

Strike while the Iron is Hot: Optimal Monetary Policy with a Nonlinear Phillips Curve

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Motivation

- ▶ The recent inflation surge featured
 - ▶ Increase in the frequency of price changes ([Montag and Villar, 2023](#)) US
 - ▶ Increase in Phillips curve slope ([Benigno and Eggertsson, 2023](#); [Cerrato and Gitti, 2023](#)) US
- ▶ Optimal monetary policy is mainly studied in models, in which the Phillips curve is linear and the frequency is held constant ([Galí, 2008](#); [Woodford, 2003](#))
- ▶ What does optimal monetary policy look like with a nonlinear Phillips curve and endogenous variation in frequency? How should CBs respond to a large inflation surge?

What do we do?

- ▶ We use the standard state-dependent pricing model of [Golosov and Lucas \(2007\)](#)
- ▶ Solve it [nonlinearly](#) using a new algorithm over the sequence space
- ▶ [Positive analysis](#) under a Taylor rule
 - ▶ Trace the responses to shocks of different sizes
 - ▶ Assess the nonlinearity of the Phillips curve
- ▶ [Normative analysis](#): Ramsey optimal policy
 - ▶ Optimal long-run inflation
 - ▶ Trace the optimal responses to shocks
 - ▶ Characterize the (nonlinear) targeting rule after *large* cost-push shocks

What do we find?

- ▶ In this model the Phillips curve is **nonlinear**: it gets steeper as frequency increases
- ▶ In response to **small shocks**, optimal monetary policy is similar to the one under Calvo
- ▶ In response to efficiency shocks, there is **divine coincidence**, as in Calvo
- ▶ Different responses to small and large **cost-push shocks**. Optimal policy leans aggressively against inflation when frequency rises: “it strikes while the iron is hot”

Literature

- ▶ Nonlinear Phillips curve (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023)
 - ▶ Microfounded by state-dependent price setting
(Golosov and Lucas, 2007; Gertler and Leahy, 2008; Auclert et al., 2022)
 - ▶ In the presence of large aggregate shocks (Karadi and Reiff, 2019; Alvarez and Neumeyer, 2020; Costain et al., 2022; Alexandrov, 2020; Blanco et al., 2024)
- ▶ Optimal policy in a menu cost economy
 - ▶ Optimal inflation target (Burstein and Hellwig, 2008; Adam and Weber, 2019; Blanco, 2021)
 - ▶ Small shocks, large shocks, optimal nonlinear target rule (comp. Galí, 2008; Woodford, 2003)
 - ▶ Focus on aggregate shocks (unlike Caratelli and Halperin, 2023, who study *sectoral* shocks)

Overview of the model

- ▶ Heterogeneous-firm DSGE model with fixed costs of price-adjustment
- ▶ Households: consume a Dixit-Stiglitz basket of goods, and work
- ▶ Firms: produce differentiated goods using labor only and are subject to aggregate TFP shocks and idiosyncratic “quality” shocks. They have market power and set prices optimally subject to a [fixed cost \(Goloso and Lucas, 2007\)](#)
- ▶ Monetary policy: either follows Taylor rule or set optimally to maximize household welfare under [commitment](#)

Households

- ▶ A representative household consumes (C_t) , supplies labor hours (N_t) and saves in one-period nominal bonds (B_t) .
- ▶ The household's problem is:

$$\max_{C_t, N_t, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(C)_t - \nu N_t$$

$$\text{s.t. } P_t C_t + B_t + T_t = R_{t-1} B_{t-1} + W_t N_t + D_t,$$

where P_t is the price level, R_t is the gross nominal interest rate, W_t is the nominal wage, T_t are lump sum transfers and D_t are profits

Consumption and labor

- Aggregate consumption C_t and the price level are defined as:

$$C_t = \left\{ \int [A_t(i)C_t(i)]^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}, \quad P_t = \left[\int_0^1 \left(\frac{P_t(i)}{A_t(i)} \right)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

where $A_t(i)$ is product quality, ϵ is the elasticity of substitution.

- Labor supply condition and Euler equation are given by:

$$W_t = vP_tC_t, \quad 1 = \mathbb{E}_t \left[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{R_t}{\Pi_{t+1}} \right]$$

Monopolistic producers

- ▶ Production of good i is given by $Y_t(i) = A_t \frac{N_t(i)}{A_t(i)}$, where quality follows a random walk

$$\log(A_t(i)) = \log(A_{t-1}(i)) + \varepsilon_t(i), \quad \varepsilon_t(i) \sim N(0, \sigma_t^2)$$

- ▶ Firms real profits

$$\frac{D_t(i)}{P_t} = \frac{P_t(i)}{P_t} Y_t(i) - (1 - \tau_t) \frac{W_t}{P_t} N_t(i)$$

- ▶ Firms face a fixed cost η to update prices

Quality-adjusted relative prices

- ▶ Let $p_t(i) \equiv \log(P_t(i)/(A_t(i)P_t))$ be the quality-adjusted log relative price
- ▶ Real profit then is

$$\Pi(p_t(i), w_t, A_t) \equiv \frac{D_t(i)}{P_t} = C_t e^{p_t(i)(1-\epsilon)} - C_t(1 - \tau_t) \frac{w_t}{A_t} e^{p_t(i)(-\epsilon)}$$

where w_t is the real wage.

- ▶ When nominal price $P_t(i)$ stays constant, $p_t(i)$ evolves: $p_t(i) = p_{t-1}(i) - \sigma_t \varepsilon_t(i) - \pi_t$

Pricing decision

- ▶ Let $\lambda_t(p)$ be the price-adjustment probability
- ▶ Value function is

$$\begin{aligned} V_t(p) &= \Pi(p, w_t, A_t) \\ &+ \mathbb{E}_t [(1 - \lambda_{t+1}(p - \sigma_{t+1}\varepsilon_{t+1} - \pi_{t+1})) \Lambda_{t,t+1} V_{t+1}(p - \sigma_{t+1}\varepsilon_{t+1} - \pi_{t+1})] \\ &+ \mathbb{E}_t [\lambda_{t+1}(p - \sigma_{t+1}\varepsilon_{t+1} - \pi_{t+1}) \Lambda_{t,t+1} (\max_{p'} V_{t+1}(p') - \eta w_{t+1})]. \end{aligned}$$

- ▶ The adjustment probability is

$$\lambda_t(p) = I[\max_{p'} V_t(p') - \eta w_t > V_t(p)]$$

where $I[\cdot]$ is the indicator function.

Monetary Policy

- ▶ The central bank either sets policy optimally, or follows a Taylor rule:

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1 - \rho_r) [\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^e)] + \varepsilon_{r,t} \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2)$$

- ▶ Shocks: employment subsidy (τ_t), TFP (A_t), volatility (σ_t)

$$\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_{A,t} \quad \varepsilon_{A,t} \sim N(0, \sigma_A^2)$$

$$\tau_t - \tau = \rho_\tau(\tau_{t-1} - \tau) + \varepsilon_{\tau,t} \quad \varepsilon_{\tau,t} \sim N(0, \sigma_\tau^2),$$

$$\log(\sigma_t/\sigma) = \rho_\sigma \log(\sigma_{t-1}/\sigma) + \varepsilon_{\sigma,t} \quad \varepsilon_{\sigma,t} \sim N(0, \sigma_\sigma^2)$$

Aggregation and market clearing

- Aggregate price index

$$1 = \int e^{p(1-\epsilon)} g_t(p) dp,$$

- Labor market equilibrium

$$N_t = \frac{C_t}{A_t} \underbrace{\int e^{p(-\epsilon)} g_t(p) dp}_{\text{dispersion}} + \eta \underbrace{\int \lambda_t(p - \sigma_t \varepsilon - \pi_t) g_{t-1}(p) dp}_{\text{frequency}},$$

Law of motion of the price density

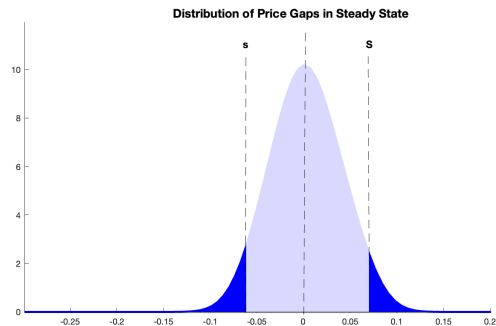
$$g_t(p) = \begin{cases} (1 - \lambda_t(p)) \int g_{t-1}(p + \sigma\varepsilon + \pi_t) d\xi(\varepsilon) & \text{if } p \neq p_t^*, \\ (1 - \lambda_t(p_t^*)) \int g_{t-1}(p_t^* + \sigma\varepsilon + \pi_t) d\xi(\varepsilon) + \\ \int_{\underline{p}}^{\bar{p}} \lambda_t(\tilde{p}) \left(\int g_{t-1}(\tilde{p} + \sigma\varepsilon + \pi_t) d\xi(\varepsilon) \right) d\tilde{p} & \text{if } p = p_t^*. \end{cases}$$

Calibration

Households			
β	$0.96^{1/12}$	Discount rate	Golosov and Lucas (2007)
ϵ	7	Elasticity of substitution	Golosov and Lucas (2007)
γ	1	Risk aversion parameter	Midrigan (2011)
v	1	Utility weight on labor	Set to yield $w = C$
Price setting			
η	3.6%	Menu cost	Set to match 8.7% of frequency
σ	2.4%	Std dev of quality shocks	and 8.5% size in Nakamura and Steinsson (2008)
Monetary policy			
ϕ_π	1.5	Inflation coefficient in Taylor rule	Taylor (1993)
ϕ_y	0.5/12	Output gap coefficient in Taylor rule	Taylor (1993)
ρ_i	$0.75^{1/3}$	Smoothing coefficient	
Shocks			
ρ_A	$0.95^{1/3}$	Persistence of the TFP shock	Smets and Wouters (2007)
ρ_τ	$0.9^{1/3}$	Persistence of the cost-push shock	Smets and Wouters (2007)

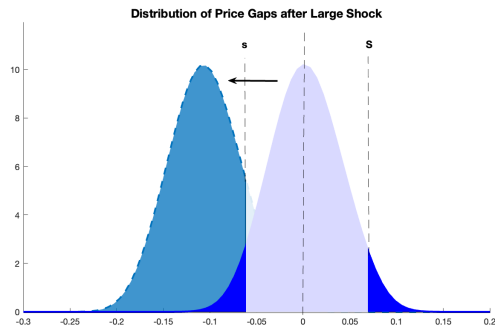
Large shocks: illustration

- ▶ Let $x_t(j) \equiv p_t(j) - p_t^*(j)$ be the price gap
- ▶ Large aggregate shock: shifts the distribution of price gaps for all firms
- ▶ Limited impact on the (s, S) bands
- ▶ Pushes a large fraction of firms outside of the inaction region
- ▶ Large increase in the frequency of price changes and hence additional flexibility of the aggregate price level (on top of “selection”)



Large shocks: illustration, cont.

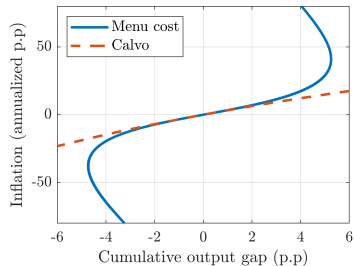
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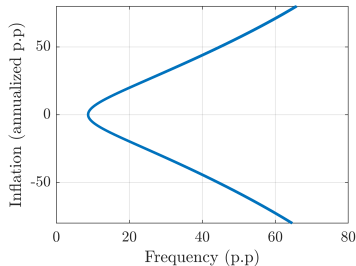
Nonlinearity of the Phillips Curve at realistic frequency (20%) US

Consider the model under a Taylor rule

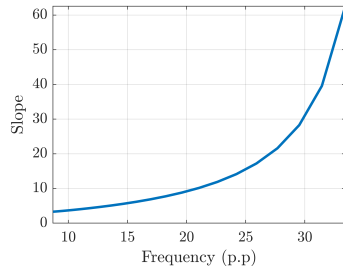
Nonlinear Phillips Curve (PC)



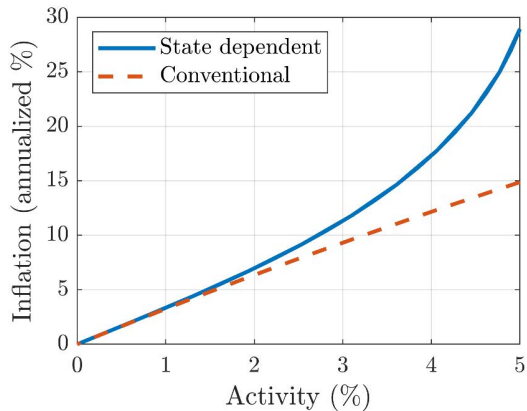
Frequency



PC slope



Nonlinearity of the Phillips Curve at realistic frequency (20%)



Normative results: The Ramsey problem ($x_t \equiv p_t - p_t^*$)

$$\max_{\{g_t^c(\cdot), g_t^0, V_t(\cdot), C_t, w_t, p_t^*, s_t, S_t, \pi_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left(C_t, \frac{C_t}{A_t} \left(\int e^{(x+p_t^*)(-\epsilon_t)} g_t^c(p) dx + g_t^0 e^{(p_t^*)(-\epsilon)} \right) + \eta g_t^0 \right)$$

subject to

$$1 = \int e^{(x+p_t^*)(1-\epsilon)} g_t^c(x) dx + g_t^0 e^{(p_t^*)(1-\epsilon)},$$

$$V_t'(0) = \Pi_t'(x) + \frac{1}{\sigma} \Lambda_{t+1} \int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \frac{\partial \phi \left(\frac{x-x'-\pi_t^*}{\sigma} \right)}{\partial x} dx' + \Lambda_{t+1} \left(\phi \left(\frac{S_{t+1} - \pi_t^*}{\sigma} \right) - \phi \left(\frac{s_{t+1} - \pi_t^*}{\sigma} \right) \right) (V_{t+1}(0) - \eta w_{t+1}),$$

$$V_t(s_t) = V_t(0) - \eta w_t,$$

$$V_t(S_t) = V_t(0) - \eta w_t,$$

$$w_t = v C_t^\gamma,$$

$$V_t(x) = \Pi(x, p_t^*, w_t, A_t) + \Lambda_{t,t+1} \frac{1}{\sigma} \int_{s_t}^{S_t} \left[V_{t+1}(x') \phi \left(\frac{(x-x') - \pi_{t+1}^*}{\sigma} \right) \right] dx' + \Lambda_{t,t+1} \left(1 - \frac{1}{\sigma} \int_{s_t}^{S_t} \left[\phi \left(\frac{(x-x') - \pi_{t+1}^*}{\sigma} \right) \right] dx' \right) [(V_{t+1}(0) - \eta w_{t+1})],$$

$$g_t^c(x) = \frac{1}{\sigma} \int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1}) \phi \left(\frac{x_{-1} - x - \pi_t^*}{\sigma} \right) dx_{-1} + g_{t-1}^0 \phi \left(\frac{-x - \pi_t^*}{\sigma} \right),$$

$$g_t^0 = 1 - \int_{s_t}^{S_t} g_t^c(x) dx.$$

Normative results: Computation

► Challenges

- Price change distribution and firms' value function are [infinite-dimensional](#) objects
- In the Ramsey problem, we need derivatives w.r.t. both

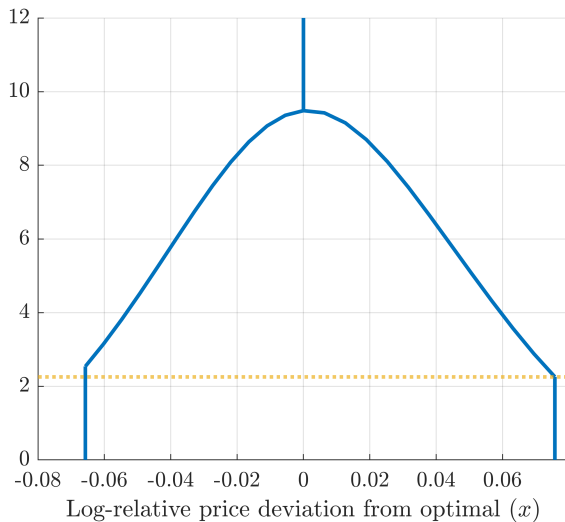
► New algorithm, inspired by [González et al. \(2024\)](#)

- Approximate distribution and value functions by piece-wise linear interpolation on grid
- [Endogenous grid points](#): (s, S) bands and the optimal reset price
- Solve in the [sequence space](#) using Dynare (Adjemian et al. (2023))

Optimal long-run inflation rate

- ▶ The Ramsey steady-state inflation rate is **slightly above zero**: $\pi^* = 0.25\%$
 - ▶ Close to the inflation rate that minimizes the steady-state frequency of price changes
- ▶ Why not exactly zero as in Calvo (1983)?
 - ▶ Asymmetry of the profit function leads to asymmetric (s,S) bands: a negative price gap is less desirable than a positive price gap of the same size
 - ▶ At zero inflation, more mass around the lower (s) band than around the higher (S) band
 - ▶ Slightly positive inflation raises p^* and pushes the mass of firms to the right inside (s,S)
 - ▶ This leads to lower frequency and lower price-adjustment costs

Steady-state price distribution (at zero inflation)



Optimal response to cost-push shocks

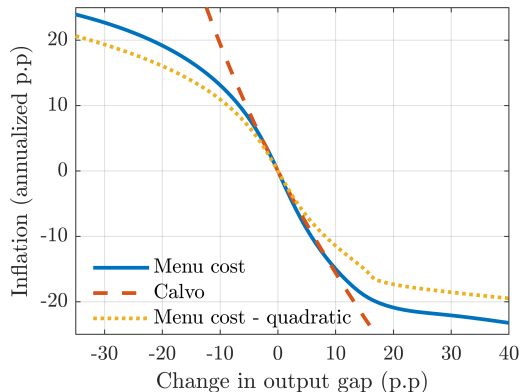
- ▶ In linearized Calvo (1983), optimal policy is a flexible **inflation targeting rule**

$$\pi_t = -\frac{1}{\epsilon} \Delta \tilde{y}_t$$

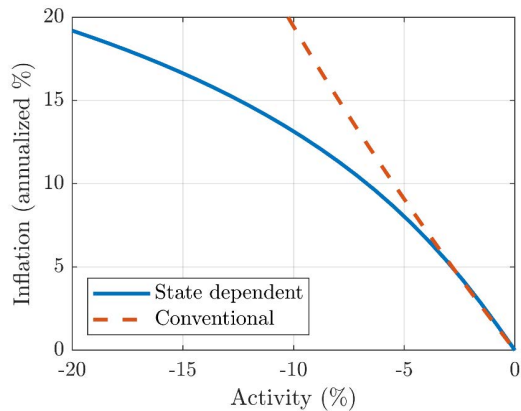
- ▶ Slope $-1/\epsilon$ is independent of the frequency of repricing or the slope of the PC
 - ▶ An increase in frequency raises the slope of the Phillips curve κ
 - ▶ But it also raises the relative weight of the *output-gap* in welfare, $\lambda = \kappa/\epsilon$
 - ▶ Why? Because more price-flexibility implies that inflation is less costly.
- ▶ For small cost-push shocks, the slope of the targeting rule in Golosov and Lucas (2007) is also $-1/\epsilon$!

Nonlinear targeting rule

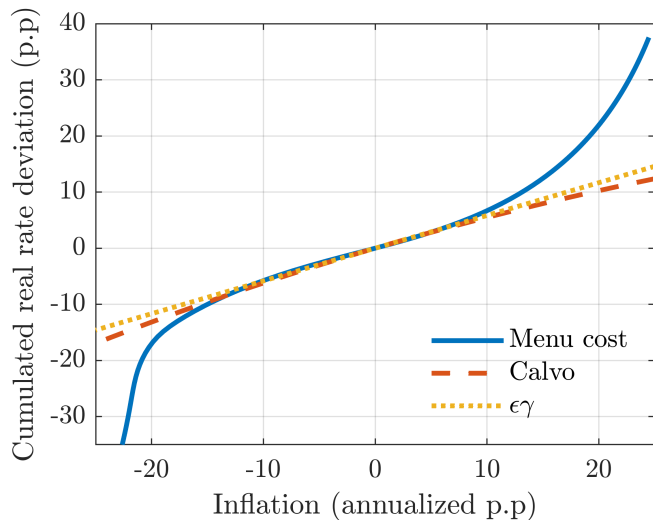
- ▶ Globally, the target rule is nonlinear
- ▶ After large shocks, the planner **stabilizes inflation more** relative to the output gap
- ▶ Why? Stabilizing inflation is cheaper due to **the lower sacrifice ratio** (higher freq.)
 - ▶ Similar results with quadratic objective
 - ▶ The nonlinearity of the targeting rule is due to the nonlinear Phillips curve



Nonlinear targeting rule

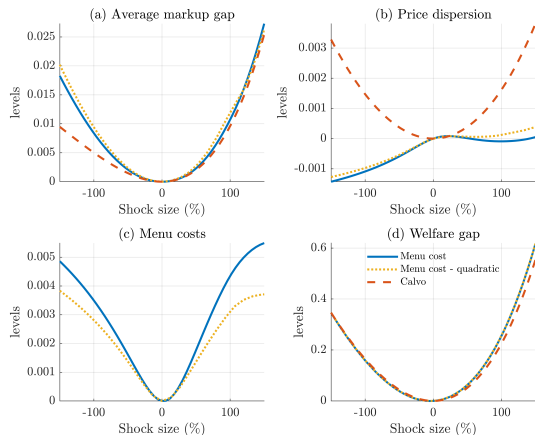


Nonlinear targeting rule for the real interest rate



Distortions

- ▶ Monopolistic competition and nominal frictions imply three distortions:
 - ▶ Inefficient markup fluctuations
 - ▶ Price dispersion
 - ▶ Price adjustment (menu) costs
- ▶ Inflation is costly
 - ▶ In GL primarily due to menu costs
 - ▶ In Calvo due to price dispersion
- ▶ A quadratic welfare in inflation and output gap is a good approximation

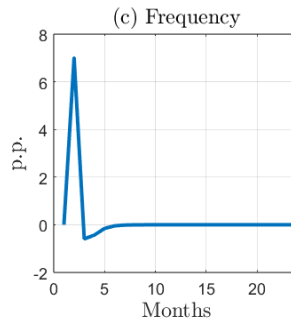
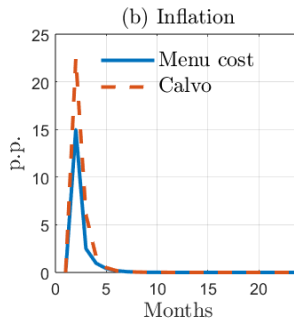
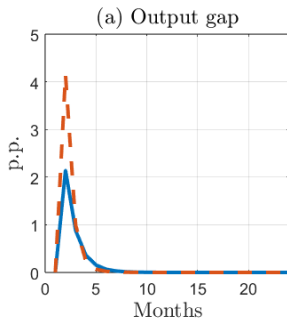


Optimal responses to efficiency shocks: “divine coincidence”

- ▶ In the standard NK model with Calvo pricing: divine coincidence holds after TFP and other shocks affecting the efficient allocation
- ▶ Optimal policy fully stabilizes inflation and closes the output gap
- ▶ We show analytically, that, after a TFP shock, [divine coincidence holds also in the menu cost model](#): inflation is fully stabilized at steady state and the output gap is closed

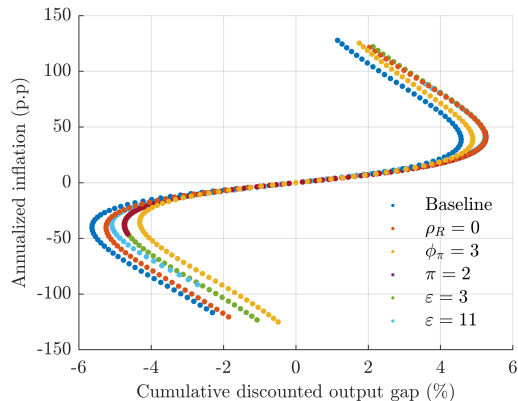
Time inconsistency

- ▶ Optimal policy without precommitment (time-0)
- ▶ Zero labor subsidy
- ▶ Weaker time inconsistency in GL than in Calvo: costlier to increase output gap

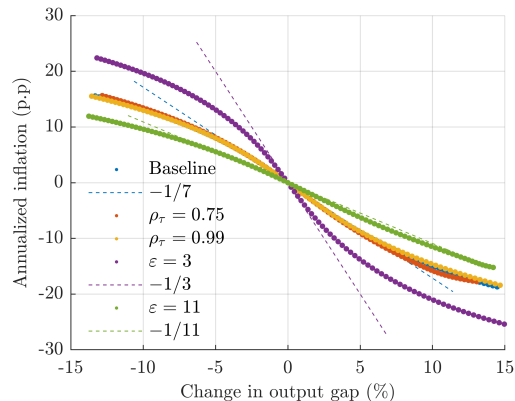


Robustness: alternative parametrization

Nonlinear Phillips Curve



Target rule

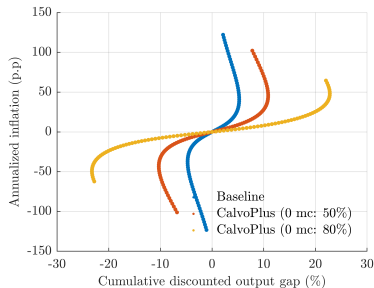


Robustness: CalvoPlus model

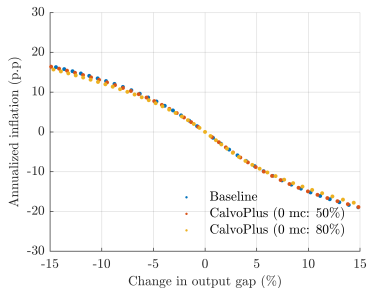
Menu cost is a random variable (Nakamura and Steinsson, 2010)

$$\tilde{\eta} = \begin{cases} \eta & \text{with prob } \alpha \\ 0 & \text{with prob } 1 - \alpha \end{cases}$$

Nonlinear Phillips Curve



Target rule

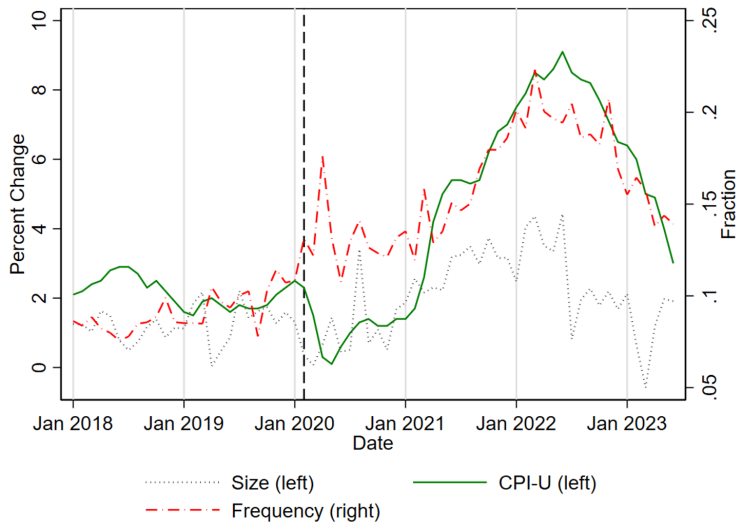


Conclusion

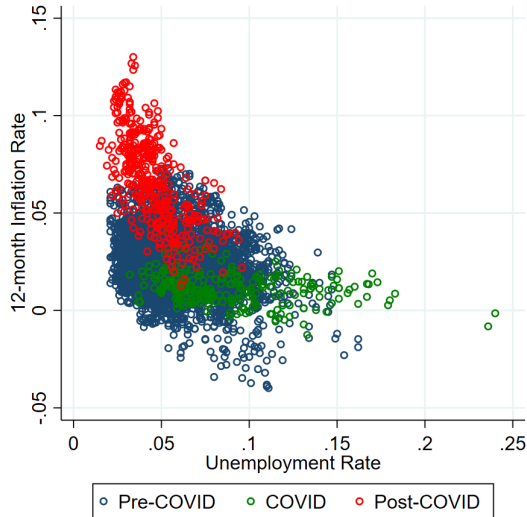
We study optimal policy in a state-dependent framework with a nonlinear Phillips curve

- ▶ Optimal long-run inflation is near zero (slightly positive)
- ▶ Divine coincidence holds for efficiency shocks
- ▶ For small cost shocks the optimal response is similar to Calvo (1983):
the lower welfare weight on inflation offsets the higher slope of the Phillips curve
- ▶ For large cost shocks, CB leans aggressively against inflation: [strike while the iron is hot!](#)
- ▶ Results robust in CalvoPlus framework

CPI and frequency of price changes in the US, Montag and Villar (2023)



Phillips correlation across US cities, Cerrato and Gitti (2023)



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