Compliant plugin

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Abstract

1 Time integration

Implicit scheme :

$$\begin{aligned}
\mathbf{v}_{n+1} &= \mathbf{v}_n + h\mathbf{M}^{-1}\left(\alpha\mathbf{f}_{n+1} + (1-\alpha)\mathbf{f}_n\right) && (1)\\
\mathbf{x}_{n+1} &= \mathbf{x}_n + h\left(\beta\mathbf{v}_{n+1} + (1-\beta)\mathbf{v}_n\right) && (2)
\end{aligned}$$

where index n denotes current values while index $n+1$ denotes next values.

$$\begin{aligned}
\Delta\mathbf{v} = \mathbf{v}_{n+1} - \mathbf{v}_n &= h\mathbf{M}^{-1}\left(\alpha\mathbf{f}_{n+1} + (1-\alpha)\mathbf{f}_n\right) && (3)\\
\Delta\mathbf{x} = \mathbf{x}_{n+1} - \mathbf{x}_n &= h(v + \beta\Delta v) && (4)
\end{aligned}$$

Constraint violation ϕ and its Jacobian $\mathbf{J}$:

$$\begin{aligned}
\mathbf{J} &= \frac{\partial\phi}{\partial\mathbf{x}} && (5)\\
\phi_{n+1} &\simeq \phi_n + \mathbf{J}\Delta x = \phi_n + h\dot{\phi} + \mathbf{J}h\beta\Delta\mathbf{v} && (6)\\
\dot{\phi}_{n+1} &\simeq \phi_{n+1} + \mathbf{J}\Delta v && (7)
\end{aligned}$$

Forces:

$$\begin{aligned}
\mathbf{f} &= \mathbf{f}_{ext} + \mathbf{J}^T\lambda && (8)\\
\lambda_i &= -\frac{1}{c_i}(\phi_i + d\dot{\phi}_i) && (9)
\end{aligned}$$

where subscript i denotes a scalar constraint.

Average Lagrange multipliers using equations 9, 6 and 7:

$$\begin{aligned}
\bar{\lambda}_i &= \alpha\lambda_{n+1} + (1-\alpha)\lambda_n\\
&= -\frac{1}{c_i}(\alpha\phi + \alpha h\dot{\phi} + \alpha h\beta\mathbf{J}\Delta\mathbf{v} + \alpha d\dot{\phi} + \alpha d\mathbf{J}\Delta v + (1-\alpha)\phi + (1-\alpha)d\dot{\phi})\\
&= -\frac{1}{c_i}(\phi + d\dot{\phi} + \alpha h\dot{\phi} + \alpha(h\beta + d)\mathbf{J}\Delta v)
\end{aligned}$$

We can rewrite the previous equation as:

$$\mathbf{J}\Delta v + \frac{1}{\alpha(h\beta+d)}\mathbf{C}\bar{\lambda} = -\frac{1}{\alpha(h\beta+d)}(\phi + (d+\alpha h)\dot{\phi}) \qquad (10)$$

where values without indices denote current values. The complete equation system is:

$$\begin{pmatrix} \frac{1}{h}\mathbf{PM} & -\mathbf{PJ}^T \\ J & \frac{1}{l}\mathbf{C} \end{pmatrix} \begin{pmatrix} \Delta v \\ \bar{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{Pf}_{ext} \\ -\frac{1}{l}(\phi + (d + \alpha h)\dot{\phi}) \end{pmatrix} \tag{11}$$

where $l = \alpha(h\beta + d)$ The system is singular due to matrix $\mathbf{P}$, however we can use $\mathbf{PM}^{-1}\mathbf{P}$ as inverse mass matrix to compute a Schur complement:

$$\begin{array}{ccc} \left(hJPM^{-1}PJ^T + \frac{1}{l}C\right)\bar{\lambda} & = & -\frac{1}{l}\left(\phi + (d + h\alpha)\dot{\phi}\right) - hJM^{-1}f_e \\ \Delta v & = & hPM^{-1}(f_e + J^T\bar{\lambda}) \\ \Delta x & = & h(v + \beta\Delta v) \end{array}$$