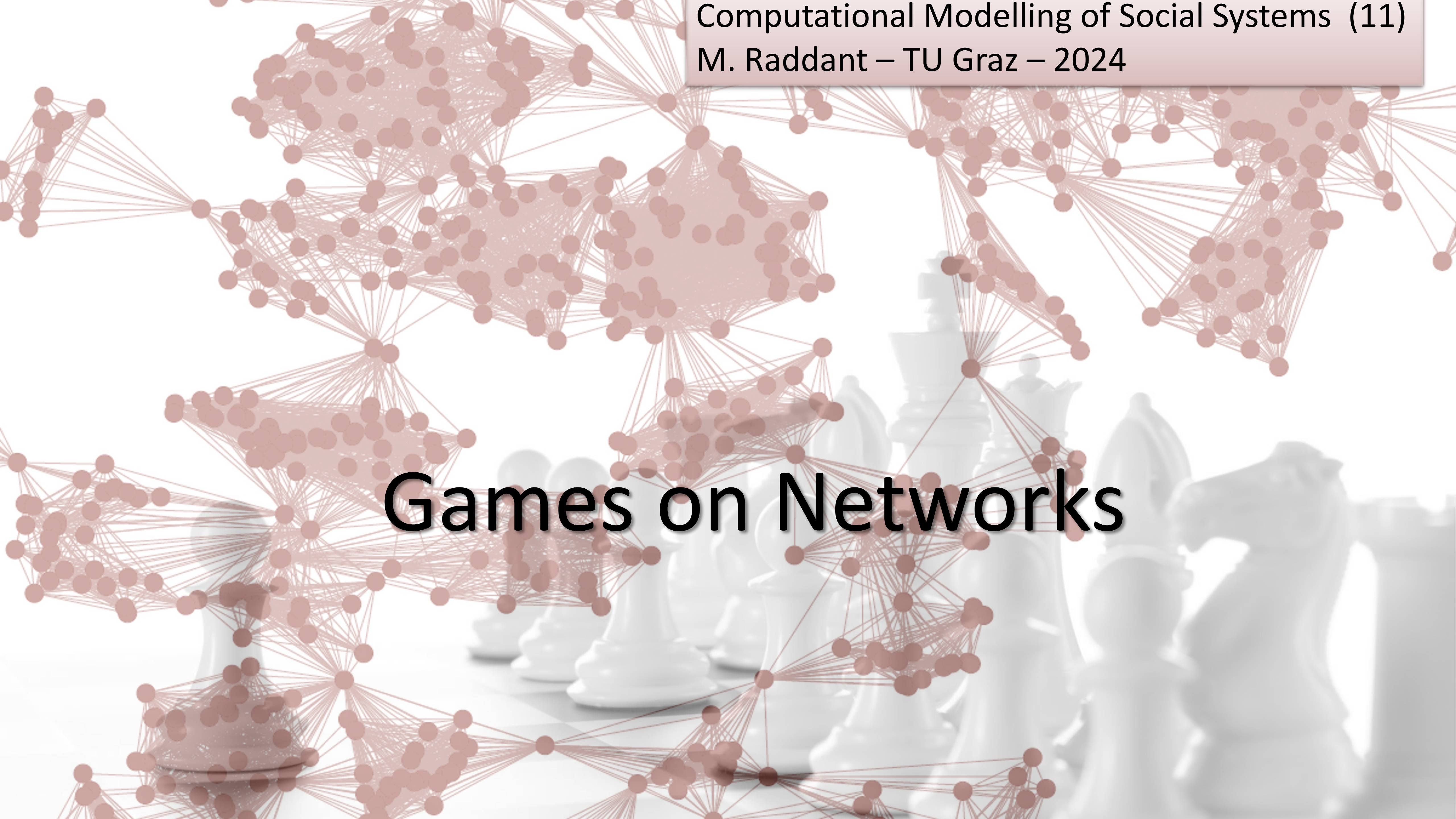


Games on Networks

Today

Network Science and Game Theory

- Game Theory basics
- Games on fixed networks
- Game Theory and network structure

Background literature for this lecture: A. Wilhite, Economic Activity on fixed Networks, in: Handbook of Computational Economics, Vol. 2, L. Tesfatsion and K.L. Judd (ed.), Elsevier 2006.

Describing games

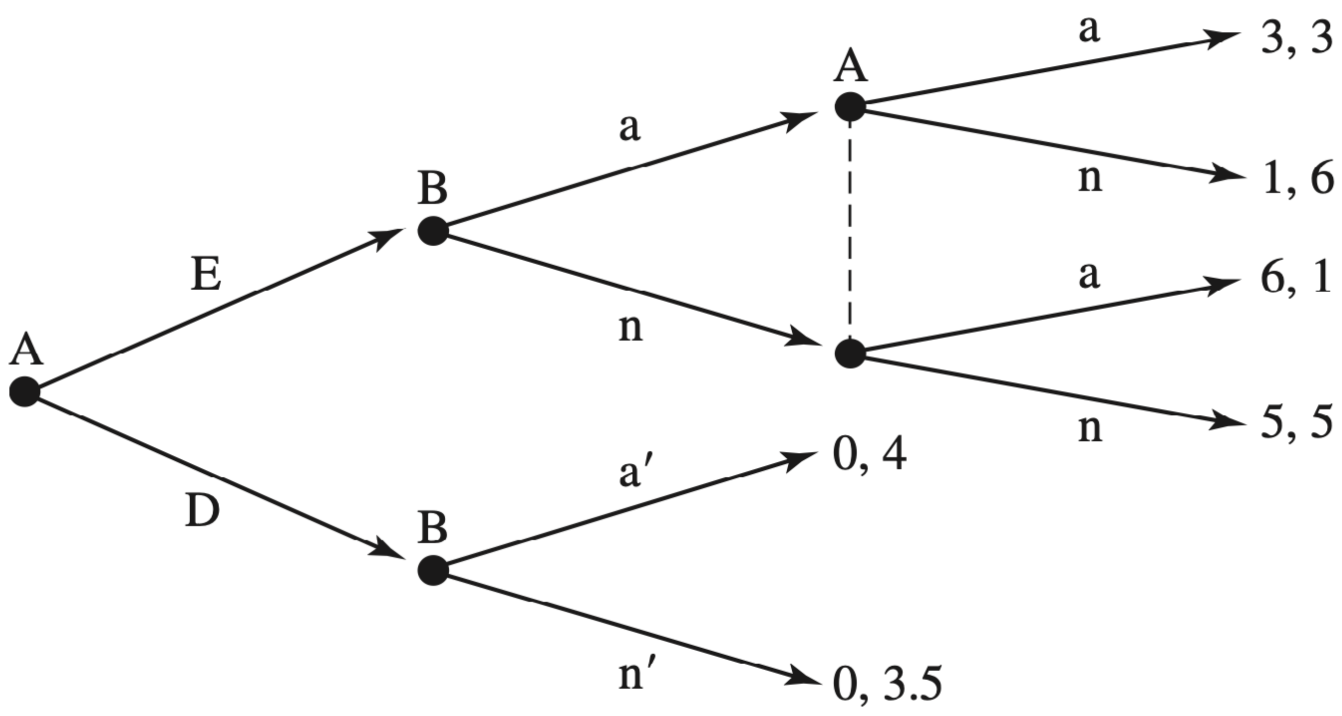
- list of players
- description of what players can do
- description of what they know when
- outcomes dependent on actions
- preferences over outcomes

Strategies

- A strategy is a complete contingent plan for a player in the game

The extensive form

Firm A decides whether to enter firm B's industry. Firm B observes this decision. If firm A enters, then the two firms simultaneously decide whether to advertise. Otherwise, firm B alone decides whether to advertise. With two firms in the market, the firms earn profits of $3 million each if they both advertise and $5 million if they both do not advertise. If only one firm advertises, then it earns $6 million and the other earns $1 million. When firm B is solely in the industry, it earns $4 million if it advertises and $3.5 million if it does not advertise. Firm A earns nothing if it does not enter.



Background literature Game Theory: J. Watson, Strategy (3rd edition), 2013, W.W. Norton

Normal-form games

Matrix representation: each row corresponds to a strategy of player 1, each column corresponds to a strategy of player 2.

Each cell of the matrix corresponds to a strategy profile and contains the payoffs for each player.

		2	
1		H	T
	H	1, -1	-1, 1
	T	-1, 1	1, -1

Matching Pennies

		2	
1		A	B
	A	1, 1	0, 0
	B	0, 0	1, 1

Coordination

		2	
1		C	D
	C	4, 4	0, 7
	D	7, 0	0.1, 0.1

Prisoners' Dilemma

Prisoners’ Dilemma, best responses

		Player 2	
		Cooperate (C)	Defect (D)
Player 1	Cooperate (C)	4, 4	0, 7
	Defect (D)	7, 0	0.1, 0.1

- Assume this game to be played once
- Consider the best responses:
    - If 1 cooperates, 2 would defect
    - If 1 defects, 2 would defect
    - the same for 2 (symmetric game)
- Hence: Cooperation is not an equilibrium

- How to achieve cooperation?

- Not possible in a one shot game!
- Possible in repeated games
- Possible by including the group payoff into the individual’s utility function (which means to change the payoffs)
- Possible if one assumes some kind of learning (or strategies like ‘tit for tat’)

Network Version

Game rules

7 different networks

Synchronous updating

Neighborhood of 5 where possible

50 runs per network

- Start: every agent is randomly assigned strategy **C** or **D**
- Each agent plays this strategy with his neighbors (and himself)
- Observe the total payoff of the neighbors and adopt the strategy that yielded the highest payoff (for the next round)

Results on the complete network

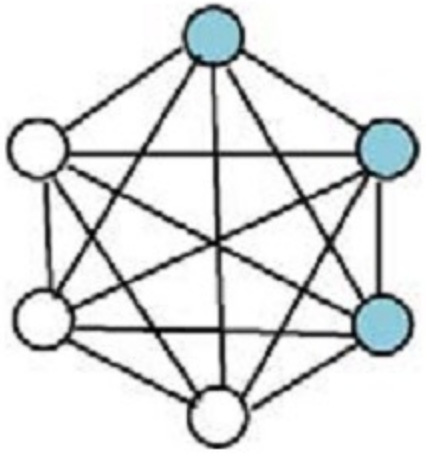


Figure from Wilhite (2006)

- Since the network is complete, just check the total payoff for **C** and **D**
- Assume ***n*** agents, among them ***m*** defectors
- Strategy **C** yields **4 (*n*-*m*)**
- Strategy **D** yields **7 (*n*-*m*) + 0.1 *m***
- As long as there is one defector, everybody will adopt **D**
☹

Results on the star network

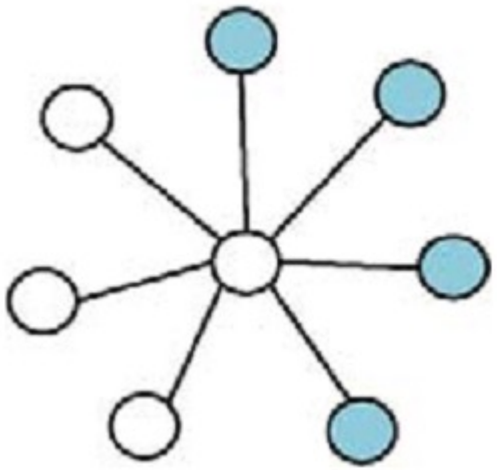


Figure from Wilhite (2006)

- Outcomes depend heavily on the center agent
- **Assume** the center agent starts with **D**
- He earns $\mathbf{7(\mathit{n}-\mathit{m})+0.1\ \mathit{m}}$
- Periphery earns **0** or **0.1**
- The periphery will adopt **D** in the next round and everybody will **defect**

In the paper the notation n/m changes!

Results on the star network (2)

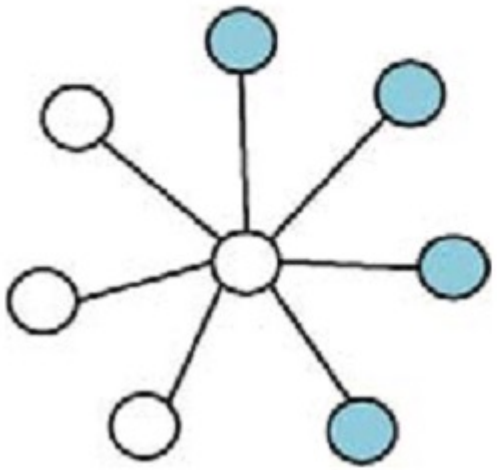


Figure from Wilhite (2006)

- **Assume** the center agent starts with **C**
- He earns **4 *(n-m)***
- Periphery earns **7** each when playing **D**
- The periphery will adopt **C** if ***n-m* ≥ 2**
- Otherwise the center and the remaining periphery agent will switch to **D**

Results on the ring

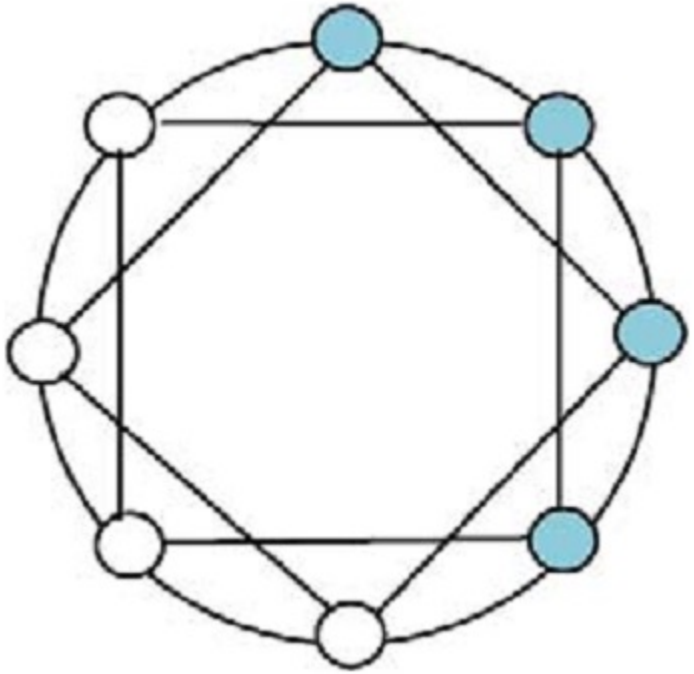


Figure from Wilhite (2006)

- Assume a neighborhood of 5
- If there is no string with more than 3 cooperators, **D** spreads
- Otherwise: **C** spreads, see the payoffs for the **C** agent at the border

… D	D	C	C	C	**C**	D	D	D	D …
7.4	14.3	12	16	16	12	14.3	7.4	0.5	0.5

- The adjacent **D** agent has a higher payoff, but the **C** agent to the left is even better
- The border **D** agent will also adopt **C**

Results on the ring (2)

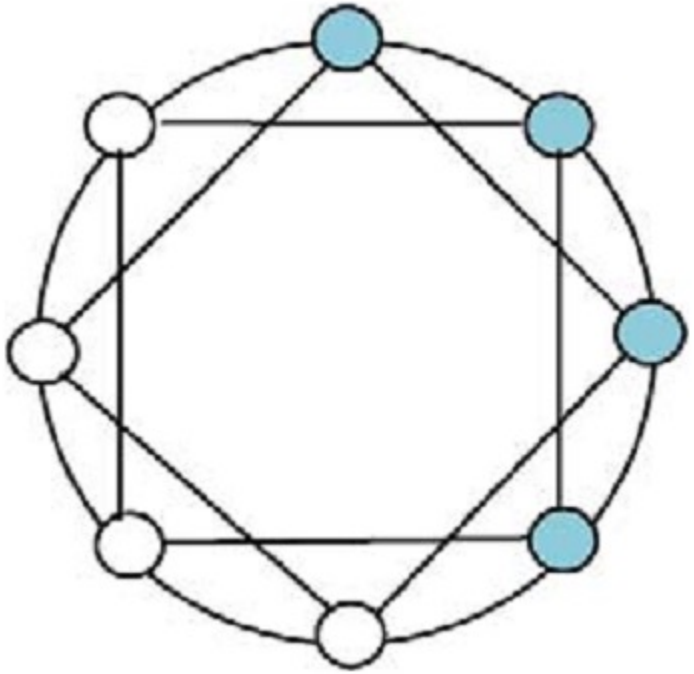


Figure from Wilhite (2006)

- Will defectors vanish?
- Assume only one is left:

… C	C	C	C	**D**	C	C	C	C	C …
20	20	16	16	28.1	16	16	20	20	20

- The 4 adjacent cooperators will switch to **D** in the next round

… C	C	D	D	D	D	**D**	C	C	C …
16	12	14.3	7.4	.5	7.4	14.3	12	16	20

- But now again the border **D** agent will switch → cycle

Small worlds

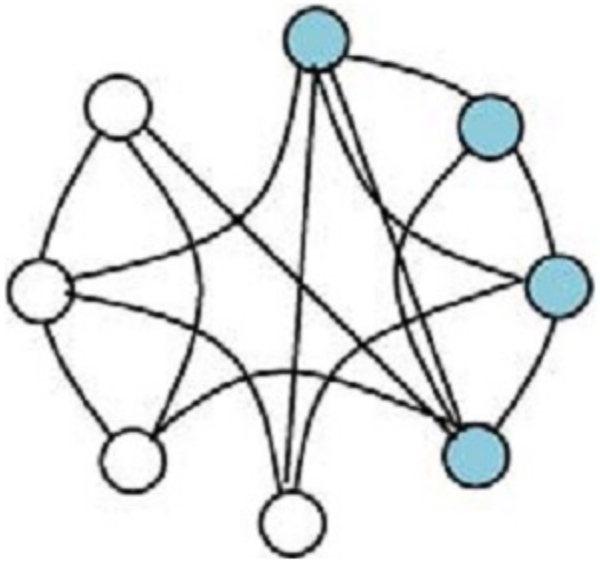


Figure from Wilhite (2006)

- The small-world network is basically a ring with re-wired shortcuts
- The shortcuts allow defectors to profit from strings of collaborators, they will split them up
- The behavior is similar to the ring, but there are more defectors

The grid

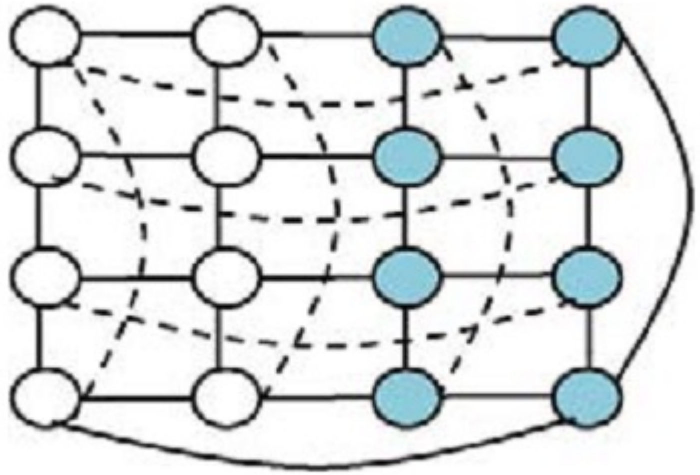


Figure from Wilhite (2006)

- Chaos!
- Following transitions:
  - One isolated **C** vanishes
  - Two or more adjacent **C**’s spread
  - One defector **D** will spread
- Results in a rather stable distribution of cooperators and defectors

The tree

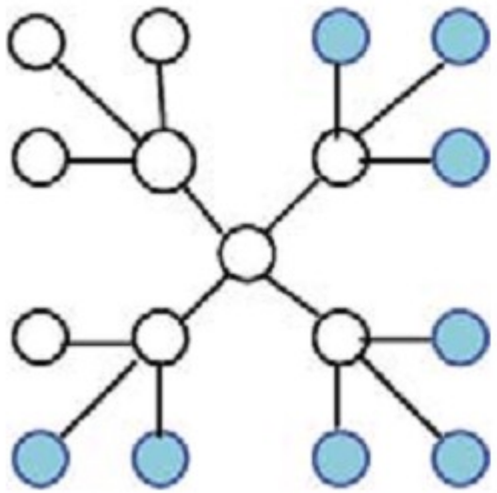


Figure from Wilhite (2006)

- A single defector will spread if not placed at the periphery
- A couple of cooperators will also spread
- The dynamics are complex:
- Cooperators which hit a defector spreading upwards will become defectors
- Defectors spread until they interfere with other defectors
- Converges to either only **C** or **D**

Scale-free

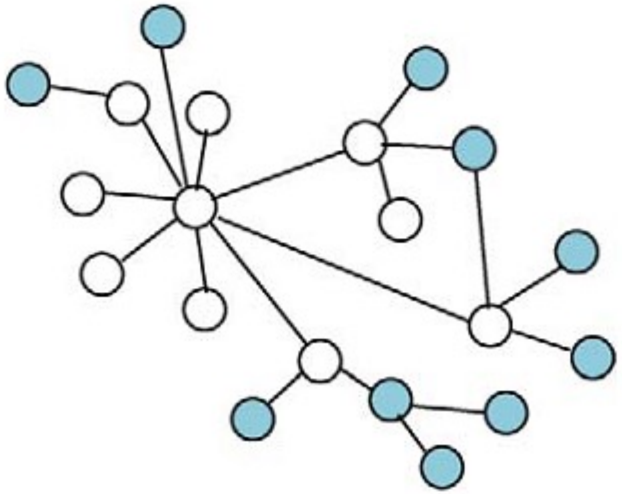


Figure from Wilhite (2006)

- Both strategies survive but cooperators are in the majority
- Hubs have a coordinating effect, since large clusters with defectors will not survive, they will mostly cooperate
- Defectors can survive in the periphery, where their cooperative neighbors will not switch to **D** themselves

Results and Limitations

- The results (convergence to only **C**/**D** or mixed equilibrium) in this game depend on the network structure
- Agents choose strategies from a local optimum
- But: agents are myopic and do not learn optimally even in a local context

Exchange in Networks

- Another interesting experiment is to study trade on these network structures (barter economy)
- Assume that two goods are distributed unevenly on the network (initial allocation of 150 / 1500 units)
- All agents ***i*** prefer to consume both goods, i.e.

$$U^i = g_1^i g_2^i$$

- Agents trade bilaterally if an exchange will increase utility for both
- We have reached an equilibrium when the marginal rates of substitution converge

$$mrs^i = \frac{U'\left(g_1^i\right)}{U'\left(g_2^i\right)} = \frac{g_2^i}{g_1^i}$$

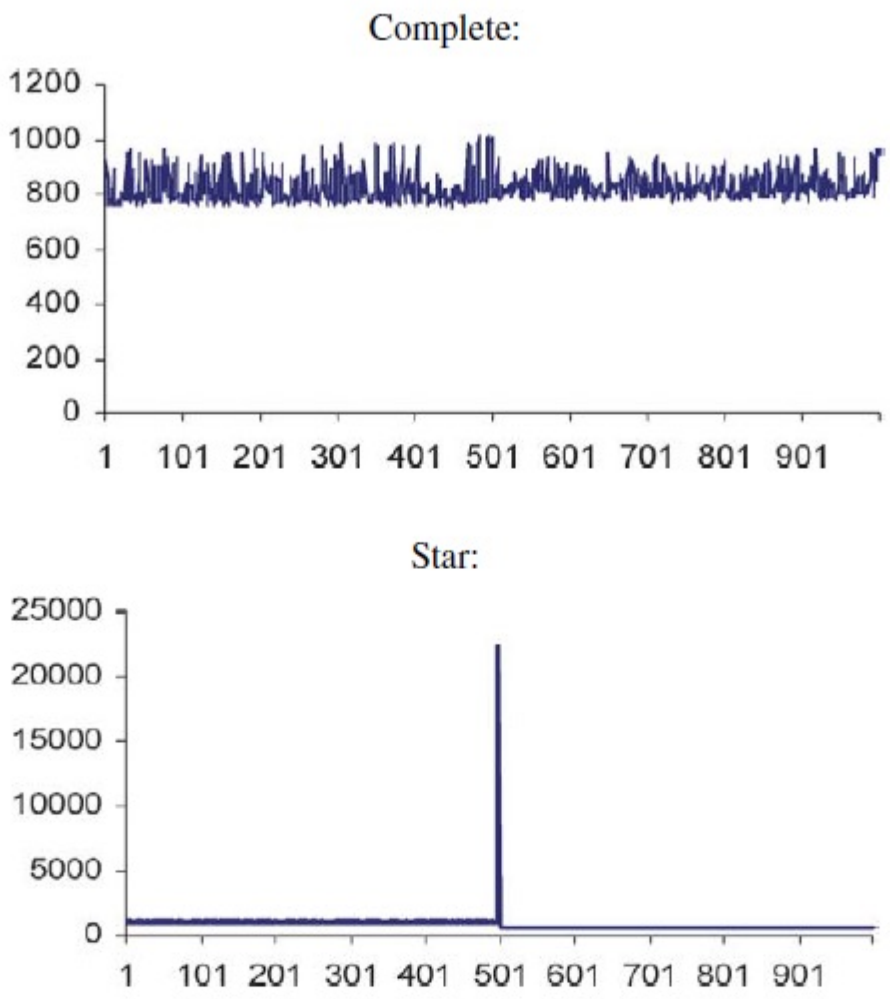
- The trading algorithm randomly picks one agent, who will then pick the neighbor with the best offer for trading
- The local price between agent ***i*** and ***j*** is given by

$$p_{i,j} = \frac{g_2^i + g_2^j}{g_1^i + g_1^j}$$

Income distribution in equilibrium

Complete network: fast convergence with high homogeneity

Star: efficient, but produces one peak



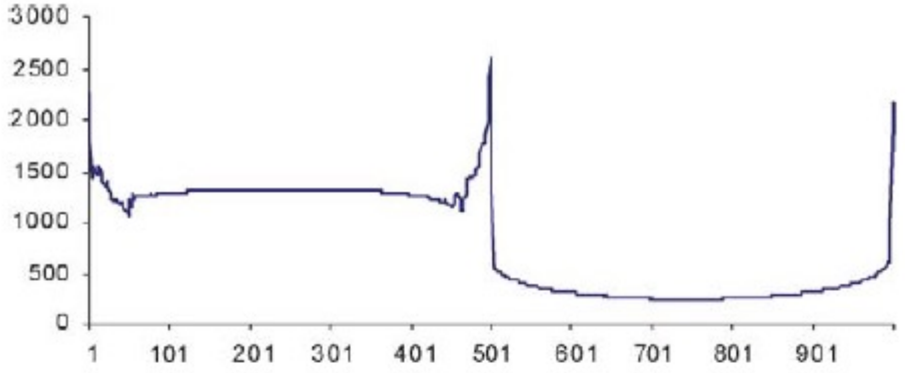
Figures from Wilhite (2006)

Income distribution in equilibrium (2)

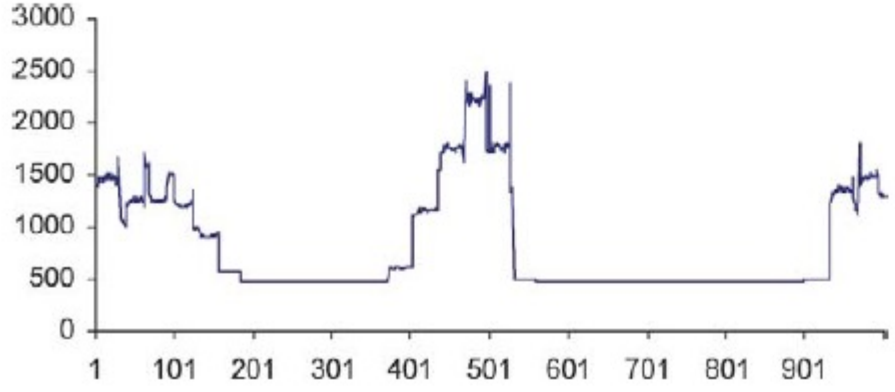
Ring: endowments in *g1* and *g2* are on different sides of the ring. The border regions earns from trade flow.

Similar results hold for the grid network.

Ring:



Grid:



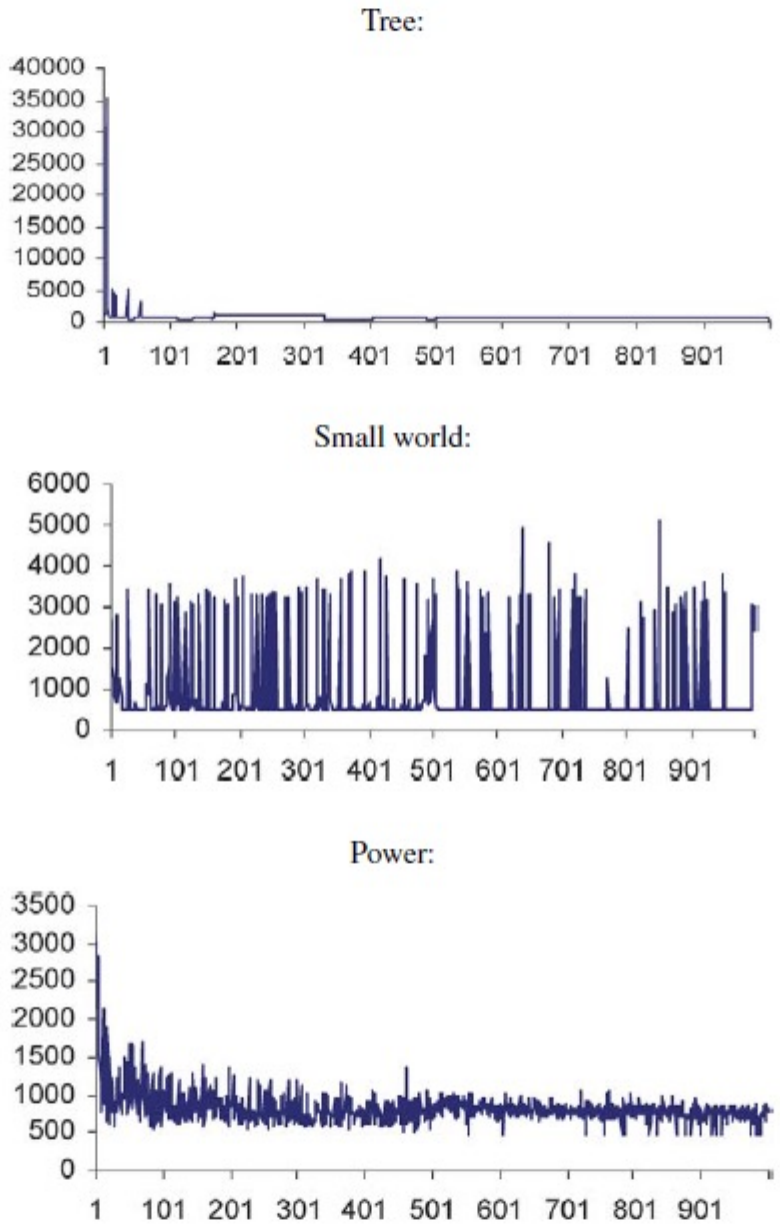
Figures from Wilhite (2006)

Income distribution in equilibrium (3)

Tree: Top level agents prosper if goods are abundant in different branches

Small world: many spikes, but lower dispersion

Scale-free: some hubs gain from their position, still rather even distribution



Figures from Wilhite (2006)

Network Formation and Games

- Instead of looking at fixed networks, a different approach is to analyze how networks emerge from games
- Assuming that a link resembles cooperation or some other game action, how does the network look like, when agents play a repeated game?

- These games can be cooperative or non-cooperative games
- Links might be costly or free
- Connections might be unidirectional or bidirectional
- Payoffs can be aggregated or counted as a one shot game
- Agents might discount future payoffs
- Agents might or might not learn from the past

More games on networks...

- In most papers the described games are based on individual incentives of the agents (opposed to centrally planned networks)
- Jackson and Wolinsky (1996) let agents play a game where each side has to agree to have a link. Such games normally do not result in stable networks (often even empty networks).
- Bala and Goyal (2000) assume a non-cooperative game, where a link is costly and there is some kind of learning

Jackson, M. and A. Wolinsky (1996), A Strategic Model of Economic and social Networks, Journal of Economic Theory 71, 44-74.

- Specifically, assume that all agents have pieces of information, which they can trade by forming a link (‘communicating’)
- Each agent chooses a strategy, describing to which of the other agents he links
- Each agents receives a payoff dependent on the information it accesses minus the costs of forming the links
- Information can also be accessed indirectly, through ‘friends of friends’
- One-way and two-way flow is analyzed

- This game yields non-trivial results if the costs are high enough to prohibit linking to everybody, and low enough to assure that links to other connected agents might be profitable
- Each round a random agent with some probability ‘rethinks’ his strategy by choosing the best response to all other players’ strategies
- Omitting further details, lets have a look at some outcomes of this game

- Results for the one-way flow version:
- The ring and the star are the only stable Nash networks

FIGURE 3A.—The star and the wheel (one-way model).

Bala, V. and S. Goyal (2000), A Noncooperative Model of Network Formation, Econometrica 68(5), 1181-1229

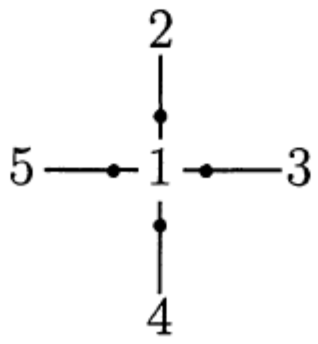


FIGURE 5A.—Center-sponsored.

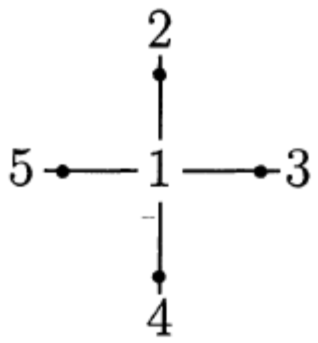


FIGURE 5B.—Periphery-sponsored.

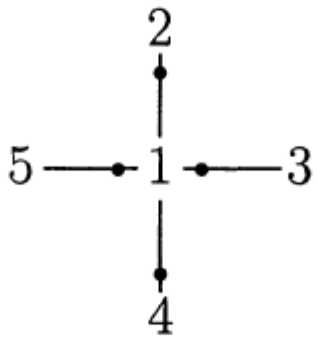
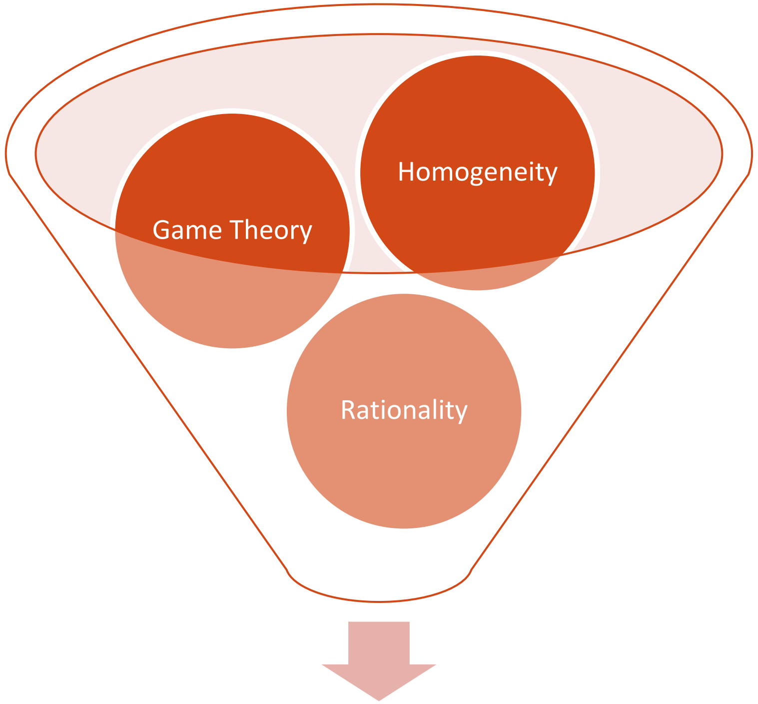


FIGURE 5C.—Mixed-type.

For the two-way flow model the outcome are star-like networks which differ in who is paying for the link

Figure from Bala and Goyal (2000)

Results and Problems



Realistic Structure?

- Economic and social activity can be modelled with networks
- Problem 1: Fixed networks are unrealistic
- Problem 2: Networks that emerge when applying traditional economic concepts or Game Theory are highly stylized
- Conjecture: Networks do not emerge out of a rational best response game or alike
☹