$$u_t - 9u_{xx} = x + t \qquad 0 < x < 1 \qquad 0 < t$$

$$u(x, 0) = 1 - \cos \pi x \qquad 0 \le x \le 1$$

$$u(0, t) = 0 \qquad u(1, t) = 2 \qquad t \ge 0$$

$$u(x, t) = v(x, t) + 2x$$

$$u_t = v_t \quad , \quad u_{xx} = v_{xx} \implies v_t - 9v_{xx} = x + t \implies v(x, t) = \sum_{n=1}^{\infty} G_n(t) \sin n\pi x$$

$$\implies \sum \dot{G}_n \sin n\pi x + 9n^2\pi^2 \sum G_n \sin n\pi x = \sum \left(\dot{G}_n + 9n^2\pi^2 G_n \right) \sin n\pi x = x + t$$

$$\implies \boxed{\; 2\int_0^1 (x+t)\sin(n\pi x)\, dx = \dot{G}_n + 9n^2\pi^2 G_n \;} \rightarrow \text{linear first-order ODE}$$

$$h = \int 9n^2\pi^2 \, dt = 9n^2\pi^2 t$$

$$\implies G_n = e^{-9n^2\pi^2 t}\left[2\int e^{9n^2\pi^2 t}\int_0^1 (x+t)\sin(n\pi x)\, dx\, dt + C \right]$$

حصر تحيا