$$u_t - 9u_{xx} = x + t \qquad 0 < x < 1 \qquad 0 < t$$

$$u(x,0) = 1 - \cos \pi x \qquad 0 \le x \le 1$$

$$u(0,t) = 0 \qquad u(1,t) = 2 \qquad t \ge 0$$

$$u(x,t) = v(x,t) + 2x$$

$$u_t = v_t \;,\; u_{xx} = v_{xx} \Rightarrow v_t - 9v_{xx} = x + t \Rightarrow v(x,t) = \sum_{n=1}^{\infty} G_n(t) \sin n\pi x$$

$$\Rightarrow \sum \dot{G}_n \sin n\pi x + 9n^2\pi^2 \sum G_n \sin n\pi x = \sum \left(\dot{G}_n + 9n^2\pi^2 G_n\right) \sin n\pi x = x + t$$

$$\Rightarrow \boxed{2 \int_0^1 (x+t) \sin(n\pi x)\, dx = \dot{G}_n + 9n^2\pi^2 G_n} \rightarrow \text{linear first-order ODE}$$

$$h = \int 9n^2\pi^2\, dt = 9n^2\pi^2 t$$

$$\Rightarrow G_n = e^{-9n^2\pi^2 t} \left[2 \int e^{9n^2\pi^2 t} \int_0^1 (x+t) \sin(n\pi x)\, dx\, dt + C \right]$$

آناهیتا صبری

اگر در معادله‌ی موج، $f(x)\equiv 0$ کشش $g(x)$ به صورت است؟

$$u(x,t)=\frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,ds$$

$$\begin{cases} u(0,t)=0 \Rightarrow \int_{-ct}^{ct} g(s)\,ds=0 \Rightarrow G(ct)=G(-ct) \Rightarrow & G \text{ فرد است} \\ u(l,t)=0 \Rightarrow \int_{l-ct}^{l+ct} g(s)\,ds=0 \Rightarrow G(l+ct)=G(l-ct) & G \text{ با تناوب } 2l \end{cases}$$

$$\star\ G'=g$$