PDEs of First Order

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* general form: $F(x, y, u, u_x, u_y) = 0$
  ↳ for $u(x,y)$

* general solution: $f(\phi(x,y,u), \psi(x,y,u)) = 0$

* quasi-linear PDE: $a(x,y,u)\,u_x + b(x,y,u)\,u_y = c(x,y,u)$

Special Cases:

1. transport equation: $u_t + v\,u_x = 0$ where v is a constant

   → solution: $u(x,t) = f(x - vt)$

   where f is any differentiable function of one variable

② $u_x + p(x,y)\,u_y = 0$

solution: $u(x,y) = f(\psi)$

where $\psi(x,y) = C$ is the solution of $\dfrac{dy}{dx} = p(x,y)$

and f is any differentiable function of one variable

③ more generalized: $\boxed{u_x + p(x,y)\,u_y = c(x,y,u)}$

solution: $f(\phi, \psi) = 0$

where $\phi = c_1$ is the solution of $\dfrac{dy}{dx} = p(x,y)$

and $\psi = c_2$ is the solution of $\dfrac{du}{dx} = c(x,y,u)$