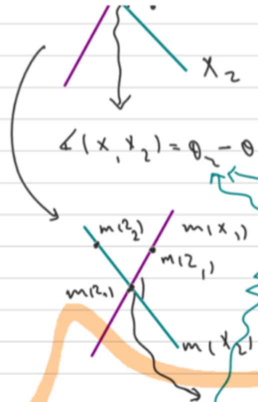
$$\frac{\cdots}{Re(z_2-z_0)} =: s_2$$



$$\theta_1 = Tg^{-1}s_1 \,,\quad \theta_2 = tg^{-1}s_2$$

$$\measuredangle(x_1,x_2) = \theta_2-\theta_1 = Tg^{-1}s_2 - tg^{-1}s_1$$

$\frac{az+b}{m(z)}$

$a = r_0 e^{i\theta_0}$

اگر $m(z) = az+b$

$m(x_1)$ شیب : $\frac{Im(\underbrace{m(z_1)}_{az_1+b}-\underbrace{m(z_0)}_{az_0+b})}{Re(m(z_1)-m(z_0))} =: T_1 = \frac{Im(a(z_1-z_0))}{Re(a(z_1-z_0))}$

$m(x_2)$ شیب : $\frac{Im(\underbrace{m(z_2)}_{az_2+b}-\underbrace{m(z_0)}_{az_0+b})}{Re(m(z_2)-m(z_0))} =: T_2 = \frac{Im(a(z_2-z_0))}{Re(a(z_2-z_0))}$ $=?$

$$\measuredangle(m(x_1),m(x_2)) = \underbrace{Tg^{-1}T_1 - Tg^{-1}T_2}_{\tilde\theta_1 - \tilde\theta_2}$$

$\tilde\theta_1 = \theta_1+\theta_0$

$\tilde\theta_2 = \theta_2+\theta_0$

$$\tilde\theta_2-\tilde\theta_1 = (\theta_2+\cancel{\theta_0})-(\theta_1+\cancel{\theta_0}) = \theta_2-\theta_1 \;=?$$

$$\frac{Im(z_1-z_0)}{Re(z_1-z_0)} = \frac{r_1\sin\theta_1}{r_1\cos\theta_1} = \tan\theta_1$$

فرض کنید $z_1-z_0=r_1e^{i\theta_1}$ و $a=r'e^{i\theta'}$ $\Leftarrow$

$$\frac{Im(az_1+b-(az_0+b))}{Re(az_1+b-(az_0+b))} = \frac{Im(a(z_1-z_0))}{Re(a(z_1-z_0))} = \frac{Im(r_1r'e^{i(\theta_1+\theta')})}{Re(r_1r'e^{i(\theta_1+\theta')})}$$

$$= \frac{r_1r'\sin(\theta_1+\theta')}{r_1r'\cos(\theta_1+\theta')} = \tan(\theta_1+\theta')$$

مشابها برای $z_2-z_0=r_2e^{i\theta_2}$ داریم:

$$\frac{Im(z_2-z_0)}{Re(z_2-z_0)} = \tan(\theta_2+\theta')$$

$$\Rightarrow [\theta_2+\theta'] - [\theta_1+\theta'] = \theta_2-\theta_1 \quad ✓$$

آنها مساوی اند