تمرین:

$$u(t) = \begin{cases} 0 & -l < t < 0 \\ E \sin \omega t & 0 < t < l \end{cases} \qquad p = 2l = \frac{2\pi}{\omega}$$

حل:

$$a_n = \frac{2}{2l} \int_{-l}^{l} u(t) \cos\left(\frac{n\pi}{l} t\right) dt = \frac{1}{l} \int_{0}^{l} E \sin\left(\frac{\pi}{l} t\right) \cos\left(\frac{n\pi}{l} t\right) dt$$

$$= \frac{E}{l} \left[\frac{-l\left(\cos(n\pi) + 1\right)}{\pi(n^2 - 1)} \right] = \frac{-E\left((-1)^n + 1\right)}{\pi(n^2 - 1)} = \begin{cases} 0 & \text{فرد } n \\ \dfrac{-2E}{\pi(n^2 - 1)} & \text{زوج } n \end{cases}$$

$$b_n = \frac{2}{2l} \int_{-l}^{l} u(t) \sin\left(\frac{n\pi}{l} t\right) dt = \frac{1}{l} \int_{0}^{l} E \sin\left(\frac{\pi}{l} t\right) \sin\left(\frac{n\pi}{l} t\right) dt$$

$$= \frac{E}{l} \left[\frac{-l \sin(n\pi)}{\pi(n^2 - 1)} \right] = 0$$

$$\Rightarrow \boxed{u(t)_{\text{سری فوریه}} = \frac{E}{\pi} - \frac{2E}{\pi} \sum_{k=1}^{\infty} \frac{\cos\left(\frac{2k\pi}{l} t\right)}{4k^2 - 1}}$$

آناهیتا حیدری

سری فوریه کسینوسی؟ $f(x) = x \quad 0 < x < 2$

زوج $f^*(x) = \begin{cases} x & 0 < x < 2 \\ -x & -2 < x < 0 \end{cases} \Rightarrow l = 2$

$b_n = 0$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi}{l}x\right)dx = \int_0^2 x\cos\left(\frac{n\pi}{2}x\right)dx$$

$$= \underbrace{\frac{x\sin\left(\frac{n\pi}{2}x\right)}{\frac{n\pi}{2}}\Bigg]_0^2}_{0} - \int_0^2 \frac{\sin\left(\frac{n\pi}{2}x\right)}{\frac{n\pi}{2}}dx = 4\,\frac{\cos\left(\frac{n\pi}{2}x\right)}{n^2\pi^2}\Bigg]_0^2$$

$$= \frac{4}{n^2\pi^2}\left((-1)^n - 1\right)$$

$$= \begin{cases} 0 & n \text{ زوج} \\ \frac{-8}{n^2\pi^2} & n \text{ فرد} \end{cases}$$

$$a_0 = \int_0^2 x\,dx = 2$$

$\Rightarrow$ سری فوریه کسینوسی f روی $(0,2)$ =

$$\boxed{1 + \frac{-8}{\pi^2} \sum_{k=0}^{\infty} \frac{\cos((2k+1)x)}{(2k+1)^2}}$$

کد MATLAB:

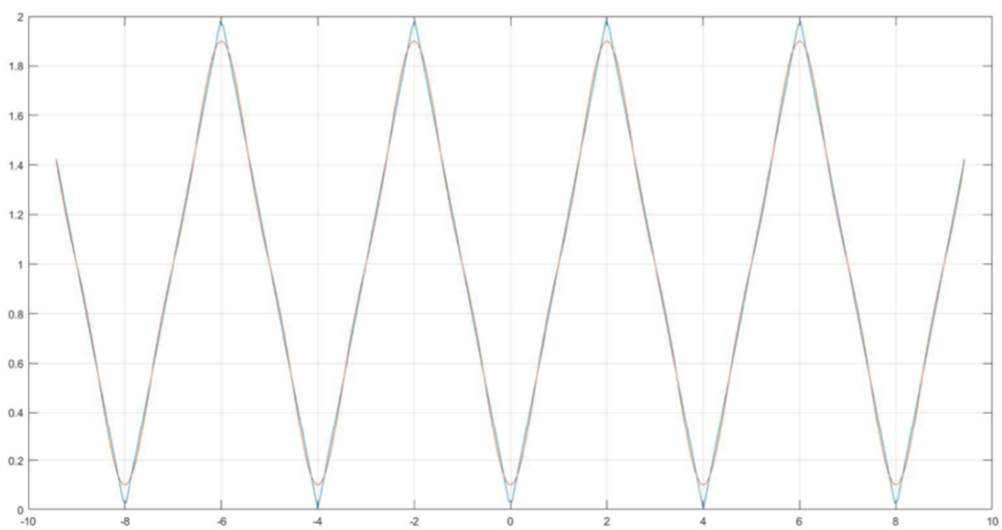
```

g=linspace(-3*pi,3*pi,300);
figure
plot(g,per(g),g,coF(g))
grid

double x
function [y]=per(x)
    pw = @(x)  (x<2).*x+(x>2).*(-x+4);
    y=pw(x-4*floor(x/4.0));
end
function [y]=coF(x)
    syms k
    y=1+symsum(cos(pi*k*x/2)*4*((-1)^k-1)/((pi^2)*(k^2)),k,1,3);
end

```

اجرای کد به ازای $k = ۳$:



اجرای کد به ازای $k = ۳۰$: