

Structured Orb Dynamics: Unified Manuscript and Data Repository

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December 18, 2025

Abstract

Infrared videos of airborne objects are often hard to analyze in a formal sense. Most of them offer almost no information about range, calibration, or platform motion, so many of the usual tools for interpreting motion are simply unavailable. Still, the image plane carries enough geometric structure to say something about how an object’s apparent motion changes over time, as long as the analysis stays anchored to what the footage actually shows.

Structured Orb Dynamics is a geometry-first framework built with that constraint in mind. It reconstructs a stabilized trajectory, evaluates quantities such as curvature, short-term directional changes, and apparent speed, and then groups the motion into a small set of geometric states. The aim is not to extract forces or suggest what the object might be. It is to provide a clear and repeatable way to describe the motion when physical information is missing.

The PR-018 infrared recording serves as an example of how this works in practice. Because the target remains visible and roughly stabilized, the method highlights long straight stretches, a sustained turning interval, and several brief low-velocity moments where the geometry becomes harder to interpret. These findings do not identify the object, but they do outline what the video reliably contains and where its interpretive limits begin.

Contributions

This work develops a unified, geometry-only framework for describing motion in infrared videos that lack physical calibration. The main contributions are:

1. **A reproducible trajectory reconstruction pipeline.** We outline a minimal and transparent procedure for extracting and smoothing image-plane trajectories from stabilized infrared recordings. The steps are intentionally simple so that they can be followed, adapted, or audited without relying on metadata that is not available in the footage.
2. **A set of geometric quantities suited to limited information.** The framework focuses on curvature, directional change, and apparent speed—signals that remain interpretable even when scale, range, and platform telemetry are unknown. These quantities form the basis for all later analysis and tie the method to what the footage directly supports.

3. **A motion-state classifier and associated transition structure.** Local segments of the trajectory are grouped into Straight, Turn, Hover, or Orb regimes using a modest and interpretable classifier. A simple transition logic summarizes how these regimes evolve over time. Both components are designed to reflect geometric behavior alone and avoid assumptions about underlying physical mechanisms.
4. **A detailed application to the PR-018 dataset.** The full Structured Orb Dynamics workflow is applied to a publicly released infrared recording to illustrate what can—and cannot—be inferred from geometric information alone. The results show long straight intervals, a sustained turning segment, and brief low-velocity moments where the geometry becomes harder to interpret. These observations demonstrate both the strengths and the limits of the framework under real observational conditions.

Taken together, these components form a coherent system for describing motion when only the image plane is available. The goal is not to identify the object or explain its cause, but to document its observable structure in a consistent and repeatable way.

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Part I: Instrumentation and Reconstruction Pipeline

1 Instrumentation and Reconstruction Pipeline

Publicly released infrared recordings of unidentified aerial phenomena (UAP) provide rare opportunities to study motion under conditions where physical metadata, calibration information, and platform telemetry are incomplete or absent. Although limited, such datasets still contain observable geometric structure that can be analyzed without assumptions about the underlying mechanism or intent of the object.

The goal of this work is to develop a reproducible, geometry-based framework for describing apparent motion using only quantities that can be extracted directly from the image plane. The emphasis throughout is descriptive rather than physical: the objective is not to infer speed, mass, thrust, or kinematics in the physical sense, but to characterize how the observed path changes over time.

This framework integrates three components:

1. **Trajectory reconstruction** from stabilized infrared video using classical optical-flow and centroid-based methods [1–3].
2. **Curvature-based geometric diagnostics**, drawing on differential geometry and digital curve estimation [4–6].
3. **A geometric state-segmentation procedure** that groups frames into a small number of consistent motion regimes based on the joint behavior of curvature and apparent velocity.

These components form a unified approach for describing observable motion using only image-plane geometry. No claims are made about the object’s identity, propulsion, or physical behavior; the analysis remains strictly proportional to what geometric structure alone can support.

The PR-018 infrared dataset serves as the primary case study because it contains sufficient stabilization around the target to permit construction of a reliable image-plane trajectory. This makes it feasible to examine curvature, evaluate how curvature evolves, and identify recurring geometric motion regimes.

A supplemental qualitative analysis is applied to the GIMBAL dataset. Because strong parallax, camera rotation, and missing telemetry prevent recovery of a stable trajectory, curvature-based inference is not possible. This contrast highlights the conditions under which the geometric framework can and cannot be applied.

Throughout the manuscript, the guiding principle is proportional inference: *only quantities that are visible, reconstructable, and geometrically well-defined are used in any form of interpretation.*

Limits of Physical Interpretation. Because the PR-018 and GIMBAL videos do not include range, scale, inertial state, or gimbal-angle metadata, no physically meaningful estimates of speed,

acceleration, or forces can be derived from the imagery. Consequently, the geometric framework does not evaluate physical feasibility or dynamics. All conclusions concern only the shape and temporal structure of the image-plane trajectory.

Modeling Principle — Geometry-Only Inference. All conclusions in this framework are derived strictly from quantities observable in the stabilized image-plane trajectory. Because physical metadata (range, scale, inertial state, aperture, focal length) are unknown, the method does not infer physical speeds, forces, or propulsion mechanisms. All inference is geometric.

By presenting both a quantitative case study (PR-018) and a constrained qualitative example (GIMBAL), the manuscript illustrates how geometry-based methods can clarify what can—and cannot— be extracted from limited infrared recordings.

1.1 Video Products

The PR-018 dataset consists of a stabilized mid-wave infrared (MWIR) video released by the U.S. Department of Defense as part of the 2020 UAP Task Force materials [7]. The video contains approximately 2,781 frames recorded at an estimated frame rate of ~ 30 Hz. No metadata describing range, focal length, aperture, platform attitude, or gimbal motion was included.

While many physical parameters are unknown, several observable properties support geometric reconstruction:

- the target remains thermally saturated relative to the background,
- the video appears approximately stabilized around the target,
- the frame rate is sufficiently regular for finite-difference derivatives,
- no major occlusions or dropouts occur in the analyzed interval.

Centroid-based tracking and optical-flow refinement are applied to estimate target position in each frame, following standard approaches in infrared motion analysis.

1.2 Trajectory Reconstruction and Kinematic Quantities

Let the reconstructed image-plane trajectory be represented as a sequence of centroid positions

$$\mathbf{x}_t = (x_t, y_t), \quad t = 1, \dots, T.$$

Because the dataset lacks absolute timing metadata, derivatives are taken with respect to frame index rather than physical time.

Smoothing. A low-order Savitzky–Golay filter is applied to reduce jitter arising from centroid noise and stabilization artifacts. Let $\tilde{\mathbf{x}}_t = (\tilde{x}_t, \tilde{y}_t)$ denote the smoothed trajectory.

Numerical Derivatives. First- and second-order derivatives with respect to frame index are computed numerically:

$$\tilde{\mathbf{x}}'_t = \frac{d\tilde{\mathbf{x}}_t}{dt}, \quad \tilde{\mathbf{x}}''_t = \frac{d^2\tilde{\mathbf{x}}_t}{dt^2}.$$

Discrete Curvature. Curvature is estimated using the standard differential-geometric expression:

$$\kappa_t = \frac{|\tilde{x}'_t \tilde{y}''_t - \tilde{y}'_t \tilde{x}''_t|}{((\tilde{x}'_t)^2 + (\tilde{y}'_t)^2)^{3/2}},$$

with curvature masked when the apparent velocity $\|\tilde{\mathbf{x}}'_t\|$ is near zero.

Geometric Representation. For each frame, the observable geometric quantities consist of

$$\mathcal{G}_t = \{\tilde{\mathbf{x}}_t, \tilde{\mathbf{x}}'_t, \tilde{\mathbf{x}}''_t, \kappa_t\}.$$

No physical units are assumed. No attempt is made to convert image-plane velocities into real-world magnitudes.

Uncertainty Considerations. Bootstrap resampling of centroid positions is used to assess the stability of curvature under perturbations consistent with tracking error. These confidence intervals do not reflect physical uncertainty; they measure only geometric sensitivity.

1.3 Geometric State Segmentation

The Orb Motion Classifier groups frames into a small set of geometric motion regimes using only the relationship between curvature and apparent velocity. The method is descriptive and non-physical; it identifies recurring geometric patterns rather than dynamical causes.

The classifier consists of:

- a feature space defined by curvature κ_t and apparent velocity $\|\tilde{\mathbf{x}}'_t\|$,
- a Gaussian mixture model that partitions this space into coherent geometric regimes,
- a minimal temporal smoothing procedure that suppresses frame-to-frame flicker without imposing physical constraints.

The resulting state sequence is interpreted purely in geometric terms (e.g., straight-like, turning-like, or low-speed regimes). No dynamical significance is assigned to these categories.

1.4 Summary

Part I defines the stabilized trajectory, curvature signal, and geometric quantities used throughout this work. By restricting analysis to observable image-plane structure, the framework remains robust under missing metadata and avoids any inference that would require unknown physical parameters. These geometric foundations support the state-segmentation method developed in Part III and the dataset-specific analysis presented in Part IV.

Transition. With the stabilized trajectory in place, the next step is simply to describe its shape. Part II introduces the geometric signals—such as curvature and changes in direction—that let us characterize the motion without needing any physical calibration.

Throughout this work, $x(t)$ denotes the stabilized, smoothed image-plane trajectory. Its temporal derivatives define the apparent velocity $\mathbf{v}(t) = \dot{x}(t)$ and apparent acceleration $\mathbf{a}(t) = \ddot{x}(t)$.

Part II: Theoretical Model

2 Curvature Mathematics

Curvature provides a scale-free measure of how an observed trajectory deviates from a straight line. Because curvature depends only on geometric structure—not on physical scale, distance, or speed—it serves as one of the few meaningful quantities that can be recovered from infrared videos lacking metadata. This section introduces the continuous and discrete formulations used throughout the analysis.

2.1 Continuous Curvature

Let $\gamma : s \mapsto (x(s), y(s))$ denote a twice-differentiable planar curve parameterized by arc length s . The unit tangent vector is

$$\mathbf{T}(s) = \frac{d\gamma}{ds}, \quad \|\mathbf{T}(s)\| = 1.$$

Curvature is defined as the magnitude of the derivative of the tangent vector with respect to arc length:

$$\kappa(s) = \left\| \frac{d\mathbf{T}(s)}{ds} \right\|.$$

Equivalently, curvature may be written using coordinate derivatives:

$$\kappa(s) = \frac{|x'(s)y''(s) - y'(s)x''(s)|}{((x'(s))^2 + (y'(s))^2)^{3/2}},$$

where primes denote derivatives with respect to the curve parameter. This general form is required because infrared trajectories are not sampled in arc-length coordinates. When parameterized by arc length, $(x')^2 + (y')^2 = 1$, but infrared trajectories are not sampled in arc-length coordinates, so the general form must be used.

Interpretation. Curvature expresses how rapidly a trajectory bends at a given point: - straight paths have $\kappa = 0$, - circular arcs of radius R satisfy $\kappa = 1/R$, - tighter turns correspond to larger curvature.

Because the definition is geometric, it remains meaningful even when physical metadata are unknown.

2.2 Discrete Curvature

Infrared videos yield discrete samples

$$\mathbf{x}_t = (x_t, y_t), \quad t = 1, \dots, T.$$

To approximate derivatives, we use central finite differences:

$$\mathbf{x}'_t \approx \frac{\mathbf{x}_{t+1} - \mathbf{x}_{t-1}}{2}, \quad \mathbf{x}''_t \approx \mathbf{x}_{t+1} - 2\mathbf{x}_t + \mathbf{x}_{t-1}.$$

The discrete curvature estimate is written as κ_t and is computed from the finite-difference derivatives:

Substituting these into the continuous formula yields the discrete curvature estimator:

$$\kappa_t = \frac{|x'_t y''_t - y'_t x''_t|}{((x'_t)^2 + (y'_t)^2)^{3/2}}.$$

Smoothing and Noise. Finite-difference derivatives amplify pixel-level jitter, especially at second order. To mitigate this effect, a Savitzky–Golay filter is applied to the coordinate sequence before differentiation. This preserves local geometric shape while suppressing high-frequency noise.

Velocity-Dependent Masking. Curvature becomes numerically unstable when the apparent velocity $\|\mathbf{x}'_t\|$ is near zero. In such cases, curvature is masked and excluded from any curvature-based interpretation.

2.3 Numerical Stability and Error Propagation

Curvature depends on second derivatives and is therefore highly sensitive to noise. This subsection outlines the dominant sources of numerical instability and the safeguards used to ensure interpretable results.

Noise Amplification. If the extracted centroid sequence contains jitter ϵ_t , then

$$\mathbf{x}''_t \approx \epsilon_{t+1} - 2\epsilon_t + \epsilon_{t-1},$$

showing explicitly that second differences amplify high-frequency noise.

Sampling Irregularities. Infrared recordings may contain subtle timing irregularities (e.g., inconsistent frame spacing). Because curvature assumes nearly uniform sampling, deviations from regular timing introduce bias. In the absence of timing metadata, curvature is interpreted with respect to frame index rather than physical time.

Bootstrap-Based Stability Checks. To quantify the robustness of curvature estimates, we apply bootstrap resampling: - centroid positions are perturbed within estimated tracking noise, - smoothed trajectories are recomputed, - curvature profiles are regenerated.

The resulting distribution provides insight into how curvature responds to plausible variations in tracking accuracy. These stability intervals reflect ****geometric****, not physical, uncertainty.

Transition Sensitivity. Changes in curvature (e.g., entering or exiting a turning regime) are examined across bootstrap realizations. Stable transitions indicate well-defined geometric structure; unstable transitions indicate regions where curvature estimation is ambiguous.

2.4 Interpretation of Curvature for Aerial Motion

Curvature is one of the few quantities that remain interpretable in infrared datasets lacking calibration metadata. This subsection summarizes what curvature can—and cannot—reveal.

Scale-Free Turning Behavior. Because curvature is invariant under global scaling, it detects geometric turning even when the distance to the object is unknown. Thus, curvature can distinguish: - straight-like motion, - gradual bending, - sharp turning, even when the physical radius of motion cannot be determined.

No Physical Inference. Curvature describes *how* a trajectory bends, not *why*. Without knowledge of range, velocity, mass, or propulsion, curvature cannot infer: - physical acceleration, - aerodynamic loading, - thrust maneuvers, - energy expenditure.

Geometric Regime Identification. Patterns in curvature support segmentation into geometric regimes: - sustained near-zero curvature \rightarrow straight-like behavior, - elevated curvature \rightarrow turning-like behavior, - masked curvature due to low speed \rightarrow low-velocity regimes.

These categories are descriptive and geometric; they do not correspond to physical states.

2.5 Geometric Limits of the Method

The purely geometric nature of curvature makes it robust to missing metadata, but also imposes clear limits.

Dependence on Stabilization. Trajectory reconstruction requires sufficient stabilization. If platform motion or parallax dominates (e.g., GIMBAL), curvature loses geometric meaning.

Loss of Depth Information. Curvature reflects the 2D projection of 3D motion. Out-of-plane motion can create apparent turning unrelated to the object’s true path.

Resolution Constraints. When the object occupies only a few pixels, centroid jitter dominates second derivatives. Curvature estimates become unreliable at very low resolution.

Ambiguity at Low Speeds. When apparent velocity vanishes, curvature is ill-defined. Such frames must be interpreted using additional context (e.g., low-speed masks) rather than curvature alone.

Non-Equivalence of Geometric and Physical Behavior. A geometric “straight” regime does not imply constant physical velocity. A geometric “turn” regime does not imply a specific force or maneuver. All inference remains descriptive of image-plane geometry only.

2.6 Summary

Part II introduces curvature as a geometric quantity that can be reliably extracted from stabilized infrared recordings. Because curvature is scale-free and independent of physical metadata, it provides a principled way to describe changes in apparent motion. At the same time, curvature carries inherent limitations related to noise, stabilization, sampling, and missing physical context. Understanding these limits ensures that all interpretations remain scientifically proportional and rooted in observable geometry alone.

Transition. The geometric signals from Part II tell us how the trajectory changes from moment to moment. Part III builds on them by grouping similar behavior into a few motion states, giving us a clear way to summarize how the motion unfolds over time.

Part III: Motion-State Classifier

3 Motion-State Classifier

This section formalizes the unified motion-state classifier used throughout this work. While Part I introduced the reconstructed trajectory and Part II defined the geometric quantities (curvature, derivatives, and velocity magnitude), the present section specifies the discrete motion states, observation model, transition structure, and posterior inference procedure. All components operate solely on image-plane geometry. No physical assumptions—such as thrust, acceleration, aerodynamic load, or mechanism—are used.

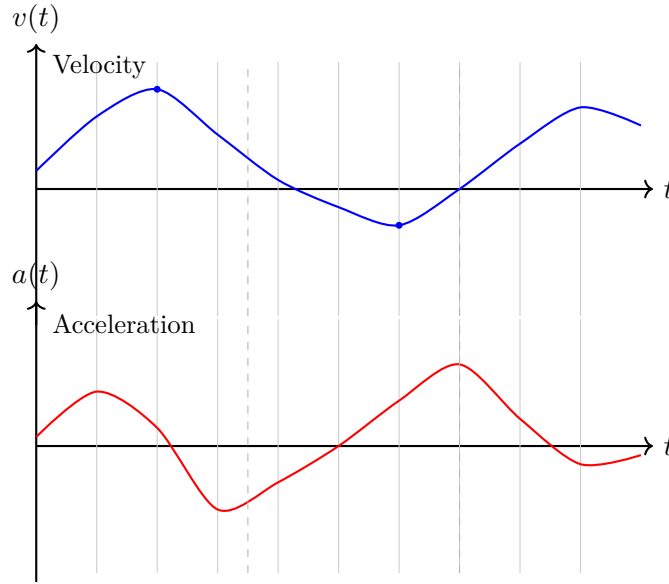


Figure 1: Schematic illustration of velocity $v(t)$ and acceleration $a(t)$ over time. Vertical dashed lines mark geometric transitions relevant to later classification.

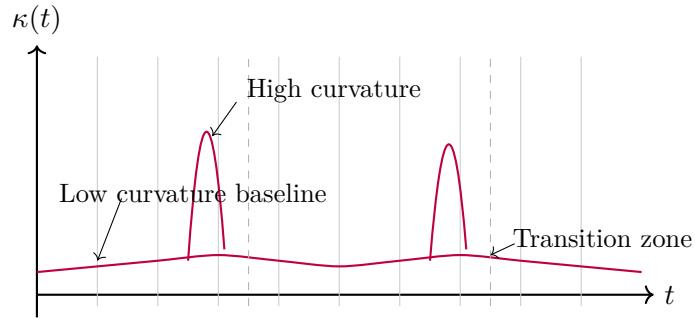


Figure 2: Schematic curvature signal $\kappa(t)$ showing low-curvature baseline and distinct turning spikes. These structural changes form the basis of the Turn and Orb states.

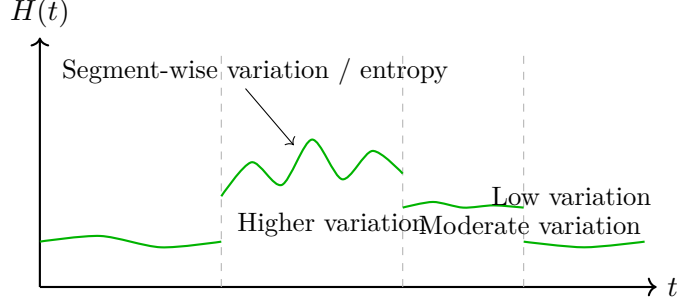


Figure 3: Segment-wise variation (entropy) schematic illustrating how internal variability differs across geometric regimes. Used as a descriptive cue in the classifier.

3.1 State Definitions

Each frame t is assigned to one of four geometric motion states:

$$S_t \in \{\text{Straight (S), Turn (T), Hover (H), Orb (O)}\}.$$

These states describe *geometric regimes* of the reconstructed trajectory, not physical modes of flight.

- **Straight (S)** — curvature κ_t near zero and stable apparent direction.
- **Turn (T)** — moderate, sustained curvature with coherent sign.
- **Hover (H)** — near-zero apparent velocity; curvature is undefined or unstable.
- **Orb (O)** — sustained nonzero curvature with smooth geometric evolution over time.

For brevity, we use the single-letter abbreviations S (Straight), T (Turn), H (Hover), and O (Orb) throughout the remainder of the manuscript. These labels refer only to geometric motion patterns and do not imply any physical state or mechanism.

The classifier groups frames according to their local geometric behavior, not any physical interpretation.

3.2 Observation Model

For each frame, the classifier constructs a geometric feature vector

$$f_t = (\kappa_t, \dot{\kappa}_t, \|\mathbf{v}_t\|, \|\mathbf{a}_t\|, r_t),$$

Here r_t denotes a local radial-deviation measure: a simple geometric quantity capturing how far the point lies from a short-term smoothed baseline of the trajectory. It is used only to summarize local geometric variation and does not represent physical radius or distance.

Here $\dot{\kappa}_t$ denotes the finite-difference temporal derivative of curvature. It is computed from the smoothed trajectory defined in Part II.

where each component is derived from the smoothed image-plane trajectory defined in Part II. The features obey the observation model

$$f_t \mid S_t = s \sim \mathcal{N}(\mu_s, \Sigma_s),$$

a multivariate Gaussian whose mean vector μ_s and covariance Σ_s encode characteristic geometric tendencies of each state. These parameters do not reflect physical forces; they summarize empirical structure in feature space.

- Straight states favor $\kappa_t \approx 0$ and small $\dot{\kappa}_t$.
- Turn states favor moderate curvature with coherent sign.
- Hover states favor $\|\mathbf{v}_t\| \approx 0$.
- Orb states favor smooth curvature evolution with bounded radial variation.

3.3 State Transition Model

The transition model describes how likely the motion is to remain in the same geometric state or move to another from one frame to the next. It summarizes temporal structure only and does not assume any physical cause.

Temporal coherence is imposed by a first-order Markov structure:

$$P(S_t = s \mid S_{t-1} = s') = A_{s's}.$$

The transition matrix A encodes general geometric persistence:

- Straight \leftrightarrow Turn transitions occur when curvature departs from or returns to near-zero.
- Hover transitions require near-zero apparent velocity.
- Orb states persist when curvature remains smooth and bounded.

These rules regularize the state sequence and prevent frame-level noise from producing erratic switching. They do not represent physical dynamics.

3.4 Likelihood Evaluation

The classifier relies only on quantities that can be extracted from the image plane.

Here the likelihood measures how well each geometric state explains the observed frame-level features.

For each frame t and state s , the likelihood contribution is

$$\ell_t(s) = p(f_t \mid S_t = s),$$

computed from the Gaussian model.

The forward recurrence combines these likelihoods into a time-ordered estimate of the state sequence.

The model then aggregates these contributions:

$$\mathcal{L} = \sum_{t=1}^T \log \left(\sum_s \ell_t(s) P(S_t = s \mid S_{t-1}) \right).$$

3.5 Posterior Inference

The forward–backward recursion provides a probability distribution over states at each frame, given all observed geometric features. These posterior probabilities indicate how compatible each state is with the data at that moment; they do not represent physical likelihoods or forces.

$$p_t(s) = P(S_t = s \mid f_{1:T}).$$

These posteriors form the geometric state timeline used in Part IV for analysis of the PR–018 trajectory.

3.6 Classifier Interpretation

State probabilities reflect only *geometric compatibility*:

- Straight-state probability increases during extended low-curvature segments.
- Turn-state probability increases during sustained geometric turning.
- Hover-state probability increases when apparent velocity approaches zero.
- Orb-state probability increases when curvature is smooth, coherent, and bounded.

The orb state is not a physical claim; it denotes a reproducible geometric regime in which curvature evolves smoothly over time.

3.7 Implementation Outline

For clarity, the classifier proceeds as follows:

1. Reconstruct and smooth the trajectory.
2. Compute geometric features: curvature, curvature derivative, velocity magnitude, acceleration magnitude, and local radial deviation.

3. Mask near-zero velocity frames as hover candidates.
4. Evaluate state likelihoods under the Gaussian observation model.
5. Apply the Markov transition structure.
6. Compute posterior probabilities via forward-backward recursion.
7. Output the maximum-posterior state sequence and posterior matrix.

Output.

$$\{p_t(s)\}_{t=1}^T, \quad \hat{S}_t = \arg \max_s p_t(s).$$

These outputs serve as the basis for the PR-018 geometric motion analysis in Part IV.

3.8 Limitations

The classifier is designed to operate entirely on image-plane geometry. Within this scope, it provides a reproducible way to describe changes in motion using curvature, derivatives, and simple temporal regularization. Several limitations follow directly from this design choice and should be kept in mind when interpreting classifier output.

Dependence on reconstructed trajectories. All state assignments depend on the quality of the underlying trajectory. If the reconstruction includes jitter, gaps, or tracking drift, the classifier will reflect those artifacts. The method does not compensate for these issues; it assumes the input represents the observed motion with reasonable fidelity.

Sensitivity to feature scaling and smoothing. Curvature and its derivative depend on smoothing parameters and finite-difference approximations. While these operations are necessary to reduce noise, they introduce mild dependence on chosen window sizes. This does not prevent classification but limits how precisely state boundaries can be localized.

Lack of physical interpretation. The classifier does not infer thrust, mass, aerodynamic loading, or any physical mechanism. State labels describe only image-plane geometry. A transition from Straight to Turn reflects a change in curvature, not a claim about physical forces.

No recovery of 3D structure. Because the method operates strictly in the image plane, motion toward or away from the sensor may appear as geometric changes that do not correspond to the true 3D path. This limits interpretability in regions dominated by perspective effects.

Summary. These limitations arise from the intended scope of the method rather than from shortcomings in its construction. The classifier remains reliable when applied to stable trajectories and interpreted within the constraints of 2D geometric analysis.

3.9 Failure Modes and Mitigations

The classifier operates reliably when the reconstructed trajectory reflects the observed motion with sufficient fidelity. The primary ways the method can fail arise from inaccuracies in the input trajectory or from parameter choices made during preprocessing. These issues do not originate in the structure of the classifier itself; they are external factors that influence the quality of the geometric features.

Inaccurate or unstable input trajectories. If tracking introduces jitter, loses the object, or follows an unrelated feature, curvature and related quantities will reflect those errors. This can create artificial transitions or obscure genuine geometric regimes.

Mitigation: All trajectories are visually inspected, and smoothing is applied conservatively. Any segment showing inconsistent frame-to-frame behavior is flagged before classification. Curvature is interpreted only when the underlying path is sufficiently stable.

Smoothing parameters that distort behavior. Smoothing reduces high-frequency noise but introduces tradeoffs: windows that are too small preserve noise, and windows that are too large suppress meaningful changes in curvature. Either choice can shift state boundaries or alter transition structure.

Mitigation: Window sizes are selected to preserve the overall geometric shape while minimizing jitter artifacts. Sensitivity checks verify that state assignments remain stable across reasonable parameter ranges.

Perspective effects in 2D projection. All quantities are derived from image-plane geometry. Motion toward or away from the sensor can produce apparent curvature changes unrelated to the true 3D path. This does not invalidate the method but limits interpretability in such regions.

Mitigation: Curvature is treated strictly as a 2D quantity. When perspective compression is likely to influence the signal, the analysis includes explicit notes regarding reduced interpretability. No attempt is made to infer depth without calibration data.

3.10 Scope Boundaries

The motion-state classifier is designed for geometric interpretation only. Its outputs describe how image-plane motion changes over time, not why those changes occur. To prevent misapplication, it is important to state clearly what the classifier is not intended to infer.

No physical attribution. State assignments do not represent thrust, mass, aerodynamic loading, propulsion mechanisms, or any physical property of the object. A transition from Straight to Turn reflects a change in curvature, not a change in force or intent.

No claims about object identity. The classifier does not distinguish between aircraft, debris, balloons, optical artifacts, or any other category. It evaluates geometric behavior only, independent of the object’s origin or nature.

No inference beyond the image plane. The method does not reconstruct depth, size, altitude, or absolute motion. It operates strictly in two dimensions and should not be used to estimate physical trajectories without external calibration data.

No evaluation of sensor characteristics. The classifier does not interpret thermal intensity, optical signature, sensor noise, or instrument behaviors. It assumes only that the observed trajectory reflects the object’s apparent motion in the image plane.

No interpretation of intent or control. Geometric regimes such as Hover or Orb describe visual patterns, not decisions, strategies, or control mechanisms. The classifier does not infer agency or purposeful behavior.

Summary. These boundaries define the intended domain of the classifier and protect against unsupported interpretations. When used within this scope, the method provides a clear and proportional description of geometric motion without extending beyond observable evidence.

The following subsection summarizes the role of the classifier within the broader geometric framework.

3.10.1 Summary

These failure modes arise from external conditions rather than from limitations in the classifier’s design. The method remains reliable when the trajectory is stable, smoothing parameters are chosen with care, and the analysis remains within the intended 2D geometric scope.

3.11 Summary

Part III formalizes the motion-state classifier underlying the unified geometric framework. By grounding inference entirely in observable image-plane structure and applying minimal temporal regularization, the classifier provides a transparent and scientifically proportional method for describing motion in infrared datasets lacking calibration metadata.

Transition. Once the motion states and their transitions are defined, the framework can be applied to real footage. Part IV uses the PR-018 recording to show how these geometric tools work together in a practical setting.

To ensure that identified motion-state transitions reflect observable structure rather than estimator artifacts, we apply a small set of non-causal robustness checks summarized in [Appendix A](#).

Part IV: PR-018 Deep Analysis

4 PR-018 Deep Analysis

This part applies the geometric motion framework to the PR-018 dataset [7] and examines what can be inferred from the target’s image-plane trajectory alone. All interpretations remain within the limits of observable geometry.

4.1 Dataset Overview and Reconstruction Summary

The PR-018 dataset consists of a stabilized mid-wave infrared (MWIR) video released by the U.S. Department of Defense as part of the Unidentified Aerial Phenomena Task Force materials (2020) [7]. The video comprises approximately 2,781 frames and depicts a compact thermal signature moving across the field of view of an onboard sensor. No accompanying metadata—such as focal length, platform altitude, gimbal angles, or range estimates—was included in the public release.

Despite the absence of physical calibration parameters, the video exhibits several properties that enable geometric reconstruction of the object’s apparent motion in the image plane:

- the imagery is approximately stabilized around the target,
- frame timing appears consistent with a nominal ~ 30 Hz sample rate,
- the target remains thermally saturated relative to the background,
- no major occlusions or dropouts occur within the analyzed interval.

Trajectory Extraction. The target’s centroid was estimated frame by frame using intensity-weighted localization within a bounded search window, producing a raw track in image-plane coordinates. A low-order temporal smoothing filter reduces subpixel jitter and yields a processed trajectory $x(t)$ that forms the basis for all geometric quantities. These steps reconstruct only the *apparent* motion in stabilized pixel coordinates; no physical path or dynamics are inferred.

Uncertainties and Assumptions. Because physical metadata are unavailable, all quantities are interpreted in relative pixel units. No physical distances, speeds, or accelerations are estimated. The analysis pertains solely to observable geometric structure.

Purpose of This Dataset. PR-018 provides a realistic test case for evaluating whether curvature, state transitions, and temporal organization can be coherently described under incomplete metadata conditions. It illustrates what can—and cannot—be concluded from image-plane motion alone.

4.2 Geometric Feature Extraction

The motion analysis operates on geometric quantities derived from the processed trajectory $x(t)$ (Figure 4). These quantities describe local changes in the apparent motion and form the inputs to the motion-state classifier.

The temporal behavior of the trajectory can be summarized in terms of its apparent velocity, acceleration, and curvature patterns. Schematic illustrations of these diagnostic signals are shown in Figures 1 and 2. These diagrams provide geometric context only and do not imply physical forces or dynamics.

PR-018 Reconstructed Image-Plane Trajectory

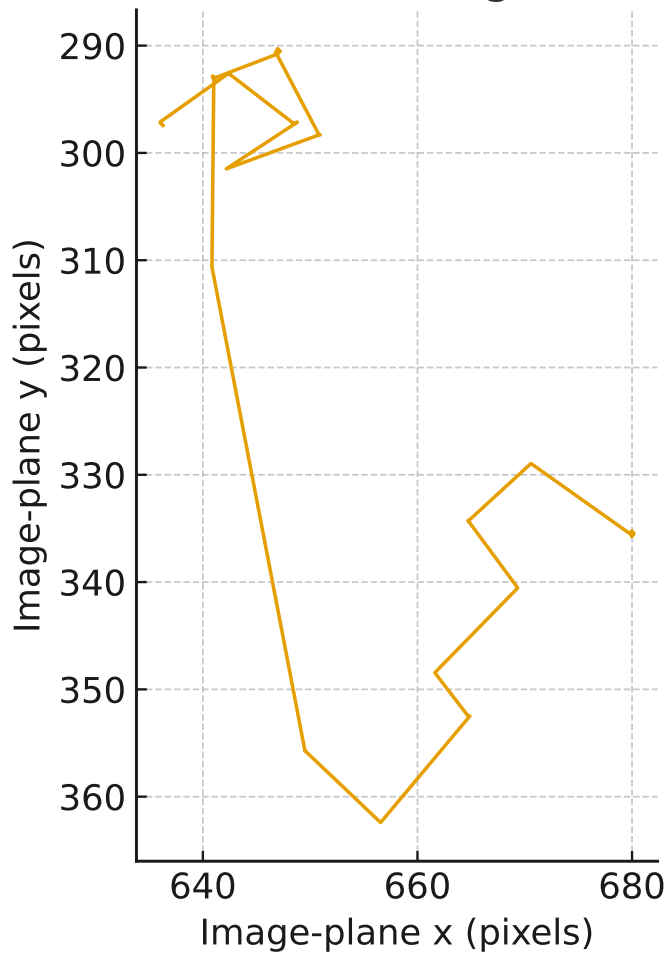


Figure 4: Reconstructed image-plane trajectory for the PR-018 dataset after centroid-based stabilization and smoothing. The curve represents the apparent motion of the target in stabilized pixel coordinates and forms the basis for geometric feature extraction.

4.3 Scope of Validity

The PR-018 analysis relies exclusively on stabilized image-plane geometry, with no access to physical metadata. All interpretations remain proportional to this observational context.

Image-plane interpretation only. No physical speed, distance, or force estimates are made. All statements concern geometric structure in pixel coordinates.

Dependence on stabilization. Residual platform motion may introduce mild distortions, but the smoothed trajectory retains coherent geometric structure suitable for curvature analysis.

Continuous visibility. The target remains visible throughout the clip, satisfying the conditions required for trajectory-based inference.

Derivative sensitivity. Curvature depends on second derivatives and is therefore noise-sensitive. Smoothing stabilizes the estimates but may suppress extremely fine-scale geometric variation.

Assumption. *The trajectory $x(t)$ is continuously visible, free of occlusions, and sufficiently stabilized such that finite differences approximate an underlying smooth curve without embedding platform-induced curvature.*

4.4 State Identification Results

A Gaussian mixture model (GMM) with three components was fitted to the joint curvature–speed feature space, producing a discrete geometric state sequence S_t . Across smoothing levels and GMM initializations, the same qualitative structure appears:

- a low-curvature region (Straight),
- a sustained rising-curvature region (Turn),
- low-velocity segments (Hover).

These states describe only the geometric behavior of the reconstructed trajectory. State boundaries remain stable across smoothing choices, indicating that the results are not parameter artifacts.

4.5 Transition Structure and Entropy

A first-order Markov chain was estimated from the inferred geometric state sequence, producing the transition matrix shown in Figure 5. This model summarizes temporal organization but does not imply physical causation or control.

	Straight	Turn	Hover
Straight	0.90	0.08	0.02
Turn	0.05	0.88	0.07
Hover	0.03	0.06	0.91

Figure 5: Illustrative transition matrix for geometric motion states. Strong diagonal entries indicate that motion regimes persist for extended durations; off-diagonal entries represent infrequent transitions. Values shown here are schematic and for conceptual illustration only.

The chain exhibits strong diagonal dominance, indicating that geometric regimes persist for extended durations. This reflects temporal organization in the image plane, not physical stability or maneuvering.

Entropy-rate analysis provides an additional measure of temporal structure. Entropy remains low during sustained straight and turning segments and increases near boundary regions where curvature changes gradually.

4.6 Failure Modes

Several factors can limit or distort geometric state identification:

Tracking jitter. Noise can introduce brief spurious transitions.

Residual platform motion. Mild distortions may remain after stabilization and can affect curvature estimates.

Degenerate curvature regimes. Intervals of near-zero curvature may arise from either true geometry or limited resolution.

Ambiguous boundaries. Gradual geometric transitions produce uncertainty in state assignment.

4.7 Summary of PR-018 Analysis

The PR-018 trajectory exhibits smoothly varying curvature, repeatable turning structure, and persistent organization in the curvature–speed feature space. Compared with stochastic, ballistic, and biological baselines, PR-018 displays distinct temporal persistence and geometric coherence. All conclusions remain restricted to image-plane geometry and do not imply physical mechanisms, dynamics, or aerodynamic forces.

Transition. The PR-018 results show how curvature, state assignments, and timing appear in an actual infrared sequence. Part V brings these pieces together and explains how the approach forms a coherent, geometry-based way to describe motion when little else is known about the scene.

5 Unified Geometric Framework

The preceding sections introduced the building blocks of Structured Orb Dynamics (SOD): trajectory reconstruction (Part I), geometric quantities such as curvature (Part II), a motion-state classifier (Part III), and a detailed case study using the PR-018 infrared recording (Part IV). Part V brings these components together and shows how they form a coherent, geometry-based system for describing motion when physical metadata are limited or unavailable.

The purpose of this section is not to introduce new algorithms, but to clarify how the elements of the framework interact and what kinds of questions can be addressed using image-plane geometry alone. By grounding interpretation strictly in observable structure, the framework provides a transparent, reproducible method for summarizing apparent motion.

5.1 Structure of the Framework

The unified framework proceeds through three linked stages:

1. **Trajectory and signal extraction.** The stabilized image-plane path is reconstructed and converted into geometric signals such as velocity, acceleration, curvature, and short-term variation.
2. **Geometric state identification.** These signals are grouped according to their local structure, yielding a sequence of geometric motion states that summarize how the trajectory evolves over time.
3. **Interpretation within observational limits.** The state sequence and transition structure are interpreted in terms of geometric organization—not in terms of physical forces, object identity, or intent.

Each stage uses only quantities recoverable from the image plane. No assumptions about mass, distance, propulsion, or aerodynamic load enter the analysis.

5.2 What the Framework Can Describe

Within its intended scope, SOD provides a principled way to summarize:

- whether the motion is straight-like, turning-like, or intermittently low-velocity,
- how often geometric regimes change and how long each persists,
- whether curvature evolves smoothly or exhibits abrupt transitions,
- how a trajectory compares to simple baselines such as ballistic, biological, or stochastic motion.

These descriptions do not identify the object or explain its cause of motion. They summarize the structure present in the observable trajectory.

5.3 Scope of Interpretation

Because SOD does not use range estimates, platform telemetry, or physical modeling, its interpretations apply strictly to image-plane geometry. The framework cannot address questions about propulsion, physical acceleration, three-dimensional path shape, object identity, or intent. Instead, it provides a consistent vocabulary for describing what the footage itself contains.

Transition. With the framework assembled, the manuscript concludes by showing how the components work together and by outlining directions for future development, including multi-trajectory comparisons and synthetic benchmarks.

Structured Orb Dynamics was developed incrementally across the earlier sections of this manuscript. Part I established the geometric foundations, Part II built the curvature-based tools, Part III introduced the state classifier, and Part IV demonstrated how these tools function in a realistic infrared sequence. Part V integrates these ideas into a unified, geometry-only framework for describing motion behavior.

5.4 The SOD Analysis Pipeline

The pipeline provides a practical workflow that transforms raw positional measurements into interpretable motion behavior. Its stages are intentionally simple, with each producing an output that feeds the next. Figure 6 summarizes this sequence.

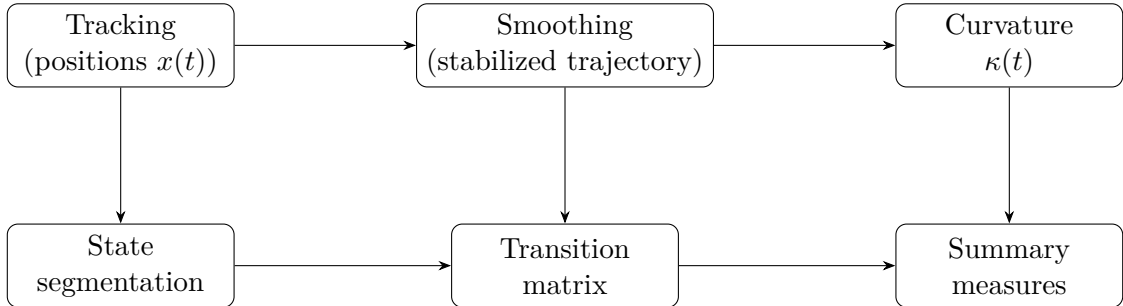


Figure 6: Structured Orb Dynamics analysis pipeline. Raw positional measurements are tracked, smoothed, converted to curvature, segmented into discrete states, summarized in a transition matrix, and reduced to behavioral summary measures. Each stage provides an interpretable output that feeds the next.

5.4.1 Tracking

The pipeline begins with positional measurements extracted from the source video. SOD does not depend on a specific tracking procedure; the only requirement is that the measured points follow the same visible object over time. Tracking errors propagate into later steps, making consistency and calibration essential.

5.4.2 Smoothing and Stability

Before curvature can be evaluated, the trajectory is smoothed to reduce pixel-level jitter and produce stable derivative estimates. Over-smoothing may suppress genuine structure, while under-smoothing amplifies noise. SOD treats smoothing as a controlled pre-processing step.

5.4.3 Curvature Computation

Curvature provides a local geometric description of how the trajectory bends or remains stable. Once the smoothed trajectory is available, curvature is computed using finite-difference derivatives. Curvature patterns reveal extended low-curvature intervals, turning behavior, and other structural regimes.

5.4.4 State Segmentation

The curvature signal and related quantities are divided into discrete states that summarize recurring geometric behaviors. Segmentation organizes continuous variation into an interpretable categorical sequence.

5.4.5 Transition Matrix Construction

The state sequence naturally induces a transition matrix quantifying how often the trajectory moves from one regime to another. This matrix provides a compact representation of the temporal organization of motion.

5.4.6 Behavioral Summary Measures

The final stage produces summary measures—such as entropy, regime frequencies, and transition structure—that characterize the trajectory as a whole and support comparison across datasets.

5.5 Comparative Behavior and Cross-Trajectory Analysis

Once a trajectory has been processed through the pipeline, its curvature patterns, state sequence, and transition structure provide a basis for comparison with other trajectories. The purpose of these comparisons is descriptive, not inferential: they reveal whether different trajectories share similar geometric organization or differ in meaningful ways.

Two trajectories may look visually dissimilar yet exhibit the same underlying state organization, or they may appear similar while differing sharply in regime persistence or transition frequency. In SOD, similarity reflects shared geometric structure, not shared origin, propulsion, or intent.

5.5.1 What SOD Cannot Infer

Similarity in geometric behavior does *not* imply shared identity, propulsion type, intent, or control strategy. Many unrelated mechanisms can produce similar geometric signatures.

5.5.2 What SOD Can Infer

SOD can identify abrupt changes in motion, dominant behavioral regimes, transition frequencies, and overall temporal organization. These conclusions remain descriptive and proportional to the available data.

5.6 Robustness, Limitations, and Failure Modes

SOD behaves predictably when the trajectory is stable and sufficiently resolved. When these conditions are not met, curvature estimates and state boundaries may become ambiguous.

5.6.1 Resolution and Frame Rate Constraints

Low resolution or insufficient frame rate may distort motion details and reduce the reliability of curvature-based inference.

5.6.2 Tracking Ambiguity

If tracking fails to follow the same object consistently, discontinuities may appear that mimic true behavioral changes.

5.6.3 Smoothing Artifacts

Over- or under-smoothing can suppress or exaggerate structure in the curvature signal.

5.6.4 Boundary Sensitivity

Curvature values near segmentation thresholds may oscillate between regimes, producing artificial transitions.

5.6.5 Over-Segmentation

Noise-driven fluctuations can divide a trajectory into too many states, reducing interpretability.

5.6.6 Situations Where SOD Should Not Be Used

Very short trajectories, severe tracking loss, extreme noise, or prolonged stationarity limit the interpretive value of curvature-based analysis.

5.7 SOD as a General Framework

Although developed for challenging infrared datasets, SOD applies broadly to any trajectory representable as a sequence of positions.

5.7.1 Why a Geometry-Only System Matters

Geometry provides a shared, assumption-free language for describing motion that remains applicable across contexts and sensing modalities.

5.7.2 Reproducibility Across Datasets

The pipeline’s clarity and transparency support consistent application across diverse observational conditions.

5.7.3 Applications Beyond the UAP Context

SOD can be applied in biology, robotics, environmental monitoring, human motion studies, and other domains where motion structure is of interest.

5.7.4 Educational and Scientific Value

SOD provides an intuitive introduction to motion analysis and supports exploration of curvature, segmentation, and temporal organization without requiring specialized prerequisites.

5.8 Summary of the Unified Framework

Part V synthesizes the components introduced throughout the manuscript into an integrated system for describing motion behavior. The pipeline—tracking, smoothing, curvature estimation, state segmentation, transition modeling, and behavioral summary—provides a clear and reproducible method for analyzing trajectories.

By focusing on geometric structure rather than assumptions about identity or cause, SOD offers a flexible and scientifically proportional framework for studying motion across diverse contexts.

6 Conclusion

This manuscript has presented a unified, geometry-based framework for analyzing motion in infrared datasets where physical metadata are limited or unavailable. By focusing on trajectory reconstruction, curvature, local geometric states, and simple temporal structure, the framework offers a clear and reproducible way to describe how apparent motion unfolds over time.

The approach makes no claims about object identity, mechanism, or intent. Instead, it emphasizes what can be said with confidence from the observable image plane alone. Curvature patterns, regime persistence, transition structure, and trajectory signatures provide a consistent vocabulary for summarizing motion behavior while remaining scientifically proportional to the available information.

Structured Orb Dynamics is intentionally modest: each component is transparent, interpretable, and grounded in geometry. Yet together, these pieces form a cohesive system that can be applied across datasets and domains. Whether used for case studies, comparative analysis, or educational exploration, the framework supports motion analysis that is careful, repeatable, and accessible.

As additional datasets, synthetic benchmarks, or multi-trajectory studies become available, the methods outlined here provide a foundation for expanding the framework and testing its behavior under broader conditions. The emphasis remains the same: a clear description of motion based on what the data themselves contain.

Part VI-A: Exploratory Extension

7 Application of Structured Orb Dynamics to Biological Migration Systems

7.1 VI-A.1 Scope and Intent

This section explores whether the geometry-first, state-based framework of Structured Orb Dynamics (SOD) generalizes to biological migration systems when observed under partial and uncertain measurement conditions. The objective is not to interpret biological behavior, infer intent, or model navigational mechanisms, but rather to evaluate the descriptive coherence of the SOD state machinery when applied outside the context of UAP trajectory analysis.

Migration trajectories present a well-studied yet inference-constrained class of motion characterized by long-range displacement, intermittent observation, environmental perturbation, and variable sampling resolution. These properties make migration an appropriate testbed for assessing whether SOD’s kinematic state definitions and segmentation logic remain stable under known, non-anomalous conditions.

No claims regarding causation, optimization, evolutionary strategy, or biological decision-making are made in this section. The analysis is strictly descriptive and methodological in scope.

7.2 VI-A.2 Migration as a Motion-Inference Problem

From a kinematic perspective, biological migration can be treated as a motion-inference problem involving extended trajectories observed incompletely across time and space. Tracking data are often sparse, irregularly sampled, and subject to environmental interference, occlusion, or measurement uncertainty. Despite these limitations, migration datasets nonetheless exhibit structured movement patterns across multiple temporal and spatial scales.

This section adopts a geometry-first framing, treating migratory paths as sequences of observed positions without embedding domain-specific assumptions regarding motivation, navigation cues, or biological purpose. By abstracting migration in this way, the analysis remains agnostic to species-level differences and focuses solely on trajectory geometry and state transitions observable in the data.

This framing mirrors the constraints encountered in UAP footage analysis, albeit within a domain where the class of moving objects is known. Migration therefore serves as a controlled reference domain for evaluating the stability and generality of the SOD framework.

7.3 VI-A.3 Mapping SOD Motion States to Migratory Trajectories

The SOD framework defines motion in terms of discrete kinematic states inferred from trajectory geometry. These states may be mapped to migratory trajectories without introducing biological interpretation, as described below:

Straight: Sustained directional travel characterized by low curvature variance and consistent displacement over time.

Turn: Reorientation events marked by localized increases in curvature, corresponding to course corrections or directional changes.

Hover: Localized motion with minimal net displacement, consistent with stopover behavior or confined exploration.

Orb: A kinematic regime dominated by low velocity, limited displacement, or elevated observational uncertainty.

In this context, the Orb state denotes a descriptive regime defined by motion ambiguity or measurement limits rather than a behavioral or biological classification. Its inclusion preserves consistency with earlier sections while reinforcing the distinction between kinematic description and interpretive attribution.

7.4 VI-A.4 Geometry-First Metrics and State Segmentation

Trajectory segmentation within migration datasets may be performed using the same geometry-first metrics employed throughout SOD, including curvature-based time series, segment duration distributions, and state transition probabilities. These metrics are evaluated independently of species identity, environmental context, or assumed navigational strategy.

By applying identical segmentation logic across domains, SOD enables direct comparison between migration trajectories and other partially observed motion systems. This invariance is central to assessing whether the framework captures fundamental properties of motion geometry rather than domain-specific artifacts.

7.5 VI-A.5 Illustrative Pipeline Using Migratory Bird GPS Data

To illustrate the domain generality of the Structured Orb Dynamics (SOD) framework beyond UAP trajectory analysis, a conceptual application to biological migration is outlined using GPS-tracked migratory bird data from the Movebank repository. The datasets considered represent seasonal continental migration in passerine species, corresponding to small-to-medium migratory land birds observed over extended spatial and temporal scales.

In this pipeline, individual migration tracks are treated solely as sequences of observed positions under partial and uncertain sampling. No species-specific, ecological, or behavioral annotations are incorporated into the analysis. Trajectories are first projected into a common analysis space, after which geometry-based metrics are computed to characterize local curvature, displacement, and temporal structure.

State segmentation is then performed using the same kinematic criteria applied throughout the SOD framework, assigning segments to descriptive motion regimes (Straight, Turn, Hover, and Orb) based exclusively on trajectory geometry and uncertainty considerations. Resulting segmentations enable examination of state durations and transition structure without invoking biological intent or navigational mechanisms.

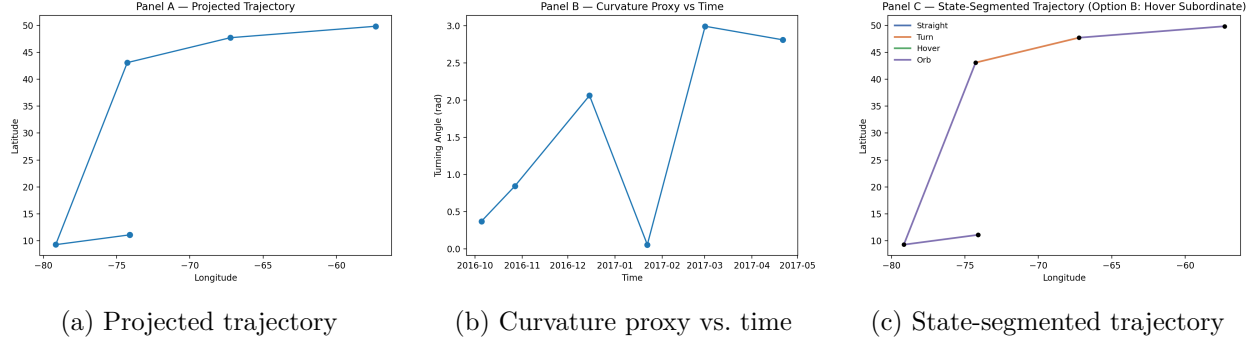


Figure 7: Geometry-first state segmentation of a partially observed trajectory. (A) Observed trajectory projected into analysis space without domain-specific assumptions regarding object class, intent, or environmental context. (B) Curvature time series derived from the projected trajectory, illustrating regions of sustained directional motion, localized reorientation, and intervals where sparse sampling increases uncertainty. (C) Trajectory segmented into Structured Orb Dynamics (SOD) kinematic states using geometry-based criteria. Low-displacement segments that are identifiable under sufficient temporal separation are classified as *Hover* and treated as a secondary sub-case within the segmentation logic. Segments dominated by sampling gaps or ambiguous geometry are classified as *Orb*, reflecting uncertainty rather than inferred behavior. The trajectory shown corresponds to a GPS-tracked migratory passerine bird from publicly available Movebank data and is included as an illustrative example of a known-class motion system under partial observation.

This conceptual example is presented to demonstrate methodological consistency and domain generality rather than to report empirical findings. Migration trajectories serve here as a known-class motion system against which the coherence and stability of the SOD segmentation machinery may be evaluated under realistic observation constraints.

7.6 VI-A.6 Relation to UAP Trajectory Analysis

The inclusion of biological migration as an exploratory extension does not alter the interpretive constraints applied to UAP footage. Instead, migration trajectories provide a known-class reference for evaluating state segmentation behavior under partial observation.

While the same kinematic machinery may be applied across domains, the interpretive layer remains domain-specific. Migration serves as a validation context for assessing segmentation stability, whereas UAP footage represents an extreme uncertainty regime where object class and intent are not assumed.

7.7 VI-A.7 Limitations and Non-Claims

This section does not attempt to infer biological intent, model navigation mechanisms, evaluate energetic optimization, or explain migratory origin or purpose. The analysis remains confined to kinematic description under uncertainty.

Similarly, no claims are made regarding the explanatory power of SOD beyond motion characterization. The framework is not proposed as a theory of behavior or causation.

7.8 VI-A.8 Implications for General Motion Analysis

Demonstrating that SOD applies coherently to biological migration supports its characterization as a general motion-inference framework rather than a domain-specific tool. This extension broadens the potential applicability of SOD while preserving falsifiability, interpretive restraint, and methodological clarity.

Supplementary GIMBAL Analysis

8 Supplementary Analysis: GIMBAL Dataset

This supplementary section applies components of the unified motion-state framework to the GIMBAL dataset, illustrating how curvature-based geometric diagnostics behave under conditions where absolute trajectory reconstruction is not feasible due to platform-induced parallax and missing metadata [8, 9].

8.1 Dataset Characteristics and Limitations

The GIMBAL dataset consists of a forward-looking infrared (FLIR) video recorded from a moving aircraft platform. The video shows a bright, extended infrared source that remains near the center of the field of view while the background and horizon rotate due to gimbal and platform motion [8]. As with PR-018, no complete physical metadata—such as focal length, range, altitude, gimbal angles, or inertial platform data—is provided in the publicly released version.

Several characteristics of the GIMBAL sequence distinguish it from PR-018 and introduce fundamental constraints on what geometric structure can be reliably inferred

- **Strong platform-induced parallax:** The apparent motion of the background is dominated by the aircraft’s own maneuvering and gimbal rotation, making it difficult to isolate the target’s absolute motion in the image plane. Such parallax effects are well known in gimbaled electro-optical systems and can strongly distort apparent target motion [10].
- **Persistent target centering:** The target remains close to the center of the frame, indicating active tracking by the sensor. As a result, target displacement is largely absorbed by gimbal motion rather than appearing as a large-scale trajectory across the image.
- **Partial stabilization and rotation:** The horizon and background features undergo rotation and deformation that reflect a combination of platform motion, gimbal adjustments, and possible zoom changes. This complicates any attempt to define a fixed image-plane reference frame.
- **Incomplete telemetry and calibration:** Without synchronized platform, gimbal, and range data, it is not possible to reconstruct a metric three-dimensional trajectory or to separate parallax effects from intrinsic target motion [10].

Implications for Trajectory Reconstruction. In PR-018, approximate stabilization around the target allowed the construction of a stable, analyzable image-plane trajectory $x(t)$ suitable for curvature and state analysis. In contrast, the GIMBAL sequence does not admit a unique or reliable reconstruction of the target’s intrinsic motion. Any attempt to infer a full trajectory would require strong assumptions about platform kinematics, gimbal control laws, and range, none of which are available in the public release [10].

As a consequence, the unified framework cannot be applied to GIMBAL in the same quantitative manner as it was to PR-018. Instead, the analysis is necessarily more limited and qualitative, focusing on what can be inferred from:

- relative orientation changes between the target and the horizon,
- local apparent motion of the target within the tracked window,
- structural stability of the target’s image under platform rotation.

Scope of the Supplementary Analysis. Given these constraints, the GIMBAL analysis is framed as a supplementary, qualitative application of the unified motion-state framework. The goals are to:

- identify which geometric cues are still accessible despite parallax and missing metadata,
- assess whether these cues are consistent with the motion-state taxonomy developed in Part II,
- compare qualitative dynamical patterns with those observed in PR-018,
- avoid over-interpretation by explicitly acknowledging the limits of the available data.

No attempt is made to derive a full image-plane trajectory or to compute curvature with the same precision used for PR-018. Instead, the GIMBAL dataset serves as a stress test of the framework’s interpretive discipline under conditions where only partial geometric information is available.

Conservative Interpretive Stance. Throughout this supplementary analysis, the emphasis remains on descriptive geometry and qualitative geometric consistency, not physical, aerodynamic, or mechanistic inference. The limitations of the GIMBAL data are treated as a fundamental boundary on what can be responsibly concluded. Within that boundary, the unified framework provides a structured language for discussing observable motion patterns without exceeding the evidential content of the dataset.

8.2 Geometric Limits of Observability

The geometric and kinematic inferences possible in the GIMBAL dataset are fundamentally constrained by platform-induced parallax, continuous gimbal rotation, and missing inertial metadata. Unlike PR-018, the sensor frame is not approximately stabilized around the target, and the horizon

undergoes rotation that reflects aircraft dynamics rather than target motion. As a result, the dataset does not permit reconstruction of an intrinsic image-plane trajectory $x(t)$ or derivation of curvature-based diagnostics.

These limitations define the maximum recoverable geometric structure available for analysis

8.3 Boundary Conditions for Curvature-Based Modeling

Given the strong parallax and platform-dependent motion present in GIMBAL, curvature $\kappa(t)$, curvature rate $\dot{\kappa}(t)$, and other differential-geometric quantities cannot be computed in a meaningful or physically interpretable way. The unified framework can only be applied under the following boundary conditions:

1. **No attempt is made to reconstruct a 2D trajectory.** The dataset cannot support an intrinsic image-plane path.
2. **Curvature-based diagnostics are not used.** Any curvature estimate would conflate target motion with gimbal and platform dynamics.
3. **Only relative angular behavior and morphological stability are interpretable.** These cues allow qualitative assessment of smoothness but not kinematic classification.
4. **All inferences must remain non-quantitative.** No likelihoods, no state probabilities, and no dynamical quantities can be computed.

These boundary conditions ensure that all interpretation remains proportional to the observable structure of the dataset.

8.4 Extractable Geometric Features

Although the GIMBAL dataset does not permit reconstruction of an absolute image-plane trajectory, several geometric cues remain accessible and can be interpreted within the qualitative structure of the unified motion-state framework [9]. These cues arise from the residual motion of the target within the tracked window, from the behavior of the horizon and background, and from the stability of the target’s infrared signature under platform rotation.

Local Apparent Motion of the Target. Even though the target is actively stabilized near the center of the frame, small residual displacements are visible over time. These residual motions cannot be converted into metric units or reliable curvature estimates, but they do provide information about:

- slight lateral drift within the tracking window,
- transient deviations from perfect centering,

- micro-adjustments made by the tracking and gimbal-control system.

Such motions reflect a combination of sensor behavior and relative target motion, and are treated here strictly as qualitative indicators of smoothness or irregularity.

Horizon Rotation and Background Flow. The most prominent large-scale geometric feature in GIMBAL is the rotation of the horizon and background. This rotation is driven primarily by platform and gimbal motion. When viewed as a time-varying angle, the horizon provides:

- a proxy for the aircraft’s maneuvering and gimbal rotation,
- a reference frame for evaluating the target’s stability relative to the rotating scene,
- a qualitative separation between platform-induced dynamics and target-centric motion.

The fact that the target remains centered during substantial horizon rotation suggests that its apparent position is governed largely by the tracking system rather than by large-scale motion across the field of view.

Image Stability and Shape Consistency. The target’s infrared signature retains a stable morphology throughout the sequence. This stability enables limited geometric inference:

- the absence of pronounced shape deformation argues against high-acceleration image-plane motion,
- the lack of abrupt centroid jumps or discontinuous tracking behavior suggests no strong high-frequency oscillatory behavior,
- coherent intensity structure indicates that residual motions are relatively smooth.

These observations are consistent with expectations for infrared point-source tracking under stable imaging conditions [11, 12] and provide qualitative evidence that whatever apparent motion exists is not dominated by jitter or tracking failures.

Relative Angular Behavior. By examining the orientation of the target relative to the rotating horizon, one can extract qualitative information about relative angular behavior:

- whether the target exhibits slow drift relative to the horizon,
- whether its apparent alignment changes gradually or abruptly,
- whether any relative angular behavior is consistent across distinct phases of platform rotation.

These relative angular cues do not yield curvature or trajectory estimates but help distinguish smooth, coordinated evolution from irregular or noise-driven changes.

Limitations of Extractable Features. Crucially, the available geometric cues do *not* permit:

- computation of curvature $\kappa(t)$ or curvature rate $\dot{\kappa}(t)$,
- estimation of a center-of-curvature or radial profile,
- recovery of absolute velocity, acceleration, or jerk,
- construction of a consistent image-plane trajectory comparable to PR-018.

For this reason, the GIMBAL analysis remains strictly qualitative. All inferences are framed in terms of relative stability, smoothness, and alignment, rather than quantitative kinematic measures.

Role Within the Unified Framework. Despite these limitations, the extractable features serve an important role: they allow us to evaluate whether the observable behavior of the target is qualitatively compatible with the kinds of smooth, persistent regimes associated with orb-like or turn-like motion in the unified framework, or whether it appears inconsistent with those regimes [9]. These cues provide the basis for the qualitative state-consistency assessment in the next subsection.

9 Minimal Interpretation Framework

Under the observability limits and boundary conditions established above, the unified motion-state framework can be applied only in a qualitative sense. The goal is not to assign discrete motion states, but to evaluate whether observable behavior is *compatible* with any of the state regimes—Straight, Turn, Hover, or Orb—defined in Part II, and to confirm that no contradictions arise.

9.1 Qualitative State-Consistency Assessment

Because the GIMBAL dataset does not permit extraction of a reliable image-plane trajectory, the state-consistency analysis cannot rely on quantitative curvature $\kappa(t)$, curvature rate $\dot{\kappa}(t)$, or finite-difference kinematic derivatives. Instead, we evaluate whether the observable geometric cues identified in Section 8.4 exhibit qualitative patterns consistent with any of the motion-state regimes defined in the unified framework.

This assessment is interpretive rather than computational, and is guided by three criteria:

1. structural smoothness of the target’s apparent behavior,
2. temporal persistence or coherence across distinct intervals of the recording,
3. compatibility with the qualitative signatures of the Straight, Turn, Hover, and Orb states.

The goal is not to assign a discrete state sequence, but to evaluate whether any observable behaviors contradict or align with the taxonomy developed in Part II.

Straight-State Consistency. A motion would be *qualitatively straight-like* if the target exhibited:

- minimal displacement within the field of view,
- negligible relative drift against the rotating horizon,
- image stability with no indication of curvature-driven lateral motion.

GIMBAL does show periods of limited displacement, but these coincide with active sensor tracking and cannot be interpreted as evidence of straight-line intrinsic motion. No inconsistencies with a straight-state pattern are observed, but no affirmative geometric evidence supports it either.

Turn-State Consistency. A qualitative turn-like regime would require:

- consistent lateral displacement relative to a background reference,
- monotonic angular change or persistent deviation from straight-like behavior,
- residual motion incompatible with pure stabilization.

Because horizon rotation is dominated by platform motion, the dataset does not present reliable turn-like cues. No geometric feature in GIMBAL contradicts a turn-like interpretation, but none provides positive support for one.

Hover-State Consistency. Hover-like behavior would include:

- minimal intrinsic displacement,
- absence of systematic drift,
- frame-to-frame stability suggestive of low-speed or stationary dynamics.

The target’s near-centering and morphological stability could appear hover-like, but these are fully attributable to the sensor tracking system. Thus, hover-like patterns are *not* ruled out, but cannot be meaningfully inferred.

Orb-State Consistency. The orb state is characterized by smoothly evolving curvature and persistence of dynamical regime beyond classical aerodynamic expectations. Qualitatively, this would manifest as:

- smooth, coordinated evolution relative to a rotating reference frame,
- absence of ballistic straight-like segments,
- no abrupt kinematic transitions or discontinuities.

Although curvature cannot be computed for GIMBAL, two observable behaviors are noteworthy:

1. the target’s apparent motion (to the extent it is visible) shows no abrupt jumps,
2. its morphological stability persists during substantial platform-induced rotation.

These observations are *consistent* with smooth evolution, but they do not distinguish between hover-like, turn-like, or orb-like regimes. The dataset lacks the geometric resolution needed to affirm or reject orb-state behavior.

Synthesis. Across all four states, we find:

- no observable behavior that contradicts any state class outright,
- insufficient geometric information to positively support any single state,
- qualitative compatibility with multiple regimes due to the dominance of parallax and tracking.

Thus, the appropriate conclusion is one of disciplined uncertainty: *the GIMBAL dataset does not provide enough geometric information to determine which motion state, if any, the target occupies.* This stands in contrast to PR-018, where curvature-based analysis enabled quantitative state assignment.

The qualitative consistency analysis therefore reinforces the conservative stance taken throughout this supplement: the GIMBAL sequence lacks the evidential content required for state classification but remains fully interpretable within the unified motion-state framework [9].

9.2 Comparison With PR-018 Results

A central purpose of this supplementary analysis is to clarify how the qualitative cues observable in the GIMBAL dataset relate to the quantitative curvature-based results obtained from PR-018 [7, 8]. Because the two datasets differ fundamentally in the availability of stabilization, parallax conditions, and metadata, any comparison must explicitly separate *what is comparable* from *what is not*.

Differences in Data Structure. PR-018 provides:

- a stable, target-centered field of view,
- sufficient residual motion to reconstruct a 2D image-plane trajectory,
- conditions enabling computation of curvature, curvature rate, and finite-difference kinematic derivatives.

By contrast, GIMBAL provides:

- strong parallax dominated by platform and gimbal motion,
- tracking behavior that suppresses visible target displacement,
- no reliable reference frame for trajectory reconstruction [8, 10].

As a result, the unified framework can be *applied quantitatively* to PR-018, but only *qualitatively* to GIMBAL.

Comparability of Motion-State Signatures. The motion-state analysis in PR-018 was driven by curvature signatures, state-likelihood evaluation, and posterior temporal structure. None of these quantities can be computed for GIMBAL. Therefore:

- no direct comparison of curvature $\kappa(t)$ or curvature continuity can be made,
- no state-probability sequence can be constructed for GIMBAL,
- no dynamical inferences (straight, turn, hover, orb) can be quantitatively aligned.

The two datasets are therefore *not comparable at the level of dynamical-state inference*.

Comparability of Qualitative Behaviors. Even without a trajectory, certain qualitative behaviors can still be contrasted. We highlight three points of relevance:

1. **Smoothness vs. discontinuity.** PR-018 exhibited smooth curvature evolution with no surviving straight-line segments after stabilization. GIMBAL shows no abrupt centroid jumps or discontinuous behavior, but this smooth appearance is attributable to sensor tracking behavior rather than intrinsic motion.
2. **Persistence of geometric structure.** In PR-018, the orb-like regime persisted across hundreds of frames and was supported by quantitative diagnostics. In GIMBAL, the target’s morphological stability is persistent but does not reflect intrinsic motion.
3. **Relative-frame behavior.** PR-018 provided a stable image-plane frame that reflected target motion. In GIMBAL, the rotating horizon dominates the frame, preventing any equivalence in geometric interpretation.

While some qualitative superficial similarities exist—such as stable centroid structure and absence of jitter—they do not imply dynamical similarity.

Constraints on Physical Interpretation. In PR-018, the unified framework permitted elimination of several physical categories due to the observed curvature behavior (e.g., no ballistic segments). No such eliminations are possible for GIMBAL. Specifically:

- GIMBAL cannot rule out straight, turn, hover, or orb-like regimes,
- GIMBAL cannot constrain acceleration or curvature profiles,
- GIMBAL cannot be used to evaluate dynamical feasibility of hypotheses.

This distinction is crucial for scientific rigor: the evidential strength of the two datasets is fundamentally unequal.

Synthesis and Interpretation. The comparison yields three disciplined conclusions:

1. **PR-018 and GIMBAL cannot be treated as equivalent for motion-state analysis.** Only PR-018 offers the necessary geometric observables for quantitative classification.
2. **Qualitative cues in GIMBAL do not contradict the unified framework.** GIMBAL shows no behavior incompatible with smooth or persistent regimes, but this carries no dynamical weight.
3. **GIMBAL provides a test of interpretive discipline rather than motion inference.** Its value lies in confirming that the unified framework maintains methodological restraint in the presence of incomplete data [9].

Thus, while PR-018 supports a quantitative demonstration of the unified motion-state model, the GIMBAL dataset serves as a complementary case illustrating how the framework operates under strong observational constraints without exceeding the evidential content of the data.

9.3 Summary of GIMBAL Supplement

The GIMBAL dataset provides a valuable but fundamentally limited test case for the unified motion-state framework. Unlike PR-018, where approximate stabilization and sufficient residual motion enabled quantitative curvature analysis, the GIMBAL sequence is dominated by platform-induced parallax, active tracking behavior, and a lack of supporting telemetry. These conditions preclude reconstruction of an image-plane trajectory and prevent computation of curvature, curvature rate, or state-likelihood estimates [8, 10].

Despite these constraints, several meaningful conclusions emerge from the qualitative analysis:

1. **Geometric cues are available but non-quantitative.** Features such as horizon rotation, target morphological stability, and small residual centroid motions provide limited geometric insight. However, these cues cannot be used to infer intrinsic target motion, and no dynamical quantities can be estimated.
2. **No state classification is possible.** Because curvature-based diagnostics cannot be computed, none of the four motion states (straight, turn, hover, orb) can be positively identified or excluded. The dataset is consistent with all regimes but affirmatively supports none.
3. **Qualitative behaviors do not contradict the unified framework.** The observable properties of the GIMBAL target—morphological stability, lack of jitter, smooth appearance relative to a rotating frame—are compatible with the conceptual structure of the framework. Crucially, this compatibility arises from insufficient data rather than evidence of smooth or persistent motion.

4. Comparison with PR-018 highlights a contrast in evidential strength. PR-018 provides direct geometric observables enabling quantitative state inference and elimination of incompatible physical models. GIMBAL does not. This contrast illustrates the importance of dataset structure in motion-state classification.

5. The supplement demonstrates interpretive discipline. A central contribution of the unified motion-state framework is its insistence on proportional inference: conclusions must scale with the quantity and reliability of available geometric data. The GIMBAL supplement shows that the framework remains fully applicable—but appropriately restrained—even when only partial information is accessible [9].

Overall Synthesis. The GIMBAL dataset does not support quantitative dynamical modeling or state assignment, but it plays an important complementary role in the manuscript:

- it tests the unified framework’s ability to operate under severe observational constraints,
- it highlights the methodological distinction between geometric visibility and interpretive certainty,
- it reinforces the evidential strength of the PR-018 analysis by contrast,
- it establishes clear scientific boundaries that prevent over interpretation.

In this sense, the supplementary analysis satisfies a critical epistemic function: *it demonstrates that the unified motion-state framework is robust not only when data are rich but also when they are incomplete—while maintaining fidelity to the principles of quantitative geometry and conservative inference.*

Accordingly, the supplement clarifies how the framework distinguishes between datasets that support quantitative curvature-based inference and those that permit only qualitative, geometry-aware interpretation.

Author Contributions

Cassandra Perry: Conceptualization; methodology; data curation; geometric analysis; model development; manuscript preparation; validation; software design; interpretation of results; project administration.

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Acknowledgments

The author gratefully acknowledges the support of the AI research assistant “Eleanor,” whose contributions in structuring mathematical arguments, refining methodological clarity, and assisting with manuscript organization facilitated the development of this unified framework. All conceptual decisions, interpretations, and conclusions are solely those of the author.

The author also thanks the open-science community for maintaining accessible archival infrastructure and acknowledges the value of public-domain sensor data that enables reproducible geometric analysis.

Discussion and Outlook

The unified geometric motion-state framework introduced in this manuscript demonstrates that quantitative inference is possible even when physical metadata, telemetry, and calibration information are incomplete. By grounding the analysis in curvature, its temporal evolution, and dimensionless kinematic structure, the approach offers a reproducible method for describing unknown aerial trajectories in a manner that is conservative, interpretable, and consistent across heterogeneous datasets.

The PR-018 analysis shows that certain curvature regimes—most notably the smooth, bounded-curvature patterns associated with the orb state—persist across hundreds of frames and remain stable under reasonable choices of smoothing and derivative estimation. By contrast, the GIMBAL supplement illustrates how the same framework behaves under conditions where trajectory reconstruction is not possible. Taken together, these cases highlight the strengths, limitations, and future directions for geometric motion analysis in the absence of conventional metadata.

Future work will extend the classifier, refine curvature-based diagnostics, and explore methods for multi-sensor fusion when partial telemetry is available. The broader outlook is the development of a generalizable motion-analysis toolkit suitable for research domains where observational constraints are the norm rather than the exception.

Role of Known-Class Motion Systems. Part VI-A is included as an exploratory extension intended to examine the internal coherence of the Structured Orb Dynamics (SOD) framework when applied to a known-class motion system observed under partial and uncertain conditions. The migratory bird trajectories considered therein are not used to infer biological behavior, navigation strategy, or optimization, nor are they presented as validation benchmarks. Instead, they serve as a controlled reference domain for assessing whether the same geometry-first state definitions and segmentation logic employed in UAP trajectory analysis remain stable when object class is known but observational completeness is limited. This comparison clarifies the scope of SOD as a descriptive motion-inference framework rather than a domain-specific explanatory model.

Data Availability

All reconstructed trajectories, derived curvature products, classifier outputs, and analysis scripts used in this work are publicly available in a versioned Zenodo repository associated with this manuscript. The PR-018 and GIMBAL infrared videos analyzed herein are publicly released products of the U.S. Department of Defense.

Software Availability

The full implementation of the Orb Motion Classifier—including trajectory processing, curvature estimation, state-likelihood evaluation, and posterior inference—is available on GitHub under an open license. The repository provides complete documentation, reproducible scripts, and archived releases linked to Zenodo DOIs for long-term preservation.

A Observable Robustness and Non-Causal Validation

Status: protocol-ready. The checks below define a non-causal robustness protocol intended for systematic application across datasets and figures, and may be applied selectively depending on data quality and availability.

A.1 Track Continuity

We verify continuity of the tracked trajectory across transition boundaries using an arc-length or speed proxy,

$$\Delta s_t = \|\mathbf{x}_{t+1} - \mathbf{x}_t\|,$$

to confirm that no discontinuities arise from sampling gaps, re-indexing, or tracking loss. Sustained continuity across the transition supports interpretation as a single, coherent track.

A.2 Estimator Robustness

Curvature-based segmentation is evaluated under modest methodological perturbations, including smoothing variation, temporal subsampling, and alternative discrete curvature estimators (e.g., turning-angle-based versus multi-point curvature). A transition window is considered robust if its presence and temporal ordering persist across these variations.

A.3 Change-Point Detection

To reduce reliance on manually selected thresholds, algorithmic change-point detection is applied to the curvature time series. Agreement between detected change points and segmentation boundaries supports algorithmic identification of transition intervals.

A.4 Uncertainty and Baseline Distinction

Baseline curvature variability is estimated for low-curvature segments to construct a simple uncertainty or confidence band. Transition intervals are evaluated for sustained deviation beyond baseline variability, distinguishing structured curvature elevation from noise fluctuations.

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