# Climate-Conscious Monetary Policy*

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**Abstract**

We study the implications of climate change and the associated mitigation measures for optimal monetary policy in a canonical New Keynesian model with climate externalities. Provided they are set at their socially optimal level, carbon taxes pose no trade-offs for monetary policy: it is both feasible and optimal to fully stabilize inflation and the welfare-relevant output gap. More realistically, if carbon taxes are initially suboptimal, trade-offs arise between core and climate goals. These trade-offs, however, are resolved overwhelmingly in favor of price stability, even in scenarios of decades-long transition to optimal carbon taxation. This reflects the untargeted and inefficient nature of (conventional) monetary policy as a climate instrument. In a model extension with financial frictions and central bank purchases of corporate bonds, we show that green tilting of purchases is optimal and accelerates the green transition. However, its effect on CO2 emissions and global temperatures is limited by the small size of eligible bonds' spreads.

KEYWORDS: Ramsey optimal monetary policy, climate change externalities, Pigouvian carbon taxes, green QE

JEL Codes: E31, E32, Q54, Q58

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# 1 Introduction

The World scientific community has come to a consensus on the need for decarbonization of the global economy in order to combat climate change, in view of the rise of global temperatures in recent decades and projections of what could happen if decisive action is not taken (see Figure 1).

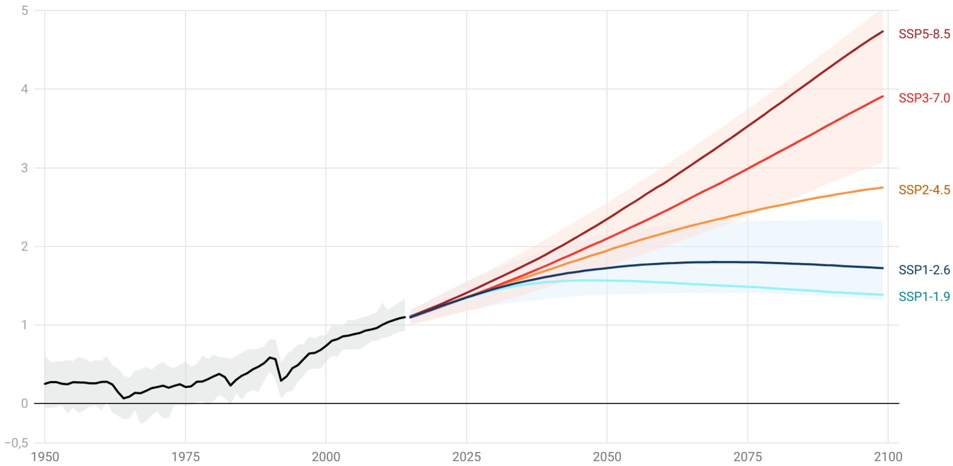


Figure 1: Global temperatures since 1950 and projections. Source: IPCC, 2021

While genuinely targeted policies, such as carbon taxation and emissions trading schemes, are widely seen as the key policy levers to mitigate climate change, there is much less agreement on what role other policy areas should play. This is especially the case for monetary policy. Different policy-makers have expressed rather opposing views on whether central banks should adopt climate change considerations in their monetary policy frameworks, given their current legal mandates (see e.g. Lagarde, 2021, and Powell, 2023).

Even if one takes the view that monetary policy should adopt climate goals, this still raises a number of crucial normative questions. How should monetary policy respond to climate change, given its obligation to pursue its core statutory goals, notably price stability? Is there a trade-off between core goals and climate goals? If so, how do these trade-offs depend on what (genuine) climate authorities are doing? And how should those trade-offs be resolved, given the monetary policy instruments at central banks' disposal?[1]

In this paper, we address the above questions using a canonical New Keynesian model extended with climate externalities. Our modelling of the latter follows Golosov et al (2014) closely: production requires the use of green and fossil energy, whereby the latter produces CO2 emissions that add to carbon concentration in the atmosphere and hence to global warming, which causes damages to the economy's productive capacity. As in Golosov et al (2014), the

[1] As argued by Hansen (2022), monetary policy tools are much less potent than fiscal ones when it comes to confronting climate change.

government can impose a tax on fossil energy production, henceforth "carbon tax". In this context, we analyze the Ramsey optimal monetary policy by a benevolent (i.e. social welfare-maximizing) central bank. Therefore, the central bank internalizes the climate externalities from fossil energy consumption. It is in this sense that we refer to monetary policy as being "climate-conscious".

We first establish analytically a benchmark result. Provided carbon taxes follow their socially optimal path at all times,[2] monetary policy does not face any trade-offs: it is both feasible and optimal to fully stabilize inflation and the *welfare-relevant* output gap (i.e. the gap between the actual and the socially efficient level of output). Thus, strict inflation targeting allows the central bank to replicate the social planner equilibrium, including the socially optimal path of CO2 emissions. The intuition for this result is simple: if carbon taxes are set at their optimal (Pigouvian) level, then all agents internalize perfectly the climate externalities from fossil energy use. This leaves nominal rigidities as the only distortion left, which the central bank can offset through a policy of strict price stability.[3]

While useful as a normative benchmark, the assumption that carbon taxes are set optimally since the very first period is unrealistic, given the rather slow pace of progress in carbon taxation and similar policies observed in practice, even in advanced economies. For this reason, we focus the remainder of our analysis on the case of a "slow green transition", which we define as a scenario in which, starting from zero, the carbon tax converges slowly towards its socially optimal path. In this case, a tension arises between price stability and climate goals. Because carbon taxes are suboptimal during the transition, the economy consumes too much fossil energy. Aware of this, the central bank has an incentive for curtailing output – bringing it closer to its socially efficient path, i.e. narrowing the welfare-relevant output gap – in order to reduce overall energy consumption, including consumption of fossil energy. However, this comes at the expense of lowering output below its *natural* (i.e. flexible-price) level and thus accepting a transitory fall in inflation below target.

In order to quantify this trade-off, we use a calibrated version of our model economy. We find that the trade-off is resolved overwhelmingly in favor of price stability. In particular, under a very slow green transition in which optimal carbon taxation is reached after 30 years, the optimal departure from strict inflation targeting is very small, of barely 10 basis points in the first quarter, and short-lived. This is mirrored by a small, short-lived fall in output below its natural level, which barely helps reduce output towards its (much lower) socially efficient level. As a result, optimal monetary policy barely affects the path of fossil energy consumption and

[2]The optimal carbon tax has the same shape as Golosov et al's (2014) well-known formula: as a proportion of output, optimal carbon taxes depend only on households' subjective discount factor and on the parameters governing the accumulation of atmospheric carbon concentration and the economic damages from global warming.

[3]As in much of the literature on optimal monetary policy in New Keynesian models, we assume that monopolistic distortions are offset by means of an appropriately chosen revenue subsidy. As a result, strict inflation targeting replicates the socially efficient equilibrium, *provided* the carbon tax equals its optimal level. This assumption allows us to isolate the effect of climate change and carbon taxes on monetary policy trade-offs from the effect of monopolistic distortions.

CO2 emissions, compared to a scenario in which monetary policy ignores climate considerations and adheres strictly to its price stability mandate.

The intuition for this result is the following. Due to its untargeted nature, conventional (interest-rate) monetary policy is a rather blunt, inefficient tool for climate-related purposes: reducing CO2 emissions requires the central bank to reduce overall (i.e. fossil, but also green) energy consumption, which in turn requires suppressing economic activity and lowering inflation below target, all of which is rather costly in social welfare terms. Faced with this unfavorable trade-off, the central bank optimally decides to deviate minimally from strict inflation targeting.

While interest-rate policy is an untargeted instrument, its effects need not be symmetric across green and fossil energy sectors. In practice, the cost of green energy sources is typically more sensitive than that of fossil energy sources to changes in interest rates, because green energy generation often involves higher upfront capital costs (see e.g. Bistline, Mehrotra and Wolfram, 2024). To capture this dimension, we extend our baseline model – in which energy production is linear in labor – to include capital in energy production. We calibrate the model to replicate evidence on the sensitivity of green and fossil energy costs to interest rates. We find that, in the slow green transition case, the optimal deviation from strict inflation targeting is even smaller than in the baseline model, thus reinforcing our main result. The reason is that, relative to strict inflation targeting, optimal monetary policy entails a slightly *lower* interest rate in the first few periods – as this allows the central bank to make green energy cheaper relative to fossil energy –, implying a smaller initial fall in inflation below its long-run target.

In practice, however, central banks have other, more targeted instruments for addressing climate change. The most prominent one is the so-called "green QE", i.e. the possibility of tilting their portfolio of corporate bond holdings in favor of "green bonds" – understood as bonds satisfying certain climate-related eligibility criteria – and to the detriment of "brown bonds".[4] To analyze optimal green tilting of QE, we extend our baseline model with a simple specification of financial frictions that allows central bank purchases of corporate bonds to affect the cost of bond financing for green and fossil energy producers and thus, through a standard cost channel, the relative price of both energy sources.

We first show that our benchmark normative result generalizes to the model with corporate QE: as long as carbon taxation is optimal, it is again feasible and optimal to fully stabilize inflation and the welfare-relevant output gap. The difference is that now optimal policy also entails "full QE", whereby the central bank absorbs as many bonds (both green and brown) as needed to offset the financial friction in both energy sectors. We then show that, in the slow green transition scenario, full *green* QE continues to be optimal, but *brown* QE has two distinct phases. Initially, the gap between actual and optimal carbon taxes is large enough that green tilting cannot raise brown bond spreads sufficiently to implement the socially optimal fossil energy price: the best the central bank can do is not to purchase any brown bonds at

[4] The ECB and the Bank of England are two prominent examples of major central banks that have explicitly incorporated green criteria in its corporate bond purchase programs; see ECB (2022) and Bank of England (2021).

all. Subsequently, once the carbon tax gap becomes sufficiently small, the central bank buys as many brown bonds as needed for brown bond spreads to reach the level necessary to implement the optimal fossil energy price. In this second phase, green tilting allows the central bank to exactly compensate for the shortfall in carbon taxation as the latter catches up with its optimal level.

Finally, we quantify how the trade-offs under slow green transition change in the presence of QE. As in the baseline model, the trade-offs continue to be resolved clearly in favor of price stability. The main difference is that optimal green tilting of QE accelerates somewhat the green transition: fossil energy use reaches its socially optimal level a year and a half earlier, compared to a climate-oblivious scenario of strict inflation targeting and no QE. However, the impact on atmospheric carbon concentration and global temperatures is very small. This reflects the fact that the effectiveness of green tilting at reducing carbon emissions depends on how much it can tighten financing conditions for brown firms and ease them for green firms. Since central banks' purchase programs typically restrict the set of eligible bonds to those with high credit quality (i.e. with investment-grade rating), their average spreads are relatively small even in the absence of central bank purchases – a fact that we incorporate in our calibration –, thus limiting the scope of green tilting for altering the relative spreads of brown vs green bonds.

In sum, our analysis suggests that, while monetary policy can play a role in confronting climate change – and *should* play it, under a welfare-maximizing criterion for optimal policy –, the extent to which it can do so is rather limited, reflecting either the untargeted, inefficient nature of its conventional instruments, or the design restrictions on its unconventional tools.

### 1.1 Related literature

Building on the seminal work by William Nordhaus on integrated climate-economy models,[5] the literature on climate change and the macroeconomy has grown considerably over the last decade. Standard environmental policies such as taxes, subsidies, and caps were studied in RBC models by Fischer and Springborn (2011), Heutel (2012) and Angelopoulos et al (2013). Optimal carbon taxation was analyzed in Golosov et al (2014), from whom we borrow our specification of climate externalities, and more recently in Barrage (2020). Annicchiarico and Di Dio (2015), Ferrari and Nispi Landi (2022), Airaudo, Pappa and Seoane (2023) and Olovsson and Vestin (2023) have all explored the macroeconomic effects of climate change mitigation policies in New Keynesian DSGE models, including the possibility of green policy-induced inflation, or "greenflation".[6]

A recent literature explores the role of monetary policy and other macroeconomic policies in New Keynesian DSGE models with climate change. Benmir and Roman (2020) assess different types of fiscal, monetary, and macroprudential policies aimed at reducing CO2 emissions. Ferrari and Pagliari (2021) explore the cross-country implications of climate-related mitigation policies

[5] See e.g. Nordhaus (2008).

[6] The macroeconomic impact of carbon taxes and other mitigation policies has also been analyzed in dedicated reports by international organizations. See e.g. IMF (2022).

in a two-country model with country-specific fiscal and monetary policies and the possibility of cooperation between them. Diluiso et al. (2020) use a model with financial frictions and climate policy to study the risks a low-carbon transition poses to financial stability and how central bank (monetary and financial) policies can be used to manage these risks. Ferrari and Nispi Landi (2021, 2023) study the effectiveness of temporary and permanent green QE, respectively, at mitigating CO2 emissions in models with environmental externalities.

In a multi-sector New Keynesian model, Del Negro, di Giovanni and Dogra (2023) analyze how the inflation-output trade-off created by climate policies depends on how flexible prices are in the "dirty" and "green" sectors relative to the rest of the economy, and on whether climate policies consist of taxes or subsidies. Using a New-Keynesian model with an energy and a goods sector, Olovsson and Vestin (2023) study the real and nominal implications of a green transition to a state with sustainable energy production, and how Taylor rule-based monetary policy should react during such transition. Fornaro, Guerrieri and Reichlin (2025) propose a New-Keynesian framework with endogenous productivity growth in the production of both "clean" and "dirty" goods, and analyze the intertemporal inflation trade-offs that the green transition poses for monetary policy. Mehrotra (2025) examines the macroeconomic cost and implications of transitioning to net zero emissions; he shows that a net zero target operates as both an anticipated negative productivity shock and a negative capital shock – such that, for monetary policy, "net zero" is a negative aggregate demand shock that lowers the natural rate of interest – and that decarbonization of US electric power generation is estimated to cost less than 0.2% of steady state consumption.

In a real model with climate externalities and financial frictions, Papoutsi, Piazzesi and Schneider (2023) study how the sectoral composition of central bank asset purchases shapes their environmental impact. They also analyze the optimal asset purchase policy. They find that if an optimal carbon tax is in place, asset purchases should focus only on minimizing financial frictions and not take climate externalities explicitly into account;[7] whereas in the absence of an optimal carbon tax, green monetary policy can improve welfare. Annicchiarico and Di Dio (2017) analyze Ramsey optimal environmental and (conventional) monetary policy in response to productivity shocks around a deterministic steady-state.

Importantly, none of the above contributions study the Ramsey optimal monetary policy, both conventional and unconventional, in a New Keynesian environment with climate externalities and a green transition. In this regard, our analysis clarifies the trade-offs that monetary policy faces between its core goals (such as price stability) and climate goals, and how such trade-offs depend on the path of carbon taxes relative to their socially optimal path.

[7]Our result on the absence of trade-offs under optimal carbon taxation in the model extention with QE can therefore be seen as a generalization of Papoutsi et al.'s (2023) above result to a nominal framework with inflation and conventional (interest-rate) monetary policy.

## 2 Baseline model

Our baseline model is a standard New Keynesian framework extended with an energy block and climate change externalities *à la* Golosov et al. (2014). The economy consists of five types of agents: households, final goods producers, energy producers, the government (which acts as the climate authority), and a monetary authority. We next describe each in turn.

### 2.1 Households

There exists a representative household that maximizes lifetime welfare,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \frac{\chi}{1+\varphi} N_t^{1+\varphi} \right], \tag{1}$$

where $C_t = \left( \int_0^1 c_{z,t}^{(\epsilon-1)/\epsilon} dz \right)^{\epsilon/(\epsilon-1)}$ is a Dixit-Stiglitz basket of consumption varieties (with $\epsilon > 1$), $N_t$ is labor supply, and $\beta \in (0,1)$ is a discount factor, subject to the following budget constraint in nominal terms,

$$\int_0^1 P_{z,t} c_{z,t} dz + B_t = R_{t-1} B_{t-1} + W_t N_t + \Pi_t + T_t,$$

where $P_{z,t}$ is the price of consumption variety $z \in [0,1]$, $B_t$ are holdings of one-period nominal debt, $R_t$ is the gross nominal interest rate, $W_t$ is the nominal wage, and $\Pi_t$ and $T_t$ are nominal lump-sum profits from firms and government subsidies, respectively. Cost minimization implies $c_{z,t} = (P_{z,t}/P_t)^{-\epsilon} C_t$ and $\int_0^1 P_{z,t} c_{z,t} dz = P_t C_t$, where $P_t = \left( \int_0^1 P_{z,t}^{1-\epsilon} dz \right)^{1/(1-\epsilon)}$ is the aggregate price index. The first-order conditions of the intertemporal problem can then be expressed as

$$\chi N_t^{\varphi} C_t = \frac{W_t}{P_t} \equiv w_t, \tag{2}$$

$$\frac{1}{C_t} = \beta R_t \mathbb{E}_t \left\{ \frac{P_t}{P_{t+1} C_{t+1}} \right\}.$$

### 2.2 Final goods producers

Each consumption variety $z \in [0,1]$ is produced by a monopolistic producer. The production function of variety-$z$ producer is

$$y_{z,t} = [1 - D(S_t)] A_t F(N_{z,t}, E_{z,t}). \tag{3}$$

where $A_t$ is exogenous total factor productivity (TFP), $N_{z,t}$ is the firm's labor demand and $E_{z,t}$ is its energy consumption. Each firm uses a combination of green and fossil energy,

$$E_{z,t} = \mathbf{E}(E_{z,t}^g, E_{z,t}^f),$$

where $E^g_{z,t}$ is green energy and $E^f_{z,t}$ is fossil energy (also known as "brown" energy). Both $F$ and $\mathbf{E}$ have constant returns to scale. The term $D(S_t)$ is the so-called *damage function*, which depends on the stock of carbon concentration in the atmosphere, $S_t$.[8] The damage function represents the key externality through which climate change affects economic activity in the model.[9]

Producer $z$'s cost minimization problem is as follows,

$$\min_{N_{z,t},\{E^g_{z,t}\}_{i=g,f}} W_t N_{z,t} + \sum_{i=g,f} p^i_t P_t E^i_{z,t} - MC_{z,t}\left[1 - D(S_t)\right] A_t F\left(N_{z,t}, \mathbf{E}(E^g_{z,t}, E^f_{z,t})\right),$$

where $p^g_t$ and $p^f_t$ are the real prices of green and fossil energy, respectively, and $MC_{z,t}$ is the Lagrange multiplier on (3), i.e. firm $z$'s nominal marginal cost. The first-order conditions are given by

$$MC_t\left[1 - D(\cdot)\right] A_t F_N\left(N_{z,t}, E_{z,t}\right) = W_t, \tag{4}$$

$$MC_t\left[1 - D(\cdot)\right] A_t F_E\left(N_{z,t}, E_{z,t}\right) \mathbf{E}_{E^i}(E^g_{z,t}, E^f_{z,t}) = p^i_t P_t, \tag{5}$$

$i = g, f$, where we use the fact that, under constant returns to scale, marginal costs are equalized across firms: $MC_{z,t} = MC_t$ for all $z$.[10] The firm's total cost can then be expressed as $W_t N_{z,t} + \sum_{i=g,f} p^i_t P_t E^i_{z,t} = MC_t y_{z,t}$.

Producers' pricing problem is standard. Each producer faces a demand curve

$$y_{z,t} = \left(P_{z,t}/P_t\right)^{-\epsilon} C_t. \tag{6}$$

Firms receive a subsidy $\tau^y$ per unit of revenue.[11] We assume Calvo (1983) pricing, with $\theta$ denoting the fraction of randomly-selected producers not adjusting their price in a given period. A producer that has the opportunity of changing its price in period $t$ chooses $P_{z,t}$ to maximize the expected future discounted stream of nominal profits over the (expected) life of the new price,

$$\sum_{t=0}^{\infty} \mathbb{E}_t \left\{ \Lambda_{t,t+s}\theta^s \left[(1+\tau^y) P_{z,t} - MC_{t+s}\right] \left(\frac{P_{z,t}}{P_{t+s}}\right)^{-\epsilon} C_{t+s} \right\},$$

[8]As explained by Golosov et al. (2014), the function $D(S_t)$ can be seen as compounding a mapping from atmospheric carbon concentration into global temperature, $T_t$, and a mapping from global warming into economic damages. Denoting the latter two mappings by $T$ and $D_T$ respectively, we therefore have $D(S_t) \equiv D_T(T(S_t))$

[9]A more general model specification could also include direct effects of climate change on household utility. See e.g. van der Ploeg and Withagen (2012) and Barrage (2020) for contributions adopting this approach. In addition, damage from climate change could affect the productivity of energy-generating sectors as well, which could be relevant for the quantitative results. We abstract from this possibility in our model.

[10]Under the assumption of constant returns to scale in $F$ and $\mathbf{E}$, the marginal products $F_N$ and $F_E$ depend only on the labor-energy ratio $N_{z,t}/E_{z,t}$, whereas $\mathbf{E}_{E^i}, i = f, g$, depend only on the green-fossil energy ratio $E^g_{z,t}/E^f_{z,t}$. Since factor prices are common to all firms, those ratios are equalized across producers, and therefore so are marginal products and the nominal marginal cost.

[11]As will become clear shortly, we introduce a revenue subsidy in order to offset the monopolistic distortion and focus the analysis on the trade-offs created by nominal rigidities and climate change externalities.

where

$$\Lambda_{t,t+s} \equiv \beta^s \frac{P_t C_t}{P_{t+s} C_{t+s}} \tag{7}$$

is the stochastic discount factor. The first-order condition is

$$\sum_{t=0}^{\infty} \mathbb{E}_t \left\{ \Lambda_{t,t+s} \theta^s \left( \left(1+\tau^y\right) P_t^* - \frac{\epsilon}{\epsilon - 1} MC_{t+s} \right) \left( \frac{P_t^*}{P_{t+s}} \right)^{-\epsilon} C_{t+s} \right\} = 0, \tag{8}$$

where $P_t^*$ is the common optimal price chosen by all time-$t$ price-setters. The overall price level follows

$$P_t = \left[ \left(1-\theta\right) \left(P_t^*\right)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon} \right]^{1/(1-\epsilon)}. \tag{9}$$

## 2.3 Energy sectors

Each type of energy is produced by a representative producer. Both types of energy are measured in Gigatons of oil equivalents (Gtoe).[12] As in Golosov et al's (2014), production in both energy sectors is linear in labor,[13]

$$E_t^i = A_t^i N_t^i,$$

for $i = g, f$, where $A_t^i$ is sector-specific exogenous productivity. Each type of energy is sold in a perfectly competitive market at real price $p_t^i$. Fossil-fuel energy production is subject to a per-unit tax $\tau_t^f$. The representative firm in energy sector $i$ chooses $N_t^i$ to maximize real profits,

$$\frac{\Pi^i}{P_t} = \left(p_t^i - \tau_t^i\right) A_t^i N_t^i - w_t N_t^i,$$

for $i = g, f$, with $\tau_t^g = 0$. The first-order conditions are

$$p_t^g A_t^g = w_t, \tag{10}$$

$$\left(p_t^f - \tau_t^f\right) A_t^f = w_t. \tag{11}$$

## 2.4 Climate externalities

As in Golosov et al (2014), the damage function $D\left(S_t\right)$ is such that

$$1 - D\left(S_t\right) = e^{-\gamma_t\left(S_t - \bar{S}\right)},$$

[12]In measuring energy in Gtoe we depart from Golosov et al (2014), who measure it in Gigatons of carbon (GtC) emissions. One difficulty with measuring energy of both types in GtC is that one needs to assign a carbon content to green energy sources –which by definition produce no, or almost no, emissions– when calibrating the model. This difficulty is avoided by measuring energy in Gtoe.

[13]In particular, they assume linearity in labor in the complete characterization of their model. Their key theoretical result, the closed-form solution for the optimal carbon tax, is obtained under a more general specification both for energy and final goods production technologies.

where $\gamma_t$ is an exogenously time-varying elasticity and $\bar{S}$ is pre-industrial atmospheric carbon concentration. Following also Golosov et al (2014), the law of motion for atmospheric carbon concentration is linear in past carbon emissions from fossil energy consumption,

$$S_t - \bar{S} = \sum_{s=0}^{t+T} (1 - d_s)\, \xi E_{t-s}^f, \tag{12}$$

where $\xi$ is the carbon content of fossil energy, defined as tons of carbon per ton of oil equivalent (tC/toe), such that $\xi E_t^f$ measures fossil energy consumption in gigatons of carbon (GtC).

### 2.5 Government and the monetary authority

The government's nominal budget constraint is

$$\tau^y \int_0^1 P_{z,t} y_{z,t} dz + T_t + R_{t-1} B_{t-1} = P_t \tau_t^f E_t^f + B_t.$$

Without loss of generality, we assume a balanced-budget rule. Finally, the monetary authority (the "central bank") sets the nominal interest rate $R_t$. In section 3, we will analyze optimal monetary policy, such that the central bank chooses $R_t$ so as to maximize social welfare.

### 2.6 Market clearing

Final goods market clearing requires $y_{z,t} = c_{z,t}$ for each variety $z$. We define aggregate output as $Y_t = \left( \int_0^1 y_{z,t}^{\epsilon/(\epsilon-1)} dz \right)^{(\epsilon-1)/\epsilon}$. It follows that $Y_t = C_t$. Labor market clearing requires

$$N_t = \sum_{i=g,f} N_t^i + N_t^y,$$

where $N_t^y \equiv \int_0^1 N_{z,t} dz$ is labor demand by final goods producers. Equation (3) can be expressed as $y_{z,t} = [1 - D(S_t)]\, A_t F(1, E_t/N_t^y)\, N_{z,t}$, where we use the fact that energy-labor ratios are equalized across firms at the level $E_t/N_t^y$ (where $E_t$ is total energy demand). Aggregating across firms, and using equation (6), we obtain

$$[1 - D(S_t)]\, A_t F(N_t^y, E_t) = \Delta_t Y_t, \tag{13}$$

where $\Delta_t \equiv \int_0^1 (P_{z,t}/P_t)^{-\epsilon} dz$ is an index of relative price dispersion with a law of motion

$$\Delta_t = \theta \left( \frac{P_t}{P_{t-1}} \right)^{\epsilon} \Delta_{t-1} + (1-\theta) \left( \frac{P_t^*}{P_t} \right)^{-\epsilon}. \tag{14}$$

Equation (13) implies that relative price dispersion increases the amount of labor and energy inputs needed to satisfy a certain level of aggregate consumption demand. Equations (14) and

(9) imply that nonzero inflation ($P_t/P_{t-1} \neq 1$) gives rise to relative price distortions, just as in the standard New Keynesian framework.

## 3 Optimal conventional monetary policy

We start our analysis of optimal monetary policy by establishing analytically a simple benchmark result, related to the special case in which the carbon tax is set at all times at its socially optimal level. For this purpose, we first characterize the social planner equilibrium, which also allows us to derive optimal carbon taxation.

### 3.1 Social planner equilibrium

The social planner maximizes household welfare,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \frac{\chi}{1+\varphi} \left( N_t^y + \sum_{i=g,f} N_t^i \right)^{1+\varphi} \right],$$

subject to the aggregate resource constraints,

$$[1 - D(S_t)] A_t F\left(N_t^y, \mathbf{E}(E_t^g, E_t^f)\right) = C_t, \tag{15}$$

$$A_t^i N_t^i = E_t^i \quad i = g, f,$$

and the law of motion of atmospheric carbon concentration, equation (12). As shown in Appendix A, the first-order conditions of this problem can be combined into the following three conditions for social efficiency,

$$[1 - D(S_t)] A_t F_N(\cdot) = \chi N_t^{\varphi} C_t, \tag{16}$$

$$[1 - D(S_t)] A_t F_E(\cdot) \mathbf{E}_{E^g}(\cdot) = \frac{\chi N_t^{\varphi} C_t}{A_t^g}, \tag{17}$$

$$\begin{aligned} [1 - D(S_t)] A_t F_E(\cdot) \mathbf{E}_{E^f}(\cdot) &= \frac{\chi N_t^{\varphi} C_t}{A_t^f} + C_t \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \xi (1 - d_s) \gamma_{t+s} \frac{Y_{t+s}}{C_{t+s}} \right\} \\ &= \frac{\chi N_t^{\varphi} C_t}{A_t^f} + Y_t \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \xi (1 - d_s) \gamma_{t+s} \right\}, \end{aligned} \tag{18}$$

where the second equality in (18) uses the fact that $C_t = Y_t$ for all $t$.

Equation (16) is the standard efficiency condition in the basic New Keynesian model (except for the presence of the climate externality factor, $1 - D(S_t)$). It requires that the marginal product of labor equals its marginal utility cost ($\chi N_t^{\varphi}$) expressed in consumption units (i.e. rescaled by marginal consumption utility, $1/C_t$). Equation (17) is the analogous efficiency condition for green energy: it requires that its marginal contribution to the production of final goods equals the marginal utility cost of producing it (again, expressed in consumption units).

Equation (18) is the corresponding efficiency condition for fossil energy. It differs from its green energy counterpart in the presence of the second term in the right-hand side, which is Golosov et al's (2014) well-known formula for the marginal externality damage of carbon emissions.[14] The latter term captures the expected present-discounted value of the future economic damages produced by an additional unit of fossil energy consumption. Fossil energy consumption produces carbon emissions today in the amount $\xi$. These emissions add to atmospheric carbon concentration at each future date $t+s$, $s \geq 0$, in an amount determined by the carbon depreciation structure, $1-d_s$. This in turn reduces future output in the amount $\gamma_{t+s}Y_{t+s}$, or $\gamma_{t+s}Y_{t+s}C_{t+s}^{-1}$ once expressed in utils, which simplifies to $\gamma_{t+s}$ given that in this model all output is consumed. Finally, these externality damages are expressed in units of today's consumption by dividing them by the time-$t$ marginal utility of consumption, $C_t^{-1} = Y_t^{-1}$, such that they are proportional to current output.

### 3.2 Flexible-price equilibrium

We now return to the decentralized economy. As customary in analyses of optimal monetary policy in New Keynesian models, it is useful to investigate the properties of equilibrium under flexible prices, because it represents the equilibrium achieved under sticky prices when the monetary authority follows a policy of strict inflation targeting. Under flexible prices ($\theta = 0$), equation (8) becomes

$$P_{z,t} = (1+\tau^y)^{-1} \frac{\epsilon}{\epsilon - 1} MC_t.$$

Therefore, all firms choose the same price, $P_{z,t}/P_t = 1$ for all $z$, and relative price distortions are eliminated, $\Delta_t = 1$. The real marginal cost is then given by $MC_t/P_t = (1+\tau^y)\frac{\epsilon-1}{\epsilon}$. Combining this with (2), (4), (5), (10) and (11), we obtain

$$(1+\tau^y)\frac{\epsilon-1}{\epsilon}\left[1-D\left(S_t\right)\right]A_t F_N\left(\cdot\right) = \chi N_t^{\varphi} C_t, \tag{19}$$

$$(1+\tau^y)\frac{\epsilon-1}{\epsilon}\left[1-D\left(S_t\right)\right]A_t F_E\left(\cdot\right)\mathbf{E}_{E^g}\left(\cdot\right) = \frac{\chi N_t^{\varphi} C_t}{A_t^g}, \tag{20}$$

$$(1+\tau^y)\frac{\epsilon-1}{\epsilon}\left[1-D\left(S_t\right)\right]A_t F_E\left(\cdot\right)\mathbf{E}_{E^f}\left(\cdot\right) = \frac{\chi N_t^{\varphi} C_t}{A_t^f} + \tau_t^f. \tag{21}$$

[14] The only difference between our formula and theirs is the presence of the parameter $\xi$, i.e. the carbon content of fossil energy. This stems from the fact that – as explained before – we measure energy in Gtoe instead of in GtC (the measure used in Golosov et al 2014). Thus, transforming fossil energy consumption into carbon emissions requires multiplying the former by its carbon content.

Comparing the above three equations with the social efficiency conditions (16) to (18), it follows that setting

$$\tau^y = \frac{\epsilon}{\epsilon - 1} - 1, \tag{22}$$

$$\tau_t^f = Y_t \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s (1 - d_s) \xi \gamma_{t+s} \right\} \equiv \tau_t^{f*}, \tag{23}$$

allows the flexible-price equilibrium to replicate the social planner equilibrium.[15] Equation (22) shows a well-known result in normative New Keynesian theory: setting the revenue subsidy rate $\tau^y$ equal to the net price markup under flexible prices, $\frac{\epsilon}{\epsilon-1} - 1$, allows policy-makers to offset the monopolistic distortion. Equation (23) reflects Golosov et al.'s (2014) key theoretical result: if the carbon tax is set equal to the marginal externality damage of carbon emissions, then the decentralized economy replicates the social planner equilibrium. Intuitively, if the carbon tax equals $\tau_t^{f*}$ (also known as the optimal Pigouvian tax), then all agents perfectly internalize the climate externality from fossil energy use. In our model, the same result is true for the flexible-price equilibrium, provided equation (22) is also satisfied.

### 3.3 Optimal monetary policy: the case of optimal carbon taxation

We are now ready to obtain our main analytical result. As in the standard New Keynesian framework, in our model a policy of strict inflation targeting, $\pi_t \equiv P_t/P_{t-1} = 1$ for all $t \geq 0$, allows the decentralized economy to replicate the flexible-price equilibrium. To see this, notice from equation (9) that such a policy ensures that newly-set optimal prices always equal the overall price level, $P_t^* = P_t$, as in the flexible-price equilibrium. Intuitively, if the price level is held constant by monetary policy, those firms that have the chance of resetting their price in a given period have no reason for actually changing it. Equation (14) then implies $\Delta_t = \theta \Delta_{t-1} + 1 - \theta$, such that relative price dispersion at any given time depends only on past dispersion.

In what follows, we make two maintained assumptions. First, we assume that there is no initial price dispersion ($\Delta_{-1} = 1$), such that from $t = 0$ onwards price dispersion arises only to the extent that there is nonzero inflation. Under a zero (net) inflation policy, we thus have $\Delta_t = 1$ for all $t \geq 0$, as in the flexible price equilibrium. Second, we assume that the revenue subsidy satisfies condition (22) in all scenarios. These assumptions allow us to isolate monetary policy trade-offs from the influence of initial price dispersion and monopolistic distortions, and thus focus our analysis on how climate externalities and carbon taxation affect those trade-offs, which is the key objective of our paper.[16]

From the above and the discussion in section 3.2, it follows that, provided the carbon tax

[15] Notice that, given $\Delta_t = 1$, equation (13) also replicates its social planner counterpart, equation (15).

[16] As shown by Benigno and Woodford (2005) and Woodford (2003), monopolistic distortions give the central bank a reason for transitorily deviating from a zero inflation policy under the time-0 optimal monetary policy commitment, with inflation converging asymptotically to its optimal long-run value of zero. For an analysis of optimal monetary policy in the presence of initial price dispersion, see Yun (2005).

equals the optimal Pigouvian tax in equation (23), a policy of strict inflation targeting allows the central bank to also replicate the social planner equilibrium. In particular, output replicates its socially efficient level, which we may denote by $Y_t^*$, such that the *welfare-relevant output gap* is fully stabilized: $Y_t/Y_t^* = 1$ for all $t \geq 0$. Thus, the optimal monetary policy is strict inflation targeting.

We summarize the above discussion in the following proposition. A formal proof can be found in Appendix B, which also lays out the general optimal monetary policy commitment problem.

**Proposition 1** *Provided the carbon tax is set at its socially optimal level (equation 23) at all times, the optimal monetary policy is strict inflation targeting: $\pi_t = 1$. This policy allows the decentralized economy to replicate the social planner equilibrium.*

The intuition for this key result is simple: if the carbon tax is set at its socially optimal level, then all agents internalize perfectly the negative externality from carbon emissions. This leaves nominal price rigidity as the only distortions left, which can be offset by the central bank through a strict inflation targeting policy. In sum, as long as climate authorities set them at the optimal level, *carbon taxes pose no trade-off for monetary policy*: it is both feasible and optimal to fully stabilize inflation and the welfare-relevant output gap.

# 4 Quantitative analysis

While Proposition 1 provides a useful normative benchmark, the assumption of optimal carbon taxation from the very first period may be seen as unrealistic, given the sluggish pace of progress in carbon taxation and other mitigation policies worldwide. Therefore, we next turn our attention to the more realistic case in which carbon taxation is suboptimal, which creates trade-offs between core and climate goals. We start by calibrating the model.

## 4.1 Calibration

**Preferences.** As is standard in the New Keynesian literature, we set the (inverse) labor supply elasticity $\varphi$ to 1. We normalize initial labor supply to 1, which requires setting the scale parameter of labor disutility $\chi$ to 1.[17] The elasticity of substitution across final goods varieties, $\epsilon$, is also set to a standard value of 7. As in Nordhaus (2008), we assume a household net discount rate of 1.5% a year, which in our quarterly model implies $\beta = 0.985^{1/4}$.

**Technology.** We assume a CES functional form for the final-goods production technology,

$$F(N_t, E_t) = [\alpha (E_t)^{\delta} + (1-\alpha)(N_t)^{\delta}]^{1/\delta},$$

[17] See Appendix C for further details.

and a CES function for the energy basket,

$$E = \mathbf{E}(E^g, E^f) = [\omega\,(E^g)^\rho + (1-\omega)\,(E^f)^\rho]^{1/\rho}. \tag{24}$$

As in Golosov et al (2014), we set $\alpha = 0.04$, which corresponds approximately with the energy share of World GDP. Based on the empirical evidence in Papageorgiou et al (2017), we set $\rho$ to 0.65, which implies an elasticity of substitution between green and fossil energy, $1/(1-\rho)$, close to 3. The remaining parameters are calibrated as follows. Under the above functional forms, the relative price of green energy is given by [18]

$$\frac{p_0^g}{p_0^f} = \frac{\partial F/\partial E^g}{\partial F/\partial E^f} = \frac{\omega}{1-\omega}\left(\frac{E_0^f}{E_0^g}\right)^{(1-\rho)}.$$

Using data from BP, we obtain levels of World annual consumption of fossil and green energy in 2019 of 11.70 and 3.28 Gtoe, respectively,[19] such that $E_0^f = 11.70/4$ and $E_0^g = 3.28/4$. Using data from the International Renewable Energy Agency, we estimate a relative price of green over fossil energy of $p_0^g/p_0^f = 0.54$.[20] Given these targets, we solve for $\omega$ as

$$\omega = \frac{1}{1 + (p_0^f/p_0^g)(E_0^f/E_0^g)^{1-\rho}}.$$

The productivity factors of both energy sectors, $A_0^g$ and $A_0^f$, are then calibrated to match $E_0^g$ and $E_0^f$ (see Appendix C for further details).[21]

**The carbon process and the damage function.** In order to calibrate the carbon content of fossil energy ($\xi$), we take the carbon content of each of the three sources of fossil energy (coal, gas, and oil) and average them using as weights the share of each source in fossil energy consumption.[22] This yields $\xi = 0.879$ tC/toe.

[18]Notice that $\frac{\partial F}{\partial E^i} = \frac{\partial F}{\partial E}\frac{\partial \mathbf{E}}{\partial E^i} = \alpha(\frac{F}{E})^{(1-\delta)}\omega_i\left(\frac{E}{E^i}\right)^{(1-\rho)}, i = f, g$, with $\omega_g = \omega = 1-\omega_f$.

[19]We take data on World consumption of different energy sources in 2019 in Terawatt-hour (TWh) from BP's Statistical Review of World Energy, and transform them into Gtoe using conversion factors from the International Energy Agency's (IEA) unit converter. We then calculate green energy as the sum of energy from wind, nuclear, hydropower, traditional biomass, biofuels and other renewable sources; and fossil energy as the sum of energy from coal, gas and oil.

[20]We use the levelized cost of energy (LCOE) as a proxy for the price of energy of different sources. Using global LCOE estimates for 2019 from IRENA (2020), we calculate the price for green energy as the weighted average of LCOEs for biomass, hydroelectric, solar and wind energy, using as weights their share in World consumption of all these energy sources in the BP data. This gives a price of 0.061 USD per kWh. According to the same source, LCOE estimates for fossil fuel-generated power range from 0.05 to 0.177 USD/kWh; we take the simple average of both numbers, yielding 0.113 USD/kWh. Taking the ratio of green and fossil LCOEs gives a relative price of 0.537.

[21]For simplicity, we assume that productivity is constant in both energy sectors: $A_t^i = A_0^i$ for all $t \geq 0$, $i = f, g$. We make the same assumption for TFP in final goods production, $A_t$.

[22]We take the carbon content of oil and coal from Golosov et al (2014): 0.846 tC/ton for oil and 0.716 tC/ton for coal (anthracite). As noted by these authors, one ton of oil equivalents (toe) is 1.58 tons of coal, so the carbon content of coal expressed in tC/toe is $0.716 \times 1.58 = 1.131$. The carbon content of natural gas is 0.0153 tC/GJ (IPCC 2006, table 1.3), or equivalently 0.641 tC/toe (after using the conversion factor GJ/toe, equal to 41.87 according to the IEA Unit Converter). We then weight these contents in tC/toe by the share of each source in

We assume the following depreciation structure for atmospheric carbon concentration,

$$1 - d_s = \phi_0 \left(1 - \phi\right)^s ,$$

for $t \geq 0$, where $1 - \phi_0$ is the share of carbon emissions into the atmosphere that exits it within the same quarter (into the biosphere and the surface oceans), and $\phi$ is the rate at which the remaining share disappears from the atmosphere. Our two-parameter specification is a special case of Golosov et al.'s (2014), and is motivated by our need to have a well-defined terminal steady state in the model for computational purposes. [23] We calibrate both parameters such that our carbon depreciation structure mimics Golosov et al's as closely as possible over a 300-year horizon. Figure 2 shows how our $1 - d_s$ function compares with theirs.

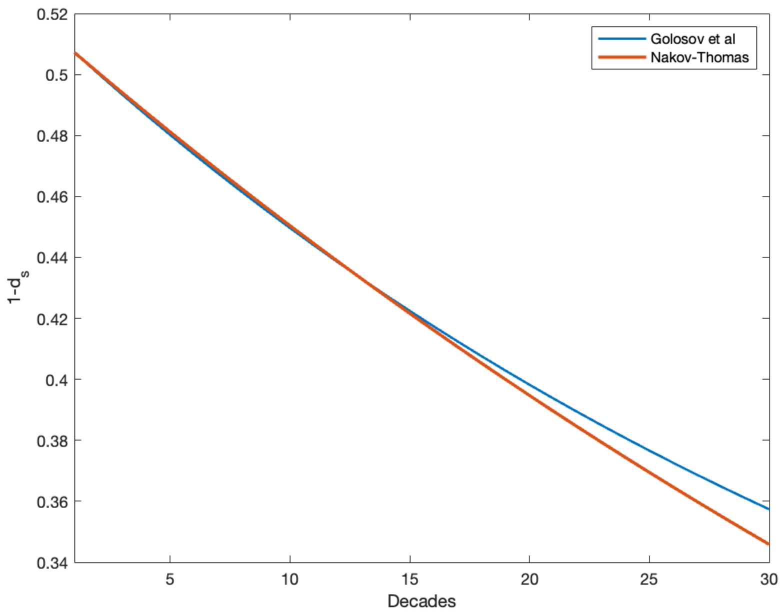


Figure 2: Carbon decay structure

As in Golosov et al (2014), we set the pre-industrial stock of atmospheric carbon concentration $\bar{S}$ to 581 GtC and the stock in the present $S_0$ to 802 GtC.

As regards the damage function, we follow Golosov et al (2014) in setting the elasticity $\gamma$ to $2.4 \times 10^{-5}$. Given the values of $\bar{S}$ and $S_0$, the economic damage from global warming in the

total fossil energy consumption in 2019 in the BP data (32% coal, 29% gas, 39% oil).

[23] Golosov et al (2014) use the three-parameter specification $1 - d_s = \phi_L + (1 - \phi_L)\phi_0 \left(1 - \phi\right)^s$, where $\phi_L$ is the share of carbon emitted into the atmosphere that stays in it forever. Under this specification, the model does not have a well-defined terminal steady state, at least as long as carbon emissions do not converge to zero, because atmospheric carbon concentration $S_t$ grows unboundedly with new emissions as time goes by. Because our model is forward-looking, we need to have a terminal steady state in order to be able to solve it. This is achieved by setting $\phi_L$ to zero. Figure 11 in Appendix F shows the convergence of the carbon stock to its "net zero" steady state both under our baseline calibration and under alternative calibrations.

| | Description | Value | Target/Source |
|---|---|---|---|
| New Keynesian block | | | |
| $\beta$ | Household discount factor | $0.985^{1/4}$ | Nordhaus (2008) |
| $\theta$ | Calvo parameter | 0.75 | Price adj. freq. 1 yr |
| $\epsilon$ | Elasticity of substitution | 7 | Standard |
| $\varphi$ | (inv) elasticity labor supply | 1 | Standard |
| Energy & climate block | | | |
| $\alpha$ | Energy share of output | 0.04 | Golosov et al (2014) |
| $\rho$ | (1-inv) elast subst $g$ vs $f$ | $1-1/2.86$ | Papageorgiou et al (2017) |
| $\delta$ | (1-inv) elasticity subst L vs E | $1-1/0.4$ | Boringer and Rivers (2021) |
| $\gamma$ | Elasticity damage function | 0.000024 | Golosov et al (2014) |
| $\phi_0, \phi$ | carbon depreciation structure | 0.51, 0.00033 | Golosov et al deprec. |
| $\omega$ | weight of green energy | 0.2571 | $p^g/p^f = 0.54$ |
| $A^f$ | productivity fossil sector | 290.33 | $E^f = 11.7$ Gtoe |
| $A^g$ | productivity green sector | 537.65 | $E^g = 3.28$ Gtoe |
| $\xi$ | carbon content fossil energy (tC/toe) | 0.879 | IPCC (2006) tables |
| $\bar{S}, S_0$ | Atmosph. carbon concentr. (GtC) | 581, 802 | Golosov et al (2014) |
| QE extension | | | |
| $\kappa_f$ | Spread sensitivity, brown bonds | 0.0813 | Impact CSPP anncmnt |
| $\kappa_g$ | Spread sensitivity, green bonds | 0.5373 | Impact CSPP anncmnt |
| $\bar{B}^f$ | Min. private absorption brown bonds | 0.00512 | Brown bond spreads |
| $\bar{B}^g$ | Min. private absorption green bonds | 0.00076 | Green bond spreads |
| $\psi_f = \psi_g = \psi$ | Bond leverage of energy firms | 5 | Lever. CSPP-elig. issuers |

Table 1: Calibration

present amounts to $\gamma(S_0 - \bar{S}) = 0.53\%$.

**Price stickiness.** Finally, we set the Calvo parameter $\theta$ to 0.75, such that prices change on average once a year, which roughly corresponds with empirical evidence for the euro area.

The first two panels of Table 1 summarize the calibration of the baseline model.

## 4.2 Results

We turn to the quantitative results for the case of a "slow" green transition. "Slow" means that the carbon tax is assumed to be phased in gradually, starting from zero and reaching its optimal level after 30 years. This gradualism in climate policy in principle provides ample scope for activist monetary policy. We will measure the extent of monetary activism by the deviations from strict inflation targeting (zero inflation), which as proved in Section 3 is the optimal monetary policy when carbon taxes follow their optimal path from time zero.

Figure 3 shows the evolution of the variables of interest. The red solid lines display transition dynamics under the optimal carbon tax; as explained before, in this scenario all real variables replicate their paths in the social planner equilibrium. The latter is characterized by constant levels for all variables,[24] with the exceptions of atmospheric carbon concentration, which in-

[24]To see this, rescale by $1 - D(S_t)$ all social-planner equilibrium conditions featuring that term and use $C_t = Y_t$

creases as a result of (socially optimal) positive net carbon emissions, and of output, which declines as a result of rising damages from global warming.

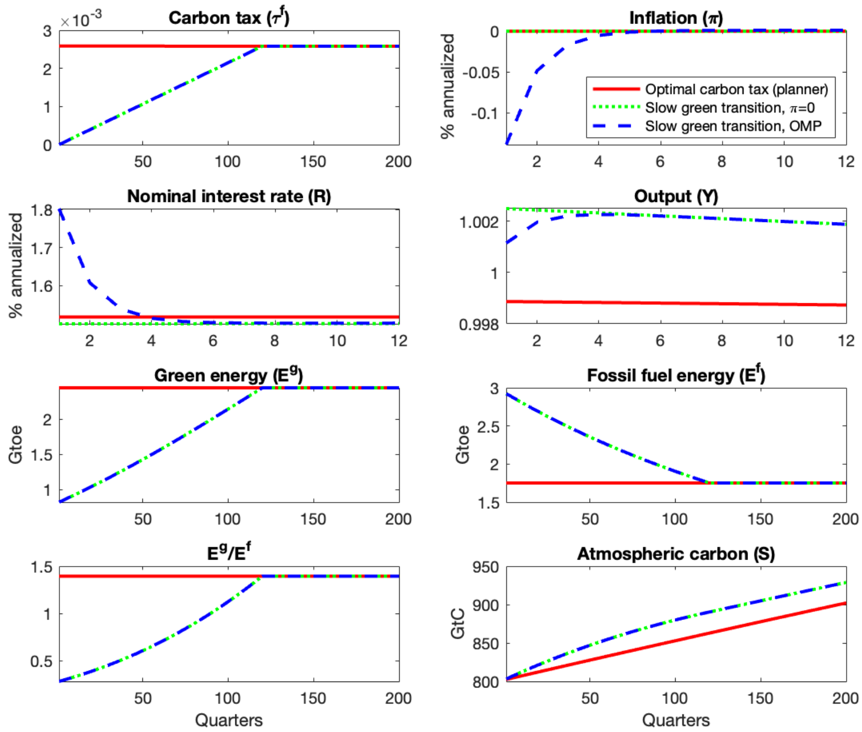


Figure 3: Transitions in baseline model

The green dotted lines show the dynamics under the slow green transition, assuming also that monetary policy ignores climate externalities and sticks to strict inflation targeting (zero inflation), such that output replicates its natural (flexible-price) level. In this case, output starts out above its socially efficient level (i.e. the welfare-relevant output gap is positive), reflecting the lower levels of carbon taxation. Low carbon taxation implies that fossil energy is too cheap, implying inefficiently high (low) levels of fossil (green) energy use. This in turn produces a faster accumulation of carbon in the atmosphere, and hence a faster increase in economic damages from global warming. Along the transition, output falls more quickly than (and hence converges towards) its efficient counterpart, reflecting both rising carbon taxation

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to obtain

$$A_t F(N_t^y, E(E_t^g, E_t^f)) = Y_t/[1 - D(S_t)] \equiv Y_t^D,$$

$$A_t \partial F(\cdot)/\partial N_t^y = \chi N_t^\phi Y_t^D,$$

$$A_t \partial F(\cdot)/\partial E_t^i = \chi N_t^\phi Y_t^D / A_t^i + 1_{i=f} \mathbb{E}_t \sum\nolimits_{s=0}^{\infty} \beta^s (1 - d_s) \xi \gamma_{t+s},$$

$i = f, g$, where $Y_t^D$ is gross-of-damages output. These four equations, together with $A_t^i N_t^i = E_t^i, i = f, g$, jointly determine the path of $N_{t\,i=y,f,g}^i, E_{t\,i=f,g}^g, Y_t^D$. Absent any change in TFP factors $(A_t, A_t^i, i = f, g)$, the former six variables are constant. Since $Y_t = [1 - D(S_t)]Y_t^D$, it follows that $Y_t$ evolves in proportion to $1 - D(S_t)$.

and a faster increase in climate-related damages.

The blue dashed lines show the dynamics under the optimal monetary policy. The benevolent, climate-conscious central bank understands that, during the green transition, carbon taxes are suboptimally low and CO2 emissions excessively high. Aware of this, and compared to the zero inflation scenario, it implements a tighter interest rate path in order to reduce aggregate demand and thus overall energy consumption, including fossil energy consumption. This way, output comes closer to its socially efficient path, i.e. the welfare-relevant output gap becomes narrower. In return, the central bank accepts a fall in output below its *natural* level and hence a fall in inflation below its long-run target (zero). However, the deviation from strict inflation targeting is very small (around 10 basis points in the first period) and short-lived (barely a year). Therefore, the optimal policy can be characterized essentially as price stability. Fossil energy use does fall, although compared to the scale of its reduction along the transition, the effect is indistinguishable from zero. Correspondingly, the path of atmospheric carbon concentration is essentially unaffected.

Why is optimal monetary policy not more climate-activist? The reason is that interest rate policy affects green and fossil energy use in basically the same way. Therefore, it is a rather blunt, untargeted, inefficient instrument for reducing carbon emissions. The central bank could always achieve a larger reduction in carbon emissions, but this would come at the expense of a deep recession – with the resulting fall in consumption – and a severe fall in inflation below target, all of which would be excessively costly in social welfare terms.

### 4.3 Optimal responses to shocks

Our analysis so far has focused on how optimal monetary policy trades off core vs. climate goals along a slow green transition. However, it is also interesting to analyze to what extent climate externalities affect the optimal monetary policy response to shocks. To address this question, we add standard supply and demand shocks and compare the economy's response to them under optimal monetary policy in our model vs. in the standard New Keynesian model, in which there are no climate change considerations.

In particular, we consider a cost-push shock in the form of a transitory reduction in the sales subsidy $\tau^y$. The shock is such that the sales subsidy drops down to half its steady-state value $(1/(\epsilon-1))$, then reverts back to steady-state with quarterly persistence 0.65. The dashed red lines in Figure 4 show the response of key variables in the standard New Keynesian model and confirm a well-known result: following a cost-push shock, Ramsey optimal monetary policy raises the policy rate in order to drive output below its natural level (i.e. the inflation-relevant output gap falls) and this way mitigate the initial increase in inflation. The solid green lines show the responses to the same cost-push shock in our model with climate externalities.[25] As

[25]In particular, since in our model the economy experiences transition dynamics even in the absence of shocks, we display the *differences* between the paths of the variables with the cost-push shock and their paths without the shock, in order to isolate its effects.

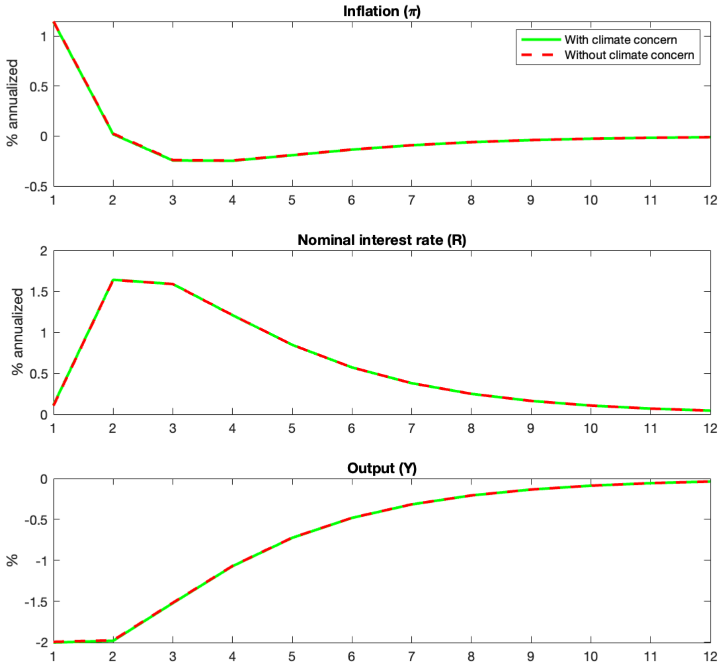


Figure 4: Optimal responses to a cost-push shock

can be seen, the responses are nearly indistinguishable from those in the standard model.

We now consider an expansionary demand shock in the form of a transitory increase in government consumption.[26] The shock is such that government spending initially rises by 1% of GDP and then reverts to its (zero) steady-state level with persistence 0.65. The dashed red lines of Figure 5 confirm a well-known prescription from the standard New Keynesian model: in the face of an increase in government spending, optimal monetary policy follows strict inflation targeting. To this end, it raises the policy rate so as to offset any increase in the inflation-relevant output gap, i.e. output increases by exactly the same amount as in the flexible-price equilibrium. As in the case of the cost-push shock, the responses in our model with climate externalities (solid green lines) are virtually the same as in the standard model.

In Appendix G, we show that increasing the magnitudes of the shocks to subsidies (x2) and government spending (x5) simply scales up the responses of output, inflation, and the interest rate proportionally. More importantly, such responses continue to be identical with and without climate externalities.

In sum, according to our simple model, climate change considerations do not seem to affect how optimal monetary policy reacts to supply (cost-push) or demand (government spending) shocks. This result reflects once again the fact that, in our model, interest-rate policy is not well suited for tackling climate externalities, such that how the central bank uses this tool to

[26] Government spending ($G_t$) enters the model via the resource constraint $Y_t = C_t + G_t$.

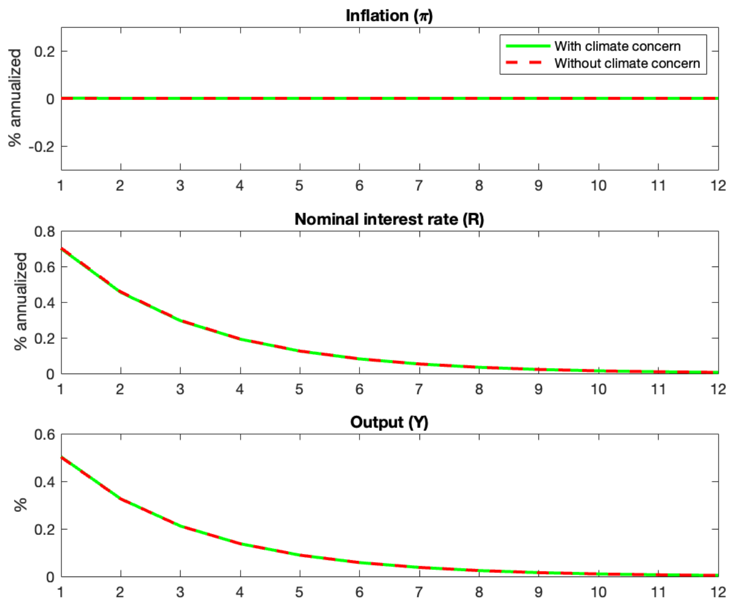


Figure 5: Optimal responses to a government spending shock

address standard supply and demand shocks is barely affected by climate considerations.

Notice finally that, for computational reasons, we have restricted our analysis to one-time shocks. However, it is plausible to consider that, in reality, climate change may endogenously increase the frequency and/or severity of weather events and the associated supply shocks. Understanding how monetary policy would optimally address the trade-offs between climate and price stability goals in such a context is an important question that we leave for future research.

## 4.4 Capital accumulation

Our baseline analysis assumes that labor is the only input in energy production. However, policy discussions of the energy transition note that large amounts of capital need to be deployed to replace fossil fuel energy production with clean energy production. Moreover, as noted in Bistline *et al.* (2023), interest rates have a disproportionate effect on the cost of electric power generation from clean energy relative to fossil fuel energy. This implies that interest-rate policy need not be neutral in terms of how it affects green vs fossil energy prices. In order to incorporate these considerations, in this section we extend our model to include capital in energy production. Here we briefly describe the main changes relative to the baseline model, while Appendix D provides a full derivation of the extended model.

We assume that, in addition to labor, energy producers in each sector $i = g, f$ use capital specific to that sector, according to a Cobb-Douglas technology,

$$E_t^i = A_t^i \left(K_t^i\right)^{\alpha_i} \left(N_t^i\right)^{1-\alpha_i}, \quad i = g, f,$$

where $K_t^i$ is sector-$i$-specific capital. Energy firms rent capital from households at a sector-specific real rate $r_t^i$. Households purchase capital from competitive capital goods producers at sector-specific real price $p_{K,t}^i$. The law of motion of sector-specific capital is

$$K_{t+1}^i = I_t^i + (1-\delta_i) K_t^i, \quad i = g, f,$$

where $I_t^i$ is sector-specific investment and $\delta_i$ is the sector-specific depreciation rate. Capital is produced by sector-specific capital producers that linearly transform final goods into capital at an exogenous rate $A_{K,t}^i$; as shown in the appendix, this implies that the equilibrium price of capital is simply $p_{K,t}^i = 1/A_{K,t}^i$. Aggregate real output equals

$$Y_t = C_t + \sum_{i=f,g} p_{K,t}^i I_t^i.$$

**Optimal carbon taxation.** The appendix shows that our normative result in Proposition 1 carries over to the model extension with capital, provided the carbon tax is set to

$$\tau_t^f = C_t \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s (1-d_s) \xi \frac{Y_{t+s}}{C_{t+s}} \gamma_{t+s} \right\}. \tag{25}$$

The formula above generalizes equation (23) to the model with capital, the only difference being that the optimal carbon tax depends on the entire future path of the consumption rate, $C_t/Y_t$, which no longer equals one.

**Calibration.** Following Mehrotra (2025), we set the depreciation rate of capital in both energy sectors ($\delta_g$, $\delta_f$) to 0.025. In order to calibrate the elasticity of energy production with respect to capital ($\alpha_g$, $\alpha_f$), we proceed as follows. We assume for simplicity that real capital prices are constant over time.[27] As shown in Appendix D, energy prices must then satisfy

$$p_{t+1}^i = \frac{p_{K,t}^i K_{t+1}^i \left[(R_t/\pi_{t+1} - 1) + \delta_i\right] + w_{t+1} N_{t+1}^i}{E_{t+1}^i} + \tau_{t+1}^i, \tag{26}$$

$i = f, g$, with $\tau_{t+1}^g = 0$. Equation (26) implies that, *ceteris paribus*, a higher level of capital costs per unit of energy produced ($p_{K,t}^i K_{t+1}^i / E_{t+1}^i$) implies a higher sensitivity of next period's energy prices to changes in interest rates $R_t$. The appendix shows that the *ceteris paribus*

[27] In particular, we assume that capital producers' productivity is constant, $A_{K,t}^i = \bar{A}_K^i$ for all $t \geq 0$, which implies $p_{K,t}^i = 1/A_{K,t}^i = 1/\bar{A}_K^i$ for all $t \geq 0$. We further normalize capital prices to 1.

semi-elasticity of energy prices with respect to $R_t$ can be approximated by

$$\frac{\partial p_{t+1}^i / p_{t+1}^i}{\partial R_t} \approx \frac{\alpha_i}{(1/\beta - 1) + \delta_i}, \tag{27}$$

$i = f, g$. We then use estimates in IAE (2020) of LCOEs of different power generation technologies for different levels of discount rates in order to construct empirical proxies for the interest rate semi-elasticities of green and fossil energy prices.[28] The resulting interest rate semi-elasticity for green energy prices, $\frac{\partial p_{t+1}^g / p_{t+1}^g}{\partial R_t} = 11.1$, is much higher than for fossil energy prices, $\frac{\partial p_{t+1}^g / p_{t+1}^g}{\partial R_t} = 2.0$, consistently with the fact that upfront capital costs have a much higher weight in the overall cost of green energy.[29] Given these semi-elasticities and our calibrated values for $\{\delta_i\}_{i=g,f}$ and $\beta$, we use equation (27) to obtain $\alpha_g = 0.32$ and $\alpha_g = 0.06$.

**Results.** We simulate the same three scenarios as in the baseline model: optimal monetary policy under optimal carbon taxation (which now requires $\tau_t^f$ to satisfy equation 25 at all times); and both strict inflation targeting and optimal monetary policy under slow green transition (with $\tau_t^f$ converging linearly from zero to optimal over a 30-year period). As shown in Figure 10 in Appendix D, our main result from the baseline model carries over to the case with capital accumulation: even in a scenario of decades-long transition to optimal carbon taxation, optimal monetary policy continues to be very close to strict inflation targeting, such that the path of carbon emissions remains essentially unaffected.

In fact, the optimal deviation of inflation from its long-run target is even smaller than in the baseline model, reaching less than 10 bp. To understand why, in Figure 6 we show the *differences* in the paths of key variables under optimal monetary policy relative to strict inflation targeting. In contrast to the baseline model, the nominal interest rate is initially *lower* under optimal policy than under strict inflation targeting, and so is the real *ex ante* interest rate, $R_t/\pi_{t+1} - 1$ (panel a). From equation (26), a lower interest rate in the first period allows the central bank to lower the price of both green and fossil fuel energy in the following period, but it lowers green energy prices by *more*, given their higher interest rate sensitivity (panels f and i). This allows the central bank to reduce the relative price of green vs fossil energy, and thus favor green over fossil energy consumption through substitution effects (panels d and g). After only a few periods, however, the central bank abandons its efforts to alter relative energy prices and instead returns to the optimal prescription from the baseline model: reducing fossil energy consumption by *raising* the nominal interest rate relative to its zero-inflation path. This carries

[28] IAE (2020) provides technology-specific LCOEs for three different discount rates (3%, 7%, and 10%); see https://www.iea.org/reports/projected-costs-of-generating-electricity-2020. For each technology, we compute the semi-elasticity of the corresponding LCOE as the discount rate increases from 3% to 7%. We then construct green energy and fossil energy semi-elasticities as weighted averages of the semi-elasticities of green sources (wind, solar, hydro, biomass and nuclear) and fossil fuel sources (coal and gas/CCGT), using the same weights as those used to calculate the empirical targets for green and fossil energy prices in the baseline model.

[29] As mentioned above, equation (27) approximates the *ceteris paribus* semi-elasticities of energy prices. However, we have verified that the exact, general equilibrium semi-elasticities following a transitory shock to the policy rate (equal to 11.1 and 2.6, respectively) are actually very close to their empirical targets.

over to the real interest rate (the relevant one for aggregate demand), which rises too above its zero-inflation path after a few periods. Thus, the inflation-relevant output gap (i.e. output relative to its zero-inflation level), which depends on the entire future path of real interest rates, is initially positive under optimal policy but quickly turns negative. Since inflation depends on the discounted future path of the output gap, optimal inflation falls by less than in the baseline model in the first period but follows from then on a very similar path.

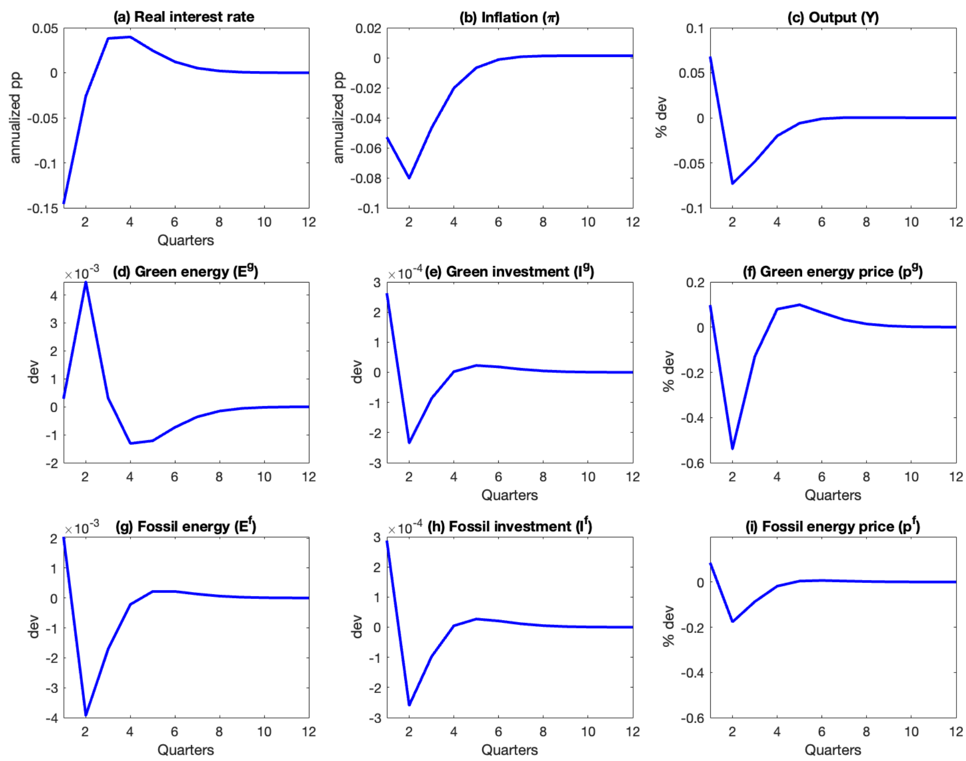


Figure 6: Transition paths in deviations from zero-inflation equilibrium, model with capital

# 5 Green QE

So far we have restricted our analysis to conventional (interest-rate) monetary policy, with the aim of showing in the simplest and most transparent way to what extent the existence of climate externalities creates trade-offs for monetary policy. In practice, some central banks use other instruments in order to pursue climate-related goals. Among the latter, a prominent role is played by "green QE", whereby central banks tilt their bond portfolios in favor of "green bonds" that satisfy certain climate-related eligibility criteria, and against "brown bonds". In this section, we extend our baseline model by introducing a simple financial friction that allows central bank purchases of corporate bonds to play a role in affecting equilibrium allocations.

### 5.1 A simple model of green (and brown) QE

Assume that a fraction $\psi_i$ of energy producers' operating costs must be pre-financed with working capital loans. In particular, these firms are assumed to issue short-term bonds at the start of the period. Bonds of sector-$i$ energy firms are issued at a price $1/R_t^i$ –which is taken as given by firms– and have unit face value. Therefore, the number of bonds issued by each sector-$i$ firm in period $t$ equals $\frac{\psi_i w_t N_t^i}{1/R_t^i} = \psi_i R_t^i w_t N_t^i$, which is also the face value to be repaid.[30] We adopt the timing convention that bond repayments are due at the end of the period – i.e. bonds are intra-period. This assumption allows us to preserve the static nature of energy prices in our baseline model, which simplifies the algebra, but is otherwise essentially innocuous.[31] Thus, the net bond return $R_t^i - 1$ can also be interpreted as the bond spread relative to the frictionless intra-period net return (zero).

Under these assumptions, the maximization problem of firms in energy sector $i = f, g$ becomes

$$\max_{N_t^i} \left(p_t^i - \tau_t^i\right) A_t^i N_t^i - \left[1 + \psi_i \left(R_t^i - 1\right)\right] w_t N_t^i,$$

again with $\tau_t^g = 0$. The first-order conditions are now given by

$$p_t^g A_t^g = \left[1 + \psi_g \left(R_t^g - 1\right)\right] w_t, \tag{28}$$

$$(p_t^f - \tau_t^f) A_t^f = \left[1 + \psi_f \left(R_t^f - 1\right)\right] w_t. \tag{29}$$

The above expressions differ from their baseline model counterparts (equations 10 and 11) in the presence of the terms $\psi_i \left(R_t^i - 1\right)$, i.e. the product of the leverage factor $\psi_i$ and the bond spread, $R_t^i - 1$. These terms create a wedge between the marginal revenue product of labor (net of carbon taxes, in the case of sector $f$) and its marginal (non-financial) cost, $w_t$. *Ceteris paribus*, these financial wedges raise the real price of both types of energy.

Corporate bonds are purchased by households and the central bank. Let $B_t^i$ denote the market value of sector-$i$ bond purchases by the household. Following Andrés, López-Salido and Nelson (2004), Chen, Curdia and Ferrero (2012) and Gertler and Karadi (2013), among others, we assume that households incur transaction costs when adjusting their corporate bond portfolio. In particular, the household budget constraint is now

$$P_t C_t + B_t + \sum_{i=g,f} B_t^i \left(1 + \zeta_t^i\right) = R_{t-1} B_{t-1} + \sum_{i=g,f} R_t^i B_t^i + W_t N_t + \Pi_t + T_t,$$

where $\zeta_t^i$ is a transaction cost per sector-$i$ bond. Notice that, since corporate bonds are intra-period, their repayments $\sum_{i=g,f} R_t^i B_t^i$ accrue within the period. As in Gertler and Karadi (2013),

[30] Our baseline (no QE) model can be nested by setting $\psi_i = 0$.

[31] Alternatively, we could assume that bonds repayments are due at the beginning of the following period. This would make the firm's problem dynamic and, in particular, it would introduce time $(t+1)$-dated terms in equations (28) and (29) below. However, such modification would leave our numerical results essentially unchanged.

we assume quadratic transaction costs of the form

$$\zeta_t^i = \frac{\kappa_i}{2} \frac{\left(B_t^i - \bar{B}^i\right)^2}{B_t^i},$$

$i = g, f$, with $\kappa_i > 0$. The first-order conditions for $\{B_t^i\}_{i=g,f}$ are given by

$$R_t^i = 1 + \kappa_i \left(B_t^i - \bar{B}^i\right), \tag{30}$$

$i = g, f$. All other household first-order conditions are as in the baseline model. Market clearing for sector-$i$ bonds requires that household demand equals firms' supply minus bonds absorbed by the central bank,

$$B_t^i = \psi_i w_t N_t^i - B_t^{i,cb}, \tag{31}$$

for $i = g, f$, where $B_t^{i,cb} \in [0, \psi_i w_t N_t^i]$ is the market value of the central bank's purchases of sector-$i$ bonds. Therefore, central bank purchases of bonds of a given energy sector reduce the amount of such bonds to be absorbed by the private sector (households). From equation (30), this allows the central bank to reduce bond spreads for that sector and thus (from equations 28 and 29) lower the real price of that type of energy, $p_t^i$.

## 5.2 Optimal monetary policy with QE: analytical results

We now show that our key normative result from Section 3 carries over to an environment in which the central bank toolkit includes bond purchases.

**Optimal monetary policy under optimal carbon taxation.** Provided the carbon tax is set at its socially optimal level ($\tau_t^f = \tau_t^{f*}$, where the optimal Pigouvian carbon tax continues to be given by equation 23), it is trivial to show that optimal monetary policy combines strict inflation targeting ($\pi_t = 1$, as in our baseline model) with central bank purchases of corporate bonds in the amount necessary to fully eliminate bond spreads for *both* sectors: $R_t^g = R_t^f = 1$. Under this configuration of monetary policy, equations (28) and (29) collapse to their baseline model counterparts (equations 10 and 11) and the decentralized equilibrium replicates the social-planner allocation. From equations (30), eliminating corporate spreads requires reducing households' bond absorption to $B_t^i = \bar{B}^i$, which from equation (31) requires the central bank to absorb all bond supply over and above $\bar{B}^i$: $B_t^{i,cb} = \psi_i w_t N_t^i - \bar{B}^i$,$i = g, f$, for all $t \geq 0$. For brevity (and lack of a better name), we may refer to this policy as "full QE".

The above result can be summarized by the following proposition, which extends Proposition 1 to the model with QE.

**Proposition 2** *Consider the model extension with corporate bond purchases. Provided the carbon tax is set at its socially optimal level at all times, optimal monetary policy combines strict inflation targeting ($\pi_t = 1$) and full QE, such that bond spreads of both energy sectors are eliminated: $R_t^g = R_t^f = 1$. This policy allows the decentralized economy to replicate the social planner* *equilibrium.*

The intuition for this result is again very simple. The extended model contains two additional frictions: the financial wedges affecting green and fossil energy prices ($\psi_i(R_t^i - 1), i = f, g$). Provided carbon taxes are socially optimal, all the central bank needs to do in order for energy prices to follow their optimal paths is to fully offset the financial frictions by eliminating green and brown bond spreads ($R_t^i = 1, i = f, g$). The climate externality is thus fully offset, leaving nominal rigidities as the only distortion, which the central bank addresses through strict inflation targeting.

**Optimal monetary policy under suboptimal carbon taxation.** We now consider the implications of the more realistic case of suboptimal taxation for optimal monetary policy.[32] We focus on the case of the "slow green transition" analyzed in the previous sections, such that $\tau_t^f < \tau_t^{f*}$ for an initial period, after which $\tau_t^f = \tau_t^{f*}$. It is again optimal for the central bank to do full *green* QE, i.e. to eliminate green bonds' spread ($R_t^g = 1$) at all times by absorbing their supply over and above $\bar{B}^g$: $B_t^{g,cb} = \psi_g w_t N_t^g - \bar{B}^g$ for all $t \geq 0$.

As regards fossil energy sector bonds – henceforth "brown bonds" – optimal QE policy aims at tightening financing conditions for that sector as much as needed in order for the market price of fossil energy to replicate its socially optimal level. The latter is the level of $p_t^f$ that solves equation (11) evaluated at the optimal Pigouvian carbon tax: $p_t^f = \tau_t^{f*} + \frac{w_t}{A_t^f}$. However, such an outcome is not always feasible. To see this, we solve for $p_t^f$ in equation (29) and equate the resulting expression to the socially optimal price,

$$\tau_t^f + \left[1 + \psi_f \left(R_t^f - 1\right)\right] \frac{w_t}{A_t^f} = \tau_t^{f*} + \frac{w_t}{A_t^f}.$$

Solving the latter equation for the brown bond spread yields $R_t^f - 1 = \frac{1}{\psi_f} \frac{\tau_t^{f*} - \tau_t^f}{w_t / A_t^f}$. However, equation (30) for $i = f$ and the fact that households cannot hold more brown bonds than the amount supplied by firms ($B_t^f \leq \psi_f w_t N_t^f$) imply an upper bound on the brown bond spread,

$$R_t^f - 1 \leq \kappa_f (\psi_f w_t N_t^f - \bar{B}^f). \tag{32}$$

This allows us to obtain the following optimal rule for the brown bond spread,

$$R_t^f - 1 = \min \left\{ \kappa_f \left( \psi_f w_t N_t^f - \bar{B}^f \right), \frac{1}{\psi_f} \frac{\tau_t^{f*} - \tau_t^f}{w_t / A_t^f} \right\}. \tag{33}$$

Equation (33) can be translated into an optimal rule for brown bond purchases ("brown QE")

[32] Appendix E lays out the general optimal monetary policy problem in the model with QE.

by combining it with (30) and (31) for $i = f$, yielding

$$B_t^{f,cb} = \max\left\{0, \psi_f w_t N_t^f - \bar{B}^f - \frac{1}{\kappa_f \psi_f} \frac{\tau_t^{f*} - \tau_t^f}{w_t / A_t^f}\right\}.$$

We can now characterize optimal brown QE in a slow green transition. Provided the initial gap between actual and optimal carbon taxes $(\tau_t^{f*} - \tau_t^f)$ is large enough that $\frac{1}{\psi_f} \frac{\tau_t^{f*} - \tau_t^f}{w_t / A_t^f} > \kappa_f(\psi_f w_t N_t^f - \bar{B}^f)$, in the initial phase of the transition the central bank cannot make brown bond spreads high enough because it hits its short-selling constraint $(B_t^{f,cb} \geq 0)$, and the best it can do is to maximize the spread on brown bonds by holding none of them. Once the carbon tax gap becomes small enough that $\frac{1}{\psi_f} \frac{\tau_t^{f*} - \tau_t^f}{w_t / A_t^f} \leq \kappa_f(\psi_f w_t N_t^f - \bar{B}^f)$, QE policy can fully replicate the first-best allocation by purchasing as many brown bonds as needed in order for their spread to reach its socially optimal level. In other words, the central bank allows the financial friction affecting fossil energy firms to do the job of making fossil energy sufficiently expensive, thus compensating for the shortfall in carbon taxation. Once the carbon tax gap is closed $(\tau_t^{f*} = \tau_t^f)$, the central bank implements "full brown QE" by purchasing brown bonds in the amount $\psi_f w_t N_t^f - \bar{B}^f$, thus eliminating brown bond spreads $(R_t^f = 1)$.

An implication of the preceding analysis is that, in the initial phase of the slow green transition, monetary policy cannot fully offset the climate externality and hence faces a trade-off between stabilizing inflation and the welfare-relevant output gap, similar to the one in the baseline model. Once the carbon tax gap becomes sufficiently narrow, the transition enters a second phase in which QE policy is able to achieve the socially optimal prices for both green and fossil energy, thus fully offsetting the climate externality. From that point onwards, there is no longer a trade-off between core and climate goals, allowing the central bank to replicate the first-best equilibrium.[33] The next subsection analyzes numerically the slow green transition scenario.

### 5.3 Numerical analysis

**Calibration of the new parameters.** In order to calibrate the sensitivity of bond spreads to central bank purchases, $\kappa_i = \frac{d(R_t^i)}{d(B_t^{i,cb})}$, we first note that such sensitivity can be expressed as $\frac{d(R_t^i)}{d(B_t^{i,cb})/B_t^{i,s}} \frac{1}{B_t^{i,s}}$, where $B_t^{i,s} \equiv \psi_i w_t N_t^i$ is sector $i$'s bond supply. Todorov (2020) estimates that the ECB's initial announcements on its corporate sector purchase program (CSPP) in March and April 2016 lowered yields of eligible bonds by a combined 52 basis points (bp). We thus target $4 \times d(R_i) = 0.5\%$, $i = f, g$, in our quarterly model.[34] Since its implementation, the CSPP

[33] Strictly speaking, the absence of trade-offs in the second phase of the green transition requires the absence of relative price distortions once that phase starts: $\Delta_{t^*-1} = 1$, where $t^*$ denotes the time at which the central banks' short-selling constraint ceases to bind. The first phase $(t < t^*)$ will typically involve some transitory nonzero inflation and hence some price dispersion. However, in our numerical simulations $\Delta_{t^*-1}$ is indistinguishable from 1.

[34] Todorov's (2020) study does not distinguish between green and brown issuers. However, since the original CSPP annnouncements did not make any explicit distinction between green and brown issuers, it is plausible to assume that the yield impact was similar for both types of issuers.

has absorbed approximately 30% of eligible bonds.[35] We hence target $d(B_t^{i,cb})/B_t^{i,s} = 0.3$. Given the value of each sector's initial bond supply ($B_0^{i,s}$, $i = f, g$) implied by our calibration, we then obtain $\kappa_f = 0.024$ and $\kappa_g = 0.160$.

We calibrate the minimum level of bond absorption by private investors, $\{\bar{B}^i\}_{i=f,g}$, by targeting euro area energy sector bond spreads before the introduction of the CSPP. We use Bloomberg data as of 31 December 2015 on yields of all outstanding bonds issued by firms in the energy and utilities sectors,[36] and calculate each bond's spread relative to the same-maturity OIS rate. We define as "green" and "brown" bonds those issued by firms with emissions intensity (greenhouse gas emissions per sales) above and below the median, respectively. This results in average spreads somewhat below 1.5% for both bond types.[37] Based on this, we target $4 \times (R_i - 1) = 1.5\%$, $i = f, g$, in our quarterly model. Using equation (30), we then solve for $\bar{B}^i = B_t^i - (R_t^i - 1)/\kappa_i$, $i = f, g$.

Finally, using also Bloomberg data on wage costs and bond debt outstanding for the same euro area energy sector issuers, we set the leverage factor $\psi_f = \psi_g = \psi$ to 5.[38]

The last panel of Table 1 displays the calibrated values of these new parameters.

**Results.** Figures 7 and 8 compare, for the extended model with QE, the dynamics under optimal carbon taxes (equivalently the social-planner allocation, in the case of the real variables; red solid lines) and under the slow green transition, both in a climate-oblivious scenario with strict inflation targeting and no QE (green dotted lines), and the optimal climate-conscious monetary policy (dashed blue lines).

As shown in Figure 7, and in accordance with the analytical results in the previous subsection, the optimal monetary policy involves full green QE (such that green bond spreads are eliminated) and two different phases for brown QE. For most of the transition, the gap between actual and optimal carbon taxation is too big to be compensated for by green tilting of QE. The best the central bank can do is not to purchase any brown bonds and let their spread reach its maximum level (the one consistent with private investors absorbing the entire supply). In this first phase, the central bank implements 100% green tilting. Once the carbon tax gap becomes sufficiently small, the central bank starts purchasing brown bonds, in the amount necessary in order for fossil energy prices to exactly replicate their optimal level. This second phase, which lasts only a few quarters, is therefore characterized by partial green tilting. Once optimal carbon taxation

[35] ICMA estimates a universe of CSPP eligible bonds at end June 2022 with a nominal value of EUR 1,250 bn, which coupled with ECB holdings data implies that 28% of eligible bonds were held by the program.

[36] We include the following sectors (in Bloomberg's classification): Coal Operations, Exploration and Production or Integrated Oils, Oil and Gas Services and Equipment, Pipeline, Refining and Marketing, Renewable Energy, Power Generation and Utilities.

[37] In particular, 1.44% for green bonds, and 1.26% for brown bonds.

[38] In the model, $\psi_i$ is the ratio between the value of outstanding bonds ($\psi_i w_t N_t^i$) and personnel costs ($w_t N_t^i$). We calculate the ratio of bond debt outstanding and personnel expenditures for the sample of bond-issuing energy firms as a whole (equivalently, the weighted average of the same ratio across firms, using as weights each firm's share of total personnel costs), which yields a ratio of 5.05. Notice that we have assumed a common leverage factor for green ($i = g$) and fossil energy firms ($i = f$). As it turns out, this is innocuous: using the same emissions intensity-based classification as before, we find almost identical leverage factors for green (5.01) and brown issuers (5.06).

is reached, green tilting stops and the central bank implements full brown QE too in order to eliminate brown bond spreads.

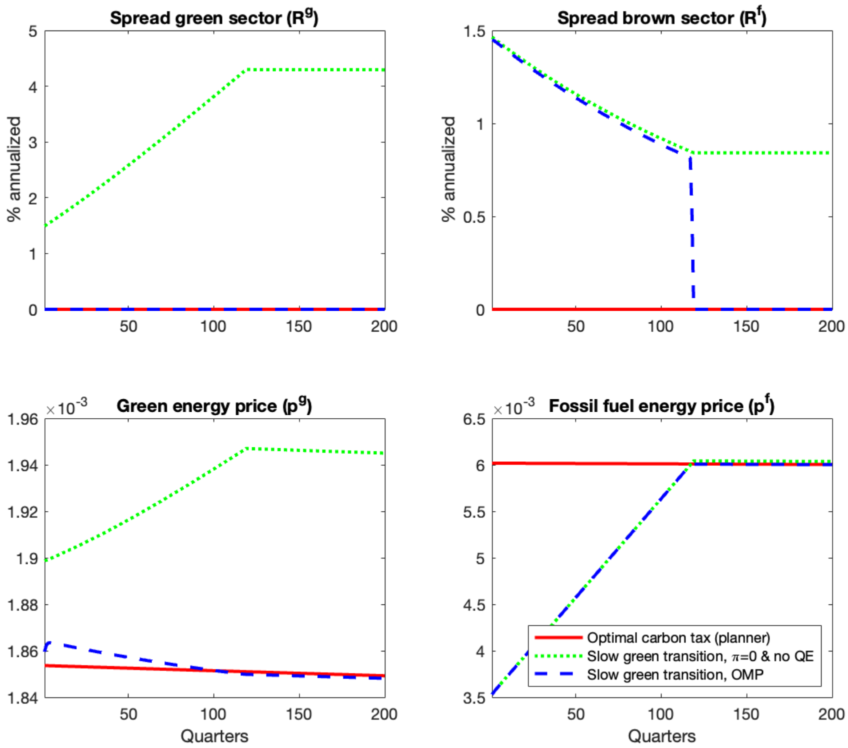


Figure 7: Transitions in green QE model: spreads

Figure 8 shows the corresponding macroeconomic dynamics. Compared to the baseline model (Figure 3), the main difference is that optimal QE allows the central bank to accelerate the green transition somewhat: compared to the zero inflation, no QE policy, green energy consumption reaches its socially efficient level earlier. Figure 7 reveals that this is not the result of more expensive fossil energy. Instead, the elimination of green bond spreads implies cheaper green energy, leading final goods producers to substitute away from fossil energy and into clean one. However, the reduction in fossil energy use is again too small to make much of a difference for atmospheric carbon concentration (not shown). Finally, the optimal departure from strict inflation targeting is again minimal, as the central bank avoids a more forceful increase in nominal interest rates and a larger drop in output below its natural level.

Why is green tilting of QE relatively ineffective at lowering carbon emissions? In our model, the effectiveness of green tilting depends on the size of energy firms' bond spreads in the absence of QE, which determines the extent to which the relative financing conditions of green vs brown energy firms can be affected by climate-oriented asset purchases. Our calibration is consistent with the fact that the ECB, and most other major central banks, restrict their purchases to bonds with high credit quality, as reflected e.g. in investment-grade rating. As a result, the

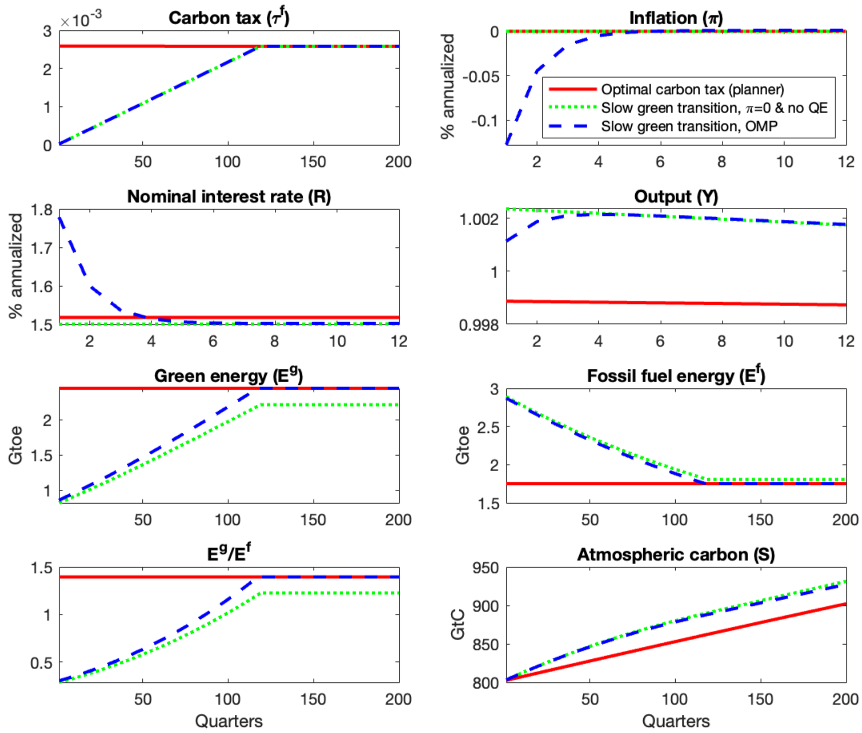


Figure 8: Transitions in green QE model: macro

spreads of bonds eligible under the CSPP and similar corporate bond purchase programs are relatively small to begin with, even in the absence of QE. This substantially limits the scope of green QE tilting for altering brown vs green firms' financing conditions and hence the relative price of dirty vs clean energy.

## 5.4 Sensitivity Analysis

Finally, we analyze how our baseline results change for alternative calibrations of key parameters, in the context of our full model with both conventional monetary policy and green QE. We focus on the elasticity of substitution between energy and labor, $1/(1-\delta)$, as well as two critical determinants of the severity of climate externalities and therefore of optimal carbon taxation, namely the elasticity of the damage function, $\gamma$, and households' (and the social planner's) discount factor, $\beta$.

**Elasticity of substitution between labor and energy.** The first three rows of Table 2 show key equilibrium objects for different values of the elasticity of substitution: the baseline value (0.4), one (i.e. the Cobb-Douglas case) and 0.2, respectively. As shown in the third column, changes in this elasticity barely affect the maximum optimal deviations from strict inflation targeting, which remain very small (less than 15 annualized basis points); see also the upper left plot in Figure 6. This shows that, regardless of how easily final goods producers can substitute energy for labor and vice versa, the central bank does not find it optimal to deviate much from its core (price stability) mandate. This reflects again the inefficient nature of interest-rate policy as a climate tool and the limited scope of green QE for affecting the path of carbon emissions under current eligibility criteria for central bank bond purchases.

However, a lower (higher) elasticity of substitution does imply a faster (slower) pace of reduction in fossil fuel energy use (bottom left plot) and hence in carbon emissions; this in turn implies a flatter (steeper) path of atmospheric carbon concentration (bottom right plot) and hence of global temperatures. The intuition is the following. The rise in carbon taxation along the green transition makes firms substitute away from fossil energy and into green energy *or* labor. When the energy-labor elasticity is low, it is costly to substitute towards labor, so firms substitute more towards green energy, and therefore substitute away from fossil energy more quickly. Thus, the green transition is faster and stronger under a low substitution elasticity. The opposite is true with a high energy-labor elasticity.

**Damage from climate change.** The fourth row of Table 2 shows results when we raise by threefold the elasticity of economic damages from atmospheric carbon concentration. As a result, whereas under the baseline calibration an increase in carbon concentration from current levels by 200 GtC (roughly the increase projected by the model over the next 90 years) would lower World GDP by almost 0.5%, under this more extreme calibration economic damages would amount to nearly 1.5% of global GDP.

The first consequence of this calibration is that the socially optimal carbon tax (which is linear in $\gamma$, according to equation 23) becomes three times larger, such that optimal carbon tax revenues go up from 0.76% to 2.27% of GDP. As shown by the bottom-row plots in Figure 6, this implies a faster reduction in fossil energy use and a flatter path of atmospheric carbon concentration, which plateaus (i.e. net carbon emissions reach zero) after about 30 years. Also, the optimal maximum deviation from strict inflation targeting becomes larger, almost 40 basis points (annualized). The intuition is simple: in the face of much more severe climate externalities, the central bank is willing to strike a more balanced trade-off between its price stability mandate and climate goals.

**Discount factor: a Paris Agreement-like calibration.** We next consider the implications of raising households' (and the benevolent policymakers') discount factor, $\beta$. This could be justified on the grounds that, when it comes to climate change, those living today become much more aware of how current climate policies (and lack thereof) affect future generations.[39] One way to calibrate such a scenario is to relate our model to the emissions-reduction goals contained in the so-called Paris Agreement.[40] In particular, we ask the question: how forward-looking should households be for the model-implied optimal path of carbon concentration to imply net zero emissions (i.e. $S_t - S_{t-1} = 0$) by 2050, or equivalently after 30 years (if one normalizes 2020 to $t = 0$ and $\tau_0 = 0$). We find that the required (quarterly) discount factor is $\beta = 0.99912$, which implies an annual discount rate of 0.4% (compared to 1.5% in our baseline calibration).

As shown in the last row of Table 2, this Paris Agreement-like calibration actually mimics closely the high-damage calibration. In particular, optimal carbon tax revenues are almost three times larger than in the baseline calibration. Reflecting households' stronger concerns about the well-being of future generations in a context of rising global temperatures, the benevolent central bank is again willing to compromise more on its core goals, by accepting a larger deviation from strict inflation targeting (of almost 45 annualized basis points) in order to pursue its climate goals more actively.

**Price stickiness.** In our baseline model, we assumed a Calvo parameter $\theta = 0.75$, which implies that prices change on average once a year. We now test the sensitivity of our main results with a lower ($\theta = 0.60$) and higher ($\theta = 0.95$) degree of price stickiness. As expected, with a higher degree of rigidity ($\theta = 0.95$), the response to inflation is muted and more drawn out in the future than with a lower degree of rigidity ($\theta = 0.60$). Consequently, the motive to restrain output is amplified (attenuated) when the Calvo parameter is higher (lower) than in the baseline (see figure (9)) .

**Welfare gains from adopting climate goals.** Finally, we analyze the welfare implications of the central bank adopting climate goals in its monetary policy-making. The last column of Table 2 shows the welfare gains, as a percent of life-time consumption, from the central bank following the Ramsey optimal, climate-conscious monetary policy as opposed to sticking to strict

[39] As is well known, the assumption of infinitely-lived households used here is a stand-in for the fact that current generations care about their offspring's welfare, and so on.

[40] The Paris Agreement is a legally binding international treaty on climate change adopted at the UN Climate Change Conference (COP21) in Paris in December 2015.

| Calibration | C-tax rev (% GDP) | Max infl dev (pp) | Max y-gap (%) | Net em's in 2050 | $S(t)$ reduct in 2050 | Welfare gain (% Cons) |
|---|---|---|---|---|---|---|
| Baseline | 0.7570 | -0.1276 | 0.3319 | 0.4887 | -2.1045 | 0.0023 |
| Cobb-Douglas | 0.7570 | -0.1148 | 0.3225 | 0.7168 | -0.7473 | 0.0011 |
| ES = 0.2 | 0.7570 | -0.1367 | 0.1745 | -0.1944 | -6.8273 | 0.0098 |
| Higher $\gamma$ (x3) | 2.2709 | -0.3918 | 0.8185 | 0.0322 | -4.1229 | 0.0155 |
| Higher $\beta$ | 2.5655 | -0.4432 | 0.9049 | -0.0119 | -4.3389 | 0.0132 |
| Lower $\theta$ | 0.7570 | -0.1621 | 0.3397 | 0.4887 | -2.1031 | 0.0023 |
| Higher $\theta$ | 0.7570 | -0.0348 | 0.2724 | 0.4888 | -2.1220 | 0.0023 |

Table 2: Sensitivity Analysis

Notes: "C-tax rev" are carbon tax revenues as a % of GDP under optimal carbon taxation (equivalently, tax revenues as a % of GDP at the end of the green transition). "Max infl dev" is the maximum deviation of inflation from zero (i.e. from strict inflation targeting) in annualized percentage points (pp). "Max y-gap" is the maximum welfare-relevant output gap ($Y/Y^*$) in %. "Net em's in 2050" are net carbon emissions ($S_t - S_{t-1}$) in 2050, i.e. 30 years after time 0 (to which we normalize the year 2020). "S(t) reduct in 2050" is the stock of atmospheric carbon concentration in 2050 under optimal monetary policy relative to the same stock under strict inflation targeting. "Welfare gain" is the welfare gain from optimal monetary policy relative to strict inflation targeting, expressed in % of life-time consumption.

inflation targeting. Consistent with the above discussion, the scenarios that justify a higher optimal carbon taxation (whether due to higher economic damages from climate change, or to households being more forward-looking), and therefore larger deviations from strict price stability, also imply larger welfare gains from deviating from the latter policy. However, it is important to notice that such welfare gains remain very small (in fact, second order) in all cases. In this sense, our welfare calculations synthesize in a single number the basic message suggested by our analysis thus far, namely that monetary policy's ability to improve climate outcomes, whether through conventional instruments or (under current design restrictions) through unconventional tools such as corporate bond purchases, is rather limited.

## 6 Conclusions and directions for future research

This paper has provided a normative analysis of monetary policy in a canonical New Keynesian model with climate change externalities and carbon taxation. Our model is deliberately simple, with the aim of clarifying as transparently as possible the trade-offs that monetary policy faces between core and climate goals, given its tools and the path of other mitigating measures, such as carbon taxes. Thus, some caveats are in order, which also suggest directions for further research.

First, we have treated the world economy as a single jurisdiction in terms of both climate and monetary policies. While some international coordination of climate policies exists in practice (notably through the annual United Nations Conference of Parties), progress is far from being perfectly homogeneous. Therefore, it would be interesting to extend this analysis to a multi-

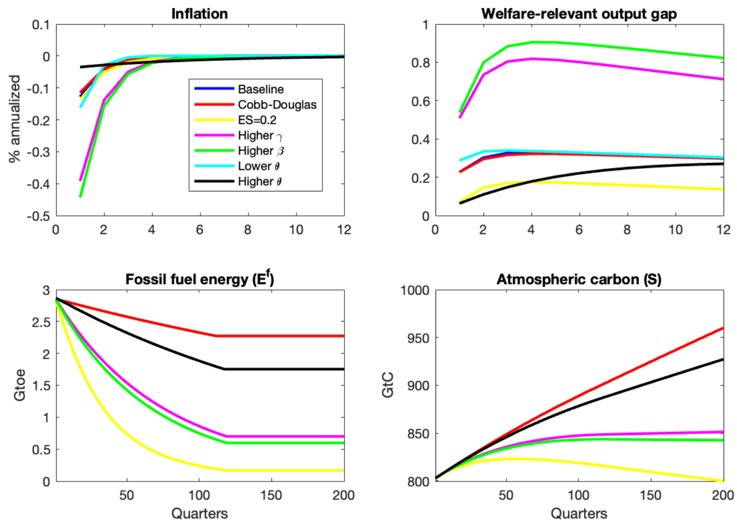


Figure 9: Sensitivity Analysis

region framework with asymmetric policies.

Second, we assume exogenous production technologies, implying fixed (and low, under reasonable calibrations) elasticities of substitution between different inputs. In models with directed technical change, elasticities of substitution are low in the short run but potentially high in the long run (see e.g. Hassler, Krusell and Olovsson, 2021). While our main results would largely hold in a model with this feature, given the short-term nature of the monetary policy trade-offs, allowing for endogenous technical change is likely to be important under scenarios of severe procrastination by climate authorities, in which the time needed for carbon taxes to reach their socially optimal level extends far beyond the horizon considered here.

Finally, our analysis has focused on economic damages that reduce production and on the financing costs of green and fossil energy sectors as key channels through which climate change and monetary policy interact. However, climate change may also interact with monetary policy by affecting the type and distribution of shocks to the economy, e.g. by raising the frequency and/or severity of weather events and the associated supply shocks. Understanding how the trade-offs between climate and price stability goals would change in a context in which climate change endogenously alters the frequency and distribution of future shocks is an important question that we leave for future research.

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# Appendix

## A. Social planner equilibrium

The Lagrangian of the social planner's problem is given by

$$\begin{aligned}&\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0\left\{log(C_t)-\frac{\chi}{1+\varphi}\left(N_t^y+\sum_{i=g,f}N_t^i\right)^{1+\varphi}\right.\\&+\lambda_t^y\left[\left[1-D\left(S_t\right)\right]A_tF\left(N_t^y,\mathbf{E}(E_t^g,E_t^f)\right)-C_t\right]\\&\left.+\sum_{i=g,f}\lambda_t^i[A_t^iN_t^i-E_t^i]+\zeta_t[S_t-\sum_{s=0}^{t+T}\left(1-d_s\right)\xi E_{t-s}^f]\right\}.\end{aligned}$$

The first-order conditions with respect to $C_t, N_t^y, \{N_t^i\}_{i=f,g}, E_t^g, E_t^f$ and $S_t$ are given respectively by

$$\lambda_t^y=1/C_t, \tag{34}$$

$$\lambda_t^y\left[1-D\left(S_t\right)\right]A_t\frac{\partial F\left(\cdot\right)}{\partial N_t^y}=\chi N_t^{\varphi}, \tag{35}$$

$$\lambda_t^iA_t^i=\chi N_t^{\varphi},\quad i=g,f, \tag{36}$$

$$\lambda_t^y\left[1-D\left(S_t\right)\right]A_t\frac{\partial F\left(\cdot\right)}{\partial E_t^g}=\lambda_t^g, \tag{37}$$

$$\lambda_t^y\left[1-D\left(S_t\right)\right]A_t\frac{\partial F\left(\cdot\right)}{\partial E_t^f}=\lambda_t^f+\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^s\left(1-d_s\right)\xi\zeta_{t+s}\right\}, \tag{38}$$

$$\zeta_t=\lambda_t^yD'\left(S_t\right)A_tF\left(N_t^y,\mathbf{E}(E_t^g,E_t^f)\right). \tag{39}$$

Combining all the above conditions, and using $D'\left(S_t\right)=\gamma_t\left[1-D\left(S_t\right)\right]$, we obtain

$$\left[1-D\left(S_t\right)\right]A_t\frac{\partial F\left(\cdot\right)}{\partial N_t^y}=\chi N_t^{\varphi}C_t,$$

$$\left[1-D\left(S_t\right)\right]A_t\frac{\partial F\left(\cdot\right)}{\partial E_t^g}=\frac{\chi N_t^{\varphi}C_t}{A_t^g},$$

$$\begin{aligned}\left[1-D\left(S_t\right)\right]A_t\frac{\partial F\left(\cdot\right)}{\partial E_t^f}&=\frac{\chi N_t^{\varphi}C_t}{A_t^f}+C_t\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s\left(1-d_s\right)\xi\frac{Y_{t+s}}{C_{t+s}}\gamma_{t+s}\\&=\frac{\chi N_t^{\varphi}C_t}{A_t^f}+Y_t\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s\left(1-d_s\right)\xi\gamma_{t+s},\end{aligned}$$

where the second equality uses the fact that $C_t=Y_t$ for all $t$.

## B. General optimal monetary policy problem and proof of Proposition 1

Let $p_t^* \equiv P_t^*/P_t, \pi_t \equiv P_t/P_{t-1}$. Using (7), we can then write firms' optimal price decision (equation 8) as

$$p_t^* = \frac{V_t^{num}}{V_t^{den}},$$

where

$$\begin{aligned} V_t^{num} &\equiv \sum_{t=0}^{\infty} (\beta\theta)^s \, \mathbb{E}_t \left\{ \left( \frac{P_{t+s}}{P_t} \right)^{\epsilon} \frac{\epsilon/(\epsilon-1)}{1+\tau^y} mc_{t+s} \right\} \\ &= \frac{\epsilon/(\epsilon-1)}{1+\tau^y} mc_t + \beta\theta \mathbb{E}_t \left\{ \pi_{t+1}^{\epsilon} V_{t+1}^{num} \right\}. \end{aligned}$$

$$V_t^{den} \equiv \sum_{t=0}^{\infty} (\beta\theta)^s \, \mathbb{E}_t \left\{ \left( \frac{P_{t+s}}{P_t} \right)^{\epsilon-1} \right\} = 1 + \beta\theta \mathbb{E}_t \left\{ \pi_{t+1}^{\epsilon-1} V_{t+1}^{den} \right\}.$$

The Lagrangian of the Ramsey optimal monetary policy problem is

$$\begin{aligned} &\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \Bigg\{ \log(Y_t) - \frac{\chi}{1+\varphi} \left( \sum_{i=y,g,f} N_t^i \right)^{1+\varphi} \\ &+\lambda_t^y \left[ (1-D(S_t)) A_t F\left( N_t^y, \mathbf{E}(E_t^g, E_t^f) \right) - \Delta_t Y_t \right] \\ &+ \sum_{i=g,f} \lambda_t^i \left[ A_t^i N_t^i - E_t^i \right] + \zeta_t \left[ S_t - \sum_{s=0}^{t+T} (1-d_s) \xi E_{t-s}^f \right] \\ &+\lambda_t^w \left[ \chi \left( \sum_{i=y,g,f} N_t^i \right)^{\varphi} Y_t - w_t \right] + \lambda_t^{p^f} \left[ \frac{w_t}{A_t^f} + \tau_t^f - p_t^f \right] \\ &+\lambda_t^{p^g} \left[ \frac{w_t}{A_t^g} - p_t^g \right] + \lambda_t^N \left[ mc_t (1-D(S_t)) A_t \frac{\partial F(\cdot)}{\partial N_{z,t}} - w_t \right] \\ &+ \sum_{i=g,f} \lambda_t^{E^i} \left[ mc_t (1-D(S_t)) A_t \frac{\partial F(\cdot)}{\partial E_{z,t}^i} - p_t^i \right] \\ &+\lambda_t^{\Delta} \left[ \theta \pi_t^{\epsilon} \Delta_{t-1} + (1-\theta)(p_t^*)^{-\epsilon} - \Delta_t \right] \\ &+\lambda_t^{\pi} \left[ (1-\theta)(p_t^*)^{1-\epsilon} + \theta \pi_t^{\epsilon-1} - 1 \right] + \lambda_t^{p^*} \left[ \frac{V_t^{num}}{V_t^{den}} - p_t^* \right] \\ &+\lambda_t^{num} \left[ \frac{\epsilon/(\epsilon-1)}{1+\tau^y} mc_t + \beta\theta E_t \pi_{t+1}^{\epsilon} V_{t+1}^{num} - V_t^{num} \right] \\ &+ \lambda_t^{den} \left[ 1 + \beta\theta E_t \pi_{t+1}^{\epsilon-1} V_{t+1}^{den} - V_t^{den} \right] \Bigg\} \end{aligned}$$

The first-order conditions are

$$\frac{1}{Y_t} + \lambda_t^w \chi \left( \sum_{i=y,g,f} N_t^i \right)^{\varphi} = \lambda_t^y \Delta_t, \tag{$Y_t$}$$

$$\chi N_t^{\varphi} = \lambda_t^{y}\left[1-D\left(S_t\right)\right]A_t\frac{\partial F\left(\cdot\right)}{\partial N_t^{y}} + \lambda_t^{w}\chi\varphi N_t^{\varphi-1}Y_t \tag{40}$$

$$+\left[\lambda_t^{N}\frac{\partial^2 F\left(\cdot\right)}{\partial N_{z,t}^2} + \sum_{i=g,f}\lambda_t^{E^i}\frac{\partial^2 F\left(\cdot\right)}{\partial E_{z,t}^{i}\partial N_t^{y}}\right]\left[1-D\left(S_t\right)\right]A_t,$$

$$\chi N_t^{\varphi} = \lambda_t^{i}A_t^{i} + \lambda_t^{w}\chi\varphi N_t^{\varphi-1}Y_t, \tag{$N_t^i, i=f,g$}$$

$$0 = \lambda_t^{y}\left[1-D\left(S_t\right)\right]A_t\frac{\partial F\left(\cdot\right)}{\partial E_t^{i}} - \lambda_t^{i} - \mathbf{1}_{i=f}\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^{s}\left(1-d_s\right)\xi\zeta_{t+s}\right\} \tag{41}$$

$$+\left[\lambda_t^{N}\frac{\partial F^2\left(\cdot\right)}{\partial N_{z,t}\partial E_t^{i}} + \lambda_t^{E^i}\frac{\partial F^2\left(\cdot\right)}{\partial\left(E_{z,t}^{i}\right)^2} + \lambda_t^{E^{j\neq i}}\frac{\partial F^2\left(\cdot\right)}{\partial E_{z,t}^{i}\partial E_{z,t}^{j\neq i}}\right]\left[1-D\left(S_t\right)\right]A_t,$$

$$\zeta_t = \lambda_t^{y}D'\left(S_t\right)A_tF\left(\cdot\right) + \left[\lambda_t^{N}\frac{\partial F\left(\cdot\right)}{\partial N_{z,t}} + \sum_{i=g,f}\lambda_t^{E^i}\frac{\partial F\left(\cdot\right)}{\partial E_{z,t}^{i}}\right]mc_tD'\left(S_t\right)A_t, \tag{$S_t$}$$

$$\lambda_t^{w} + \lambda_t^{N} = \sum_{i=g,f}\lambda_t^{p^i}/A_t^{i}, \tag{$w_t$}$$

$$0 = \lambda_t^{p^i} + \lambda_t^{E^i}, \tag{$p_t^i, i=f,g$}$$

$$0 = \left[\lambda_t^{N}\frac{\partial F\left(\cdot\right)}{\partial N_{z,t}} + \sum_{i=g,f}\lambda_t^{E^i}\frac{\partial F\left(\cdot\right)}{\partial E_{z,t}^{i}}\right]\left[1-D\left(S_t\right)\right]A_t + \lambda_t^{num}\frac{\epsilon/\left(\epsilon-1\right)}{1+\tau^{y}}, \tag{$mc_t$}$$

$$\lambda_t^{y}Y_t = \beta\theta\mathbb{E}_t\left\{\lambda_{t+1}^{\Delta}\pi_{t+1}^{\epsilon}\right\} - \lambda_t^{\Delta}, \tag{$\Delta_t$}$$

$$\lambda_t^{p^*} = \lambda_t^{\Delta}\left(1-\theta\right)\left(-\epsilon\right)\left(p_t^*\right)^{-\epsilon-1} + \lambda_t^{\pi}\left(1-\theta\right)\left(1-\epsilon\right)\left(p_t^*\right)^{-\epsilon}, \tag{$p_t^*$}$$

$$0 = \lambda_t^{\Delta}\theta\epsilon\pi_t^{\epsilon-1}\Delta_{t-1} + \lambda_t^{\pi}\theta\left(\epsilon-1\right)\pi_t^{\epsilon-2} + \lambda_{t-1}^{num}\theta\epsilon\pi_t^{\epsilon-1}V_t^{num} + \lambda_{t-1}^{den}\theta\left(\epsilon-1\right)\pi_t^{\epsilon-2}V_t^{den}, \tag{$\pi_t$}$$

$$\lambda_t^{num} = \lambda_t^{p^*}\frac{1}{V_t^{den}} + \lambda_{t-1}^{num}\theta\pi_t^{\epsilon}, \tag{$V_t^{num}$}$$

$$\lambda_t^{den} = -\lambda_t^{p^*}\frac{V_t^{num}}{\left(V_t^{den}\right)^2} + \lambda_{t-1}^{den}\theta\pi_t^{\epsilon-1}. \tag{$V_t^{den}$}$$

**Proof of Proposition 1.** We now show that, under the maintained assumption that equation (22) holds, and provided (23) is satisfied, the zero inflation equilibrium satisfies the above conditions. Let $\pi_t = p_t^* = \Delta_t = \frac{V_t^{num}}{V_t^{den}} = mc_t = 1$ for all $t$. Then the first-order conditions become

$$\frac{1}{Y_t} + \lambda_t^{w}\chi\left(\sum_{i=y,g,f}N_t^{i}\right)^{\varphi} = \lambda_t^{y}, \tag{42}$$

$$\chi N_t^{\varphi} = \lambda_t^{y}\left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial N_t^{y}} + \lambda_t^{w} \chi \varphi N_t^{\varphi-1} Y_t \tag{43}$$

$$+ \left[\lambda_t^{N} \frac{\partial^2 F\left(\cdot\right)}{\partial N_{z,t}^2} + \sum_{i=g,f} \lambda_t^{E^i} \frac{\partial^2 F\left(\cdot\right)}{\partial E_{z,t}^{i} \partial N_t^{y}}\right]\left[1 - D\left(S_t\right)\right] A_t,$$

$$\chi N_t^{\varphi} = \lambda_t^{i} A_t^{i} + \lambda_t^{w} \chi \varphi N_t^{\varphi-1} Y_t, \tag{44}$$

$$0 = \lambda_t^{y}\left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^{i}} - \lambda_t^{i} - \mathbf{1}_{i=f} \mathbb{E}_t\left\{\sum_{s=0}^{\infty} \beta^{s}\left(1 - d_s\right) \xi \zeta_{t+s}\right\} \tag{45}$$

$$+ \left[\lambda_t^{N} \frac{\partial F^2\left(\cdot\right)}{\partial N_{z,t} \partial E_t^{i}} + \lambda_t^{E^i} \frac{\partial F^2\left(\cdot\right)}{\partial\left(E_{z,t}^{i}\right)^2} + \lambda_t^{E^{j \neq i}} \frac{\partial F^2\left(\cdot\right)}{\partial E_{z,t}^{i} \partial E_{z,t}^{j \neq i}}\right]\left[1 - D\left(S_t\right)\right] A_t,$$

$$\zeta_t = \lambda_t^{y} D'\left(S_t\right) A_t F\left(\cdot\right) + \left[\lambda_t^{N} \frac{\partial F\left(\cdot\right)}{\partial N_{z,t}} + \sum_{i=g,f} \lambda_t^{E^i} \frac{\partial F\left(\cdot\right)}{\partial E_{z,t}^{i}}\right] D'\left(S_t\right) A_t, \tag{46}$$

$$\lambda_t^{w} + \lambda_t^{N} = \sum_{i=g,f} \lambda_t^{p^i} / A_t^{i}, \tag{47}$$

$$0 = \lambda_t^{p^i} + \lambda_t^{E^i}, \tag{48}$$

$$0 = \left[\lambda_t^{N} \frac{\partial F\left(\cdot\right)}{\partial N_{z,t}} + \sum_{i=g,f} \lambda_t^{E^i} \frac{\partial F\left(\cdot\right)}{\partial E_{z,t}^{i}}\right]\left[1 - D\left(S_t\right)\right] A_t + \lambda_t^{num}, \tag{49}$$

$$\lambda_t^{y} Y_t = \beta \theta E_t \lambda_{t+1}^{\Delta} - \lambda_t^{\Delta}, \tag{50}$$

$$\lambda_t^{p^*} = -\left(1 - \theta\right)\left[\lambda_t^{\Delta} \epsilon + \lambda_t^{\pi}\left(\epsilon - 1\right)\right],$$

$$0 = \theta\left[\lambda_t^{\Delta} \epsilon + \lambda_t^{\pi}\left(\epsilon - 1\right)\right] + \lambda_{t-1}^{num} \theta \epsilon V_t^{num} + \lambda_{t-1}^{den} \theta\left(\epsilon - 1\right) V_t^{den},$$

$$\lambda_t^{num} = \lambda_t^{p^*} \frac{1}{V_t^{den}} + \lambda_{t-1}^{num} \theta,$$

$$\lambda_t^{den} = -\lambda_t^{p^*} \frac{1}{V_t^{den}} + \lambda_{t-1}^{den} \theta.$$

It is trivial to show that the solution to the last four equations is

$$\lambda_t^{p^*} = \lambda_t^{num} = \lambda_t^{den} = 0,$$

$$\lambda_t^{\pi} = -\lambda_t^{\Delta} \frac{\epsilon}{\epsilon - 1}. \tag{51}$$

We conjecture that

$$\lambda_t^{w} = \lambda_t^{N} = \lambda_t^{p^i} = \lambda_t^{E^i} = 0,$$

for $i = f, g$. It is trivial to show that equations (47) to (49) are then satisfied. Also, equations

(42) to (46) become

$$\lambda_t^y = \frac{1}{Y_t}, \tag{52}$$

$$\chi N_t^{\varphi} = \lambda_t^y \left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial N_t^y},$$

$$\chi N_t^{\varphi} = \lambda_t^i A_t^i, \quad i = g, f$$

$$\lambda_t^i + \mathbf{1}_{i=f} E_t \{ \sum_{s=0}^{\infty} \beta^s \left(1 - d_s\right) \xi \zeta_{t+s} \} = \lambda_t^y \left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^i}, \quad i = g, f \tag{53}$$

$$\zeta_t = \lambda_t^y D'\left(S_t\right) A_t F\left(\cdot\right).$$

The latter seven equations are identical to equations (34) to (39) in the social planner problem. Using (50) and (52), we can solve for $\lambda_t^{\Delta}$ as follows,

$$\lambda_t^{\Delta} = \beta\theta E_t \lambda_{t+1}^{\Delta} - 1 = \frac{-1}{1 - \beta\theta},$$

such that, from (51), we obtain

$$\lambda_t^{\pi} = \frac{1}{1 - \beta\theta} \frac{\epsilon}{\epsilon - 1}.$$

This completes our proof.

## C. Calibration procedure

Let

$$F\left(N, E\right) = \left(\alpha E^{\delta} + \left(1 - \alpha\right) N^{\delta}\right)^{1/\delta}, \tag{54}$$

which converges to Cobb-Dougles (unit elasticity of substitution) as $\delta \to 0$.[41] Therefore

$$\frac{\partial F\left(\cdot\right)}{\partial N} = \left(1 - \alpha\right) \left(\frac{F}{N}\right)^{1-\delta},$$

$$\frac{\partial F\left(\cdot\right)}{\partial E} = \alpha \left(\frac{F}{E}\right)^{1-\delta}.$$

Equilibrium conditions featuring $F$,

$$F_t = \left(\alpha E_t^{\delta} + \left(1 - \alpha\right) \left(N_t^y\right)^{\delta}\right)^{1/\delta},$$

$$\left[1 - D\left(S_t\right)\right] A_t F_t = \Delta_t Y_t,$$

$$mc_t \left[1 - D\left(S_t\right)\right] A_t \left(1 - \alpha\right) \left(\frac{F_t}{N_t^y}\right)^{1-\delta} = w_t,$$

[41] The elasticity of substition is given by $ES = \frac{1}{1-\delta}$. Therefore, unit elasticity of substitution (Cobb-Douglas) requires $\delta = 0$.

$$mc_t\left[1-D\left(S_t\right)\right]A_t\alpha\left(\frac{F_t}{E_t}\right)^{1-\delta}=p_t^i,\quad i=f,g.$$

In this case, as in the Cobb-Douglas case, the relative price of green energy is given by $\frac{p_0^g}{p_0^f}=\frac{\omega}{1-\omega}\left(\frac{E_0^f}{E_0^g}\right)^{(1-\rho)}$. So given data for $\frac{p_0^g}{p_0^f}$ and for $\{E_0^i\}_{i=f,g}$ (in GtC), we can solve for $\omega$ as

$$\omega=\frac{1}{1+(p_0^f/p_0^g)\left(E_0^f/E_0^g\right)^{(1-\rho)}}. \tag{55}$$

We can then use (24) to compute $E_0$.

We assume that $N_0$ is normalized to 1, such that $w_0=\chi(1)^{\varphi}C_0=\chi Y_0$. We guess $\chi=1$. With CES goods-production technology, the equilibrium conditions (20) and (21) become

$$\frac{\chi Y_0}{A_0^g}=\left[1-D\left(S_0\right)\right]A_0\alpha\left(\frac{F\left(N_0^y,E_0\right)}{E_0}\right)^{1-\delta}\omega_i\left(\frac{E_0}{E_0^i}\right)^{1-\rho},$$

$i=f,g$, with $\omega_g=\omega=1-\omega_f$. Using $Y_0=\left[1-D\left(S_0\right)\right]A_0F\left(1,E_0\right)$ and cancelling terms, we obtain

$$\frac{\chi}{A_0^g}=\frac{\alpha}{E_0}\left(\frac{E_0}{F\left(N_0^y,E_0\right)}\right)^{\delta}\omega_i\left(\frac{E_0}{E_0^i}\right)^{1-\rho},$$

$i=f,g$. *Guessing a value for* $N_0^y$ (e.g. $1-\alpha$), these equations can be solved for $\{A_0^i\}_{i=f,g}$ as

$$A_0^g=\frac{\chi E_0}{\alpha\omega_i}\left(\frac{F\left(N_0^y,E_0\right)}{E_0}\right)^{\delta}\left(\frac{E_0^i}{E_0}\right)^{1-\rho}.$$

We can then solve for $\{N_0^i\}_{i=f,g}$ as $N_0^i=E_0^i/A_0^i$, $i=f,g$. Total energy sector labor then equals

$$\sum_{i=f,g}N_0^i=\frac{\alpha}{\chi}\left(\frac{E_0}{F\left(1,E_0\right)}\right)^{\delta}\sum_{i=f,g}\omega_i\left(\frac{E_0^i}{E_0}\right)^{\rho}=\frac{\alpha}{\chi}\left(\frac{E_0}{F\left(N_0^y,E_0\right)}\right)^{\delta},$$

where the second equality follows from (24). The equilibrium condition (19) can be expressed as

$$\chi Y_0=(1-\alpha)\left[1-D\left(S_0\right)\right]A_0\left(\frac{F\left(N_0^y,E_0\right)}{N_0^y}\right)^{1-\delta}\Leftrightarrow N_0^y=\frac{1-\alpha}{\chi}\left(\frac{N_0^y}{F\left(N_0^y,E_0\right)}\right)^{\delta},$$

where the last equation follows from (54). Total labor input then equals

$$N_0^y+\sum_{i=f,g}N_0^i=\frac{1-\alpha}{\chi}\left(\frac{N_0^y}{F\left(N_0^y,E_0\right)}\right)^{\delta}+\frac{\alpha}{\chi}\left(\frac{E_0}{F\left(N_0^y,E_0\right)}\right)^{\delta}=\frac{1}{\chi},$$

which equals 1 (as per our normalization) if and only if $\chi=1$, which verifies our guess. Since we have already solved for $\{N_0^i\}_{i=f,g}$, we can solve for $N_0^y$ as $N_0^y=1-\sum_{i=f,g}N_0^i$. We then iterate until convergence in $N_0^y$.

## D. Extension to capital in energy production

**Energy producers.** Assume that, in addition to labor, energy producers in each sector $i = g, f$ use capital specific to that sector, according to a Cobb-Douglas technology,

$$E_t^i = A_t^i \left(K_t^i\right)^{\alpha_i} \left(N_t^i\right)^{1-\alpha_i}, \quad i = g, f, \tag{56}$$

where $K_t^i$ is sector-$i$-specific capital, $i = g, f$. Capital is rented from households at sector-specific real rate $r_t^i$. The representative firm in energy sector $i$ maximizes

$$\frac{\Pi^i}{P_t} = \left(p_t^i - \tau_t^i\right) A_t^i \left(K_t^i\right)^{\alpha_i} \left(N_t^i\right)^{1-\alpha_i} - w_t N_t^i - r_t^i K_t^i,$$

for $i = g, f$, with $\tau_t^g = 0$. The first-order conditions with respect to labor and capital are, respectively,

$$\left(p_t^i - \tau_t^i\right) A_t^i \left(1 - \alpha_i\right) \left(K_t^i / N_t^i\right)^{\alpha_i} = w_t, \tag{57}$$

$$\left(p_t^i - \tau_t^i\right) A_t^i \alpha_i \left(N_t^i / K_t^i\right)^{1-\alpha_i} = r_t^i, \tag{58}$$

for $i = g, f$.

**Households.** Households can now also invest in capital, which they purchase from capital goods producers at sector-specific real price $p_{K,t}^i$ and depreciates at rate $\delta_i$. The household's problem is now to maximize (1) subject to the nominal budget constraint

$$P_t c_t + B_t + P_t {\textstyle\sum_{i=f,g}} p_{K,t}^i I_t^i = R_{t-1} B_{t-1} + W_t N_t + P_t {\textstyle\sum_{i=f,g}} r_t^i K_t^i + \Pi_t + T_t,$$

and the law of motion of sector-specific capital,

$$K_{t+1}^i = I_t^i + \left(1 - \delta_i\right) K_t^i, \quad i = g, f, \tag{59}$$

where $I_t^i$ is sector-specific investment. Letting $\lambda_{K,t}^i$ denote the Lagrange multiplier associated to constraint (59), the first-order conditions for $I_t^i$ and $K_{t+1}^i$ can be expressed as follows,

$$C_t^{-1} p_{K,t}^i = \lambda_{K,t}^i, \tag{60}$$

$$\lambda_{K,t}^i = \beta \mathbb{E}_t \left[C_{t+1}^{-1} r_{t+1}^i + \lambda_{K,t+1}^i \left(1 - \delta_i\right)\right], \tag{61}$$

respectively, for $i = g, f$. All other household first-order conditions are as in the baseline model.

**Capital producers.** Capital is produced by perfectly competitive sector-specific capital producers. Analogously to households' consumption demand, the representative producer of sector-$i$-specific capital uses a Dixit-Stiglitz combination of final goods, which it transforms linearly into capital goods,

$$I_t^i = A_{K,t}^i \left( \int_0^1 \left( I_{z,t}^i \right)^{(\epsilon-1)/\epsilon} dz \right)^{\epsilon/(\epsilon-1)},$$

$i = f, g$, where $I_t^i$ are new units of sector-$i$ capital, $I_{z,t}^i$ is investment demand for final good variety $z \in [0,1]$ and $A_{K,t}^i$ is a capital-specific exogenous technology factor. Cost minimization implies $I_{z,t}^i = (P_{z,t}/P_t)^{-\epsilon} I_t^i / A_{K,t}^i$ and $\int_0^1 P_{z,t} I_{z,t}^i dz = P_t I_t^i / A_{K,t}^i$. Nominal profits can then be expressed as $\Pi_{K,t}^i = P_t p_{K,t}^i I_t^i - \int_0^1 P_{z,t} I_{z,t}^i dz = P_t \left( p_{K,t}^i I_t^i - I_t^i / A_{K,t}^i \right)$. Profit maximization with respect to $I_t^i$ implies

$$p_{K,t}^i = \frac{1}{A_{K,t}^i}, \tag{62}$$

for $i = f, g$, such that capital producers earn zero profits.

**Final goods producers.** The producer of the variety-$z$ final good now faces consumption demand by households and investment demand by capital producers,

$$y_{z,t} = (P_{z,t}/P_t)^{-\epsilon} \left( C_t + \sum_{i=f,g} I_t^i / A_{K,t}^i \right) = (P_{z,t}/P_t)^{-\epsilon} Y_t,$$

where we use the fact that aggregate demand for final goods now equals $Y_t = C_t + \sum_{i=f,g} I_t^i / A_{K,t}^i$. Therefore, the price-setting problem of firm $z$ is now

$$\sum_{t=0}^{\infty} \mathbb{E}_t \left\{ \Lambda_{t,t+s} \theta^s \left( (1+\tau^y) P_{z,t} - MC_{t+s} \right) \left( \frac{P_{z,t}}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \right\},$$

The first-order condition (FOC) now is

$$\sum_{t=0}^{\infty} \mathbb{E}_t \left\{ \Lambda_{t,t+s} \theta^s \left( (1+\tau^y) P_t^* - \frac{\epsilon}{\epsilon - 1} MC_{t+s} \right) \left( \frac{P_t^*}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \right\} = 0,$$

which, letting $mc_t \equiv \frac{MC_t}{P_t}$, can be written as

$$p_t^* \equiv \frac{P_t^*}{P_t} = \frac{\sum_{t=0}^{\infty} \mathbb{E}_t \left\{ \beta^s \theta^s \left( \frac{P_{t+s}}{P_t} \right)^{\epsilon} \frac{\epsilon}{\epsilon-1} \frac{mc_{t+s}}{1+\tau^y} \frac{Y_{t+s}}{C_{t+s}} \right\}}{\sum_{t=0}^{\infty} \mathbb{E}_t \left\{ \beta^s \theta^s \left( \frac{P_{t+s}}{P_t} \right)^{\epsilon-1} \frac{Y_{t+s}}{C_{t+s}} \right\}} \equiv \frac{V_t^{num}}{V_t^{den}},$$

$$V_t^{num} = \frac{\epsilon/(\epsilon-1)}{1+\tau^y} mc_t \frac{Y_t}{C_t} + \beta\theta \mathbb{E}_t \{ \pi_{t+1}^{\epsilon} V_{t+1}^{num} \},$$

$$V_t^{den} = \frac{Y_t}{C_t} + \beta\theta \mathbb{E}_t \{ \pi_{t+1}^{\epsilon-1} V_{t+1}^{den} \}.$$

**Market clearing.** Finally, market clearing now requires $y_{z,t} = c_{z,t} + \sum_{i=f,g} I_{z,t}^i$ for each final good variety $z \in [0,1]$. Aggregate real output equals[42]

$$Y_t = C_t + \sum_{i=f,g} p_{K,t}^i I_t^i. \tag{63}$$

**Flexible price equilibrium.** Combining (2), (4), (5), (57), (58), (60), (61), (62) and $mc_t = (1+\tau^y)\frac{\epsilon-1}{\epsilon}$, we obtain

$$\chi N_t^{\varphi} C_t = (1+\tau^y)\frac{\epsilon-1}{\epsilon}\left[1-D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial N_{z,t}} \tag{64}$$

$$\frac{C_t^{-1}}{A_{K,t}^i} = \lambda_{K,t}^i, \quad i = f, g \tag{65}$$

$$\lambda_{K,t}^i = \beta \mathbb{E}_t \left[\chi N_{t+1}^{\varphi} \frac{\alpha_i}{1-\alpha_i} \frac{N_{t+1}^i}{K_{t+1}^i} + \lambda_{K,t+1}^i \left(1-\delta_i\right)\right], \quad i = f, g \tag{66}$$

$$\frac{\chi N_t^{\varphi} C_t}{A_t^g \left(1-\alpha_g\right) \left(K_t^g / N_t^g\right)^{\alpha_g}} = (1+\tau^y)\frac{\epsilon-1}{\epsilon}\left[1-D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_{z,t}^g}, \tag{67}$$

$$\frac{\chi N_t^{\varphi} C_t}{A_t^f \left(1-\alpha_f\right) \left(K_t^f / N_t^f\right)^{\alpha_f}} + \tau_t^f = (1+\tau^y)\frac{\epsilon-1}{\epsilon}\left[1-D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_{z,t}^f}. \tag{68}$$

**Efficient equilibrium and optimal carbon tax.** The social planner's Lagrangian is now

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left\{ \log(C_t) - \frac{\chi}{1+\varphi}\left(N_t^y + \sum_{i=g,f} N_t^i\right)^{1+\varphi} \right. \\
&+ \lambda_t^y \left[\left[1-D\left(S_t\right)\right] A_t F\left(N_t^y, \mathbf{E}(E_t^g, E_t^f)\right) - C_t - \sum_{i=f,g} \frac{I_t^i}{A_{K,t}^i}\right] \\
&+ \sum_{i=f,g} \lambda_{K,t}^{i,SP}\left[I_t^i + \left(1-\delta_i\right) K_t^i - K_{t+1}^i\right] \\
&\left. + \sum_{i=g,f} \lambda_t^i [A_t^i \left(K_t^i\right)^{\alpha_i} \left(N_t^i\right)^{1-\alpha_i} - E_t^i] + \zeta_t [S_t - \sum_{s=0}^{t+T} \left(1-d_s\right) \xi E_{t-s}^f] \right\}.
\end{aligned}$$

FOCs with respect to $C_t, N_t^y, \{N_t^i, I_t^i, K_{t+1}^i\}_{i=f,g}, E_t^g, E_t^f$ and $S_t$,

$$\lambda_t^y = 1/C_t, \tag{69}$$

$$\lambda_t^y \left[1-D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial N_t^y} = \chi N_t^{\varphi}, \tag{70}$$

$$\lambda_t^i A_t^i \left(1-\alpha_i\right) \left(K_t^i / N_t^i\right)^{\alpha_i} = \chi N_t^{\varphi}, \quad i = g, f, \tag{71}$$

[42] Nominal output of final goods equals $\int_0^1 P_{z,t} c_{z,t} dz + \sum_{i=f,g} \int_0^1 P_{z,t} I_{z,t}^i dz = P_t C_t + \sum_{i=f,g} P_t p_{K,t}^i I_t^i \equiv P_t Y_t$, where we have used the fact that $\int_0^1 P_{z,t} I_{z,t}^i dz = \frac{P_t I_t^i}{A_{K,t}^i} = P_t p_{K,t}^i I_t^i$. Dividing by the price deflator $P_t$, real output $Y_t$ is therefore given by (63).

$$\frac{\lambda_t^y}{A_{K,t}^i} = \lambda_{K,t}^{i,SP}, \quad i = g, f, \tag{72}$$

$$\lambda_{K,t}^{i,SP} = \beta \mathbb{E}_t \left[ \lambda_{t+1}^i A_{t+1}^i \alpha_i \left(K_{t+1}^i\right)^{\alpha_i - 1} \left(N_{t+1}^i\right)^{1-\alpha_i} + \lambda_{K,t+1}^{i,SP} \left(1 - \delta_i\right) \right], \quad i = g, f, \tag{73}$$

$$\lambda_t^y \left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^g} = \lambda_t^g. \tag{74}$$

$$\lambda_t^y \left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^f} = \lambda_t^f + \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(1 - d_s\right) \xi \zeta_{t+s} \right\}. \tag{75}$$

$$\zeta_t = \lambda_t^y D'\left(S_t\right) A_t F\left(N_t^y, \mathbf{E}(E_t^g, E_t^f)\right). \tag{76}$$

Combining all the above conditions, and using $D'\left(S_t\right) = \gamma_t \left[1 - D\left(S_t\right)\right]$, we obtain

$$\left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial N_t^y} = \chi N_t^{\varphi} C_t,$$

$$\frac{C_t^{-1}}{A_{K,t}^i} = \lambda_{K,t}^{i,SP}, \quad i = g, f,$$

$$\lambda_{K,t}^{i,SP} = \beta E_t \left[ \chi N_{t+1}^{\varphi} \frac{\alpha_i}{1 - \alpha_i} \frac{N_{t+1}^i}{K_{t+1}^i} + \lambda_{K,t+1}^{i,SP} \left(1 - \delta_i\right) \right], \quad i = g, f,$$

$$\left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^g} = \frac{\chi N_t^{\varphi} C_t}{A_t^g \left(1 - \alpha_g\right) \left(K_t^g / N_t^g\right)^{\alpha_g}},$$

$$\left[1 - D\left(S_t\right)\right] A_t \frac{\partial F\left(\cdot\right)}{\partial E_t^f} = \frac{\chi N_t^{\varphi} C_t}{A_t^f \left(1 - \alpha_f\right) \left(K_t^f / N_t^f\right)^{\alpha_f}} + C_t \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(1 - d_s\right) \xi \frac{Y_{t+s}}{C_{t+s}} \gamma_{t+s}.$$

Comparing the above equations with (64) to (68), it follows that the flexible price equilibrium replicates the efficient one if and only if $\tau^y = \frac{\epsilon}{\epsilon - 1} - 1$ and

$$\tau_t^f = C_t \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left(1 - d_s\right) \xi \frac{Y_{t+s}}{C_{t+s}} \gamma_{t+s} \right\}. \tag{77}$$

The above expression generalizes the formula for the socially optimal carbon tax to the model extension with capital accumulation. Compared to the baseline formula (equation 23), expression (77) is complicated by the fact that consumption no longer equals output, as now output is also used for investment (see equation 63). Therefore, the optimal carbon tax at each time $t \geq 0$ depends on the entire future path of the consumption rate, $\{C_{t+s}/Y_{t+s}\}_{s=0}^{\infty}$. However, under our assumed carbon depreciation structure, $1 - d_s = \phi_0 \left(1 - \phi\right)^s, s \geq 0$, it is possible to express the optimal tax (rescaled by consumption) recursively as follows,

$$\frac{\tau_t^f}{C_t} \equiv \tau_t^{f,C} = \phi_0 \xi \frac{Y_t}{C_t} \gamma_t + \beta \left(1 - \phi\right) \mathbb{E}_t \left\{ \tau_{t+1}^{f,C} \right\}.$$

## Calibration

**New parameters.** We first calibrate $\{\alpha_i\}_{i=f,g}$ based on the interest-rate sensitivity of the levelized cost of energy (LCOE) of green vs fossil sources. Using (60) and (61), we can write

$$\begin{aligned} p^i_{K,t} &= \beta \frac{C^{-1}_{t+1}}{C^{-1}_t} \left[ r^i_{t+1} + p^i_{K,t+1} (1-\delta_i) \right] = \frac{1}{R_t/\pi_{t+1}} \left[ r^i_{t+1} + p^i_{K,t+1} (1-\delta_i) \right] \\ &= \frac{r^i_{t+1}}{R_t/\pi_{t+1} - (1-\delta_i)} = \frac{\left( p^i_{t+1} - \tau^i_{t+1} \right) E^i_{t+1}/K^i_{t+1} - w_{t+1} N^i_{t+1}/K^i_{t+1}}{R_t/\pi_{t+1} - (1-\delta_i)}, \end{aligned} \tag{78}$$

$i = f, g$, where in the 2nd equality we have used the consumption Euler equation under perfect foresight ($\beta \frac{C^{-1}_{t+1}}{C^{-1}_t} = \frac{\pi_{t+1}}{R_t}$), in the 3rd equality we have assumed a constant relative price of capital, $p^i_{K,t+1} = p^i_{K,t} = 1/A^i_{K,t} = 1/\bar{A}^i_K$ for all $t \geq 0$, and in the 4th equality we have used the fact that, under constant returns to scale in energy production, energy firms make zero profits, such that $r^i_t K^i_t = \left( p^i_t - \tau^i_t \right) E^i_t - w_t N^i_t$. Solving (78) for $p^i_{t+1}$, we obtain

$$p^i_{t+1} = \frac{p^i_{K,t} K^i_{t+1} \left[ R_t/\pi_{t+1} - (1-\delta_i) \right] + w_{t+1} N^i_{t+1}}{E^i_{t+1}} + \tau^i_{t+1},$$

$i = f, g$, with $\tau^g_{t+1} = 0$. *Ceteris paribus*, the semi-elasticity of time-$(t+1)$ energy prices with respect to $R_t$ is

$$\frac{\partial p^i_{t+1}/p^i_{t+1}}{\partial R_t} = \frac{p^i_{K,t}/\pi_{t+1}}{p^i_{t+1} E^i_{t+1}/K^i_{t+1}} \approx \frac{p^i_{K,t} K^i_{t+1}}{p^i_{t+1} E^i_{t+1}} = \frac{p^i_{K,t}}{r^i_{t+1}} \alpha_i = \frac{\alpha_i}{(R_t/\pi_{t+1} - 1) + \delta_i} \approx \frac{\alpha_i}{1/\beta - 1 + \delta_i},$$

where we use the fact that $\pi_{t+1} \approx 1$, $\frac{r^i_{t+1} K^i_{t+1}}{p^i_{t+1} E^i_{t+1}} = \alpha_i$ (under our assumed Cobb-Douglas energy production technology), $\frac{p^i_{K,t}}{r^i_{t+1}} = \frac{1}{R_t/\pi_{t+1} - (1-\delta_i)}$, and $R_t/\pi_{t+1} \approx 1/\beta$. Therefore, given calibrated values for $\{\delta_i\}_{i=f,g}$ and $\beta$, we can calibrate $\{\alpha_i\}_{i=f,g}$ by using empirical proxies for $\frac{\partial p^i_{t+1}/p^i_{t+1}}{\partial R_t}$.

**Recalibration of other parameters.** Given the targets for relative energy prices and energy consumption, $\omega$ is still determined by (55), and $E_0$ continues to be determined by (24). However, we need to recalibrate the TFP factors in both energy sectors, $A^i_{t \geq 0} = \bar{A}^i$, for $i = f, g$. As in the baseline model, we continue to assume that the economy initially replicates the flexible-price equilibrium and there is no monopolistic markup ($mc_0 = (1+\tau^y) \frac{\epsilon - 1}{\epsilon} = 1$), no price dispersion ($\Delta_0 = 1$), and no carbon taxation ($\tau^f_0 = 0$). We continue to set $\chi = 1$, although this generally no longer implies $N_0 = 1$. Conditions (67) and (68) for $t = 0$ become

$$\frac{\chi N^{\varphi}_0 C_0}{A^i_0 (1-\alpha_i) \left( K^i_0/N^i_0 \right)^{\alpha_i}} = \left[ 1 - D(S_0) \right] A_0 \alpha \left( \frac{F(N^y_0, E_0)}{E_0} \right)^{1-\delta} \omega_i \left( \frac{E_0}{E^i_0} \right)^{1-\rho},$$

$i = f, g$, with $\omega_g = \omega = 1 - \omega_f$. Using $Y_0 = [1 - D(S_0)]\, A_0 F(N_0^y, E_0)$, we obtain

$$\frac{\chi N_0^{\varphi} C_0}{A_0^i (1-\alpha_i) \left(K_0^i / N_0^i\right)^{\alpha_i}} = Y_0 \frac{\alpha}{E_0} \left(\frac{E_0}{F(N_0^y, E_0)}\right)^{\delta} \omega_i \left(\frac{E_0}{E_0^i}\right)^{1-\rho},$$

$i = f, g$. We guess that in equilibrium $C_0 / Y_0 \approx 1$. Therefore

$$\frac{\chi N_0^{\varphi}}{A_0^i (1-\alpha_i) \left(K_0^i / N_0^i\right)^{\alpha_i}} \approx \frac{\alpha}{E_0} \left(\frac{E_0}{F(N_0^y, E_0)}\right)^{\delta} \omega_i \left(\frac{E_0}{E_0^i}\right)^{1-\rho}. \tag{79}$$

The remainder of the procedure iterates on sectoral labor demand, $\{N_0^{i(0)}\}_{i=f,g,y}$. We start the algorithm by guessing initial values, $\{N_0^{i(0)}\}_{i=f,g,y}$ (e.g. $\alpha(1-\omega), \alpha\omega, 1-\alpha$, respectively, such that $N_0^{(0)} = \sum_{i=f,g,y} N_0^{i(0)} = 1$). Given these, and the initial conditions for capital stocks, $\{K_0^i\}_{i=f,g}$, equation (79) can be used to solve for $A_0^i$ as (we ignore the approximation error)

$$A_0^{i(0)} = \frac{\chi \left(N_0^{(0)}\right)^{\varphi} E_0}{(1-\alpha_i)\left(K_0^i / N_0^{i(0)}\right)^{\alpha_i} \alpha\omega_i} \left(\frac{F\left(N_0^{y(0)}, E_0\right)}{E_0}\right)^{\delta} \left(\frac{E_0^i}{E_0}\right)^{1-\rho}, \tag{80}$$

$i = f, g$. We can then solve for new values of $\{N_0^{i(1)}\}_{i=f,g}$ as

$$N_0^{i(1)} = \left(\frac{E_0^i}{A_0^i \left(K_0^i\right)^{\alpha_i}}\right)^{1/(1-\alpha_i)}, \quad i = g, f.$$

Using again $Y_0 = [1 - D(S_0)]\, A_0 F(N_0^y, E_0)$, and under our guess $C_0 / Y_0 \approx 1$, equation (64) can be written as

$$\chi N_0^{\varphi} \approx \frac{1-\alpha}{N_0^y} \left(\frac{F(N_0^y, E_0)}{N_0^y}\right)^{-\delta},$$

which can be used to solve for a new value of $N_0^y$ as

$$N_0^{y(1)} \approx \frac{1-\alpha}{\chi \left(N_0^{(0)}\right)^{\varphi}} \left(\frac{N_0^{y(0)}}{F\left(N_0^{y(0)}, E_0\right)}\right)^{\delta}.$$

The new values $\{N_0^{i(1)}\}_{i=f,g}$ imply new values for $\{A_0^{i(1)}\}_{i=f,g}$ according to (80). We continue iterating on $\{N_0^{i(\cdot)}, A_0^{i(\cdot)}\}_{i=f,g,y}$ until convergence.

**Transition paths**

Figure 10 shows the transition paths in the model with capital in the same three scenarios as in the baseline model: optimal monetary policy under optimal carbon taxation, and both strict inflation targeting and optimal monetary policy under slow green transition (with $\tau_t^f$ converging linearly from zero to optimal over a 30-year period). Our main result from the baseline model carries over to the case with capital: even in a scenario of decades-long transition to optimal carbon taxation, optimal monetary policy continues to be very close to strict inflation targeting, such that the path of carbon emissions remains essentially unaffected.

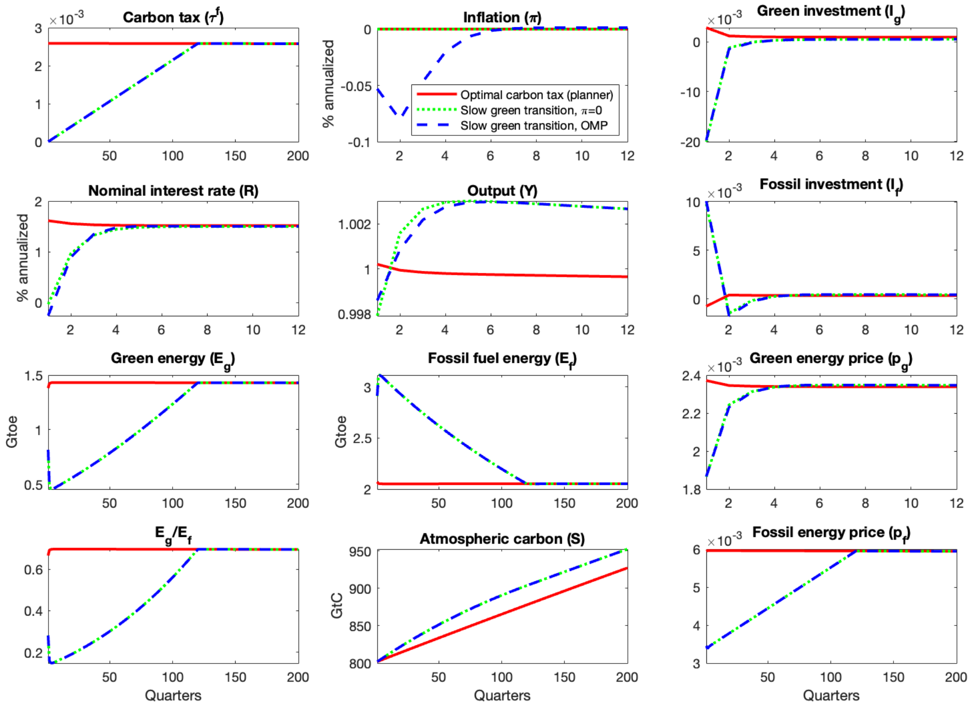


Figure 10: Transitions in the model with capital

## E. Optimal monetary policy problem with QE

Compared to the Lagrangian in the baseline model (Appendix B), the Lagrangian in the model with QE changes as follows,

$$\sum_{t=0}^{\infty} \beta^t E_0 \{ \ldots + \sum_{i=f,g} \lambda_t^{p^i} \left[ \frac{[1+\psi_i(R_t^i - 1)]w_t}{A_t^i} + \mathbf{1}_{i=f} \tau_t^f - p_t^i \right]$$
$$- \sum_{i=f,g} \mu_t^{R^i_{\min}} \left[ 1 - R_t^i \right] - \sum_{i=f,g} \mu_t^{R^i_{\max}} \left[ R_t^i - 1 - \kappa_i \left( \psi_i w_t N_t^i - \bar{B}^i \right) \right] \},$$

where $\mu_t^{R^i_{\min}}$ and $\mu_t^{R^i_{\min}}$ are the Kuhn-Tucker multipliers associated to the inequality constraints $1 \leq R_t^i$ and (32), respectively. The FOCs wrt $\{N_t^i\}_{i=f,g}$ and $w_t$ are now given by,

$$\chi N_t^{\varphi} = \lambda_t^i A_t^i + \lambda_t^w \chi \varphi N_t^{\varphi - 1} Y_t + \mu_t^{R^i_{\max}} \kappa_i \psi_i w_t, \qquad (N_t^i, i = f, g)$$

$$\lambda_t^w + \lambda_t^N = \sum_{i=g,f} \lambda_t^{p^i} \frac{1+\psi_i(R_t^i - 1)}{A_t^i} + \sum_{i=f,g} \mu_t^{R^i_{\max}} \kappa_i \psi_i N_t^i, \qquad (w_t)$$

whereas the new FOCs wrt $\{R_t^i\}_{i=f,g}$ read

$$0 = \lambda_t^{p^i} \frac{\psi_i w_t}{A_t^i} + \mu_t^{R^i_{\min}} - \mu_t^{R^i_{\max}}, \qquad (R_t^i, i = f, g)$$

$i = f, g$. Under the solution for corporate interest rates conjecture above, i.e. $R_t^g = 1$ and equation (33), the constraint $1 \leq R_t^g$ binds in periods in which the first-best equilibrium cannot be replicated, such that $\mu_t^{R^g_{\min}} > 0$, and therefore the constraint (32) for $i = g$ is slack, such that $\mu_t^{R^g_{\max}} = 0$. It follows that

$$\mu_t^{R^g_{\min}} = -\lambda_t^{p^g} \frac{\psi_g w_t}{A_t^g} > 0 \Leftrightarrow \lambda_t^{p^g} < 0,$$

which has to be verified ex post. Once the first-best equilibrium becomes feasible, it is still the case that $R_t^g = 1$, but this is actually an *interior* solution, because the central bank would *not* want to have a lower value of $R_t^g$ even if that was feasible. Therefore, both $1 \leq R_t^g$ and constraint (32) for $i = g$ are slack after that, such that $\mu_t^{R^g_{\min}} = \mu_t^{R^g_{\max}} = 0$, which in turn requires $\lambda_t^{p^g} = 0$.

Regarding $R_t^f$, under our conjectured solution, the constraint $1 \leq R_t^f$ is always slack (such that $\mu_t^{R^f_{\min}} = 0$), whereas the constraint (32) for $i = f$ binds (such that $\mu_t^{R^f_{\max}} > 0$) in periods in which the carbon tax gap is large enough that the first-best allocation is not feasible. In those periods, it follows that

$$\mu_t^{R^f_{\max}} = \lambda_t^{p^f} \frac{\psi_f w_t}{A_t^f} > 0 \Leftrightarrow \lambda_t^{p^f} > 0,$$

which must also be verified ex post. Finally, in periods in which $\tau_t^{f*} - \tau_t^f$ is small enough that the first-best is feasible, the constraint (32) is slack too, such that $\mu_t^{R^f_{\max}} = 0$, which can only be true if $\lambda_t^{p^f} = 0$.

**Summary.** To summarize, in periods in which $\frac{1}{\psi_f} \frac{\tau_t^{f*} - \tau_t^f}{w_t/A_t^f} > \kappa_f(\psi_f w_t N_t^f - \bar{B}^f)$, we have

$$R^g = 1, R^f = \kappa_f(\psi_f w_t N_t^f - \bar{B}^f), \mu_t^{R^g_{\min}}, \mu_t^{R^f_{\max}} > 0, \mu_t^{R^g_{\max}} = \mu_t^{R^f_{\min}} = 0,$$

whereas in periods in which $\frac{1}{\psi_f} \frac{\tau_t^{f*} - \tau_t^f}{w_t/A_t^f} \leq \kappa_f(\psi_f w_t N_t^f - \bar{B}^f)$, we have

$$R^g = 1, R^f = \frac{1}{\psi_g} \frac{\tau_t^{f*} - \tau_t^f}{w_t/A_t^f}, \mu_t^{R^g_{\min}} = \mu_t^{R^f_{\max}} = \mu_t^{R^g_{\max}} = \mu_t^{R^f_{\min}} = 0.$$

## F. Net zero steady state

Carbon decays extremely slowly in our model, but is not fully permanent. This implies that there is a well-defined steady state in which net emissions become zero, so that the stock of carbon in the atmosphere remains constant thereafter.

Concretely, we assume

$$1 - d_s = \phi_0 (1 - \phi)^s ,$$

for $t \geq 0$, where $1 - \phi_0$ is the share of carbon emissions into the atmosphere that exits it within the same quarter (into the biosphere and the surface oceans), and $\phi$ is the rate at which the remaining share disappears from the atmosphere. Our two-parameter specification is a special case of Golosov et al's (2014), and is motivated by our need to have a well-defined terminal steady state in our forward-looking model. We calibrate the two parameters such that our carbon depreciation structure mimics Golosov et al's as closely as possible over a 300-year horizon (see Figure 2). Figure 11 shows the convergence of atmospheric carbon concentration to its long-run steady state under the alternative scenarios considered in Table 2

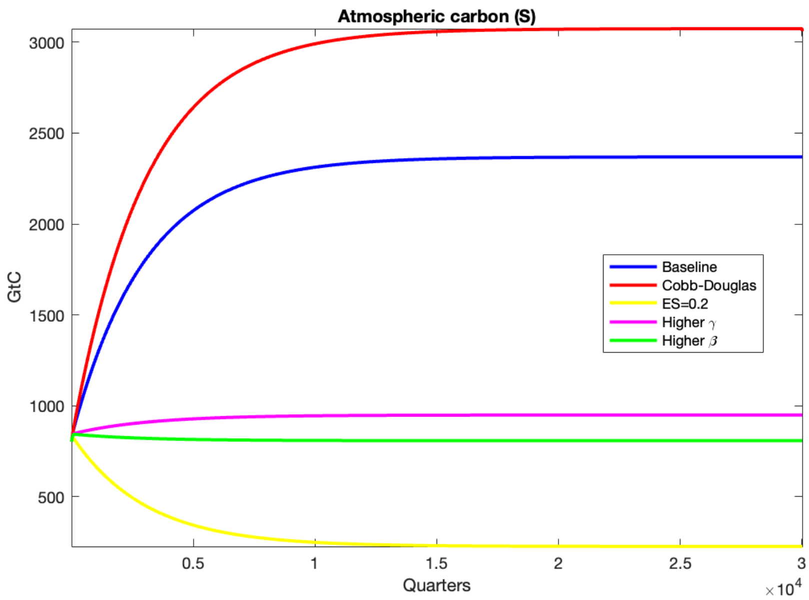


Figure 11: Steady-state carbon concentration (net zero equilibrium)

## G. Larger shocks

Here we show that doubling the size of the cost-push shock results simply in doubling the responses of inflation, output and the nominal interest rate. Similarly, increasing the shock to government spending five times results in scaling up by a factor of five the three response variables.

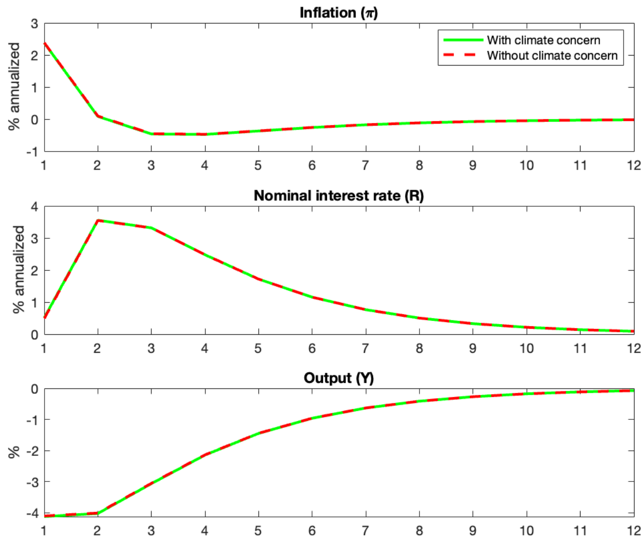


Figure 12: Optimal responses to cost-push shock of double the baseline size

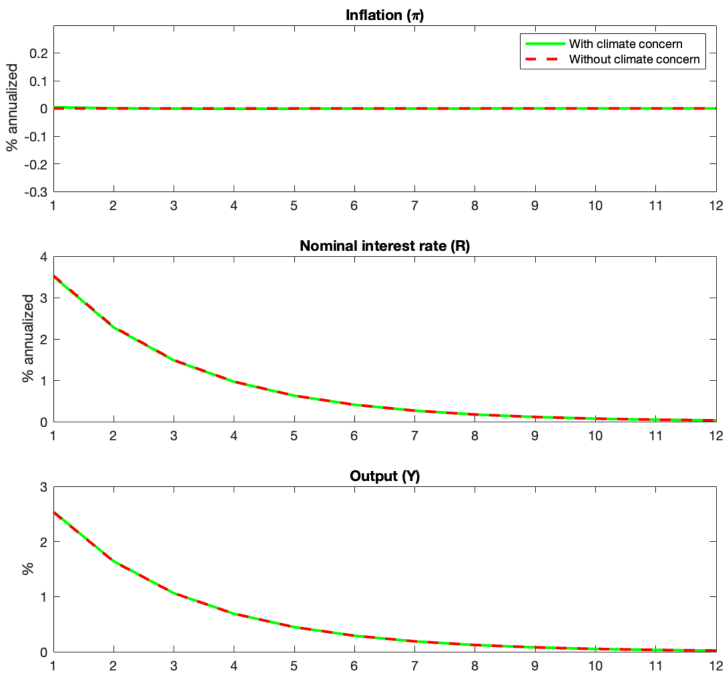


Figure 13: Optimal responses of government spending shock of five times the baseline size