

Compliant plugin

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Abstract

1 Time integration

Implicit scheme :

$$\mathbf{v}_{n+1} = \mathbf{v}_n + h\mathbf{M}^{-1}(\alpha\mathbf{f}_{n+1} + (1-\alpha)\mathbf{f}_n) \quad (1)$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + h(\beta\mathbf{v}_{n+1} + (1-\beta)\mathbf{v}_n) \quad (2)$$

where index n denotes current values while index $n+1$ denotes next values.

$$\Delta\mathbf{v} = \mathbf{v}_{n+1} - \mathbf{v}_n = h\mathbf{M}^{-1}(\alpha\mathbf{f}_{n+1} + (1-\alpha)\mathbf{f}_n) \quad (3)$$

$$\Delta\mathbf{x} = \mathbf{x}_{n+1} - \mathbf{x}_n = h(v + \beta\Delta v) \quad (4)$$

Constraint violation ϕ and its Jacobian \mathbf{J} :

$$\mathbf{J} = \frac{\partial\phi}{\partial\mathbf{x}} \quad (5)$$

$$\phi_{n+1} \simeq \phi_n + \mathbf{J}\Delta\mathbf{x} = \phi_n + h\dot{\phi} + \mathbf{J}h\beta\Delta\mathbf{v} \quad (6)$$

$$\dot{\phi}_{n+1} \simeq \dot{\phi}_{n+1} + \mathbf{J}\Delta v \quad (7)$$

Forces:

$$\mathbf{f} = \mathbf{f}_{ext} + \mathbf{J}^T\lambda \quad (8)$$

$$\lambda_i = -\frac{1}{c_i}(\phi_i + d\dot{\phi}_i) \quad (9)$$

where subscript i denotes a scalar constraint.

Average Lagrange multipliers using equations 9, 6 and 7:

$$\begin{aligned} \bar{\lambda}_i &= \alpha\lambda_{n+1} + (1-\alpha)\lambda_n \\ &= -\frac{1}{c_i}(\alpha\phi + \alpha h\dot{\phi} + \alpha h\beta\mathbf{J}\Delta\mathbf{v} + \alpha d\dot{\phi} + \alpha d\mathbf{J}\Delta v + (1-\alpha)\phi + (1-\alpha)d\dot{\phi}) \\ &= -\frac{1}{c_i}(\phi + d\dot{\phi} + \alpha h\dot{\phi} + \alpha(h\beta + d)\mathbf{J}\Delta v) \end{aligned}$$

We can rewrite the previous equation as:

$$\mathbf{J}\Delta v + \frac{1}{\alpha(h\beta + d)}\mathbf{C}\bar{\lambda} = -\frac{1}{\alpha(h\beta + d)}(\phi + (d + \alpha h)\dot{\phi}) \quad (10)$$

where values without indices denote current values. The complete equation system is:

$$\begin{pmatrix} \frac{1}{h}\mathbf{P}\mathbf{M} & -\mathbf{P}\mathbf{J}^T \\ J & \frac{1}{l}\mathbf{C} \end{pmatrix} \begin{pmatrix} \Delta v \\ \bar{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{P}\mathbf{f}_{ext} \\ -\frac{1}{l}(\phi + (d + \alpha h)\dot{\phi}) \end{pmatrix} \quad (11)$$

where $l = \alpha(h\beta + d)$ The system is singular due to matrix \mathbf{P} , however we can use $\mathbf{P}\mathbf{M}^{-1}\mathbf{P}$ as inverse mass matrix to compute a Schur complement:

$$\begin{aligned} (hJPM^{-1}PJ^T + \frac{1}{l}C) \bar{\lambda} &= -\frac{1}{l}(\phi + (d + h\alpha)\dot{\phi}) - hJM^{-1}f_e \\ \Delta v &= hPM^{-1}(f_e + J^T\bar{\lambda}) \\ \Delta x &= h(v + \beta\Delta v) \end{aligned}$$