Computational Modeling of Social Systems

Basics of spreading: Granovetter‘s threshold model

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Recap so far …

Block 1: Fundamentals of agent-based modelling

- Basics of agent-based modelling: the micro-macro gap

- Modelling segregation: Schelling's model

- Modelling cultures

Block 2: Opinion dynamics

- **Today:** Basics of spreading: Granovetter's threshold model
  - Exercise 1: Schelling's model and Pandas (session 2)
- Opinion dynamics
  - Exercise 2: Modelling culture.

To Do at your own time

Overview

1. Collective behavior
2. Granovetter's threshold model
3. Modelling online collective emotions

Check Videos on TC Tube

More is different

4 August 1972, Volume 177, Number 4047

SCIENCE

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

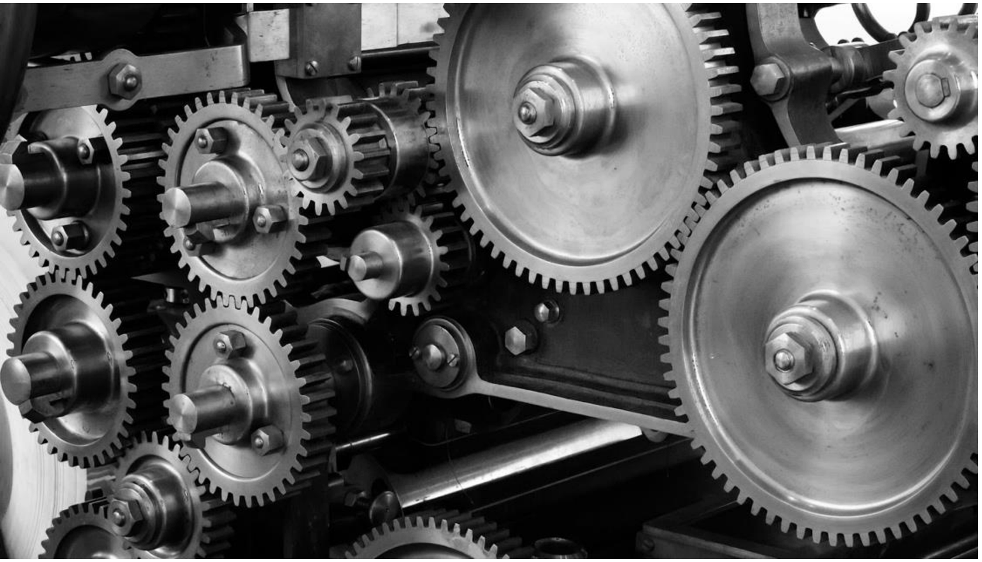
P. W. Anderson

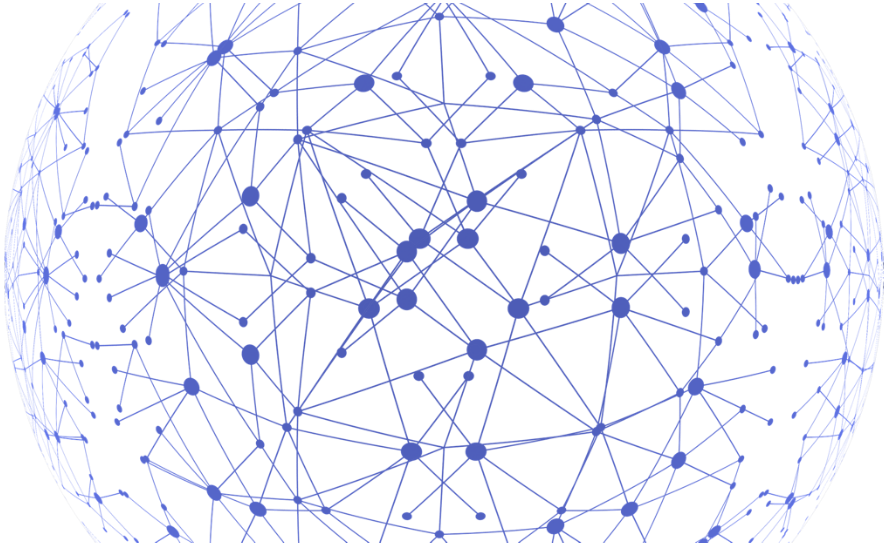
less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other. That is, it

More is different: broken symmetry and the nature of the hierarchical structure of science. Philip Anderson, Science (1972)

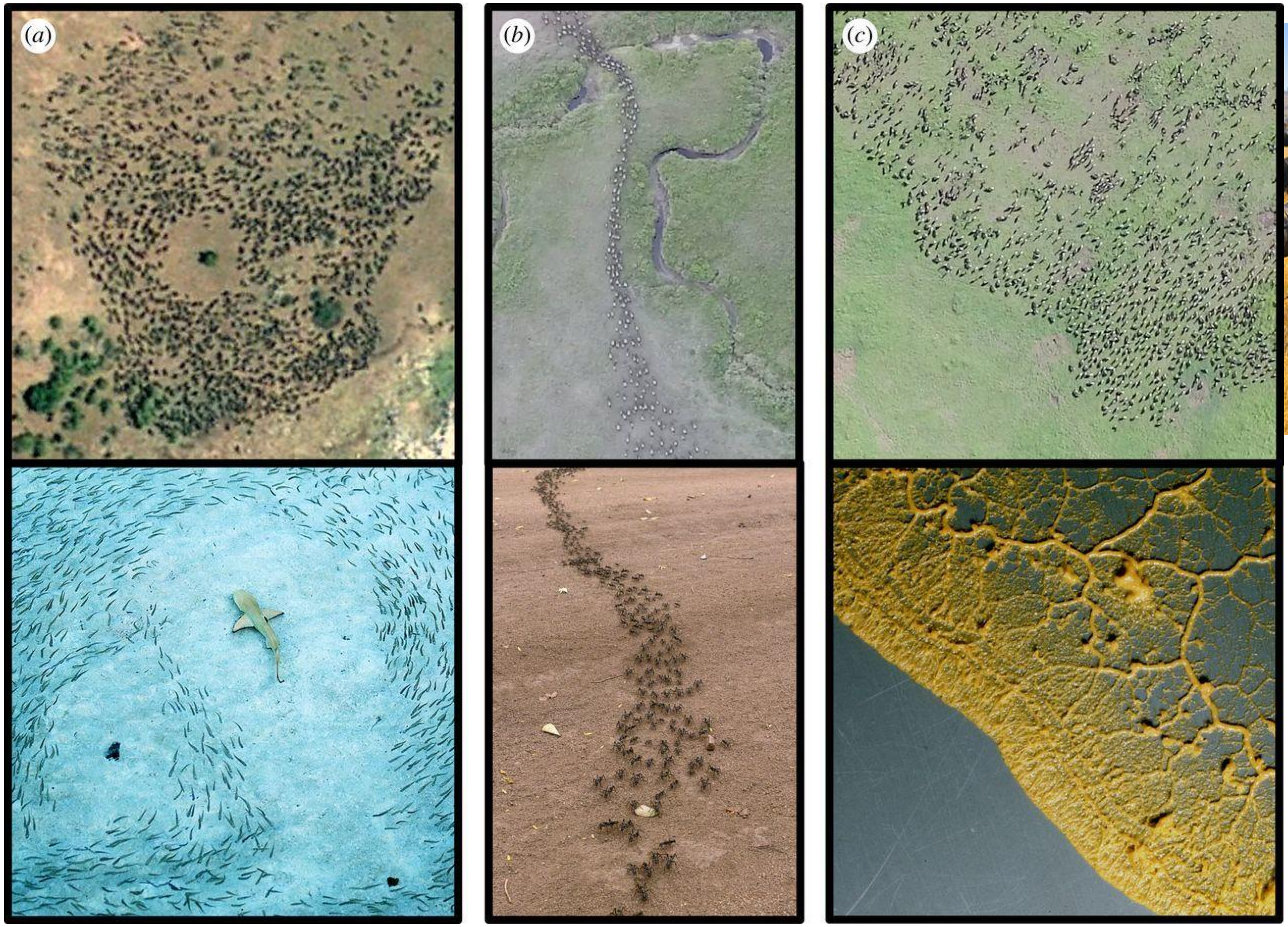
Complexity Science: Complicated versus complex





- A **complicated system** has many pieces with specific functions and well-defined relationships. It has been carefully **engineered or designed**.
- A **complex system** is composed of many particles that interact following some forces or dynamics. Its behavior follows from **natural principles**.

Collective behavior in complex systems
We can learn a lot from evolutionary biology



[Challenges and solutions for studying collective animal behaviour in the wild. Lacey Hughey et al. Philosophical Transactions of the Royal Society B (2018)]

Collective behavior in social systems

Behavioral-induced collective behavior: when the strength of interaction between individuals generates macro behavior

• Schelling's model: low tolerance triggers moves that lead to segregation
• Alxelrod's model: cultural exchange leads to larger cultures or supports the coexistence of few cultures

connectivity-induced collective behavior: when the differences between individuals or their connectivity patterns create the environment for macro behavior to emerge

• Today's case: interaction strength and average agent stay the same, only variance between agents is the driver

• More on network models: different positions in a social network are a way to induce cascades

Overview

1. Collective behavior
2. **Granovetter's threshold model**
3. Modelling online collective emotions

Collective behavior: Milgram experiment

S. MILGRAM, L. BICKMAN, AND L. BERKOWITZ

me sort of *observable action* that
tion can imitate or in some manner
. In the present study the stimulus
od on the pavement and looked up
ndow of a nearby building. This
parts of it, could be adopted by the
The passerby could simply look
building where the crowd was star-
t breaking stride, or he could make
mplete imitative action by stopping
ing alongside the crowd. Analyses
rtaken for both types of responses.
the investigators wanted to see in
ee crowds, varying in size from 1 to
, and all performing the same ob-
ction, would draw persons into their

of the observation area, stopped, and l
the sixth-floor window. This gaze was
for 60 seconds. At the end of this peric
was signaled to disperse. After the area
of the gathered crowd the procedure v
using a different size stimulus crowd. Fi
ordered trials were conducted for each
different size stimulus crowds. The stin
were composed of 1, 2, 3, 5, 10, and
Motion pictures were taken of the obse
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Binary decisions and collective behavior

Green revolution, Iran 2009

Question: Why do some movements become so big, and some die?

Binary decisions and collective behavior

The riot toy example:

• A group of individuals is part of a demonstration

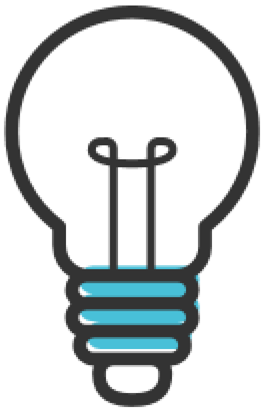
• Individuals have a threshold of how many others have to be rioting to join the riot (**Cost and benefit**)

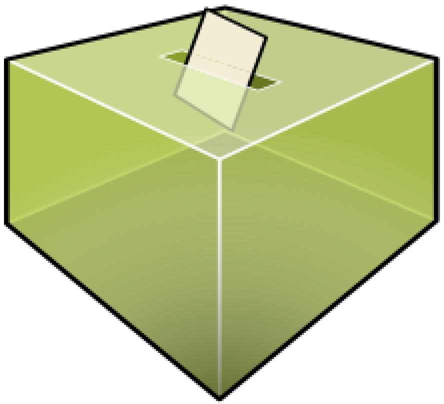
• If enough people are in the riot, individuals with lower thresholds join, too

• Proto-opinion: just participate / not participate

• Other examples with binary decisions depending on size: Diffusion of innovations, rumors, strikes, voting, leaving a party, migration

Diversity in collective behavior





- How does the distribution of preferences (thresholds) in a population affect its collective behavior?

- Knowing the preferences does not directly tell you how the population will behave, you need to analyze how the population behaves !!

- Aim: understanding groups beyond the representative "mean" member

Rational agents in collective action

Assumption: the decision to join the collective action depends on:

Risks and Benefits?

Rational agents in collective action

Assumption: the decision to join the collective action depends on:

Risk or cost of participating.

- Examples of risks and costs:
    - Risk of being jailed in riot
    - Wage loss in strike
    - Cost of technology adoption

The benefit (potential) of the action taking place.

- Examples of benefits:
    - Political change after a demonstration
    - Profit out of adopting innovation
    - Political party winning an election

An example of spreading of behaviour

https://www.youtube.com/watch?v=GA8z7f7a2Pk

https://www.youtube.com/watch?v=hO8MwBZl-Vc

Net benefit and thresholds

Net benefit = benefit – costs

• Threshold to join: Net benefit is >0

• benefits increase and costs decrease with more people in the action (monotonic or non-monotonic net benefit)

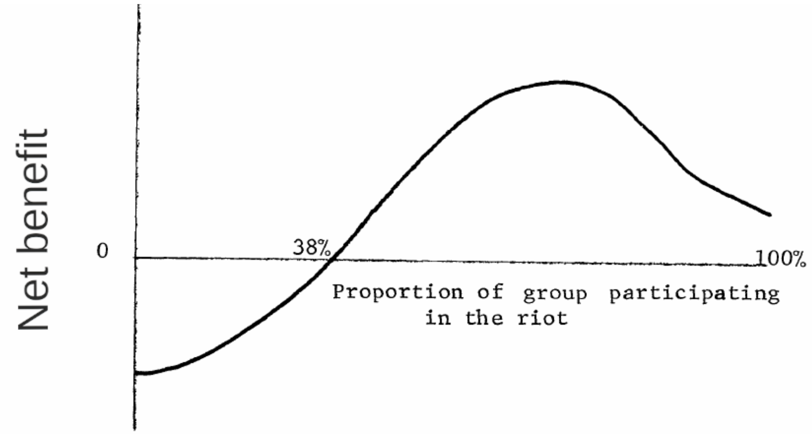
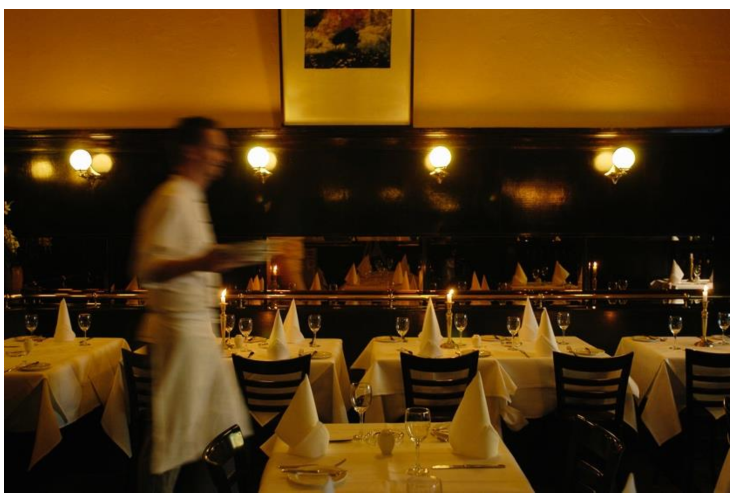


FIG. 3.—Net benefit to an individual, with threshold 38%, of joining a riot, plot
against the proportion of the group participating. (Total benefits minus total costs.)



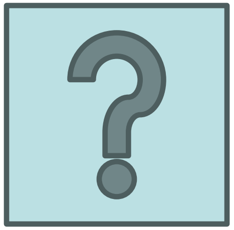
Example of net benefit function for someone with threshold of 38%

Granovetter (1978)

Toy example of Granovetter’s model

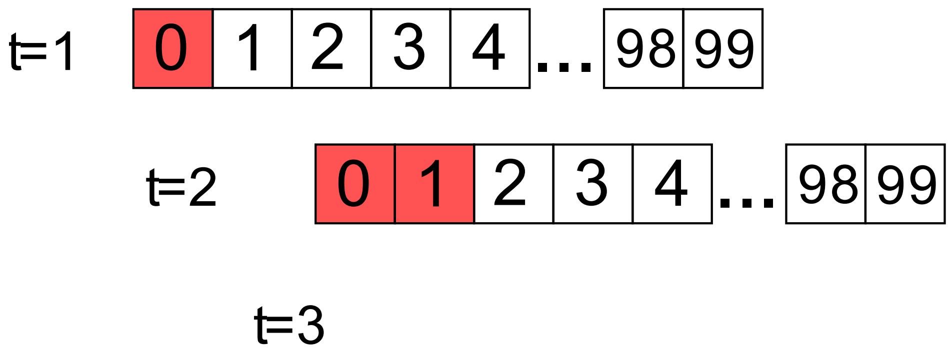
t=1 | 0 | 1 | 2 | 3 | 4 |… | 98 | 99

t=2



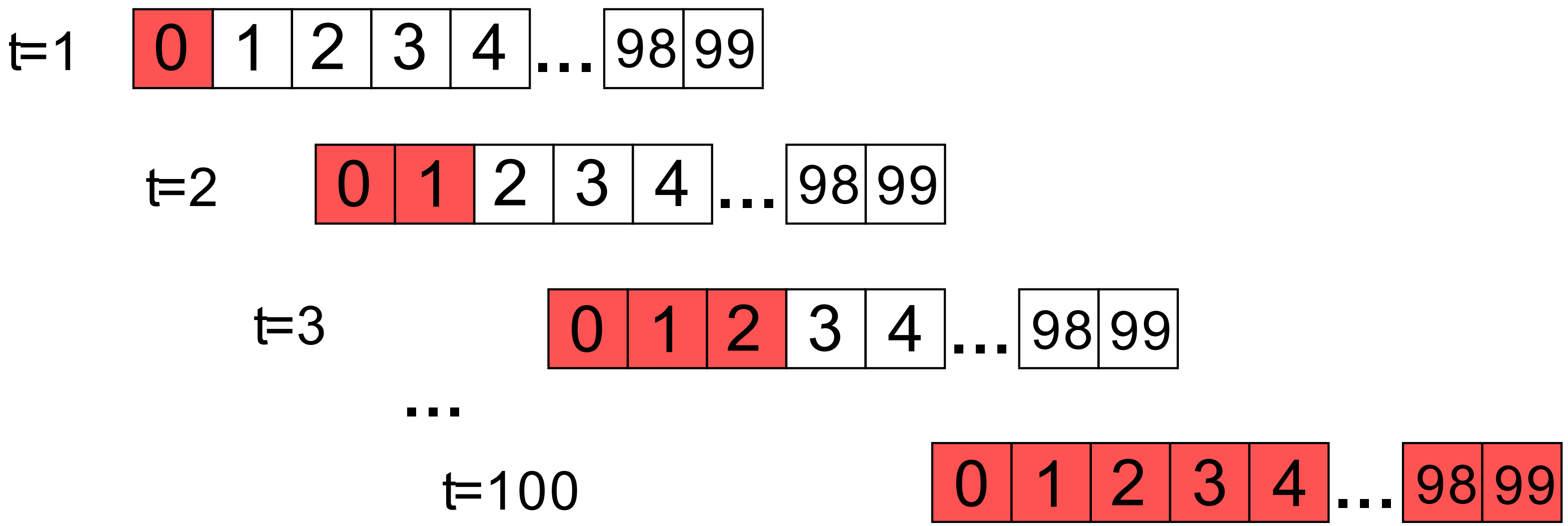
- 100 Agents
- Uniform sequence of thresholds with integer values [0,99], otherwise agents are similar
- The first agent activates, what happens next?

Toy example of Granovetter's model



- 100 Agents
- Uniform sequence of thresholds with integer values [0,99]
- The first agent activates, then the second, and so on
- One agent joins per iteration and all agents are active at the end

Toy example of Granovetter's model



- 100 Agents
- Uniform sequence of thresholds with integer values [0,99]
- The first agent activates, then the second, and so on
- One agent joins per iteration and all agents are active at the end

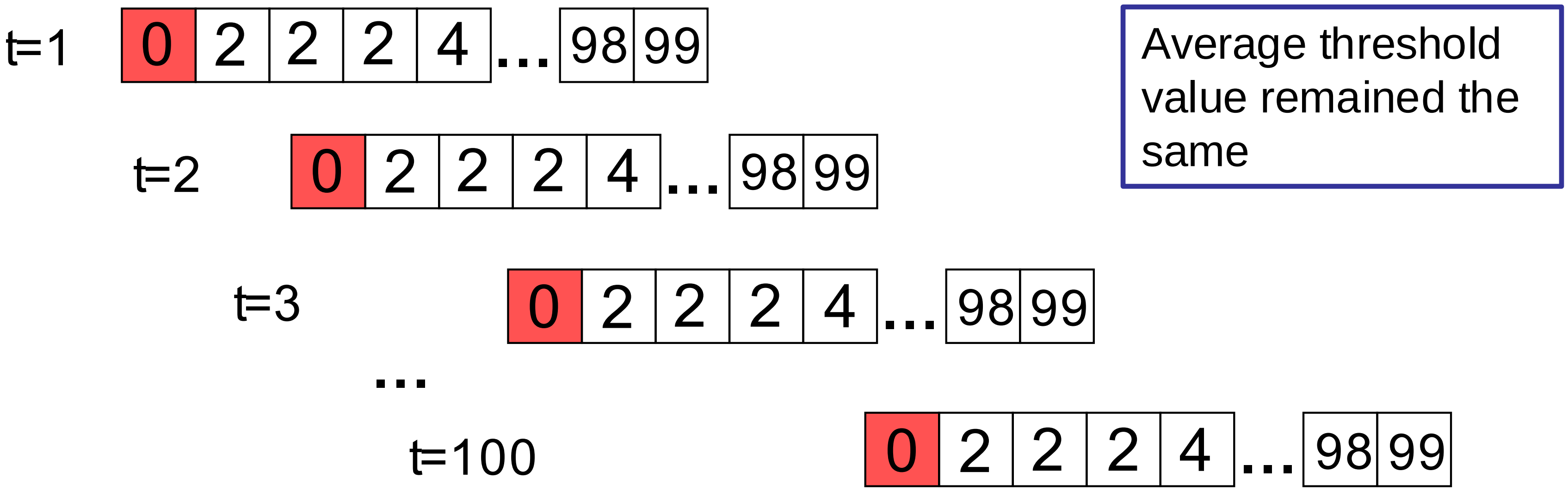
Toy example version 2

t=1	0	2	2	2	4	…	98	99

- Same example as before but agents with thresholds 1 and 3 now have threshold 2

- What happens?

Toy example version 2



- Same example as before but agents with thresholds 1 and 3 now have threshold 2
- First agent activates and the simulation ends
- Radically different outcome for minimal change in thresholds!
- **Deducing preference distributions from collective outcomes is risky**

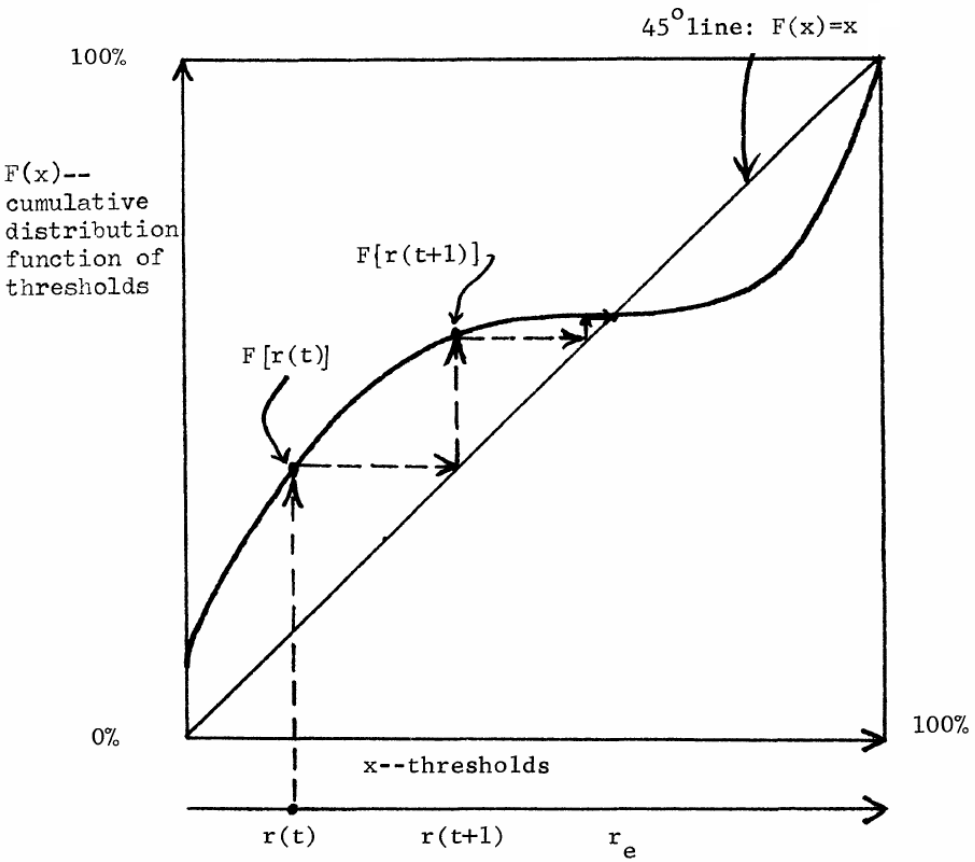
Analyzing the distribution of thresholds

- $F(x)$ is the cumulative density function (CDF) of thresholds:

$$r(t+1) = F(r(t))$$

- $r(t)$ is the number of active agents at time t
- $r(0)$: number of "instigators"
- Simulation reaches an equilibrium:

$$r(t+1)= r(t) = F(r(t))$$

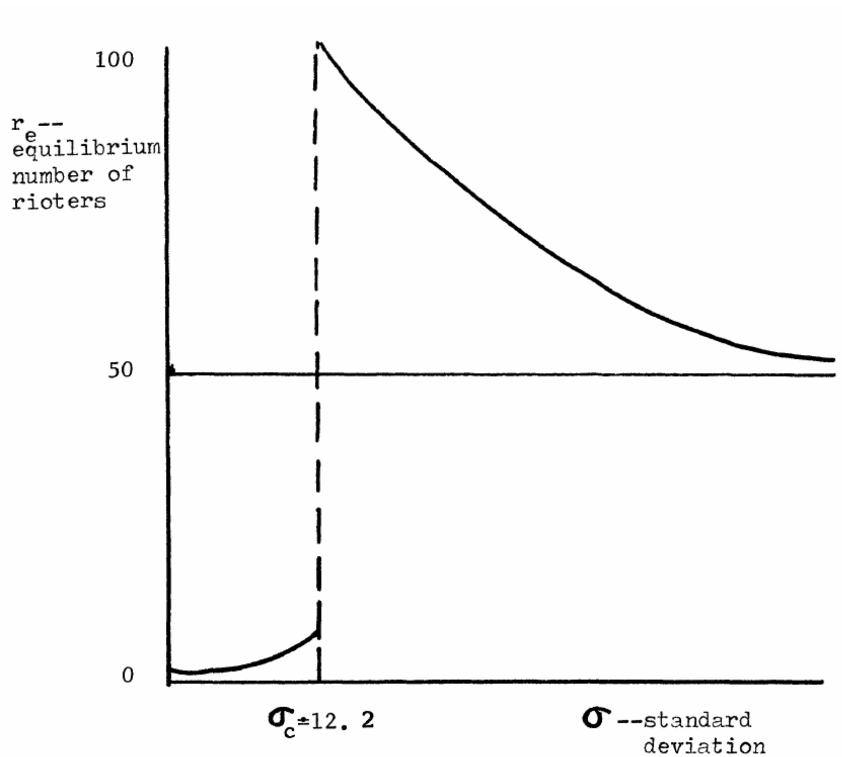


Threshold distribution

Uniform [0,1,2.., 100]

Normal distribution (mean and standard deviation)

r_e **versus** σ



- Assumption: Thresholds follow normal distribution with μ and σ

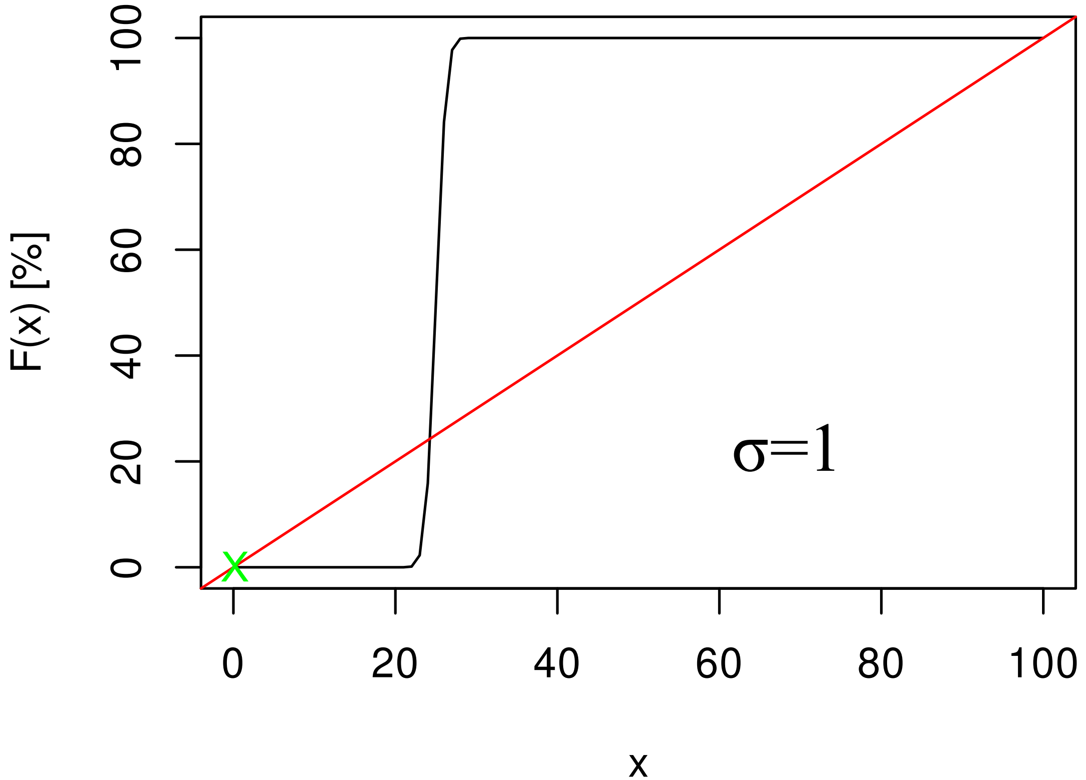
- re: equilibrium number of active agents (simulation ended)

- σ: standard deviation of distribution of thresholds

- Number of agents is constant: 100
- μ is constant: 25
- Sharp increase in r_e at a critical σ value: phase transition

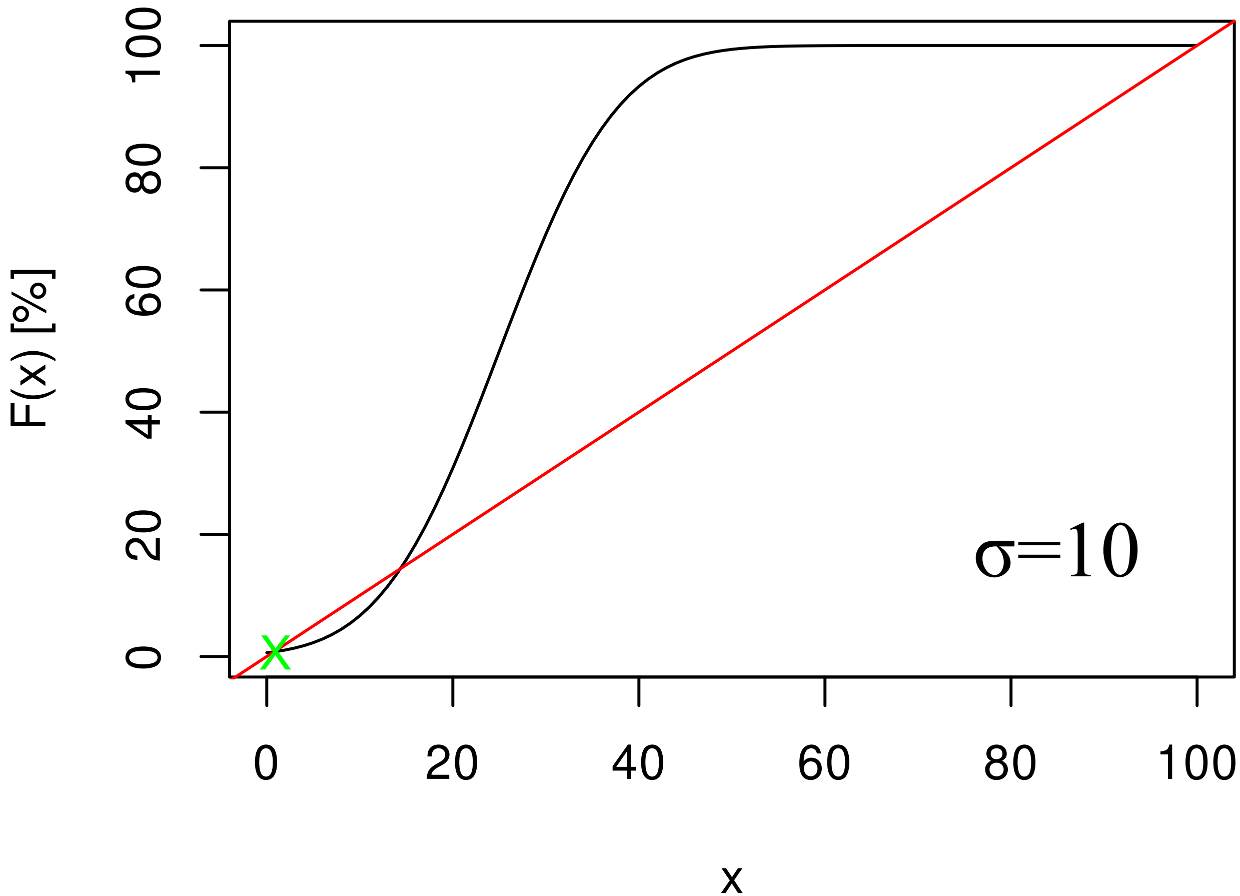
- Diversity-induced collective behavior

Equlibrium in thresholds with σ=1



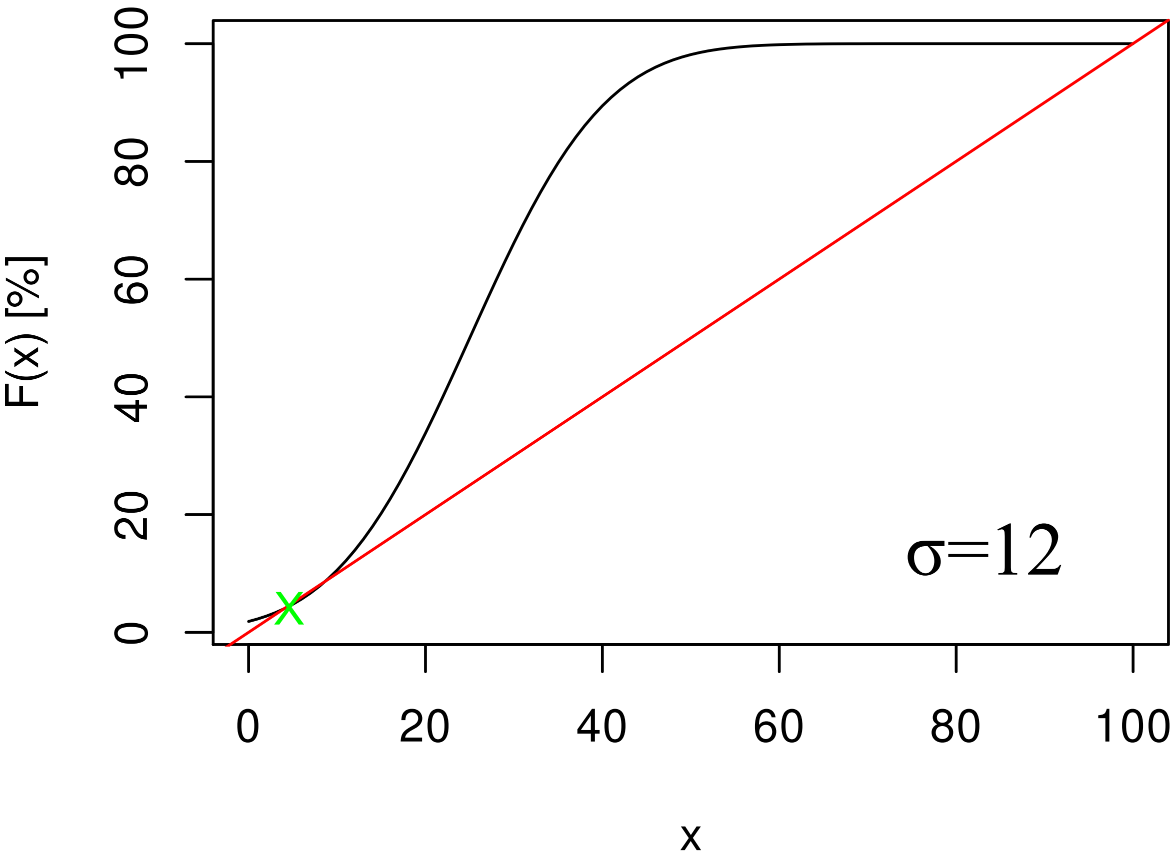
Low variance: thresholds concentrated around 25, no collective action

Equlibrium in thresholds with σ=10



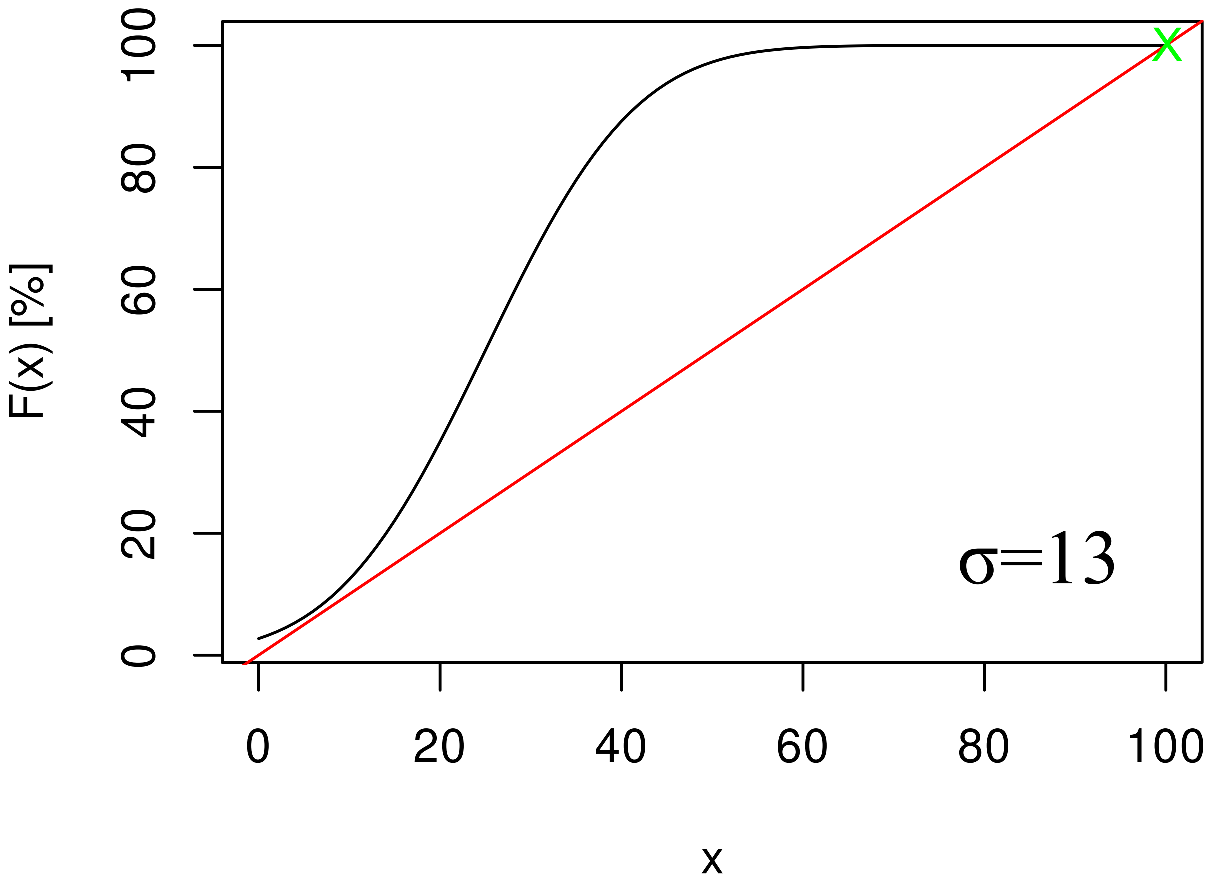
Higher variance but equilibrium is still at low value

Equlibrium in thresholds with σ=12



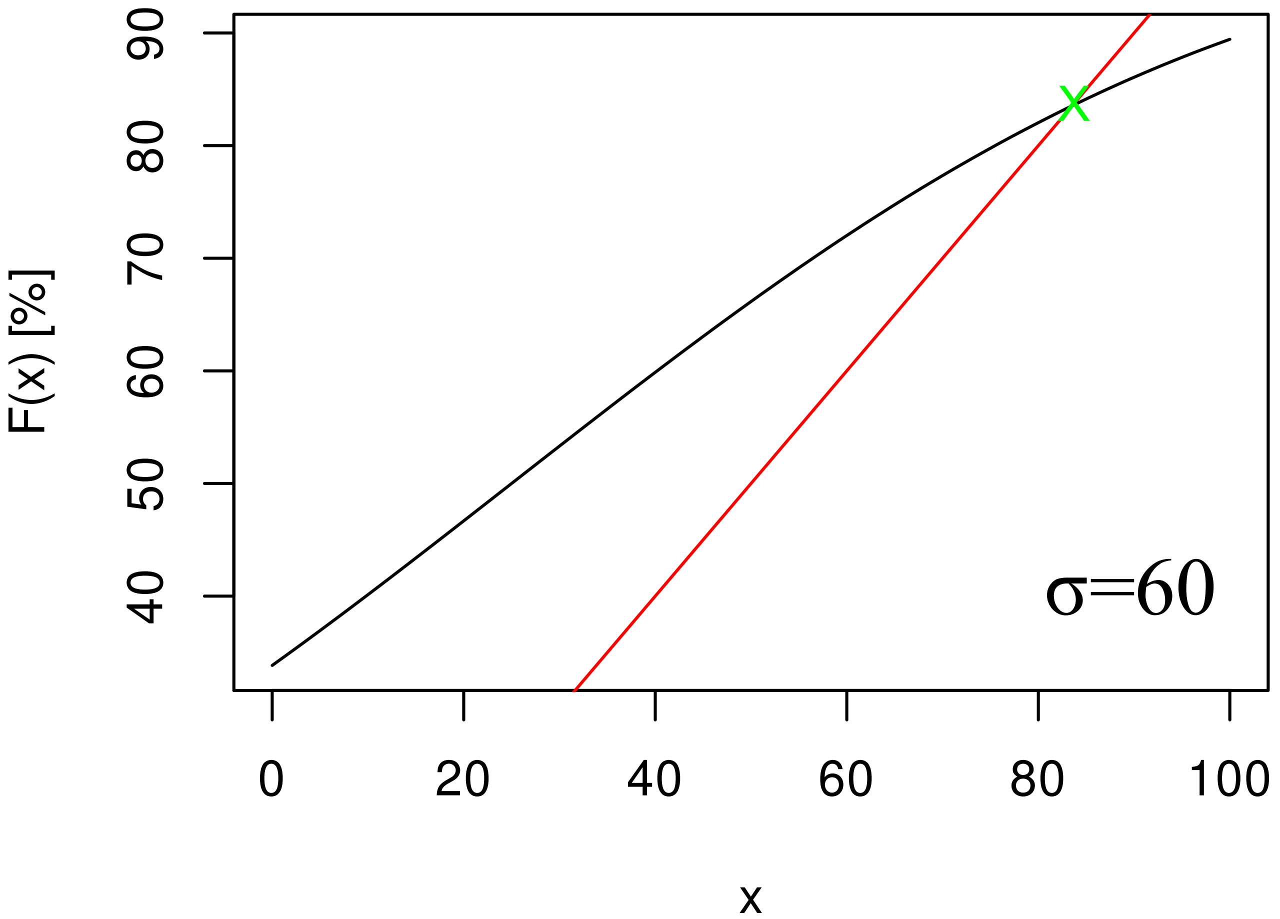
Equilibrium starts to grow to small values

Equlibrium in thresholds with σ=13



Sharp change to upper equilibrium with very high value

Equlibrium in thresholds with σ=60



Slow decrease of equilibrium point towards very high variances

Granovetter's model: take home messages

- **Modelling action as rational choice:** thresholds as points where benefits outweigh costs or risks

- **Diversity matters:** Two populations with the same average threshold have very different behaviors even if mean thresholds are the same

- **Tipping point or phase transition:** behavior changes dramatically at a narrow range of standard deviation of thresholds

- **Size effects:** small changes in threshold sequences can be important. When the population is small, you have a probability of very different outcomes. Inferring the preferences from the outcome is very hard and/or misleading.

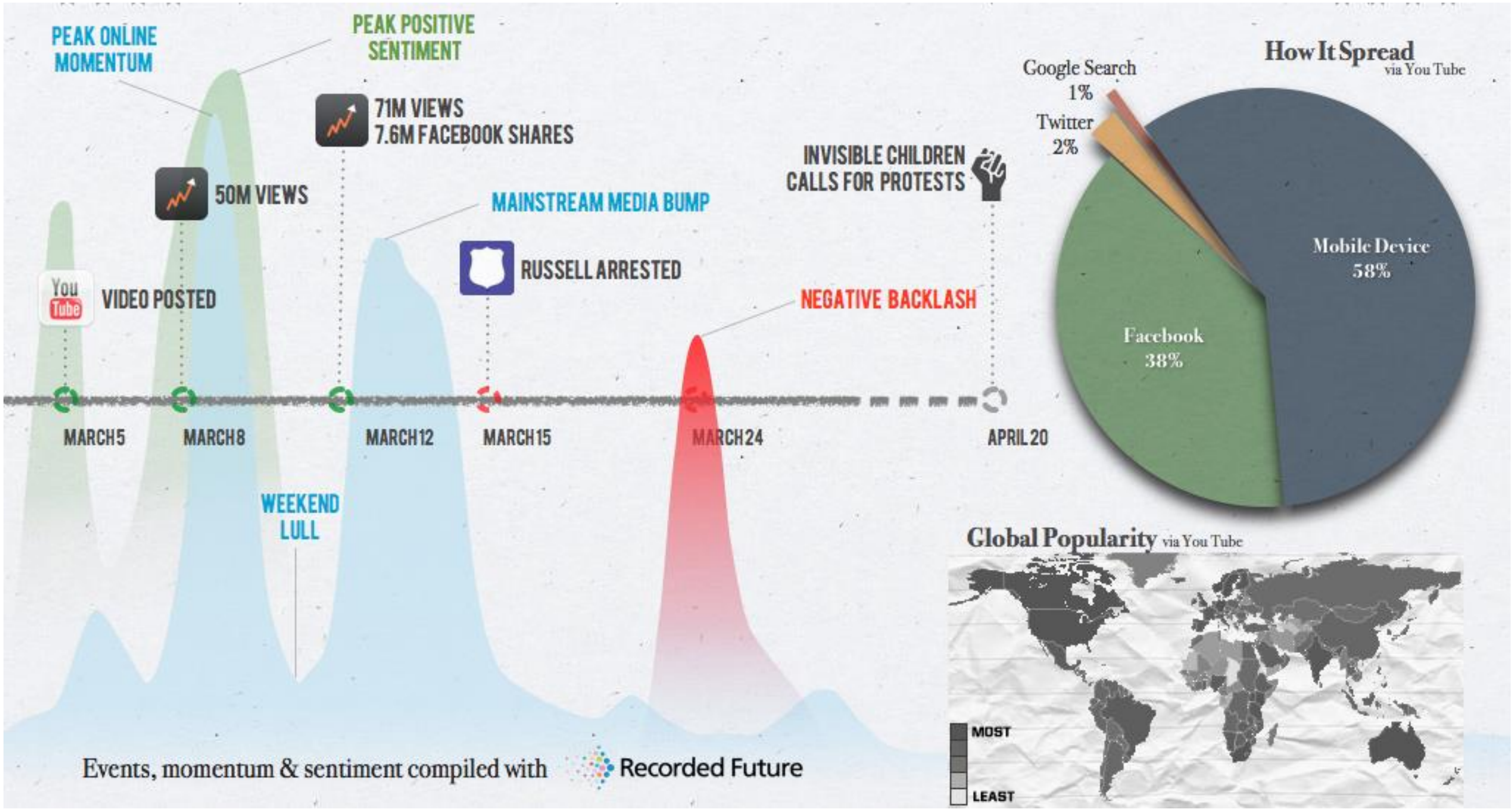
Collective emotions

Collective emotions: Emotional states shared by a large amount of people at the same time

[Collective Emotions, Christian von Scheve and Mikko Salmela, Oxford University Press (2013)]

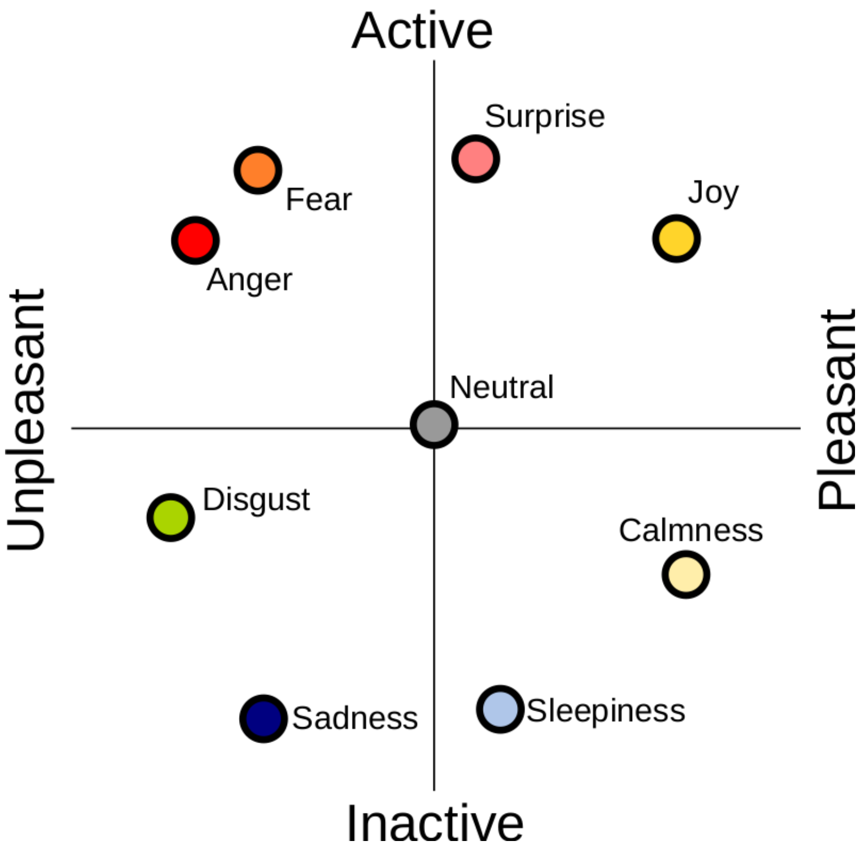
Emotion, behaviour, adoption

Collective emotions on social media



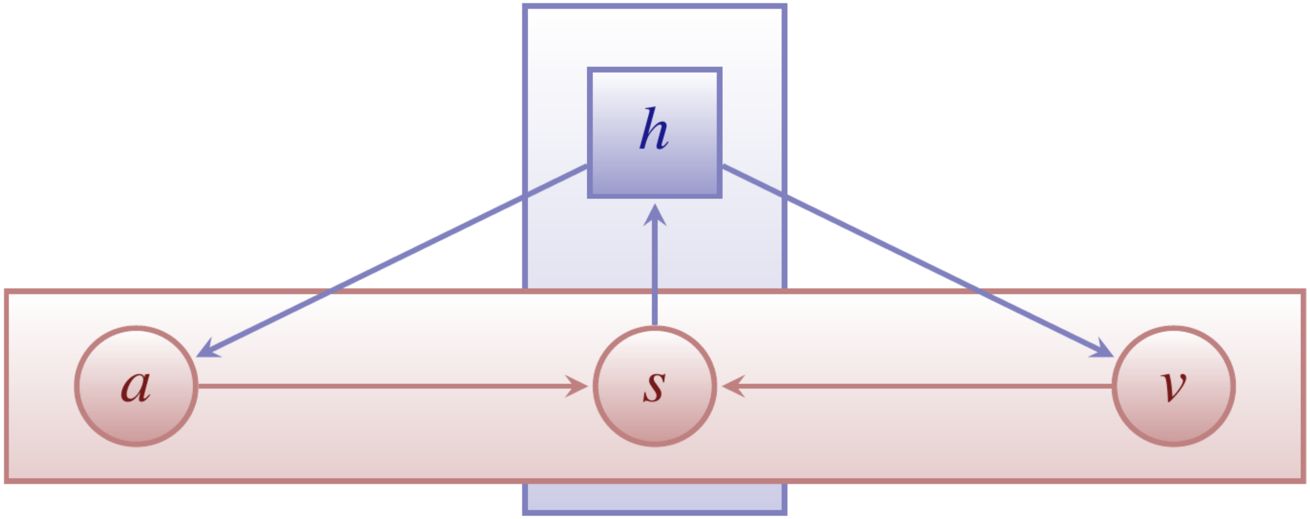
[Kony 2012 Timeline [Infographic], Chris Holden, The Huffington Post]

Quantifying emotions: valence and arousal



- **Valence:** the degree of pleasure experienced in an emotion
  - Explains the most variance from positive/pleasant to negative/unpleasant
- **Arousal:** the level of activity associated with an emotion
  - Explains less variance than valence but it is informative to differentiate emotions

The Cyberemotions modelling framework



- Horizontal: agent design
  - v, a: internal valence and arousal emotional state of the agent
  - s: visible emotional expression as measured (e.g. pos/neg/neu)
- Vertical: interaction between agents
  - h is a communication field averaging recent expression of agents
  - Agent's emotions change with time and the value of h

Valence and arousal dynamics

$$\frac{\delta v_i(t)}{\delta t} = -\gamma_{vi}(v_i(t) - b) + \mathcal{F}_v(h, v_i(t)) + A_{vi}\xi_v(t)$$

$$\frac{\delta a_i(t)}{\delta t} = -\gamma_{ai}(a_i(t) - d) + \mathcal{F}_a(h, a_i(t)) + A_{ai}\xi_a(t)$$

•b,d: baselines of valence and arousal
•γ_v, γ_a: relaxation tendency of valence and arousal towards baselines
•ξ_v, ξ_a: stochastic components of valence and arousal dynamics

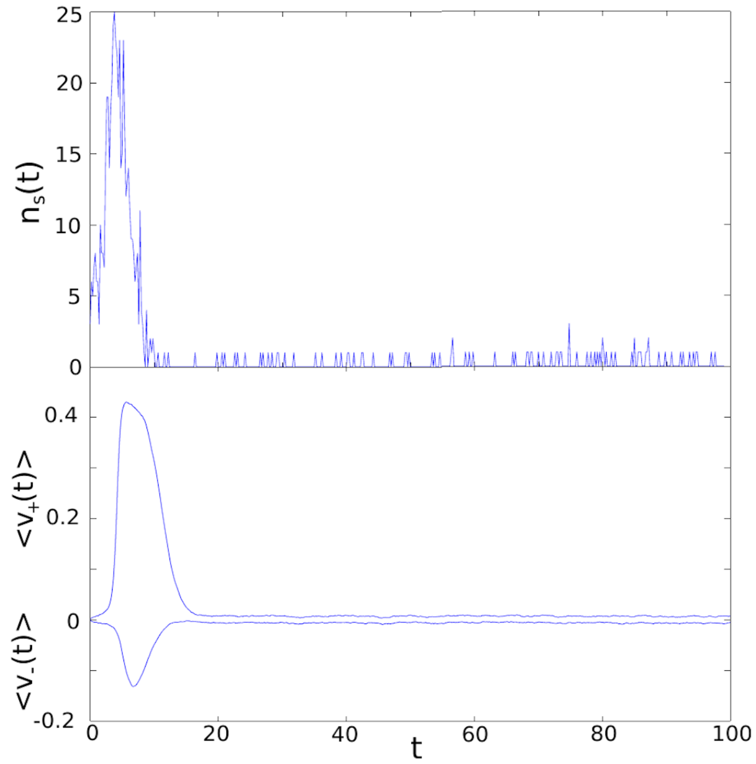
Examples of field influence functions

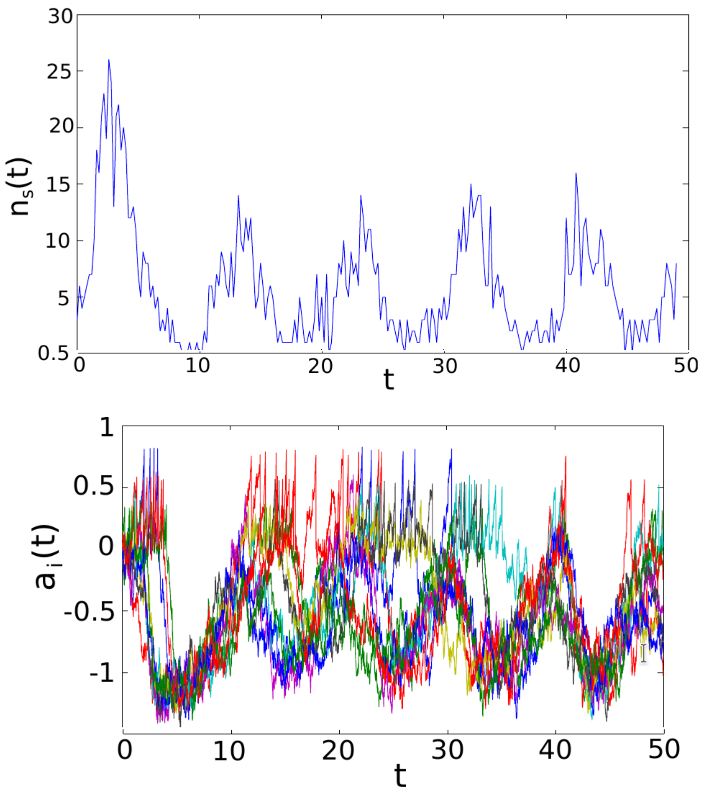
$$\mathcal{F}_v(h, v_i(t)) = h * \left(\sum_{k=0}^{3} b_k v_i(t)^k \right) = h * (b_0 + b_1 v_i(t) + b_2 v_i(t)^2 + b_3 v_i(t)^3)$$

$$\mathcal{F}_a(h, a_i(t)) = |h| * \left(\sum_{k=0}^{3} d_k a_i(t)^k \right) = |h| * (d_0 + d_1 a_i(t) + d_2 a_i(t)^2 + d_3 a_i(t)^3)$$

- Field influence functions as products of a polynomial of valence and arousal
- Approximation to some unknown function to fit empirically
- Valence depends on h (pos/neg) and arousal on |h| (absolute value)

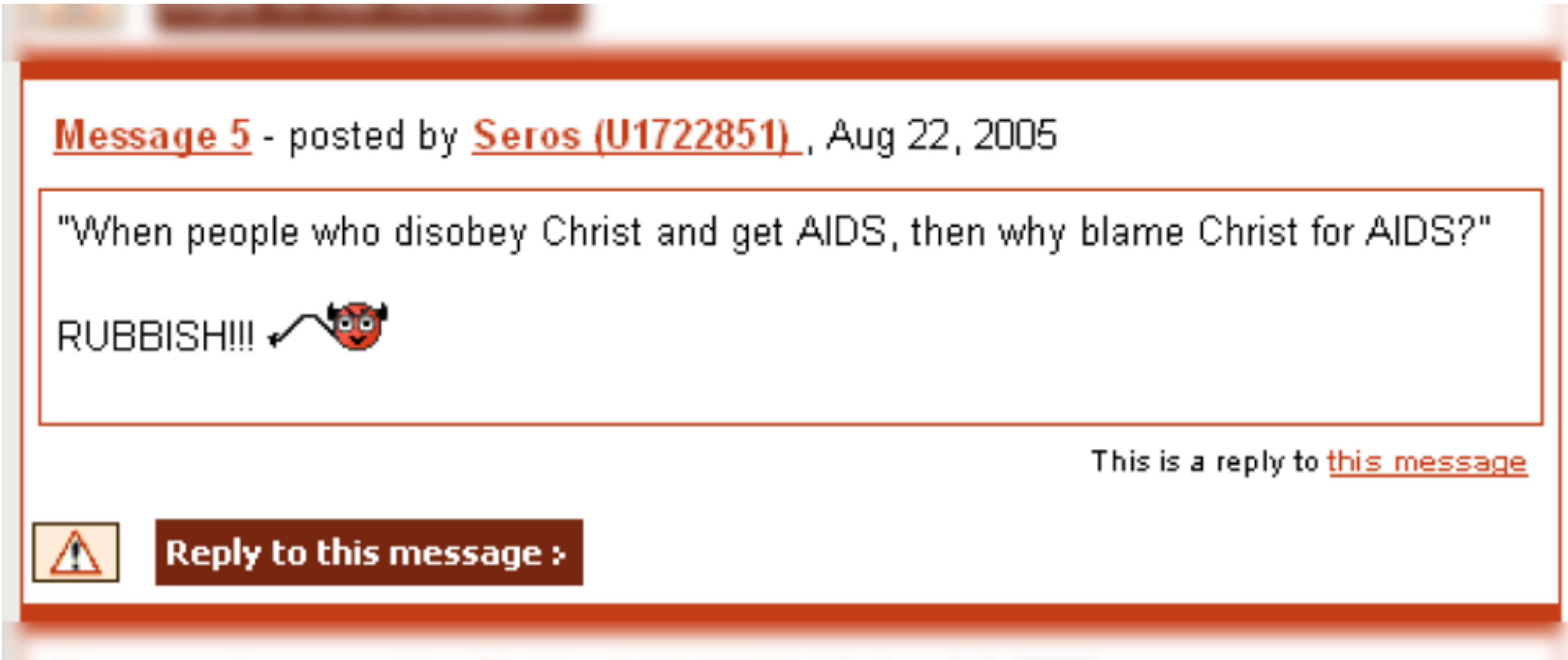
Behavior in simulations





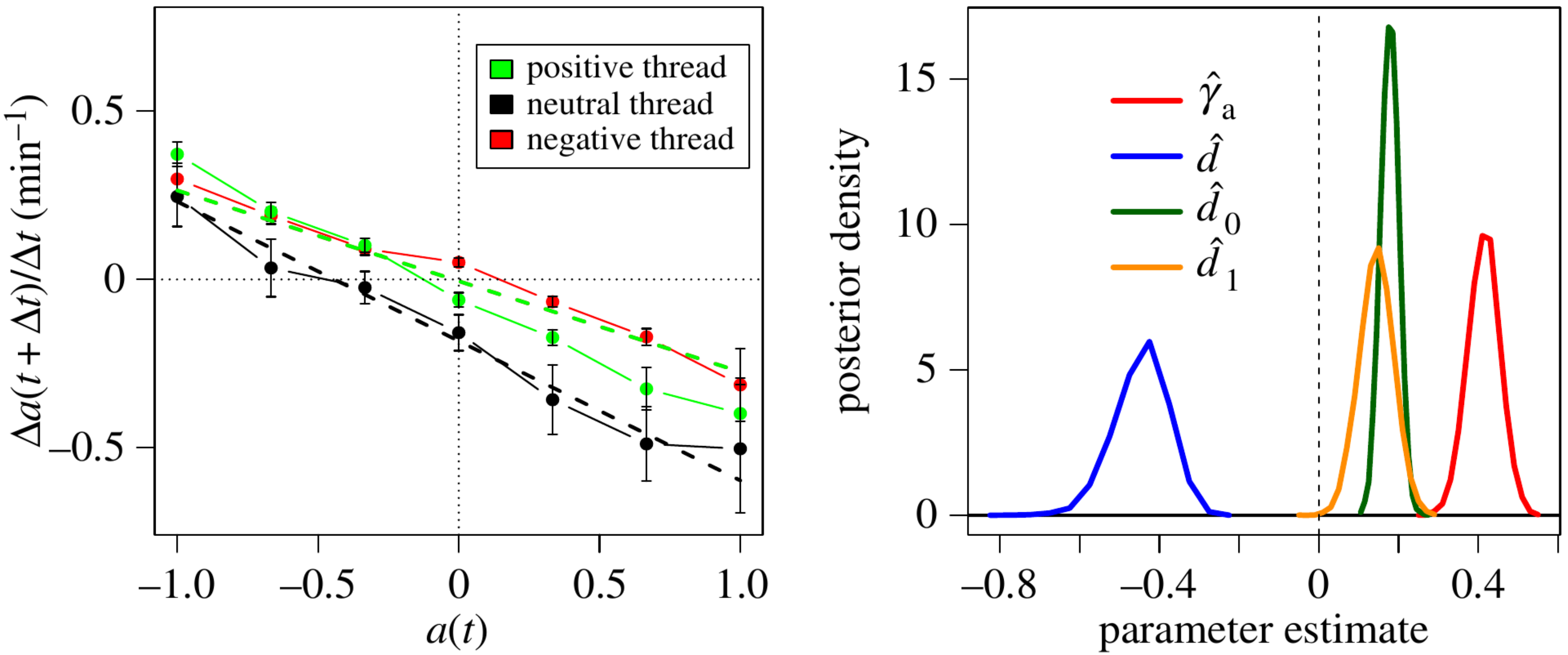
An agent-based model of collective emotions in online communities. Frank Schweitzer, David Garcia. The European Physical Journal B, 2010

Calibration experiment setup



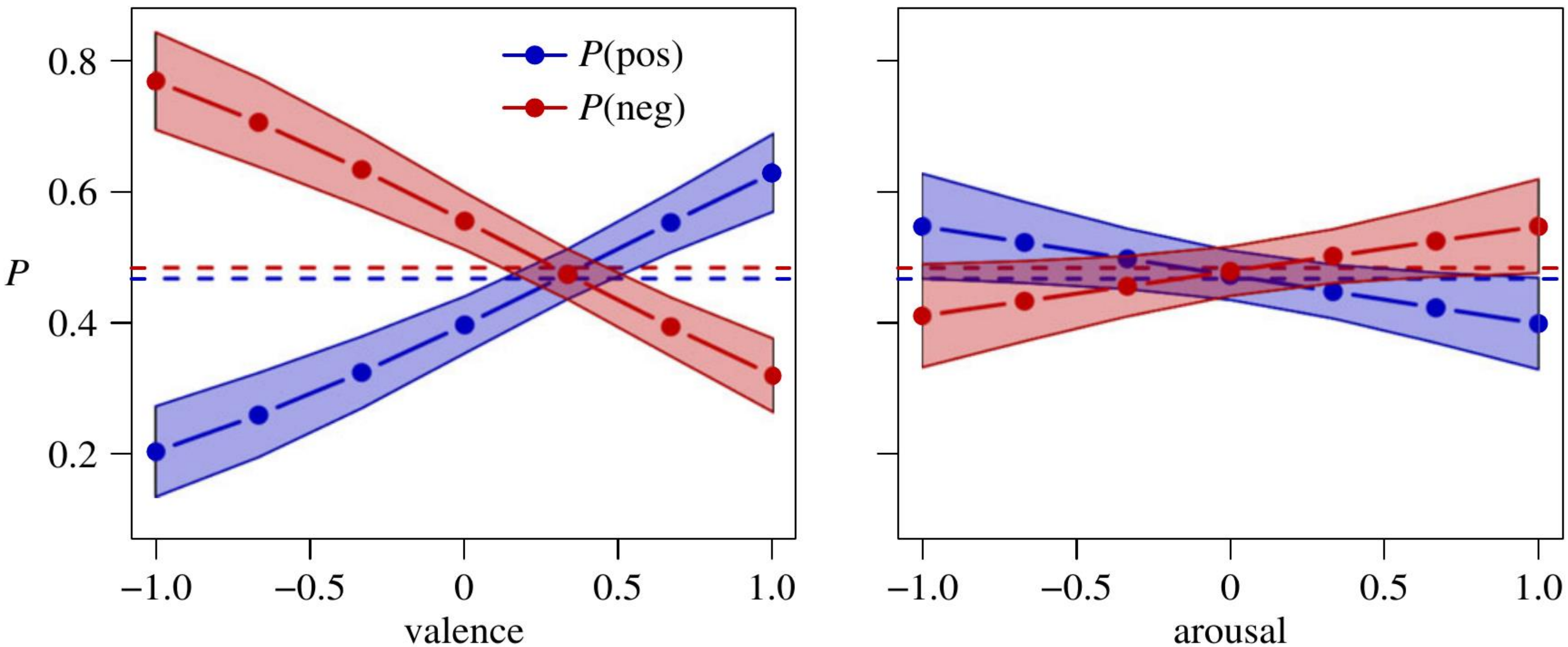
- Study 1: reading pos/neg/neu threads and self-reports at home
- Study 2: reading pos/neg/neu threads and self-reports in the lab
- Study 3: reply to pos/neg/neu threads and self-reports before/after
- Self-reports include valence and arousal ratings and intention to participate in discussion and to continue reading the thread

Arousal trigger results



•Dynamics well fitted by a linear function with intercept shift depending on thread abs value (|h|). Natural decay of arousal with γ_a=0.41[min−1]

Empirical expression function



•Probability of post being classified as positive vs not positive and negative vs not negative in logistic regression (SentiStrength output with threshold)
•Function of valence but independent of arousal

Summary

- Collective behavior in social systems
  - Complex versus complicated systems
  - Interaction versus diversity-induced collective behavior

- Granovetter's threshold model
  - Interaction in a well-mixed system given preferences or thresholds
  - Macro outcomes can vary a lot for small changes in threshold values
  - Variance in thresholds leads to aggregated activation

- Modelling online collective emotions
  - The Cyberemotions modelling framework
  - Activation dynamics based on arousal thresholds
  - Calibrating an emotions model with experiments

Quiz

- For a given σ, does μ change the outcome in Granovetter's model?
- How can you find the fraction of active agents at t=0 in Granovetter's model?