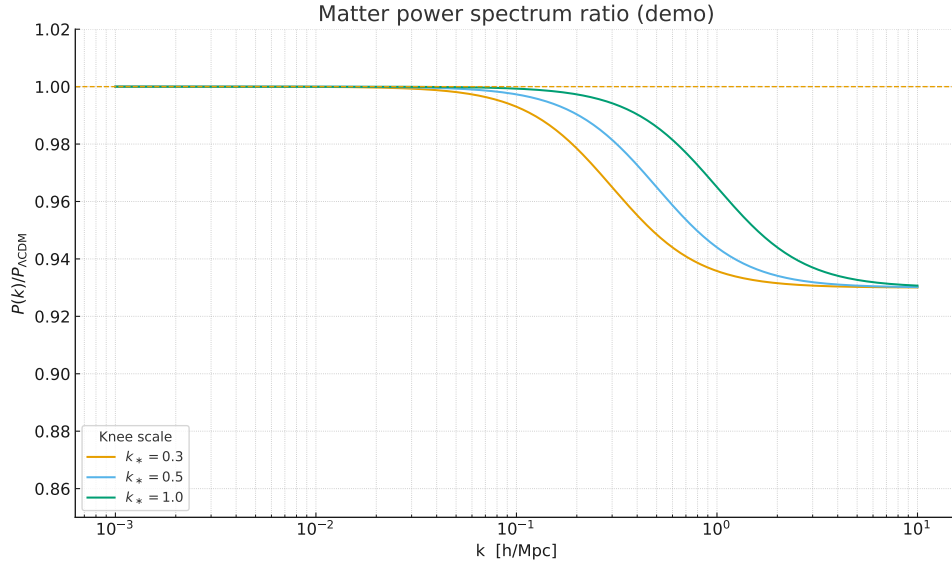[3] A. De Felice and S. Tsujikawa, *f(R) theories*, Living Rev. Rel. **13**, 3 (2010).

FIG. 2. Ratio of matter power spectrum $P(k)/P_{\Lambda\mathrm{CDM}}$ for illustrative transition scales k_*. The presence of a knee at $k \sim k_*$ is a key observational signature.
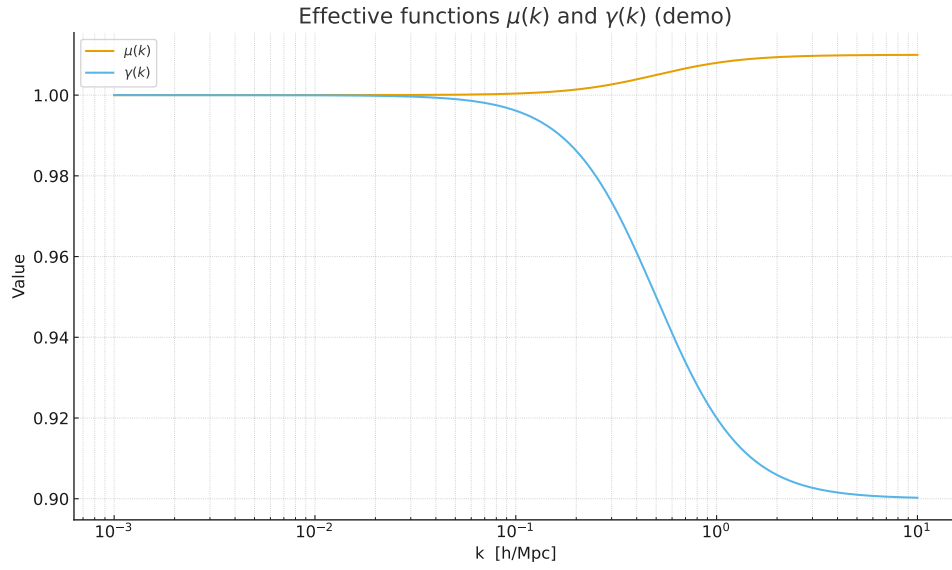

FIG. 3. Effective modification functions $\mu(k)$ and $\gamma(k)$ for pilot parameters (L_c, τ_I, β_I). The deviations from unity occur around the transition scale k_*.

[4] Euclid Collaboration, *Euclid mission overview and science goals*, (refs.).
[5] LSST Dark Energy Science Collaboration, *Science requirements and forecasts*, (refs.).
[6] DESI Collaboration, *The DESI experiment: design, commissioning, and early results*, (refs.).