Strike while the Iron is Hot:

Optimal Monetary Policy with a Nonlinear Phillips Curve

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The views here are those of the authors only, and do not necessarily represent the views of their employers.

Motivation

- The recent inflation surge featured
  - Increase in the frequency of price changes (Montag and Villar, 2023) US
  - Increase in Phillips curve slope (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023) US
- Optimal monetary policy is mainly studied in models, in which the Phillips curve is linear and the frequency is held constant (Galí, 2008; Woodford, 2003)
- What does optimal monetary policy look like with a nonlinear Phillips curve and endogenous variation in frequency? How should CBs respond to a large inflation surge?

What do we do?

- We use the standard state-dependent pricing model of Golosov and Lucas (2007)
- Solve it nonlinearly using a new algorithm over the sequence space
- Positive analysis under a Taylor rule
  - Trace the responses to shocks of different sizes
  - Assess the nonlinearity of the Phillips curve
- Normative analysis: Ramsey optimal policy
  - Optimal long-run inflation
  - Trace the optimal responses to shocks
  - Characterize the (nonlinear) targeting rule after *large* cost-push shocks

What do we find?

- In this model the Phillips curve is nonlinear: it gets steeper as frequency increases
- In response to small shocks, optimal monetary policy is similar to the one under Calvo
- In response to efficiency shocks, there is divine coincidence, as in Calvo
- Different responses to small and large cost-push shocks. Optimal policy leans aggressively against inflation when frequency rises: "it strikes while the iron is hot"

Literature

- Nonlinear Phillips curve (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023)
  - Microfounded by state-dependent price setting

    (Golosov and Lucas, 2007; Gertler and Leahy, 2008; Auclert et al., 2022)
  - In the presence of large aggregate shocks (Karadi and Reiff, 2019; Alvarez and Neumeyer, 2020; Costain et al., 2022; Alexandrov, 2020; Blanco et al., 2024)
- Optimal policy in a menu cost economy
  - Optimal inflation target (Burstein and Hellwig, 2008; Adam and Weber, 2019; Blanco, 2021)
  - Small shocks, large shocks, optimal nonlinear target rule (comp. Galí, 2008; Woodford, 2003)
  - Focus on aggregate shocks (unlike Caratelli and Halperin, 2023, who study *sectoral* shocks)

Overview of the model

- Heterogeneous-firm DSGE model with fixed costs of price-adjustment
- Households: consume a Dixit-Stiglitz basket of goods, and work
- Firms: produce differentiated goods using labor only and are subject to aggregate TFP shocks and idiosyncratic "quality" shocks. They have market power and set prices optimally subject to a fixed cost (Golosov and Lucas, 2007)
- Monetary policy: either follows Taylor rule or set optimally to maximize household welfare under commitment

Households

- A representative household consumes (C_t), supplies labor hours (N_t) and saves in one-period nominal bonds (B_t).

- The household's problem is:

$$\max_{C_t,N_t,B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(C)_t - \nu N_t$$

$$\text{s.t. } P_t C_t + B_t + T_t = R_{t-1}B_{t-1} + W_t N_t + D_t,$$

where P_t is the price level, R_t is the gross nominal interest rate, W_t is the nominal wage, T_t are lump sum transfers and D_t are profits

Consumption and labor

- Aggregate consumption C_t and the price level are defined as:

$$C_t = \left\{ \int \left[A_t(i) C_t(i)\right]^{\frac{\epsilon - 1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon - 1}}, \quad P_t = \left[\int_0^1 \left(\frac{P_t(i)}{A_t(i)} \right)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

where $A_t(i)$ is product quality, ϵ is the elasticity of substitution.

- Labor supply condition and Euler equation are given by:

$$W_t = \upsilon P_t C_t, \quad 1 = \mathbb{E}_t \left[\beta \frac{u'\left(C_{t+1}\right)}{u'\left(C_t\right)} \frac{R_t}{\Pi_{t+1}} \right]$$

Monopolistic producers

- Production of good i is given by $Y_t(i) = A_t \frac{N_t(i)}{A_t(i)}$, where quality follows a random walk

$$log(A_t(i)) = log(A_{t-1}(i)) + \varepsilon_t(i), \quad \varepsilon_t(i) \sim N(0, \sigma_t^2)$$

- Firms real profits

$$\frac{D_t(i)}{P_t} = \frac{P_t(i)}{P_t} Y_t(i) - (1 - \tau_t) \frac{W_t}{P_t} N_t(i)$$

- Firms face a fixed cost η to update prices

Quality-adjusted relative prices

- Let $p_t(i) \equiv \log\left(P_t(i)/(A_t(i)P_t)\right)$ be the quality-adjusted log relative price
- Real profit then is

$$\mathit{\Pi}(p_t(i), w_t, A_t) \equiv \frac{D_t(i)}{P_t} = C_t e^{p_t(i)(1-\epsilon)} - C_t(1-\tau_t)\frac{w_t}{A_t}e^{p_t(i)(-\epsilon)}$$

where w_t is the real wage.

- When nominal price $P_t(i)$ stays constant, $p_t(i)$ evolves: $p_t(i) = p_{t-1}(i) - \sigma_t \varepsilon_t(i) - \pi_t$

Pricing decision

- Let $\lambda_t(p)$ be the price-adjustment probability
- Value function is

$$\begin{aligned} V_t(p) &= \Pi(p, w_t, A_t) \\ &+ \mathbb{E}_t\left[\left(1-\lambda_{t+1}\left(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}\right)\right)\Lambda_{t,t+1}V_{t+1}(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1})\right] \\ &+ \mathbb{E}_t\left[\lambda_{t+1}\left(p-\sigma_{t+1}\varepsilon_{t+1}-\pi_{t+1}\right)\Lambda_{t,t+1}\left(\max_{p'} V_{t+1}\left(p'\right)-\eta w_{t+1}\right)\right]. \end{aligned}$$

- The adjustment probability is

$$\lambda_t(p) = I[\max_{p'} V_t\left(p'\right) - \eta w_t > V_t(p)]$$

where $I[\cdot]$ is the indicator function.

Monetary Policy

- The central bank either sets policy optimally, or follows a Taylor rule:

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1-\rho_r)\left[\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^e)\right] + \varepsilon_{r,t} \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2)$$

- Shocks: employment subsidy (τ_t), TFP (A_t), volatility (σ_t)

$$\begin{aligned}
\log(A_t) =& \rho_A \log(A_{t-1}) + \varepsilon_{A,t} \quad \varepsilon_{A,t} \sim N(0, \sigma_A^2) \\
\tau_t - \tau =& \rho_\tau(\tau_{t-1} - \tau) + \varepsilon_{\tau,t} \quad \varepsilon_{\tau,t} \sim N(0, \sigma_\tau^2), \\
\log(\sigma_t/\sigma) =& \rho_\sigma \log(\sigma_{t-1}/\sigma) + \varepsilon_{\sigma,t} \quad \varepsilon_{\sigma,t} \sim N(0, \sigma_\sigma^2)
\end{aligned}$$

Aggregation and market clearing

- Aggregate price index

$$1 = \int e^{p(1-\epsilon)} g_t(p)\, dp,$$

- Labor market equilibrium

$$N_t = \frac{C_t}{A_t} \underbrace{\int e^{p(-\epsilon)} g_t(p)\, dp}_{\text{dispersion}} + \eta \underbrace{\int \lambda_t(p - \sigma_t \varepsilon - \pi_t) g_{t-1}(p) dp}_{\text{frequency}},$$

Law of motion of the price density

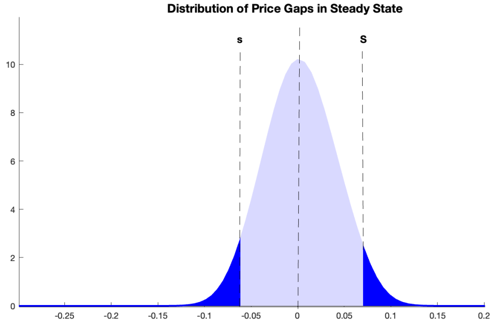
$$g_t(p) = \begin{cases} (1-\lambda_t(p)) \int g_{t-1}(p+\sigma\varepsilon+\pi_t) d\xi(\varepsilon) & \text{if } p \neq p_t^*, \\ (1-\lambda_t(p_t^*)) \int g_{t-1}(p_t^*+\sigma\varepsilon+\pi_t) d\xi(\varepsilon) + & \\ \qquad \int_{\underline{p}}^{\overline{p}} \lambda_t(\tilde{p}) \left(\int g_{t-1}(\tilde{p}+\sigma\varepsilon+\pi_t) d\xi(\varepsilon) \right) d\tilde{p} & \text{if } p = p_t^*. \end{cases}$$

Calibration

		Households	
β	$0.96^{1/12}$	Discount rate	Golosov and Lucas (2007)
ϵ	7	Elasticity of substitution	Golosov and Lucas (2007)
γ	1	Risk aversion parameter	Midrigan (2011)
υ	1	Utility weight on labor	Set to yield $w = C$
		Price setting	
η	3.6%	Menu cost	Set to match 8.7% of frequency
σ	2.4%	Std dev of quality shocks	and 8.5% size in Nakamura and Steinsson (2008)
		Monetary policy	
ϕ_π	1.5	Inflation coefficient in Taylor rule	Taylor (1993)
ϕ_y	0.5/12	Output gap coefficient in Taylor rule	Taylor (1993)
ρ_i	$0.75^{1/3}$	Smoothing coefficient	
		Shocks	
ρ_A	$0.95^{1/3}$	Persistence of the TFP shock	Smets and Wouters (2007)
ρ_τ	$0.9^{1/3}$	Persistence of the cost-push shock	Smets and Wouters (2007)

Large shocks: illustration

- Let $x_t(j) \equiv p_t(j) - p_t^*(j)$ be the price gap
- Large aggregate shock: shifts the distribution of price gaps for all firms
- Limited impact on the (s,S) bands
- Pushes a large fraction of firms outside of the inaction region
- Large increase in the frequency of price changes and hence additional flexibility of the aggregate price level (on top of "selection")



Large shocks: illustration, cont.

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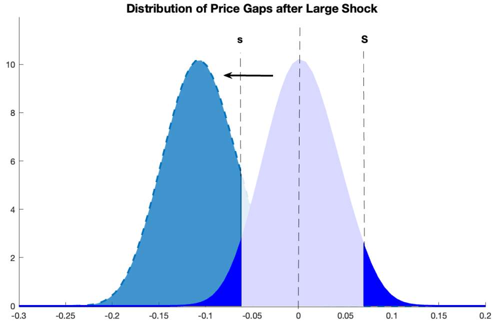
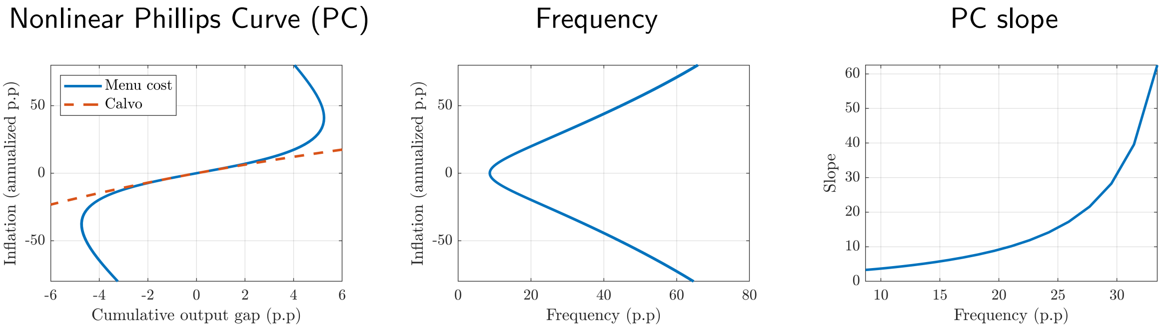


Nonlinearity of the Phillips Curve at realistic frequency (20%) US

Consider the model under a Taylor rule



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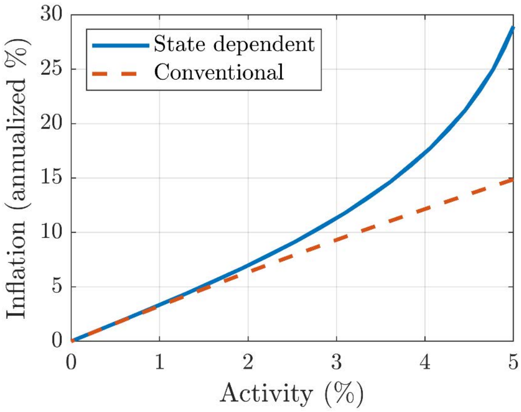


Normative results: The Ramsey problem ($x_t \equiv p_t - p_t^*$)

$$\max_{\{g_t^c(\cdot), g_t^0, V_t(\cdot), C_t, w_t, p_t^*, s_t, S_t, \pi_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, \frac{C_t}{A_t}\left(\int e^{(x+p_t^*)(-\epsilon_t)} g_t^c(p)\,dx + g_t^0 e^{(p_t^*)(-\epsilon)}\right) + \eta g_t^0\right)$$

subject to

$$\begin{aligned}
1 &= \int e^{(x+p_t^*)(1-\epsilon)} g_t^c(x)\,dx + g_t^0 e^{(p_t^*)(1-\epsilon)}, \\
V_t'(0) &= \Pi_t'(x) + \frac{1}{\sigma}\Lambda_{t+1}\int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x')\frac{\partial\phi\left(\frac{x-x'-\pi_t^*}{\sigma}\right)}{\partial x}dx' + \Lambda_{t+1}\left(\phi\left(\frac{S_{t+1}-\pi_t^*}{\sigma}\right) - \phi\left(\frac{s_{t+1}-\pi_t^*}{\sigma}\right)\right)(V_{t+1}(0) - \eta w_{t+1}), \\
V_t(s_t) &= V_t(0) - \eta w_t, \\
V_t(S_t) &= V_t(0) - \eta w_t, \\
w_t &= \upsilon C_t^{\gamma}, \\
V_t(x) &= \Pi(x, p_t^*, w_t, A_t) + \Lambda_{t,t+1}\frac{1}{\sigma}\int_{s_t}^{S_t}\left[V_{t+1}(x')\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right]dx' + \Lambda_{t,t+1}\left(1 - \frac{1}{\sigma}\int_{s_t}^{S_t}\left[\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right]dx'\right)[(V_{t+1}(0) - \eta w_{t+1})], \\
g_t^c(x) &= \frac{1}{\sigma}\int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1})\phi\left(\frac{x_{-1}-x-\pi_t^*}{\sigma}\right)dx_{-1} + g_{t-1}^0\phi\left(\frac{-x-\pi_t^*}{\sigma}\right), \\
g_t^0 &= 1 - \int_{s_t}^{S_t} g_t^c(x)dx.
\end{aligned}$$

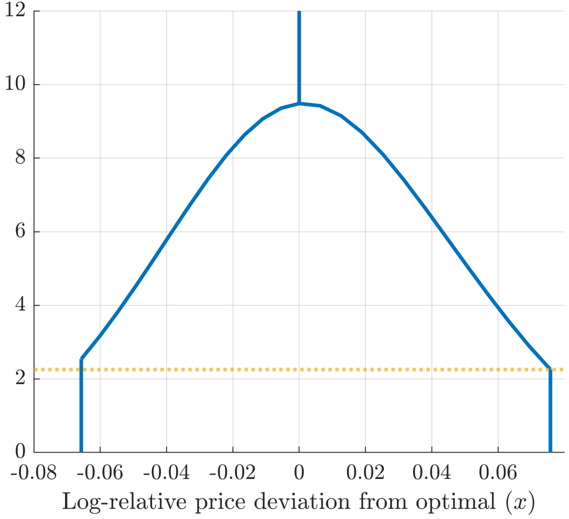
Normative results: Computation

- Challenges
  - Price change distribution and firms' value function are infinite-dimensional objects
  - In the Ramsey problem, we need derivatives w.r.t. both
- New algorithm, inspired by González et al. (2024)
  - Approximate distribution and value functions by piece-wise linear interpolation on grid
  - Endogenous grid points: (s,S) bands and the optimal reset price
  - Solve in the sequence space using Dynare (Adjemian et al. (2023))

Optimal long-run inflation rate

- The Ramsey steady-state inflation rate is slightly above zero: $\pi^* = 0.25\%$
  - Close to the inflation rate that minimizes the steady-state frequency of price changes
- Why not exactly zero as in Calvo (1983)?
  - Asymmetry of the profit function leads to asymmetric (s,S) bands: a negative price gap is less desirable than a positive price gap of the same size
  - At zero inflation, more mass around the lower (s) band than around the higher (S) band
  - Slightly positive inflation raises p^* and pushes the mass of firms to the right inside (s,S)
  - This leads to lower frequency and lower price-adjustment costs

Steady-state price distribution (at zero inflation)



Optimal response to cost-push shocks

- In linearized Calvo (1983), optimal policy is a flexible inflation targeting rule

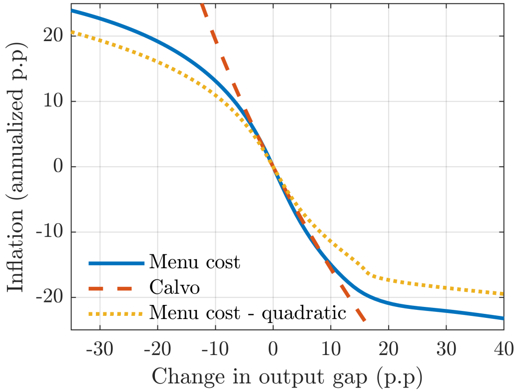
$$\pi_t = -\frac{1}{\epsilon}\Delta\tilde{y}_t$$

- Slope $-1/\epsilon$ is independent of the frequency of repricing or the slope of the PC

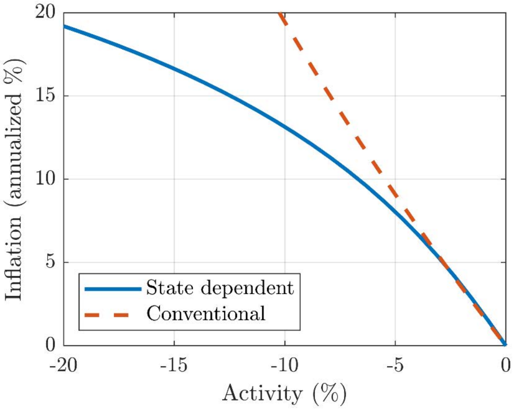
- For small cost-push shocks, the slope of the targeting rule in Golosov and Lucas (2007) is also $-1/\epsilon$!

Nonlinear targeting rule

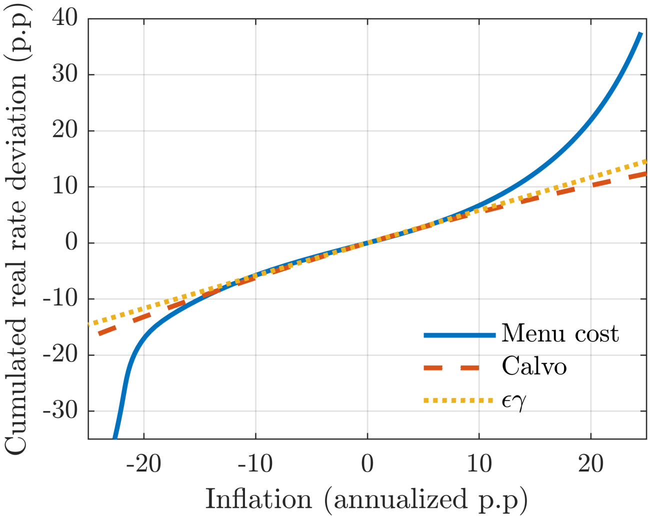
- Globally, the target rule is nonlinear
- After large shocks, the planner stabilizes inflation more relative to the output gap
- Why? Stabilizing inflation is cheaper due to the lower sacrifice ratio (higher freq.)
  - Similar results with quadratic objective
  - The nonlinearity of the targeting rule is due to the nonlinear Phillips curve



Nonlinear targeting rule

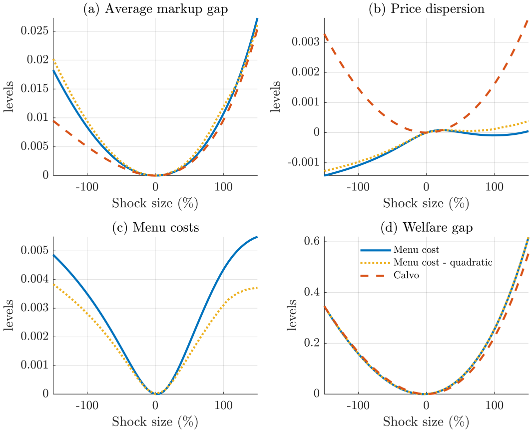


Nonlinear targeting rule for the real interest rate



Distortions

- Monopolistic competition and nominal frictions imply three distortions:
    - Inefficient markup fluctuations
    - Price dispersion
    - Price adjustment (menu) costs
- Inflation is costly
    - In GL primarily due to menu costs
    - In Calvo due to price dispersion
- A quadratic welfare in inflation and output gap is a good approximation

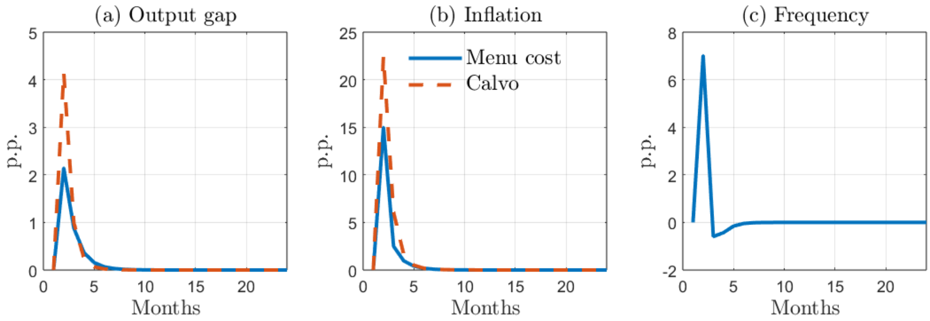


Optimal responses to efficiency shocks: "divine coincidence"

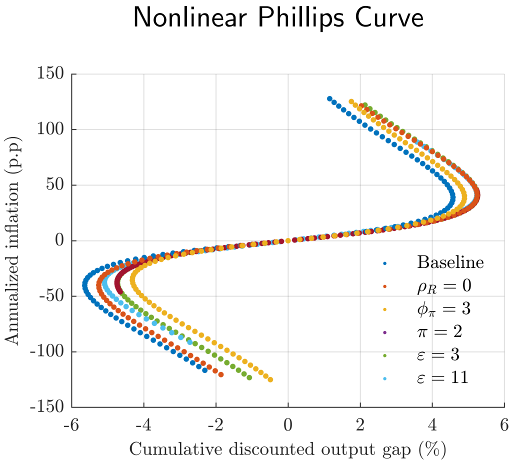
- In the standard NK model with Calvo pricing: divine coincidence holds after TFP and other shocks affecting the efficient allocation
- Optimal policy fully stabilizes inflation and closes the output gap
- We show analytically, that, after a TFP shock, divine coincidence holds also in the menu cost model: inflation is fully stabilized at steady state and the output gap is closed

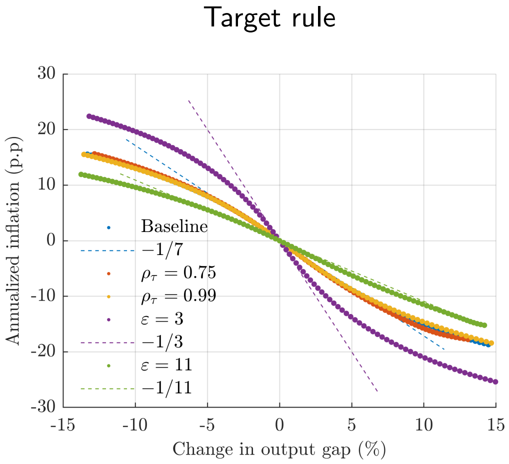
Time inconsistency

- Optimal policy without precommitment (time-0)
- Zero labor subsidy
- Weaker time inconsistency in GL than in Calvo: costlier to increase output gap



Robustness: alternative parametrization



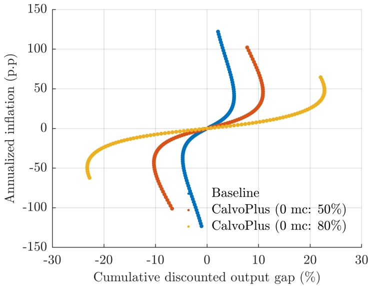


Robustness: CalvoPlus model

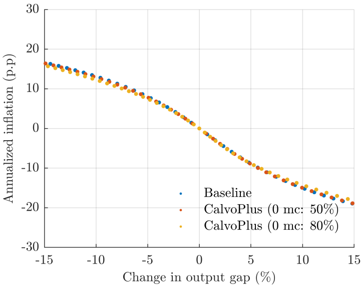
Menu cost is a random variable (Nakamura and Steinsson, 2010)

$$\tilde{\eta} = \begin{cases} \eta & \text{with prob } \alpha \\ 0 & \text{with prob } 1 - \alpha \end{cases}$$

Nonlinear Phillips Curve



Target rule

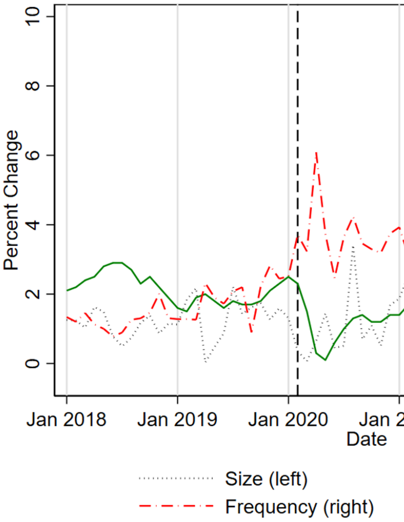


Conclusion

We study optimal policy in a state-dependent framework with a nonlinear Phillips curve

- Optimal long-run inflation is near zero (slightly positive)
- Divine coincidence holds for efficiency shocks
- For small cost shocks the optimal response is similar to Calvo (1983): the lower welfare weight on inflation offsets the higher slope of the Phillips curve
- For large cost shocks, CB leans aggressively against inflation: strike while the iron is hot!
- Results robust in CalvoPlus framework

CPI and frequency of price changes in the US, Montag and Villar (2023)



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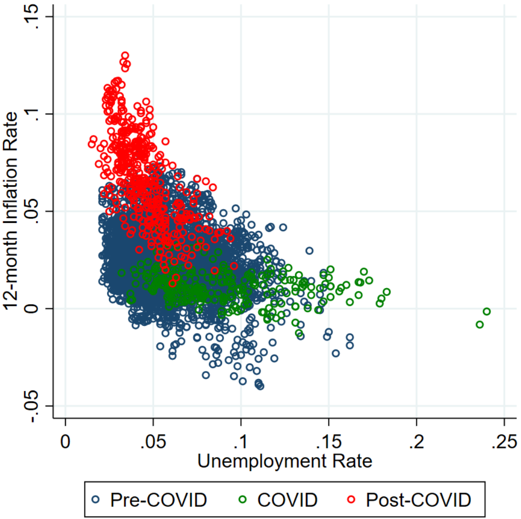
Phillips correlation across US cities, Cerrato and Gitti (2023)



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