تمرین:

$$u(t)=\begin{cases}0 & -l<t<0\\ E\sin\omega t & 0<t<l\end{cases}\qquad p=2l=\frac{2\pi}{\omega}$$

حل:

$$a_n=\frac{2}{2l}\int_{-l}^{l}u(t)\cos\left(\frac{n\pi}{l}t\right)dt=\frac{1}{l}\int_{0}^{l}E\sin\left(\frac{\pi}{l}t\right)\cos\left(\frac{n\pi}{l}t\right)dt$$

$$=\frac{E}{l}\left[\frac{-l\left(\cos(n\pi)+1\right)}{\pi(n^2-1)}\right]=\frac{-E\left((-1)^n+1\right)}{\pi(n^2-1)}=\begin{cases}0 & n \text{ فرد}\\ \dfrac{-2E}{\pi(n^2-1)} & n \text{ زوج}\end{cases}$$

$$b_n=\frac{2}{2l}\int_{-l}^{l}u(t)\sin\left(\frac{n\pi}{l}t\right)dt=\frac{1}{l}\int_{0}^{l}E\sin\left(\frac{\pi}{l}t\right)\sin\left(\frac{n\pi}{l}t\right)dt$$

$$=\frac{E}{l}\left[\frac{-l\sin(n\pi)}{\pi(n^2-1)}\right]=0$$

$$\Rightarrow\quad \boxed{u(t)_{\text{سری فوریه}}=\frac{E}{\pi}-\frac{2E}{\pi}\sum_{k=1}^{\infty}\frac{\cos\left(\frac{2k\pi}{l}t\right)}{4k^2-1}}$$

آناهیتا میری