Business Cycles with Pricing Cascades*

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Abstract

Business cycles often feature large shocks to specific sectors, accompanied by pronounced inflationary swings, driven by a growing share of price-adjusting firms. Rationalizing such phenomena requires enhancing our modeling toolkit. We do that by building a novel non-linear dynamic general equilibrium framework containing a disaggregated production economy with networks and optimal decisions on the timing and size of price adjustments. The interaction of our model ingredients creates equilibrium *cascades*: large movements in aggregates trigger additional price adjustment decisions on the extensive margin. Crucially, networks may dampen or amplify cascades, depending on the *type of shock* driving the business cycle. When faced with large *demand* shocks, such as monetary interventions, networks *dampen* cascades, thus slowing down price adjustment decisions and giving central banks substantial power to stimulate the real economy with limited inflationary consequences. In contrast, under aggregate or sector-specific *supply* shocks, networks *amplify* cascades, leading to fast increases in the frequency of repricing and large inflationary swings. Applied to Euro Area data, we show that it is the novel interaction of networks with pricing cascades that allows us to quantitatively match the surges in inflation and the repricing frequency in the post-Covid era.

JEL Classification: E31, E32

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1 Introduction

The dynamics of aggregate prices and quantities over the business cycle has long been a central theme in economics. Recent events, such as the Covid pandemic and the Russian invasion of Ukraine, have brought renewed attention to the topic, along with new evidence on the cyclical properties of key macro-variables. First, following a prolonged period of stability, we have witnessed the possibility of *large* inflationary swings in advanced economies, marked by persistent double-digit rates of price growth in the US, the UK, and the Euro Area. All while the movements in aggregate activity have been milder and transitory. Second, we have learned that much of the inflationary surge has come from rising *frequency* at which firms adjust their prices (Montag and Villar, 2023; Cavallo et al., 2024). Third, large granular shocks, hitting specific *sectors* such as energy and agriculture, have caused significant consequences for the rest of the economy, despite their relatively small share in aggregate activity. Although informative, such evidence cannot be analyzed using existing theoretical frameworks—however influential—that rely on linearized single-sector setups with a constant frequency of adjustment (Woodford, 2004; Galí, 2015). This discrepancy calls for a new framework for studying aggregate prices and quantities over the business cycle, and this paper develops one.

Our novel dynamic general equilibrium framework features a previously unexplored combination of three ingredients. First, a multi-sector structure with a fully unrestricted input-output architecture, allowing to capture empirically realistic production networks. Second, firms making pricing decisions in an optimal state-dependent manner, so that both the extensive and the intensive margins of adjustment are endogenous. Third, a fully non-linear solution strategy, tracing out the response of the economy to arbitrarily large shocks, either aggregate or sector-specific. The *interaction* of our three ingredients delivers a novel theoretical mechanism, namely pricing *cascades*: large movements in aggregates trigger possibly self-reinforcing adjustment decisions at the extensive margin. Crucially, networks can dampen or amplify cascades, depending on the *type of shock* hitting the economy. For demand shocks, such as monetary interventions, networks dampen cascades, leading to muted price responses with a near-constant frequency of adjustment in equilibrium. In contrast, networks amplify cascades following either aggregate or sectoral supply shocks, with strongly non-linear price responses led by the extensive margin. As a result, the novel mechanism of pricing cascades allows our framework to produce realistic monetary non-neutrality, while simultaneously generating substantial inflationary surges following reasonably-sized and structurally-interpretable shocks. When estimated to Euro Area data, the interaction of networks with cascades allows our model to jointly match the surges in inflation and the repricing frequency in the post-Covid era.

The shock-dependent interaction between production networks and pricing cascades is the key novel channel that is unique to our model. The precise workings of the mechanism, as well as its key quantitative implications, are as follows. Under *demand* shocks networks *dampen* cascades by shrinking the magnitudes of desired price changes, hence making firm-level adjustment decision less likely. In an economy with networks, a demand shock affects both the wage, as well as the price of intermediate inputs. As long as markups are countercyclical under demand shocks, the price of intermediates moves by less than the wage, generating smaller changes in marginal costs relative to an economy without networks. Smaller movements in marginal costs attenuate deviations between actual and desired prices, making it less likely that the firm opts for the costly adjustment decision. Such cascades dampening effect has aggregate consequences, namely smaller changes in the overall fraction of adjusters, muted response of aggregate inflation, and stronger response of real GDP. In the version of our model estimated to 39 sectors of the Euro Area economy, the aggregate consequences of cascades dampening are substantial. Following a large expansion in money supply (+10%), the economy with fixed menu costs and networks features a rise in the fraction of adjusters by 0.18, compared to an increase by 0.35 without the linkages. As for monetary non-neutrality, a 1% expansion leads to an impact GDP rise of 0.85% with networks and 0.78% without networks; for the larger shock (+10%), the gap widens: 6% under networks, as opposed to 2.5% without. Hence, cascades dampening leads to substantial additional monetary non-neutrality, which persists even for large shocks and even in the economy with fixed menu costs.

In contrast, networks *amplify* cascades following aggregate or sector-specific *supply shocks*. For example, consider an exogenous contraction in aggregate total factor productivity (TFP). At first, it leads to a one-for-one increase in firm-level marginal costs; on top of that, as long as prices of intermediate inputs also go up in equilibrium, it leads to a further surge in costs. Relative to the economy without networks, such double whammy of marginal cost increases further expands deviations between actual and desired price changes, making the price adjustment decision more likely. Such cascades amplification is not specific to aggregate supply shocks. To see that, consider a TFP shock to a sector that acts as a major supplier of intermediate inputs to the rest of the economy. On top of impacting the shocked sector itself, it also affects marginal costs and hence the desired price changes in the downstream industries, thus setting off cascades there as well. Quantitatively, the amplification of supply-driven cascades by networks has considerable aggregate consequences. Following a large aggregate TFP contraction (-10%), the fraction of adjusters rises by 0.65, as opposed to 0.28 without networks. The latter creates a strong non-linearity in the behavior of aggregate CPI inflation. While a -1% TFP contraction generates 0.59% inflation on impact, a -10% contraction leads to 17% inflation, a surge driven by the sharp increase in the equilibrium adjustment frequency. Beyond aggregate disturbances, we find that large shocks to sectors such as "Chemicals and chemical products", "Food and beverages" and "Crops and animal production" can have a disproportionally large effect on the economy-wide adjustment frequency, and hence generate sharp non-linear inflationary surges.

As a further quantification of the role played by the interaction of networks with cascades, we subject our model to the key structural shocks experienced by the Euro Area economy in the (post-)Covid years (2020-2024), and compare the model-implied dynamics of aggregate inflation and adjustment frequency to that observed in the data. In particular, we feed in four series, corresponding to aggregate demand, aggregate labor wedge, as well as the sectoral dynamics of energy and food prices. We find that the model successfully matches the five percentage point increase in the aggregate repricing frequency, as well as the aggregate inflation surge up to 11% at the peak. In contrast, an otherwise identical model without networks generates at most a one percentage point increase in aggregate repricing frequency, as well as an aggregate inflation surge to only 5% at the peak. As for an economy with networks but time-dependent pricing, it generates no change in adjustment frequency by construction, with a maximum of 7% inflation. These results highlight the quantitative importance of our novel theoretical channel – the interaction of networks with pricing cascades – for explaining aggregate business cycle dynamics.

It also follows that our model is able to generate the empirically-observed surges in inflation and adjustment frequency with structurally-interpretable shocks that are as large as in the data, without compromising the potency to produce realistic non-neutrality of money. Recent work by L'Huillier and Phelan (2023) and Blanco et al. (2024b) stresses that models, which produce reasonable non-neutrality of money and Phillips Curve slope struggle to generate double-digit inflation surges without appealing to shocks that are either unreasonably large or lack a clear structural interpretation, such as cost-push or markup shocks. At the same time, models that match inflation and frequency movements with reasonably-sized shocks imply that money is close to neutral. In this sense, our novel theoretical channel, working through the interaction of production networks and pricing cascades, offers a resolution to this long-standing and first-order issue in the literature without deviating from a conventional price-setting setup.

Contribution to the literature Our paper contributes to at least **three** broad strands of the literature. **First**, we add to the vast literature on state-dependent pricing in macroeconomics; see Costain and Nakov (2024) for a recent survey. Under state-dependent pricing, the probability of a price change is endogenous and affected by idiosyncratic and aggregate shocks, in contrast to time-dependent models such as Taylor (1979) or Calvo (1983). Our main contribution is to the literature on general equilibrium implications of state-dependent pricing, marked by the works of Golosov and Lucas (2007), Gertler and Leahy (2008) and Midrigan (2011) in the context of single-sector models with small aggregate shocks and fixed menu costs.[1] This framework has been further explored analytically by Alvarez et al. (2016), Alvarez and Lippi (2022) and Alvarez et al. (2022), whose results provide model-based sufficient statistics linking the dynamics of macro aggregates to pricing moments that can be measured in firm-level data. Subsequent work also considers one-sector models with state-dependent pricing subjected to *large* aggregate shocks, such as the studies by Karadi and Reiff (2019), Cavallo et al. (2024), Blanco et al. (2024a,b) and Karadi et al. (2024).[2] As for multi-sector models with state-dependent pricing, the seminal work by Nakamura and Steinsson (2010) studies the transmission of monetary shocks in a setup with heterogeneous pricing and roundabout production. More recent papers by Carvalho and Kryvtsov (2021) and Caratelli and Halperin (2023) consider multi-sector frameworks with heterogeneous state-dependent pricing, but without an explicit input-output structure.

We contribute to this literature by developing the first general equilibrium model, which combines a multi-sector setup with a fully general input-output structure, state-dependent pricing and arbitrarily large aggregate and sector-specific shocks. Moreover, we estimate the model to 39 sectors of the Euro Area economy, fully matching the sector-specific pricing moments as well as the input-output structure.

Second, our paper is related to the growing literature on production networks and their role in connecting micro shocks and frictions with aggregate business cycle fluctuations. The seminal work by Acemoglu et al. (2012) considers a flexible-price efficient setup and shows how production networks can amplify sector- or firm-specific shocks to create aggregate volatility. Subsequent work by Baqaee and Farhi (2020) and Bigio and La'O (2020) provides general first-

[1] There are also papers that consider models with menu costs that are random rather than fixed (Dotsey et al., 1999; Nakamura and Steinsson, 2010), or where the price change probability is a *smoothly* increasing function of the gain from adjustment(Caballero and Engel, 2007; Costain and Nakov, 2011), instead of the *step function* it is in the fixed menu cost model.

[2] See also the work of Alexandrov (2020) for the effect of trend inflation on the transmission of large shocks in state-dependent pricing models.

order aggregation results for microeconomic shocks and distortions in economies with inefficiencies and networks. As for the propagation of large shocks in flexible-price network economies, it is studied in Baqaee and Farhi (2019). A separate strand of this literature analyzes linearized models with production networks and *time-dependent* pricing, both positively (Pasten et al., 2020; Ghassibe, 2021; Afrouzi and Bhattarai, 2023) and normatively (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023).

We contribute to this literature by showing that the *interaction* of production networks and state-dependent pricing creates a novel source of non-linearity in aggregate business cycles, through pricing *cascades* created by large aggregate or sector-specific shocks. Our results stress that both the source of the shock, either demand- or supply-side, as well as its sectoral original matter for aggregate fluctuations and the degree of non-linearity. In case of sectoral supply shocks, the position of the industry in the network can matter over an above its equilibrium size, against the network-irrelevance results established in the prior literature (Hulten, 1978).

Third, we contribute to the literature that aims to explain the observed time series of aggregate activity and inflation through the lens of general equilibrium models subjected to structural shocks. The seminal work by Smets and Wouters (2003, 2007) estimates a rich dynamic stochastic general equilibrium model with full-information Bayesian techniques, obtaining a decomposition of key macroeconomic aggregates in the United States and the Euro Area in terms of aggregate structural shocks. While hugely influential, a key criticism of the approach points to the excessive importance of wage and price markup shocks in decompositions, while such disturbances have ambiguous microfoundations (Chari et al., 2009). Subsequent work by Galí et al. (2012) addresses the criticism by enriching the estimated model with involuntary unemployment. More recent papers consider multi-sector setups with networks, allowing for a role of industry-specific shocks in explaining aggregate fluctuations. In particular, Rubbo (2024) finds that in the US, sectoral shocks account for most of the deflation and subsequent inflation in the immediate aftermath of Covid, while aggregate factors explain most of the price surge in 2021 and beyond. In a multi-country study allowing for international spillovers, Di Giovanni et al. (2023) attribute inflation to 2020 to supply chain bottlenecks, while assigning a major role to both aggregate shocks and energy prices in the subsequent periods.

With our quantitative exercise for the Euro Area in the (post-)Covid era, we contribute by showing that one can explain both the surge in inflation *and* and the rise in the aggregate adjustment frequency with four structural shocks: aggregate demand, aggregate labor wedge, as well as sectoral shocks to energy and food prices. Crucially, we find that network amplification of pricing cascades following the sector-specific commodity price shocks is critical for the quantitative fit.

Roadmap The remainder of the paper is structured as follows. Section 2 outlines the optimization problem faced by each type of agent in the economy and the numerical strategy to solve the equilibrium dynamics. Section 3 explains the key model mechanisms in a simplified version of our setup. Section 4 outlines our procedure for estimating the structural parameters of the model to match key sectoral micro-pricing moments for the Euro Area. Section 5 shows our quantitative results for monetary shocks. Section 6 turns to quantitative results for aggregate and sector-specific TFP shocks. Section 7 considers extensions to our baseline results. Section 8 describes our quantification exercise, where we assess the ability of our model to explain the aggregate dynamics of inflation and repricing frequency in the Euro Area. Section 9 concludes.

2 Model

We begin by introducing our theoretical model, which presents a novel combination of three key ingredients. First, it features a number of sectors populated by firms interconnected by production networks, which facilitate trade in intermediate inputs, both within and across sectors. Second, firms make optimal pricing decisions subject to menu costs. Third, we allow for both aggregate, sector-specific and firm-level shocks, and present a numerical strategy that allows to compute the economy-wide equilibrium dynamic response to an arbitrarily large disturbance of any origin.

2.1 Overview

Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is populated by three (types of) agents: households, firms and the government. There is a continuum of identical households, each consuming output and supplying labor. Firms are subdivided into N sectors, indexed by $i \in \{1, 2, ..., N\}$, each sector containing a continuum of monopolistically competitive firms of measure one; we use Φ_i to denote the set of all firms in sector i. The government consists of the central bank, which conducts policy by setting money supply, and the fiscal authority, which collects taxes from firms and rebates them to households in a lump-sum fashion.

2.2 Households

The representative household chooses a sequence of consumption, labor supply, and one-period nominal bond holdings to maximize expected lifetime utility:

$$\max_{\{C_t, L_t, B_t\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{1}$$

subject to the period-by-period budget constraint

$$P_t^C C_t + \mathbb{E}_t\{\Lambda_{t,t+1} B_{t+1}\} \quad \leq \quad B_t + W_t L_t + \sum_{i=1}^{N} \int_0^1 D_{i,t}(j) dj + T_t, \tag{2}$$

where C_t is consumption, L_t is labor supply, B_t is the level of nominal bond holdings, T_t is the level of lump-sum transfers from the government, $D_{i,t}(j)$ are the dividends received lump-sum from firm j in sector i at time t, $\Pi_t^C = \left(P_t^C / P_{t-1}^C\right)$ is the gross CPI inflation rate, W_t is the nominal wage and $\Lambda_{t,t+1}$ is the nominal stochastic discount factor of the household.

Total final consumption C_t is given by an aggregator over sector-specific varieties:

$$C_t = \mathcal{D}(C_{1,t}, ..., C_{N,t}) \tag{3}$$

where $\mathcal{D}(\cdot)$ is homogeneous of degree one and non-decreasing in each of the arguments. The household chooses consumption of each of the sector-specific varieties to minimize total expenditure $\sum_i P_{i,t} C_{i,t}$, subject to the aggregator in (3). The minimal cost of assembling such a basket of sectoral varieties aggregating to $C_t = 1$ pins down the consumption price index as $P_t^C = \mathcal{P}^C(P_{1,t}, ..., P_{N,t})$, where $\mathcal{P}^C$ is homogeneous of degree one and non-decreasing in each of the arguments.

Sectoral final consumption $C_{i,t}$ is in turn given by the following aggregator over firm-specific varieties:

$$C_{i,t} = \left\{ \int_0^1 [\zeta_{i,t}(j) C_{i,t}(j)]^{\frac{\epsilon-1}{\epsilon}} dj \right\}^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

where $\epsilon > 1$ is the within-sector elasticity of substitution, $C_{i,t}(j)$ is the final demand for the output of firm $j \in [0, 1]$ in sector i at time t, and $\zeta_{i,t}(j)$ is a firm-specific idiosyncratic *quality* process. The quality process follows a random walk in logs:

$$\log \zeta_{i,t}(j) = \log \zeta_{i,t-1}(j) + \sigma_i \varepsilon_{i,t}(j), \tag{5}$$

where $\varepsilon_{i,t}(j)$ is an *i.i.d.* Gaussian innovation with mean zero and standard deviation of one. The final demand for firm j in sector i is given by:

$$C_{i,t}(j) = \zeta_{i,t}(j)^{\epsilon-1} \left(\frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon} C_{i,t}, \tag{6}$$

and the sectoral price index of sector i is given by:

$$P_{i,t} = \left[\int_0^1 \left(\frac{P_{i,t}(j)}{\zeta_{i,t}(j)} \right)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}. \tag{7}$$

The representative household is also subject to a cash-in-advance constraint, which requires that the nominal money holdings are sufficient to cover the aggregate nominal final demand:

$$P_t^C C_t \leq M_t. \tag{8}$$

The aggregate money supply process $\{M_t\}_{t\geq 0}$ is set by the central bank, and agents treat this process as exogenous. An alternative is to consider a central bank which conducts monetary policy by setting the nominal interest rate according to a Taylor rule; we consider such an extension in Section 7.1.

We now specify the functional forms for household preferences. First, for household preferences over aggregate consumption and labor supply, we use the log-linear preferences of Golosov and Lucas (2007):

Assumption 1 (Golosov-Lucas preferences)**.** *The utility function over consumption and labor supply is log-linear:* $u(C_t, L_t) = \log C_t - L_t$*.*

Under such preferences, we obtain the following intra-temporal labor supply condition: $\frac{W_t}{P_t^C} = C_t$. When combined with the cash-in-advance constraint (8), it implies that the nominal wage equals money supply in every period: $W_t = M_t$. In addition, the nominal stochastic discount factor satisfies: $\Lambda_{t,t+1} = \beta \frac{P_t^C C_t}{P_{t+1}^C C_{t+1}} = \beta \frac{M_t}{M_{t+1}}$.

As for aggregation across final consumption of sectoral varieties, in the baseline model we assume it to take the Cobb-Douglas form:

Assumption 2 (Consumption aggregation)**.** *The consumption aggregator* $\mathcal{D}(\cdot)$ *is given by:*

$$\mathcal{D}(C_{1,t}, ..., C_{N,t}) = \iota^C \prod_{i=1}^{N} C_i^{\overline{\omega}_{i,t}^C}, \tag{9}$$

where $\iota^C \equiv \prod_{i=1}^N \overline{\omega}_i^{C - \overline{\omega}_i^C}$ *is a normalization term and* $\sum_i \overline{\omega}_i^c = 1$, $\overline{\omega}_i^c \geq 0, \forall i$.

Under this assumption, the equilibrium sectoral final consumption shares are constant over time: $\omega_{i,t}^C \equiv \frac{P_{i,t}C_{i,t}}{P_t^C C_t} = \overline{\omega}_i^C$. In Section 7.3 we consider a more general CES aggregator over sectoral consumption varieties.

2.3 Firms: production

The production function of firm j in sector i is given by:

$$Y_{i,t}(j) = \frac{1}{\zeta_{i,t}(j)} \times A_{i,t} \times \mathcal{F}_i \left[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j) \right], \tag{10}$$

where $\mathcal{F}_i(\cdot)$ is homogeneous of degree one and non-decreasing in inputs; $L_{i,t}(j)$ is the labor used by firm j in sector i at time t, $X_{i,k,t}(j)$ is intermediate inputs bought by firm j in sector i from sector k at time t. In addition, $A_{i,t}$ is an exogenous sector-specific total factor productivity process, while $\zeta_{i,t}(j)$ is the firm-level idiosyncratic quality process introduced in (5).

The intermediates demand $X_{i,k,t}(j)$ is in turn an aggregator over intermediates bought from each firm in sector k:

$$X_{i,k,t}(j) = \left\{ \int_0^1 \left[\zeta_{k,t}(j') X_{i,k,t}(j,j') \right]^{\frac{\epsilon-1}{\epsilon}} dj' \right\}^{\frac{\epsilon}{\epsilon-1}}, \tag{11}$$

where $X_{i,k,t}(j,j')$ is intermediates bought by firm j in sector i from firm j' in sector k, which satisfies the following demand condition in equilibrium: $X_{i,k,t}(j,j') = \zeta_{k,t}(j')^{\epsilon-1} \left(\frac{P_{k,t}(j')}{P_{k,t}} \right)^{-\epsilon} X_{i,k,t}(j)$.

Each firm chooses its labor and intermediate inputs in order to minimize the total cost of production, subject to the production technology in (10). The latter delivers the following marginal cost function for firm j in sector i at time t:

$$MC_{i,t}(j) = \zeta_{i,t}(j) \times \mathcal{Q}_i(W_t, P_{1,t}, ..., P_{N,t}; A_{i,t}) \tag{12}$$

where $\mathcal{Q}_i(\cdot)$ is the common component of the marginal cost index for all firms within a sector, which strictly falls in $A_{i,t}$ and is homogeneous of degree one and non-decreasing in the prices of all inputs.

In our baseline model, we assume that production technology takes a Cobb-Douglas form for all firms in all sectors:

Assumption 3 (Production technology)**.** *The production technology $\mathcal{F}_i(\cdot)$ for a firm j in sector i is given by:*

$$\mathcal{F}_i[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j)] = \iota_i L_{i,t}(j)^{\overline{\alpha}_i} \prod_{k=1}^{N} X_{i,k,t}(j)^{\overline{\omega}_{ik}}, \tag{13}$$

where $\iota_i \equiv \overline{\alpha}_i^{-\overline{\alpha}_i} \prod \overline{\omega}_{ik}^{-\overline{\omega}_{ik}}$ is a normalization term and $\overline{\alpha}_i + \sum_i \overline{\omega}_{ik} = 1$, $\overline{\alpha}_i$, $\overline{\omega}_{ik} \geq 0, \forall i$.

Under this assumption, the equilibrium labor cost shares and the input-output cost shares are constant over time and the same for all firms within a sector: $\alpha_{i,t} \equiv \frac{W_t L_{i,t}(j)}{MC_{i,t}(j)Y_{i,t}(j)} = \overline{\alpha}_i$, $\omega_{i,k,t} \equiv \frac{P_{k,t}X_{i,k,t}(j)}{MC_{i,t}Y_{i,t}(j)} = \overline{\omega}_{ik}$. As with household preferences, in Section 7.3 we relax the Cobb-Douglas assumption and consider a more general CES production function.

2.4 Firms: equilibrium size

The goods market clearing condition for firm j in sector i is given by:

$$Y_{i,t}(j) = C_{i,t}(j) + \sum_{k=1}^{N} \int_0^1 X_{k,i,t}(j', j)dj'. \tag{14}$$

Aggregating up to the level of sectors, multiplying both sides by P_i and dividing by aggregate final nominal demand $P_t^C C_t$, one can express the sectoral sales share (Domar weight) $\lambda_i \equiv \frac{P_{i,t}Y_{i,t}}{P_t^C C_t}$ as:

$$\lambda_{i,t} = \omega_{i,t}^C + \sum_{k=1}^{N} \omega_{ki,t}\lambda_{k,t} \times \mu_{k,t}^{-1}, \tag{15}$$

where μ_k^{-1} is the sales-weighted harmonic average of firm-level markups in a sector $k : \mu_{k,t}^{-1} = \int_0^1 \frac{1}{\mu_{k,t}(j')} \times \frac{P_{k,t}(j)Y_{k,t}(j)}{P_{k,t}Y_{k,t}} dj$. Using the downward sloping demand condition for each firm, one can rewrite $\mu_{k,t}^{-1}$ as:

$$\mu_{k,t}^{-1} = \frac{\Delta_{k,t}}{\mathcal{M}_{k,t}}, \qquad \Delta_{k,t} \equiv (P_{k,t}/M_t)^{\epsilon} \int_0^1 \left(\frac{P_{k,t}(j')}{\zeta_{k,t}(j')M_t} \right)^{-\epsilon} dj', \qquad \mathcal{M}_{k,t} \equiv \frac{P_{k,t}}{\mathcal{Q}_{k,t}}, \tag{16}$$

where $\Delta_{k,t}$ is a measure of price dispersion within the sector and $\mathcal{M}_{k,t}$ is a measure of sectoral markup. Stacking the equation for sales shares across sectors, we can write it as:

$$\boldsymbol{\lambda}_t = \boldsymbol{\omega}_{\boldsymbol{C},t} + \tilde{\Omega}_t^T \boldsymbol{\lambda}_t \quad \Longrightarrow \quad \boldsymbol{\lambda}_t = (I - \tilde{\Omega}_t^T)^{-1} \boldsymbol{\omega}_{\boldsymbol{C},t} \tag{17}$$

where $\tilde{\Omega}_t$ is a $N \times N$ matrix whose $[i,j]$ entry is given by $[\tilde{\Omega}_t]_{i,j} = \omega_{ij,t}\left\{\frac{\Delta_{i,t}}{\mathcal{M}_{i,t}}\right\}$. Having calculated the sectoral sales shares, one obtains the sectoral total output as $Y_{i,t} = \lambda_{i,t} \times M_t/P_{i,t}$ and then the size of an individual firm as $Y_{i,t}(j) = \zeta_{i,t}(j)^{\epsilon-1}\left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\epsilon} Y_{i,t}$.

2.5 Firms: pricing

The nominal profit of firm j in sector i at time t is given by:

$$D_{i,t}(j) = [(1-\tau_{i,t})P_{i,t}(j) - MC_{i,t}(j)] \times Y_{i,t}(j), \tag{18}$$

where $\tau_{i,t}$ is an exogenous sector-specific and time-varying sales tax levied by the government.[3] Denoting by $\tilde{P}_{i,t}(j) \equiv \frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t}$ the firm's quality-adjusted *real* price and by $\tilde{P}_{i,t} \equiv \frac{P_{i,t}}{M_t}$ the sectoral *real* price index, we can write the firm-level *real* profits $\tilde{D}_{i,t}(j) \equiv \frac{D_{i,t}(j)}{M_t}$ as:

$$\begin{aligned}\tilde{D}_{i,t}(j) &= \left(\frac{P_{i,t}}{M_t}\right)^{\epsilon-1} \times \left[(1-\tau_{i,t})\frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t} - \frac{\mathcal{Q}_{i,t}}{M_t}\right] \times \left(\frac{P_{i,t}(j)}{\zeta_{i,t}(j)M_t}\right)^{-\epsilon} \times \lambda_{i,t} \\ &= \tilde{D}\left(\tilde{P}_{i,t}(j), \tau_{i,t}, \left\{\tilde{P}_{k,t}, \Delta_{k,t}, A_{k,t}\right\}_{k=1}^{N}\right).\end{aligned} \tag{19}$$

Note that keeping track of the firm-level real profits requires knowing the firm's real quality-adjusted price, the own sectoral sales tax, as well as the real sectoral prices, price dispersions and productivities of all sectors in the economy.

Resetting the *nominal* price $P_{i,t}(j)$ involves the firm paying a sector-specific and possibly time-varying menu cost $\kappa_{i,t}$ measured in units of labor. The optimal reset price maximizes the firm's value, taking into account that this new price may not change for some period of time. In particular, when the nominal price does not change, the log of quality-adjusted real price $p_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j)$ evolves according to

$$p_{i,t}(j) = p_{i,t-1}(j) + \log\left(\frac{P_{i,t-1}(i)}{\zeta_{i,t}(j)M_t}\right) - \log\left(\frac{P_{i,t-1}(j)}{\zeta_{i,t-1}(j)M_{t-1}}\right) = p_{i,t-1}(j) - \sigma_i\varepsilon_{i,t} - m_t, \tag{20}$$

where $m_t \equiv \Delta \log M_t$.

Without loss of generality, let $\eta_{i,t}(p)$ denote the probability that a firm in sector i with a quality adjusted log relative price p resets its price at t . Consider a firm with a real quality

[3]The proceeds of these taxes are then rebated to households as a lump-sum transfer $T_t = \sum_{i=1}^{N}\tau_{i,t}\int_0^1 P_{i,t}(j)Y_{i,t}(j)dj$.

adjusted price p at the end of period t, and let $p_+ \equiv (p - \sigma_i \varepsilon_{i,t+1}(j) - m_{t+1})$. Then this firm's real value at the end of period t is given by the following Bellman equation:

$$V_{i,t}(p) = \tilde{D}_{i,t}(p) + \beta \mathbb{E}_t \left[\{1 - \eta_{i,t+1}(p_+)\} V_{i,t+1}(p_+) + \eta_{i,t+1}(p_+) \left(\max_{p'} V_{i,t+1}(p') - \kappa_{i,t+1} \right) \right], \tag{21}$$

which consists of the current period real profits $\tilde{D}_{i,t}(p)$, as well as the discounted expected continuation value. The latter is computed taking into account that at time $t+1$ the nominal price does not change with probability $1 - \eta_{i,t+1}(\cdot)$, whereas with probability $\eta_{i,t+1}(\cdot)$ the firm pays the menu cost and optimally resets the nominal price.

Our formulation of the pricing problem covers a wide range of existing models of price setting, corresponding to the different functional forms of $\eta_{i,t}(\cdot)$. In the baseline setup of our model, we consider a specific functional form for the probability of adjustment function $\eta_{i,t}(\cdot)$. In particular, following Golosov and Lucas (2007), we assume that a firm adjusts if and only if the value gain from adjustment in a given period exceeds the menu cost:

Assumption 4 (*Ss* pricing)**.** *Consider a firm in sector i with the quality adjusted log relative price p at time t. Then the probability that this firm adjusts its nominal price is given by:*

$$\eta_{i,t}(p) = \mathbf{1}(L_{i,t}(p) > 0) \tag{22}$$

where $\mathbf{1}(\cdot)$ is the indicator function, and

$$L_{i,t}(p) = \max_{p'} V_{i,t}(p') - V_{i,t}(p) - \kappa_{i,t} \tag{23}$$

is the gain from adjustment (or loss from inaction), net of the menu cost.

Note that although here we specify a problem of *price* setting under nominal rigidities, our setup can automatically handle rigidities in nominal *wage* setting as well by appropriately parameterizing the input-output structure. In particular, consider a setup with a sector called the labor union (LU), such that it only uses labor in production ($\overline{\alpha}_{LU} = 1$) and moreover it is the only sector purchasing labor directly from households ($\overline{\alpha}_{-LU} = 0$). Instead, other sectors purchase labor indirectly from the labor union as an intermediate input, such that $\overline{\omega}_{i,LU}$ represents the empirical cost share of labor for sector i. Then any rigidities in the price setting of the labor union sector are isomorphic to nominal wage rigidities. Moreover, the gap between the nominal wage W_t and the price index of the labor union sector $P_{LU,t}$ has a natural interpretation as the aggregate *labor wedge*.[4]

2.6 Equilibrium definition and solution method

In addition to the goods market clearing condition in (14), the equilibrium in our economy is also characterized by clearing of the labor market:

$$L_t = \sum_{i=1}^{N} \int_0^1 L_{i,t}(j)dj + \sum_{i=1}^{N} \kappa_{i,t} \int_0^1 \eta_{i,t}(p_{i,t}(j))dj, \tag{24}$$

as well as by clearing in the market for bonds, which are in zero net supply: $B_t = 0$.

Having specified the optimality and market clearing conditions, we can now formally define the decentralized equilibrium in our economy:

Definition 1 (Equilibrium)**.** *The equilibrium is a collection of prices* $\{P_{i,t}(j)|j \in \Phi_i\}_{i=1}^N$*, allocations* $\left\{Y_{i,t}(j), L_{i,t}(j), C_{i,t}(j), \{X_{i,r,t}(j,j')|j' \in \Phi_r\}_{r=1}^N |j \in \Phi_i\right\}_{i=1}^N$*, wage* W_t *and bond holdings* B_t*, which given the realizations of firm-level quality process* $\{\zeta_{i,t}(j)|j \in \Phi_i\}_{i=1}^N$*, sectoral productivities* $\{A_{i,t}\}_{i=1}^N$*, sectoral sales tax rates* $\{\tau_{i,t}\}_{i=1}^N$ *and money supply* M_t *satisfy agent optimization and market clearing conditions in every period.*

We now briefly outline our solution strategy, which we use to compute equilibrium prices and quantities given the realizations of exogenous processes. Full details of the numerical strategy are given in Appendix C.

As a first step, we compute the steady-state of our economy, defined as the equilibrium evaluated at the point where money supply growth and sectoral TFPs are at their unconditional mean values, and the firm-level prices are in their stationary distribution. In particular, for each sector we numerically solve the stationary Bellman equation and firms' price distributions on an evenly spaced grid of log quality adjusted real prices with step size Δp, $p_j \in \left[\underline{p}, \underline{p} + \Delta p, ..., \overline{p}\right]$, $j = 1, .., J$ grid points, so that $V_j = V(p_j)$. In the algorithm, introduced in Appendix C, we jointly search across firm-level prices in each sector and sector-specific sales taxes $\{\overline{\tau}_i\}_{i=1}^N$, so that we satisfy the equilibrium conditions and obtain steady-state real sectoral price indices equal to one.

Next, we compute the non-linear responses to a sequence of monetary and TFP shocks. We operate under the assumption of perfect foresight over aggregate and sectoral exogenous shocks,

[4] More formally, the labor wedge is the gap between the nominal marginal rate of substitution across consumption and labor $P_t^C \times MRS_t^{CL} = W_t$ and the nominal cost of labor faced by firms $P_{LU,t}$.

while maintaining uncertainty over the idiosyncratic innovations. To compute the responses, we first assume that there exists a finite period T, at which the economy is back in steady state. Then, starting from a guess for the sequences of sectoral and aggregate variables, we iterate *backward* from $t = T$ to $t = 0$ to solve for the micro value functions. Having obtained the micro value functions, we iterate *forward* from $t = 0$ to $t = T$, and numerically aggregate to obtain sectoral and aggregate variables. We repeat this backward-forward iteration until convergence. Appendix C formally details the algorithm to perform the backward-forward iteration.

3 Pricing cascades and networks: formal results

We now use a simplified version of our model in order to formally introduce the notion of pricing *cascades*: large movements in aggregates creating possibly self-reinforcing price adjustment decisions at the extensive margin. Moreover, we present analytical results regarding the novel interaction of pricing cascades with networks. In particular, we formally show that networks *dampen* cascades whenever the aggregate cycle is driven by demand shocks, whereas they *amplify* cascades driven by supply shocks. We also present several examples with particular network arrangements in order to solidify the intuition behind our novel theoretical results.

3.1 Static economy

In order to obtain intuition regarding the transmission of large shocks in our model, we consider a simplified setup obtained under two additional assumptions. First, we assume the economy to be static in the sense that agents fully discount the future:

Assumption 5 (Myopia)**.** *Agents fully discount the future in their objective function, so that $\beta = 0$.*

In particular, this setting implies that any firm's value function is simply given by contemporaneous profits, and hence the optimal quality-adjusted real reset price for any firm in a sector i is given by:

$$\tilde{P}^*_{i,t} = \frac{1}{1-\tau_{i,t}} \frac{\epsilon}{\epsilon - 1} \times \frac{1}{A_i} \prod_{k=1}^{N} \tilde{P}_{i,t}^{\overline{\omega}_{ik}} = \Gamma_{i,t} \times \tilde{\mathcal{Q}}_{i,t}. \tag{25}$$

where $\Gamma_{i,t} \equiv \frac{1}{1-\tau_{i,t}} \frac{\epsilon}{\epsilon-1}$ is the (exogenous) desired markup, whose variation across time and sectors is pinned down by the movements in the sectoral tax rates $\tau_{i,t}$.

Second, we assume a specific form of time-variation of the sector-specific menu cost $\kappa_{i,t}$:

Assumption 6 (Sectoral menu costs)**.** *The sector-specific menu cost follows the following process:* $\kappa_{i,t} = \overline{\kappa}_i(1 - \tau_{i,t})[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}]^{\epsilon-1}\lambda_{i,t}$, *where* $\overline{\kappa}_i$ *is a sector-specific constant.*

The above two assumptions allow us to derive closed-form results regarding the interaction between networks, price adjustment decisions at the extensive margin, and the type of shocks hitting the economy.

The decision to change prices is based on whether the value gain from adjustment exceeds the menu cost. In the static setup, we can obtain a tractable approximation for the gain from adjustment as a function of the *price gap*, or the difference between the current and the optimal reset price:

Lemma 1 (Adjustment gains)**.** *Suppose Assumptions 1-5 hold. Let* $\tilde{p}_{i,t}(j) \equiv \log \tilde{P}_{i,t}(j) - \log \tilde{P}^*_{i,t}$ *be the price gap for a firm* j *in sector* i *at time* t*. Then the profit gain from price adjustment satisfies:*

$$\tilde{D}^*_{i,t}(j) - \tilde{D}_{i,t}(j) \quad = \quad \frac{1}{2}(\epsilon - 1)(1 - \tau_{i,t})[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}]^{\epsilon-1}\lambda_{i,t} \times [\tilde{p}_{i,t}(j)]^2 \quad + \quad \mathcal{O}[\tilde{p}_{i,t}(j)]^3 \tag{26}$$

where $\tilde{D}^*_{i,t}(j)$ *is profits at the optimal reset price,* $P_{i,t}$ *is the real sectoral price index and* $\lambda_{i,t}$ *is the sectoral sales share (Domar weight).*

To illustrate the interaction between networks and price adjustment decisions, consider the initial period ($t = 0$) in our economy. Unless specified otherwise, for notational convenience we drop the time subscript for all the variables at $t = 0$. If the firm chooses to not adjust its nominal price, then the quality-adjusted real price in the initial period is given by:

$$\log \tilde{P}_i(j) = p_{i,-1}(j) - \sigma_i\varepsilon_i(j) - m \tag{27}$$

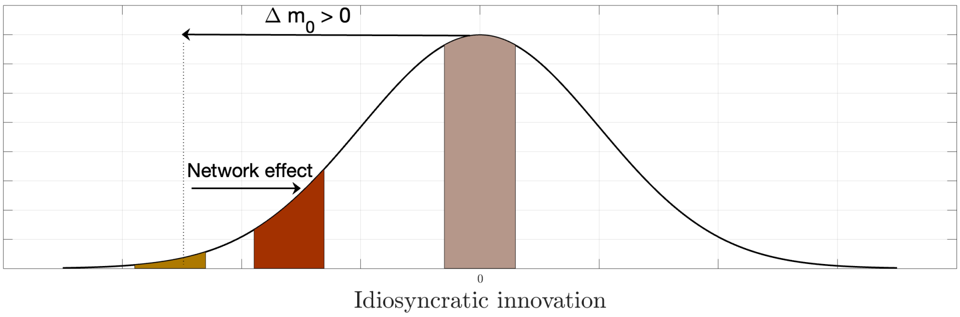
where $p_{i,-1}(j)$ is the initial (exogenous) quality-adjusted real price of firm j in sector i, $\varepsilon_i(j)$ is the realization of the firm-level quality shock in period $t = 0$, and $m \equiv \log(M/M_{-1})$ is the realization of money growth at $t = 0$. Given the expression for the optimal reset price in (25), we can write the firm-level price gap in the initial period as:

$$\tilde{p}_i(j) = -\sigma_i\varepsilon_i(j) - m - \gamma_i + a_i - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_k + (p_{i,-1}(j) - \overline{\gamma}_i). \tag{28}$$

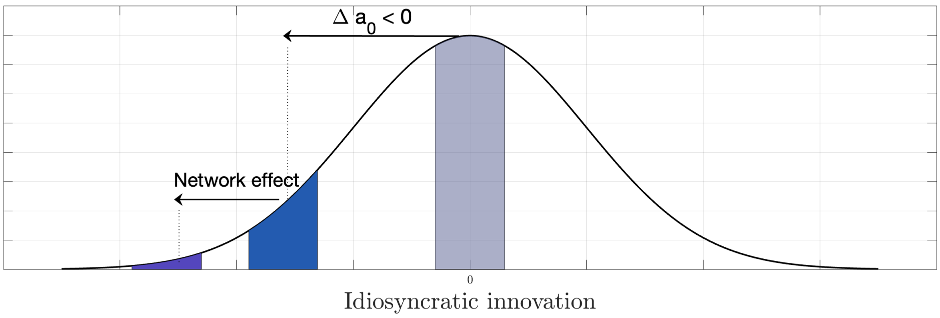
where $\overline{\gamma}_i \equiv \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$, $a_i \equiv \log A_i$ and $\gamma_i \equiv \log \Gamma_i - \overline{\gamma}_i$.

Figure 1: **Networks and inaction regions**

(a) Cascades dampening under monetary shocks



(b) Cascades amplification under TFP/markup shocks



Notes: Panel (a) considers a monetary expansion and visualizes the effect of production networks on the location of the inaction region; Panel (b) considers an aggregate TFP contraction, similarly showing the effect of production networks on the location of the inaction region.

Without loss of generality, normalize $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}$; then given the realizations of aggregate and sectoral variables, the magnitude of the price gap of the specific firm j is pinned down by the realization of its idiosyncratic quality innovation $\varepsilon_i(j)$. We can use the approximate profit gain in Lemma 1 to determine the sector-specific *inaction regions*, defining the ranges for idiosyncratic innovations under which the firm will choose not to adjust:

Lemma 2 (Inaction region)**.** *Suppose Assumptions 1-6 hold. Given the realizations of aggregate and sectoral variables and normalizing* $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$*, let* $\underline{\varepsilon}_i$ *and* $\overline{\varepsilon}_i$ *be thresholds such that a firm in sector* i *will not adjust the price if it draws an innovation in* $[\underline{\varepsilon}_i, \overline{\varepsilon}_i]$*. Then,*

$$[\sigma_i \underline{\varepsilon}_i, \quad \sigma_i \overline{\varepsilon}_i] \quad = \quad -m - \gamma_i + a_i - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_k \quad \pm \quad \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}, \tag{29}$$

where $\quad m \equiv \log(M/M_{-1}), \quad \gamma_i \equiv \log \Gamma_i - \overline{\gamma}_i, \quad a_i \equiv \log A_i \quad$ *and* $\quad \overline{\gamma}_i \equiv \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$.

Given the realizations of monetary, productivity, and desired markup shocks, which are independent of the presence of input-output linkages, we can now derive the effect of removing input-output linkages ($\overline{\omega}_{i,k} = 0, \forall i, k$) on the firm-level decision to adjust its price.

Monetary shocks Consider an increase in the money supply $m > 0$. According to Lemma 2, this increase in money supply, *ceteris paribus*, implies a leftward shift of the inaction region. In other words, more extreme (negative) realizations of idiosyncratic innovations are needed to prevent adjustment. At the same time, Lemma 2 also implies that as long as the pass-through of the money supply to sectoral prices is incomplete ($\log \tilde{P}_k < 0, \forall k$), the presence of networks attenuates the leftward shift of the inaction region for all firms that have a non-zero cost share of intermediate inputs. As a result, this weakly lowers the probability of price adjustment for any firm, creating *dampening* in price changes. Panel (a) of Figure 1 provides a graphical illustration of this mechanism.

To formalize such interaction of networks and the extensive margin under monetary shocks, the following proposition links the firm-level probability of adjustment to the monetary shock size and input-output characteristics:

Proposition 1. *Suppose Assumptions 1-6 hold, and further normalize* $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1} \frac{1}{1-\overline{\tau}_i}$. *Let* ϱ_i *be the probability that a firm in sector* i *decides to adjust its price in the initial period* $(t = 0)$*. For a monetary shock* m*, denote by* $\Delta\varrho_i(m)$ *the change in the adjustment probability relative to its steady-state value, then:*

$$\frac{1}{\phi_i}\Delta\varrho_i(m) \quad \approx \quad \Big[m \quad + \quad \overline{\mathcal{M}} \times \mathcal{C}_i \quad + \quad N \times Cov\left((\overline{\Psi} - I)^{(i)}, \log \boldsymbol{\mathcal{M}}\right)\Big]^2 \tag{30}$$

where $m \equiv \log(M/M_{-1})$ *is the monetary shock,* $\phi_i \equiv -\Phi_i''\left(\sqrt{\frac{2\kappa_i}{\epsilon-1}}\right) > 0$ *and* Φ_i *is CDF of* $\mathcal{N}(0, \sigma_i^2)$, $\overline{\mathcal{M}} \equiv \frac{1}{N}\sum_{i=1}^N \log \mathcal{M}_i$ *is the average of sectoral markup changes and* $\log \boldsymbol{\mathcal{M}} \equiv [\log \mathcal{M}_1, ..., \log \mathcal{M}_N]^T$, $\overline{\Psi}^{(i)}$ *is the i's row of the Leontief inverse matrix* $\overline{\Psi} \equiv (I - \overline{\Omega})^{-1}$*, and*

$$\mathcal{C}_i \quad \equiv \quad \sum_{j=1}^N \overline{\Psi}_{i,j} - 1 \tag{31}$$

*is the **customer centrality** of sector* i.

It follows that the firm-level probability of adjustment is a square function of a sum of three terms. First, the value of the monetary shock m, whose increases or decreases always lead to a rise in the adjustment probability. Second, the *customer centrality* of the sector to which the firm belongs, multiplied by the average sectoral markup. Third, the covariance between the total exposure to other sectors as a customer and the markups in the supplier sectors.

Therefore, as long as markups are countercyclical conditional on monetary shocks, the presence of networks leads to a reduction in firm-level adjustment probability, *ceteris paribus*. This is the formal notion of *cascades dampening*:

Corollary 1 (Cascades and demand shocks)**.** *Suppose Assumptions 1-6 hold. Consider an increase in the money supply $m > 0$. Then, as long as the pass-through of the money supply to sectoral prices is incomplete* ($\log \tilde{P}_k < 0, \forall k$)*, production networks (weakly) lower the probability of adjustment for any firm following the monetary shock.*

Aggregate supply shocks In contrast, consider a sectoral productivity deterioration $a_{i,0} < 0$. According to Lemma 2, this productivity change creates a leftward shift of the inaction region for all firms in sector i. Moreover, as long as this productivity decline leads to a rise in price indices of other sectors ($\log \tilde{P}_k > 0, \forall k$), then Lemma 2 also implies that networks further amplify the leftward shift in the inaction region for all firms in sector i, as long as the cost share of intermediates in that sector is non-zero. In other words, even more extreme (negative) realizations of idiosyncratic innovations are needed to justify non-adjustment. As a result, contrary to the case of monetary shocks, the presence of networks weakly raises the probability of price adjustment for any firm in sector i, thus *amplifying* pricing cascades. Panel (b) of Figure 1 illustrates this mechanism graphically.

We formalize this interaction between the adjustment probability and aggregate supply shocks in the proposition below:

Proposition 2. *Suppose Assumptions 1-6 hold, and further normalize $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$. Let ϱ_i be the probability that a firm in sector i decides to adjust its price in the initial period ($t = 0$). For an aggregate TFP shock a, denote by $\Delta\varrho_i(a)$ the change in the adjustment probability relative to its steady-state value, then:*

$$\frac{1}{\phi_i}\Delta\varrho_i(a) \approx \Big[a + \left(a - \overline{\mathcal{M}}\right) \times \mathcal{C}_i - N \times Cov\left((\overline{\Psi} - I)^{(i)}, \log \boldsymbol{\mathcal{M}}\right)\Big]^2 \tag{32}$$

where $a \equiv \log(A_i/A_{i,-1}), \forall i$ is the aggregate TFP shock, $\phi_i \equiv -\Phi_i''\left(\sqrt{\frac{2\kappa_i}{\epsilon-1}}\right) > 0$ and Φ_i is CDF of $\mathcal{N}(0, \sigma_i^2)$, $\overline{\mathcal{M}} \equiv \frac{1}{N}\sum_{i=1}^N \log \mathcal{M}_i$ is the average of sectoral markup changes and $\log \boldsymbol{\mathcal{M}} \equiv [\log \mathcal{M}_1, ..., \log \mathcal{M}_N]^T$, $\overline{\Psi}^{(i)}$ is the i's row of the Leontief inverse matrix $\overline{\Psi} \equiv (I - \overline{\Omega})^{-1}$, and $\mathcal{C}_i$ is is the customer centrality of sector i introduced in (31).

It follows that the firm-level probability of adjustment is a square function of a sum of three terms. First, the value of the aggregate TFP shock a, whose increases or decreases always lead to a rise in the adjustment probability. Second, the *customer centrality* of the sector to which the firm belongs, multiplied by the difference between the shock size and average sectoral markup change. Third, the covariance between the total exposure to other sectors as a customer and the markups in the supplier sectors. Therefore, as long as all sectoral prices increase after the aggregate TFP shock, the presence of networks leads to a reduction in firm-level adjustment probability, *ceteris paribus*. This is the formal notion of *cascades amplification*:

Corollary 2 (Cascades and supply shocks). *Suppose Assumptions 1-6 hold. Consider a decrease in aggregate TFP $a < 0$. Then, as long as such a shock lead to a rise in price indices of all sectors ($\log \tilde{P}_k > 0, \forall k$), production networks (weakly) increase the probability of adjustment for any firm in any sector.*

Note that an identical mechanism of cascades also applies in the case of shocks to desired markups. Following an increase in desired markups, $\gamma_i > 0$, there is a leftward shift in the inaction region, which is further moved to the left as long as the markup shock is inflationary in the aggregate ($\log \tilde{P}_k > 0, \forall k$).

Sectoral supply shocks Note that a productivity or desired markup shock in a sector k can, in principle, increase the probability of price adjustment for firms in any other sector i. This is true as long as the price indices of sectors used as suppliers by sector i ($j : \overline{\omega}_{ij} > 0$) rises following the productivity deterioration or markup increase in sector k. To formalize this idea, we first derive the cross-sensitivity between the firm-level probability of adjustment in sector i following a TFP shock to sector k:

Lemma 3. *Suppose Assumptions 1-6 hold, and further normalize $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$. Let ϱ_i be the probability that a firm in sector i decides to adjust its price in the initial period ($t = 0$). For a TFP shock specific to sector k, a_k, denote by $\Delta\varrho_i(a_k)$ the change in the adjustment frequency relative to its steady-state value, then:*

$$\frac{1}{\phi_i}\Delta\varrho_i(a_k) \quad \approx \quad \left[\overline{\Psi}_{i,k} \times a_k \quad - \quad \overline{\mathcal{M}} \times \mathcal{C}_i \quad - \quad N \times Cov\left((\overline{\Psi} - I)^{(i)}, \log \boldsymbol{\mathcal{M}}\right)\right]^2 \tag{33}$$

where $a_k \equiv \log(A_k/A_{k,-1}), \forall i$ *is the TFP shock specific to sector k,* $\phi_i \equiv -\Phi_i''\left(\sqrt{\frac{2\kappa_i}{\epsilon-1}}\right) > 0$ *and Φ_i is CDF of $\mathcal{N}(0, \sigma_i^2)$,* $\overline{\mathcal{M}} \equiv \frac{1}{N}\sum_{i=1}^{N} \log \mathcal{M}_i$ *is the average of sectoral markup changes and* $\log \boldsymbol{\mathcal{M}} \equiv [\log \mathcal{M}_1, ..., \log \mathcal{M}_N]^T$, $\overline{\Psi}^{(i)}$ *is the i's row of the Leontief inverse matrix* $\overline{\Psi} \equiv$

$(I-\overline{\Omega})^{-1}$, *and* $\mathcal{C}_i$ *is is the customer centrality of sector* i *introduced in (31).*

One can see that the adjustment probability rises in the total exposure of sector i to inputs supplied by sector k, given by the corresponding entry of the Leontief inverse matrix. At the same time, the probability falls in the customer centrality of sector i, as well as in the covariance between sector i's total exposure to inputs from other sectors and markup movements in the supplier sectors.

Having obtained the cross-sensitivities, we can now solve for the response of average adjustment probability in the economy as a whole to a TFP shock in sector k:

Proposition 3. *Suppose Assumptions 1-6 hold, set* $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}, \overline{\kappa}_i = \kappa, \sigma_i = \sigma, \forall i$ *and assume* $Cov\left((\overline{\Psi}-I)^{(i)}, \log \boldsymbol{\mathcal{M}}\right) = 0, \forall i$. *Let* $\varrho \equiv \frac{1}{N}\sum_{i=1}^{N}\varrho_i$ *be the average probability of adjustment in the initial period. For a TFP shock specific to sector* k, a_k, *denote by* $\Delta\varrho(a_k)$ *the change in the average adjustment probability relative to its steady-state value, then:*

$$\frac{1}{\phi}\Delta\varrho(a_k) \quad\approx\quad \mathcal{H}_k \times a_k^2 \quad - \quad 2\overline{\mathcal{M}} \times \left\{\overline{\mathcal{C}}\mathcal{S}_k + Cov(\overline{\Psi}_{(k)}, \boldsymbol{\mathcal{C}})\right\} \times a_k \quad + \quad \overline{\mathcal{M}}^2\overline{\mathcal{C}^2} \tag{34}$$

where $a_k \equiv \log(A_k/A_{k,-1}), \forall i$ *is the TFP shock specific to sector* k, $\phi \equiv -\Phi''\left(\sqrt{\frac{2\overline{\kappa}}{\epsilon-1}}\right) > 0$ *and* Φ *is CDF of* $\mathcal{N}(0,\sigma^2)$, $\overline{\mathcal{M}} \equiv \frac{1}{N}\sum_{i=1}^{N}\log\mathcal{M}_i$ *is the average of sectoral markup changes,* $\mathcal{C}_i$ *is is the customer centrality introduced in (31) and* $\overline{\mathcal{C}} \equiv \frac{1}{N}\sum_{i=1}^{N}\mathcal{C}_i$, $\overline{\mathcal{C}^2} \equiv \frac{1}{N}\sum_{i=1}^{N}\mathcal{C}_i^2$, $\boldsymbol{\mathcal{C}} \equiv [\mathcal{C}_1, ..., \mathcal{C}_N]^T$, $\overline{\Psi}_{(k)}$ *is the k's column of the Leontief inverse matrix and*

$$\mathcal{H}_k \quad\equiv\quad \frac{1}{N}\sum_{i=1}^{N}\overline{\Psi}_{i,k}^2, \qquad\qquad \mathcal{S}_k \quad\equiv\quad \frac{1}{N}\sum_{i=1}^{N}\overline{\Psi}_{i,k} \tag{35}$$

are, respectively, the ***supplier Herfindahl*** $(\mathcal{H}_k)$ *and* ***supplier centrality*** $(\mathcal{S}_k)$ *of sector* k.

A notable case arises in the near-complete pass-through environment, where $\overline{\mathcal{M}} \approx 0$, so that:

$$\frac{1}{\phi}\Delta\varrho(a_k) \quad\approx\quad \mathcal{H}_k \times a_k^2. \tag{36}$$

3.2 Simple examples

We now solidify the intuition behind the formal results on cascades with the aid of several examples. In particular, we return to the dynamic version of our model, but consider concrete network arrangements in order highlight the key mechanisms. To facilitate further comparability between monetary and TFP shocks, for the remainder of this subsection we assume that both follow AR(1) in levels with persistence $\rho \in (0, 1)$.

***Example 1*: roundabout production economy**

First, we consider a one-sector ($N = 1$) roundabout economy, where firms trade intermediate inputs with other firms in the same sector, as in the work of Basu (1995). Figure 2(a) illustrates such an arrangement graphically. Naturally, in the limit where we set the cost share of labor to one ($\overline{\alpha}_1 = 1$), the one-sector economy collapses to that of Golosov and Lucas (2007), where firms only use labor in production.

We use this simple example to illustrate how the presence of the network affects the response of the aggregate fraction of adjusting firms to monetary and productivity shocks of different sizes. As can be seen in the bottom panel of Figure 2(a), when there are no networks ($\overline{\alpha}_1 = 1$), monetary and productivity shocks are isomorphic in their effect on aggregate frequency. However, as soon as we add the roundabout production structure ($\overline{\alpha}_1 < 1$), the aggregate adjustment frequency responds much faster to productivity shocks relative to monetary shocks. This is because under monetary shocks, the network structure shrinks the desired price changes and the price gaps, thus *dampening* cascades, which leads to slower increases in the aggregate fraction of adjusters. In contrast, under TFP shocks, the presence of networks expands movements in desired price changes and hence the price gaps, thus *amplifying* cascades at the firm-level, leading to faster increases in the aggregate fraction of adjusters.

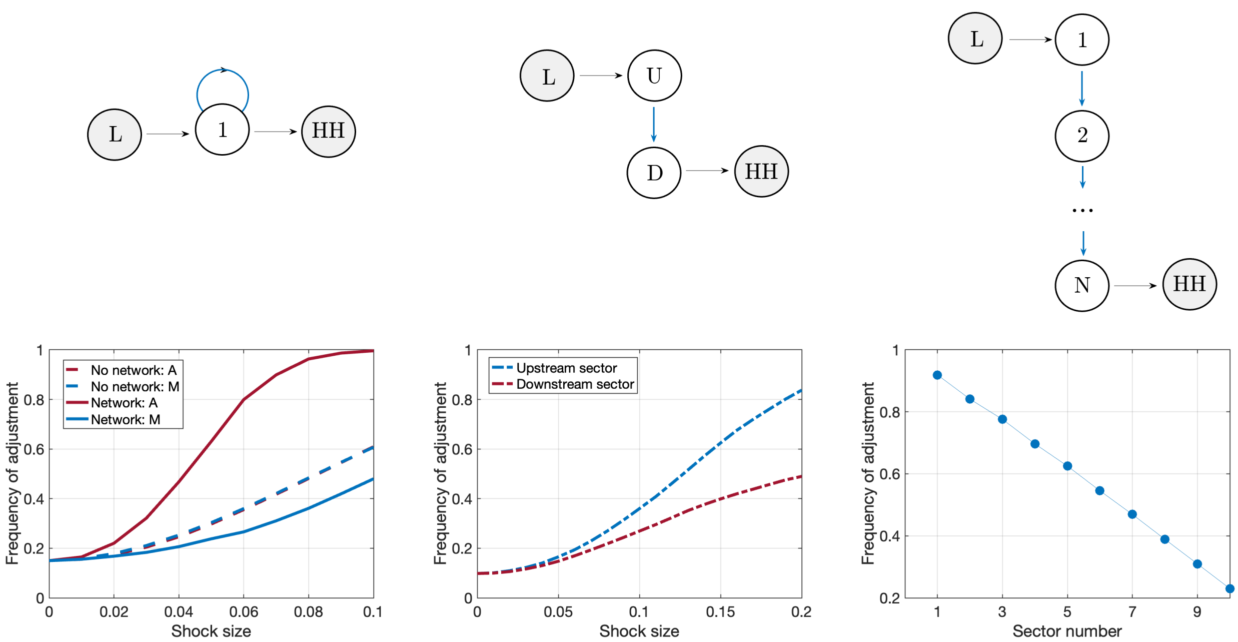
***Example 2*: two-sector vertical chain economy**

For our second example, we consider a two-sector economy ($N = 2$), which illustrates how the position of a sector in the network affects the transmission of sectoral productivity shocks to aggregate frequency. The top panel of Figure 2(b) presents the arrangement graphically: the upstream sector (U) only uses labor in production and supplies its output as an intermediate input to the downstream sector (D). Importantly, the two sectors have the same size in steady-state equilibrium, in the sense of having identical (cost-based) Domar weights. Moreover, their pricing moments are also the same in steady-state, hence their only ex ante difference comes from the position in the network.

We now consider sector-specific productivity shocks of different sizes and record their effect on the aggregate adjustment frequency. In the bottom panel of Figure 2(b) one can see that

Figure 2: **Three example economies**

(a) Roundabout production economy (b) Two-sector vertical chain economy (c) N-sector vertical chain economy



Notes: the figure shows three example economies, as well as the responses of aggregate frequency of adjustment to monetary and aggregate TFP shocks. In each example, all sectors are calibrated to have the same frequency and standard deviation of price changes in steady state.

large shocks to the upstream sector deliver faster increases in the aggregate frequency, relative to equally sized shocks to the downstream sector. This is another way in which networks amplify cascades: shocks to the upstream sector affect the marginal cost, the optimal reset price, and hence the price gaps, of the downstream sector. As a result, shocks to the upstream sector trigger extensive margin price adjustment decisions both in the upstream *and* in the downstream sector. The opposite, however, is not true: shocks to the downstream sector only affect price gaps in the downstream sector itself and do not affect price adjustment decisions in the upstream sector.

This simple example illustrates an important point: when it comes to the effect of a sector-specific shock on aggregates, the position of the shocked sector in the network can matter over and above its size. In particular, here shocks to the upstream sector have a stronger effect on aggregate adjustment frequency, even though it is as large as the downstream sector in steady state. This runs contrary to a number of established network-irrelevance results, where the presence of networks make no difference over and above its effect on equilibrium size (Hulten, 1978; Baqaee and Farhi, 2020).

***Example 3*: N-sector vertical chain economy**

With our third example, we would like to illustrate how the interaction between the network position and pricing cascades extends beyond the two-sector arrangement. In particular, as depicted in the top panel of Figure 2(c), we consider an N-sector vertical chain economy. In such a setup, Sector 1 is the most upstream sector, which uses labor to produce a good that is supplied as an intermediate to Sector 2, which then supplies intermediates to Sector 3 and so on. Sector N is the least upstream sector, as it sells everything it produces as a final good to households. As before, all the N sectors have the same steady-state pricing moments and are equally big in the sense of having identical (cost-based) Domar weights. The only relevant dimension of heterogeneity is their position in the network.

In the bottom panel of Figure 2(c), we set $N = 10$ and plot the aggregate frequency response to large (-20%) sector-specific productivity shock to each sector. One can see that the shock to the most upstream Sector 1 delivers the largest increase in aggregate frequency. Moreover, the aggregate frequency response falls monotonically as we move down the supply chain and consider increasingly less upstream sectors. As before, this represents the interaction of networks with pricing cascades: shocks to more upstream sectors affect, directly or indirectly, marginal costs and hence price gaps in a larger number of sectors, thus triggering a bigger increase in aggregate adjustment frequency.

4 Full model with Euro Area data

We now move to the quantitative analysis of our full dynamic model. In this section we outline the strategy to bring our model to the Euro Area data. In particular, we discipline the structural parameters of the model in order to make it consistent with the Euro Area economy disaggregated to 38 sectors. The household preferences and firms' production function parameters are estimated to match the observed consumption and input-output shares in the World Input-Output Tables. As for the sector-specific menu costs and variances of idiosyncratic shocks, those are estimated to fully match the observed sectoral frequencies and standard deviations of price changes in the PRISMA dataset for the Euro Area.

4.1 Parameterization and Calibration

We discipline the structural parameters of our model to the Euro Area data at monthly frequency. Table 1 summarizes our calibration.

For the aggregate parameters, the households' discount factor is set to $\beta = 0.96^{1/12}$ as in Golosov and Lucas (2007). The within-sector elasticity of substitution across varieties is $\epsilon = 3$ as in Midrigan (2011). We assume that aggregate money supply follows a random walk with drift:

$$\log M_t = \overline{\pi} + \log M_{t-1} + \varepsilon_t^M, \tag{37}$$

where $\overline{\pi}$ is the trend growth rate for money supply, which is also the equilibrium level of trend inflation; ε_t^M is an i.i.d. mean zero money growth innovation. The steady-state money growth rate is $\pi = 2\%$ per year, in line with the inflation target of the European Central Bank (ECB). As for the sectoral total factor productivities, we assume those to follow an AR(1) process:

$$\log A_{i,t} = \rho \log A_{i,t-1} + \varepsilon_{i,t}^A, \tag{38}$$

where $\rho \in (0, 1)$ is the persistence parameter and $\varepsilon_{i,t}^A$ is an i.i.d. mean zero sector-specific productivity innovation. We set the persistence of TFP processes equal to $\rho = 0.9$.

We calibrate our economy to 38 production sectors of the Euro Area economy, following the classification in the World Input-Output Database (WIOD). The final consumption shares $\{\overline{\omega}_i^C\}_{i=1}^N$ and the input-output cost shares $\{\overline{\omega}_{ik}\}_{i,k=1}^N$ are taken from the 2014 input-output tables for the Euro Area based on WIOD.[5] Regarding the sectoral cost shares of labor $\{\overline{\alpha}_i\}_{i=1}^N$, they are taken from the 2014 National Income Accounts for the Euro Area, published by the EU KLEMS database.[6] In order to capture the possibility that wages are also sticky, we introduce an auxiliary labor union sector. In particular, we assume that the labor union sector is the only one that directly purchases labor from households and then sells it to the rest of the sectors as an intermediate input. In general, we work with $N = 39$ sectors: 38 production sectors and the auxiliary labor union sector.

Unlike in Section 3.1 above, we do not allow time variation in the sectoral menu costs. Instead, we consider the more conventional fixed menu cost setup (Golosov and Lucas, 2007), allowing the menu costs to vary in the cross section only:

[5] We make use of the EMuSe Calibration Toolkit developed by Hinterlang et al. (2023), which constructs the Euro Area input-output table by combining accounts of individual countries in the WIOD.

[6] The database is at https://economy-finance.ec.europa.eu/economic-research-and-databases/economic-databases/eu-klems-capital-labour-energy-materials-and-service_en.

Assumption 6′ (Fixed menu costs)**.** *The sector-specific menu cost follows the following process:* $\kappa_{i,t} = \overline{\kappa}_i$*, where* $\overline{\kappa}_i$ *is a sector-specific constant.*

This leaves us with two parameters per sector to estimate: the menu cost $\overline{\kappa}_i$, and the standard deviation of firm-level shocks σ_i. In line with evidence in Gautier et al. (2023), we assume that the sectors "Coke and Petroleum Products" and "Mining and Quarrying" have fully flexible prices at monthly frequency. We calibrate the price setting parameters in the labor union sector to match the frequency and standard deviation of nominal wage changes in Costain et al. (2022). For the remaining 36 sectors, we estimate the parameters $\{\overline{\kappa}_i\}_{i=1}^N$ and $\{\sigma_i\}_{i=1}^N$ to match the frequency and standard deviation of price changes in each sector in the Euro area, taken from Gautier et al. (2024), in steady state.

We also parameterize two auxiliary economies, for the purpose of benchmarking them against our baseline setup. First, we estimate the firm-level pricing parameters in a counterfactual economy without input-output linkages, for the same set of sector-specific frequencies and standard deviations of price changes in steady state. Such an economy features no linkages across the 38 production sectors, which are only linked to the labor union sector instead ($\overline{\omega}_{i,LU} = 1, \forall i \neq LU$). Second, we consider an economy with input-output linkages, but featuring time-dependent price setting as in Calvo (1983). The latter setup corresponds to having constant sector-specific pricing hazards ($\eta_i(p) = \overline{\eta}_i, \forall i$) and zero menu costs ($\overline{\kappa}_i = 0, \forall i$). We therefore estimate sector-specific constant hazards and variances of idiosyncratic shocks to match the same sectoral frequencies and standard deviations of price changes as in the steady state of our baseline setup.

4.2 Sectoral characteristics

In order to better understand the cross-sectional properties of the sectors we consider in our quantitative setup, we introduce two different measures of sectoral *centrality.* First, in order to capture the full degree to which a sector is important as a buyer of intermediate inputs from the rest of the economy, we use the following *customer centrality* metric:

$$\text{Customer Centrality}_i \quad \equiv \quad \sum_{j=1}^{N} (I - \overline{\Omega})^{-1}_{ij} - 1 \tag{39}$$

where $[\overline{\Omega}]_{ij} = \overline{\omega}_{ij}$ is the matrix of input-output cost shares. Intuitively, the customer centrality measure captures the total reliance of a sector on intermediate inputs, both direct and indirect. Naturally, if a sector only uses labor in production, its customer-centrality measure collapses

Table 1: Parameter values (Euro Area, monthly)

		Aggregate parameters	
β	$0.96^{1/12}$	Discount factor (monthly)	Golosov and Lucas (2007)
ϵ	3	Goods elasticity of substitution	Midrigan (2011)
$\overline{\pi}$	0.02/12	Trend inflation (monthly)	ECB target
ρ	0.90	Persistence of the TFP shock	Half-life of seven months
		Sectoral parameters	
N	39	Number of sectors	Data from Gautier et al. (2024)
$\{\overline{\omega}_i^C\}_{i=1}^N$		Sector consumption weights	World IO Tables
$\{\overline{\omega}_{ik}\}_{i,k=1}^N$		Sector input-output matrix	World IO Tables
$\{\overline{\alpha}_i\}_{i=1}^N$		Sector labor weights	World IO Tables
		Firm-level pricing parameters	
$\{\overline{\kappa}_i\}_{i=1}^N$		Menu costs	Estimated to fit frequency, std dev.
$\{\sigma_i\}_{i=1}^N$		Std. dev. of firm-level shocks	of Δp from Gautier et al. (2024)

to zero. Table B.1 in the Appendix reports the customer centrality measure for each of the 38 production sectors. The two sectors with the largest customer centrality are "Coke and petroleum products" (4.35) and "Chemicals and chemical products" (4.25), followed by "Paper and paper products" (3.97) and "Food and beverages" (3.94), while the smallest customer centrality is in "Education" (1.55).

Second, in order to capture the degree to which a sector is important as a provider of intermediate inputs to the rest of the economy, we introduce the following *supplier centrality* metric:

$$\text{Supplier Centrality}_i \quad \equiv \quad \sum_{j=1}^{N}(I-\overline{\Omega}^T)^{-1}_{ij} - 1 \tag{40}$$

The supplier centrality measure captures the total importance of a sector as a seller, either directly or indirectly, of intermediate inputs to the rest of the economy. The value of supplier centrality for each of the 38 production sectors is reported in Table B.1. The distribution of supplier centrality features a heavy right tail, with three sectors having a disproportionally larger measure than the rest: those are "Administration and support" (7.59), "Legal, accounting, management" (6.51) and "Chemicals and chemical products" (6.18). The sector with the smallest supplier centrality is "Fishing and aquaculture" (0.11).

5 Quantitative results: monetary shocks

For our first set of quantitative results, we present the general equilibrium dynamics of our economy following monetary shocks of different sizes. First, we show that the aggregate repricing frequency response to large monetary shocks is substantially attenuated by the presence of networks, so that the effect of cascades dampening is quantitatively sizable. As a result, the economy with networks features much stronger monetary non-neutrality, which manifests in a substantial flattening of the fully non-linear Phillips Curve. Second, we study sectoral frequency and price responses, and show that, *ceteris paribus*, sectors with a larger customer centrality exhibit smaller movements in the fraction of adjusting firms and feature less size-dependence in their sectoral price responses.

5.1 Aggregate dynamics

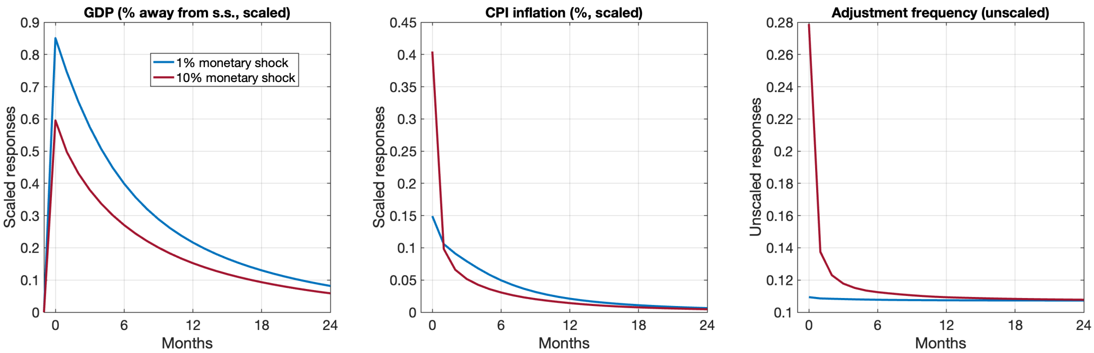
Figure 3(a) shows the scaled (per % shock) responses of aggregate CPI inflation, aggregate GDP, as well as the unscaled fraction of adjusting firms following one-time monetary shocks of two different magnitudes: $\varepsilon_0^M = 1\%$ and $\varepsilon_0^M = 10\%$. Two key features are apparent. First, the scaled response of inflation increases in the size of the monetary shock, which represents a strong *size effect*. As can be seen in the frequency panel, this happens as the fraction of adjusting firms increases rapidly with larger shocks, reaching almost 30% for the 10% monetary shock.

Second, as shown in Figure 3(b), the contribution of production networks to the magnitudes of responses differs markedly between shocks of different sizes. For the small 1% shock, networks dampen the response of inflation and, as a result, amplify the response of aggregate GDP. This is the effect of production networks known from the prior literature, which employs linearized models with time-dependent pricing: input-output linkages create pricing complementarities, dampening inflation and amplifying the consumption response. At the same time, for the large 10% shock, the amplification of the aggregate GDP response due to networks is much greater. Importantly, this is because the 10% monetary shock delivers a markedly smaller increase in the repricing frequency relative to the economy without networks, which is the cascades *dampening* effect introduced earlier.[7]

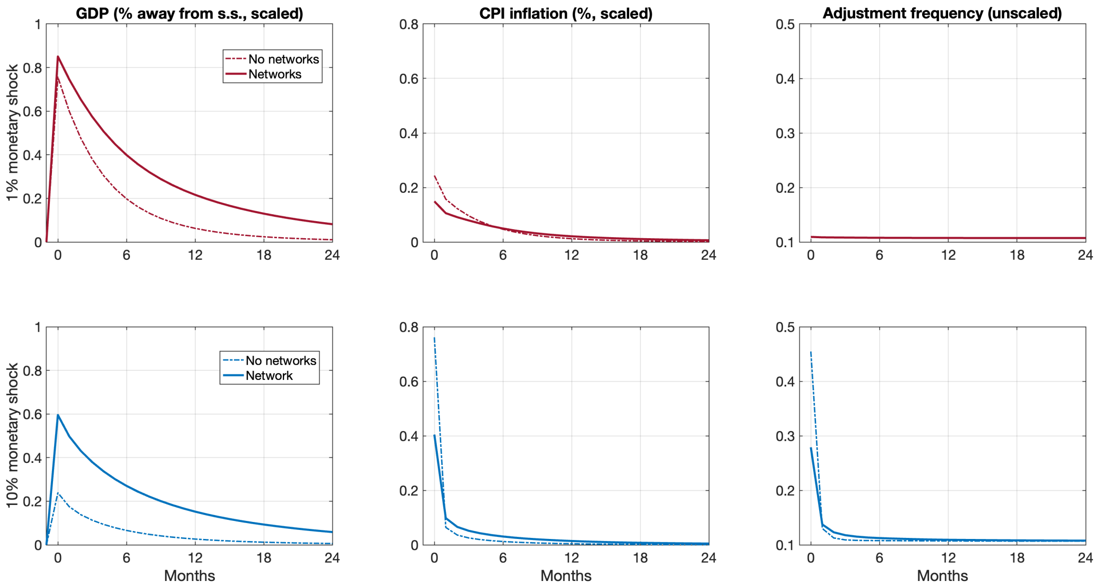
[7] In Figure E.1 we construct, for each size of the monetary shock, the difference between the output response with and without networks, as a fraction of the former. One can see that for small monetary shocks, the contribution is in the neighborhood of 10-20%. Such magnitudes are consistent with prior estimates of network contributions in linearized time-dependent setups (Ghassibe, 2021). As the size of the shock increases, however, the contribution of the network increases dramatically, reaching almost 80% for a 15% monetary expansion.

Figure 3: **Aggregate responses and network contribution under monetary shocks**

(a) Responses of aggregates to monetary shocks

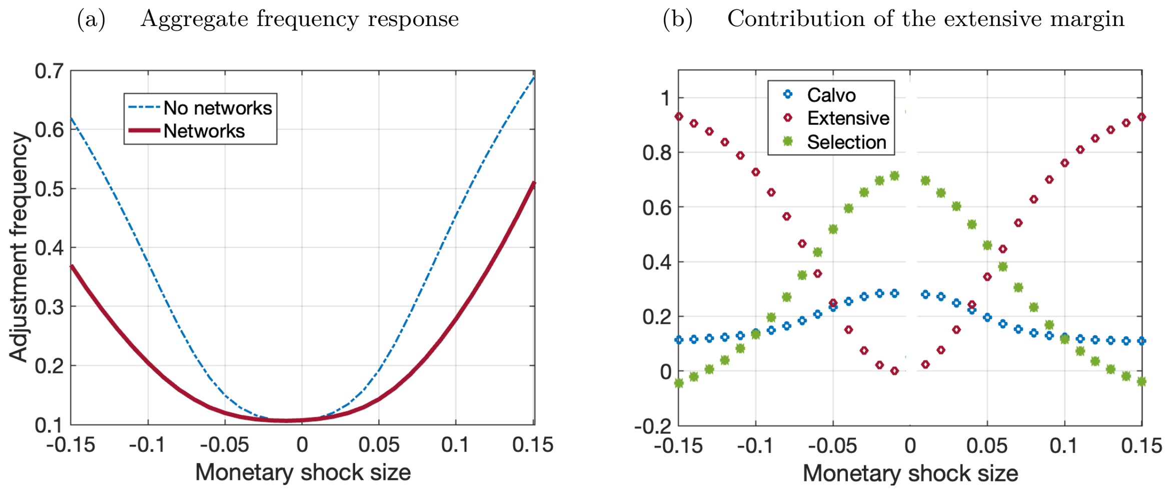


(b) Network contribution to aggregates' responses



Notes: Panel (a) shows the the responses of aggregate GDP, CPI inflation and aggregate adjustment frequency following 1% and 10% one-time monetary shocks in the baseline economy with fixed menu costs and networks; Panel (b) additionally shows the corresponding responses in the otherwise identical economy without networks.

Figure 4: **Extensive margin response to monetary shocks**



Notes: Panel (a) shows the impact responses of the aggregate adjustment frequency following monetary shocks of different sizes in the baseline economy with fixed menu costs and networks, as well as in the otherwise identical economy without networks; Panel (b) uses the decomposition in (41) to show the proportional contributions of the Calvo, Extensive and Selection components to the effect of networks on the impact response of CPI inflation to monetary shocks of different sizes.

In Figure 4(a), we further investigate the interaction between networks and the response of repricing frequency by looking at a wide range of shock sizes and signs. One can see that networks consistently dampen the response of aggregate repricing frequency to monetary shocks of all sizes that we consider. For example, following a 10% monetary expansion, the aggregate frequency rises close to 45% in the multi-sector economy without networks, but increases only to 27% in an otherwise identical economy with input-output linkages. In this sense, the aggregate consequences of cascades dampening by networks is quantitatively sizable.

We also quantify the contribution of cascades dampening to the response of aggregate inflation. To do that, we follow Costain and Nakov (2011) and Blanco et al. (2024a) in making use of the following inflation decomposition:

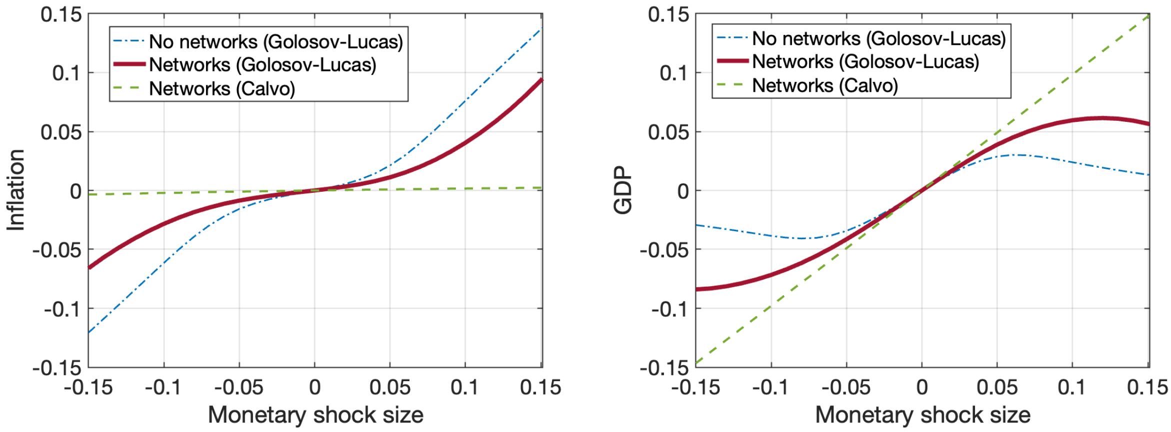
$$\Delta\pi \quad = \quad \Delta\pi^{\text{Calvo}} \quad + \quad \Delta\pi^{\text{Extensive}} \quad + \quad \Delta\pi^{\text{Selection}}, \tag{41}$$

where $\Delta\pi$ is the deviation of (net) aggregate CPI inflation rate from its steady-state value, $\Delta\pi^{\text{Calvo}}$ is the inflation component attributed purely to the change in the intensive margin of price changes (holding fixed the fraction and composition of adjusters), $\Delta\pi^{\text{Extensive}}$ is the component driven exclusively by the change in the fraction of adjusters and $\Delta\pi^{\text{Selection}}$ stands for the component driven by changes in the composition of adjusters.[8] Crucially, the decomposition

[8]Formally, inflation in sector i in the absence of the monetary shock is $\pi_i = \int \tilde{p}\eta_i(\tilde{p})\mathrm{d}g_i(\tilde{p})$, where $\tilde{p}$ is the

Figure 5: **Inflation and GDP responses to monetary shocks**

(a) Impact inflation response (b) Impact GDP response



Notes: Panels (a) and (b) show the impact responses of, respectively, CPI inflation and aggregate GDP to monetary shocks of different sizes in three economies: the baseline economy with fixed menu costs and networks, as well as the otherwise identical economies without networks and with time-dependent pricing.

holds both in the economy with and without networks, allowing us to compute the contribution of each of the three components to the difference in inflation response driven by the input-putput linkages. Figure 4(b) shows the resulting decomposition: the difference is explained mainly by the selection effect for smaller shocks, and by the extensive margin component for shocks greater than 5 percent in absolute value. Therefore, for large shocks, most of the network contribution to the slowing down of the inflation response works through the extensive margin effect, representing the *dampening* of cascades.

Not only is the cascades dampening effect important in relative terms, it also has substantial implications for the absolute magnitudes of responses in CPI inflation and aggregate GDP to large monetary interventions. In Figure 5(a), we show that as the size of the monetary shocks increases, inflation in our baseline economy rises in a non-linear fashion: a 5% shock delivers 2% inflation on impact, whereas tripling the shock to 15% delivers a five-fold increase of inflation to 10%. At the same time, the figure also shows that an otherwise identical economy without networks features inflation rising even faster with larger monetary shocks. The fact that inflation

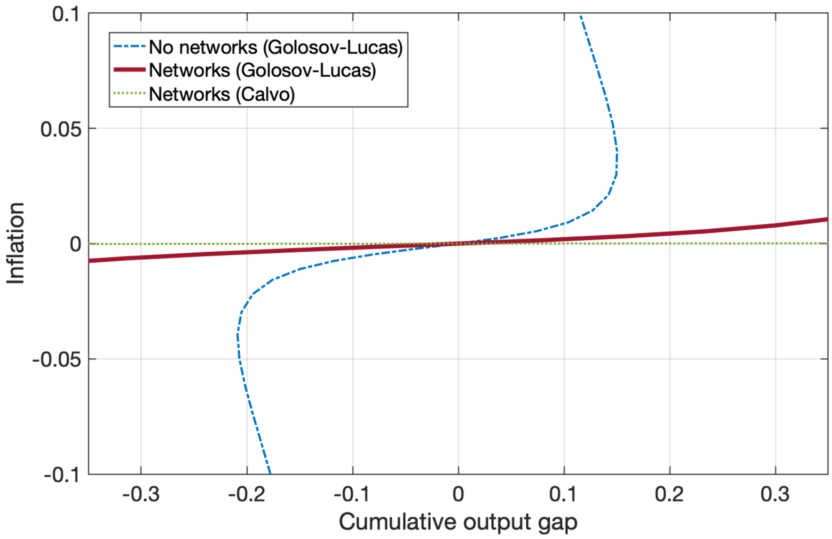
desired log price change, $\eta_i(\tilde{p})$ is the adjustment hazard, and $g_i(\tilde{p})$ is the ergodic distribution of desired price changes across firms in the sector. The monetary shock changes the desired price changes of all firms in the sector to $\tilde{p}+\delta$, where $\delta = \underline{p}^* - p^* + \Delta m$ and where $\underline{p}^*$ is the log reset price in the first period after the money shock and p^* is the log reset price in the absence of the shock. The money shock changes the inflation rate to $\underline{\pi}_i = \int (\tilde{p}+\delta)\underline{\eta_i}(\tilde{p})\mathrm{d}g_i(\tilde{p})$ where $\underline{\eta_i}(\tilde{p})$ is the new adjustment hazard after the shock. It follows that $\Delta\pi_i \equiv \underline{\pi}_i - \pi_i = \delta\int \eta_i(\tilde{p})\mathrm{d}g_i(\tilde{p}) \quad + \quad \delta\int(\underline{\eta_i}(\tilde{p}) - \eta_i(\tilde{p}))\mathrm{d}g_i(\tilde{p}) \quad + \quad \int \tilde{p}(\underline{\eta_i}(\tilde{p}) - \eta_i(\tilde{p}))\mathrm{d}g_i(\tilde{p})$. Multiplying both sides by the final consumption share $\overline{\omega}_i^C$ and summing across sectors, one obtains the decomposition for aggregate CPI inflation in (41).

rises relatively more slowly in the economy with networks reflects mainly the slower response of the fraction of adjusters, as documented in Figure 4. In order to quantify the importance of nonlinearity and state-dependent pricing, in Figure 5 (a) we also consider a version of our model with time-dependent pricing (Calvo, 1983), calibrated to match the sectoral frequencies of adjustment in steady state. Under such a time-dependent setup, even when solved fully non-linearly, inflation is rising more slowly as the monetary shock gets larger. The latter reflects the contribution of both the selection effect (for smaller shocks) and the extensive margin effect (for larger shocks) in delivering faster pricing increases in the state-dependent pricing model.

Figure 5(b) shows that in the baseline economy with networks, the aggregate consumption response is hump-shaped in the size of monetary shocks and is maximized following a 12% monetary expansion, delivering an increase of almost 6%. At the same time, the equivalent economy without networks has its consumption response maximized following a 5% monetary shock, corresponding to a smaller increase of just over 3%. The higher maximal response of consumption under networks, as well as the fact that it occurs following a larger monetary shock, reflect the slower response of the fraction of adjusters, once again, as documented in Figure 4. Figure 5(b) also shows the responses in the alternative setup with time-dependent Calvo (1983) pricing. With time-dependent pricing, one can see that even for very large shocks and a non-linear solution, the time-dependent setup has aggregate consumption rise quasi-linearly in the size of the monetary shock. Moreover, the non-linear time-dependent pricing results deviate substantially from the non-linear state-dependent solutions.

Figure 6 illustrates the trade-off between GDP stimulus and inflation under monetary interventions of different sizes. In particular, the figure traces out a non-linear "Phillips curve" in the cumulative output gap–CPI inflation space, under different model configurations. In the network-based baseline economy, a cumulative output stimulus up to 5% or so can be achieved with little inflationary response, reflecting a locally flat Phillips curve. However, in a counterfactual economy without networks, the Phillips curve is steeper for small shocks and low output gap values. This suggests a global "flattening" of the Phillips curve due to networks, and more specifically the cascades dampening effect. Moreover, once the shocks are sufficiently large, the Phillips curve without networks becomes backward bending, with a maximum possible cumulative output stimulus of around 15%. This happens because, under very large shocks, the fraction of adjusters increases much faster in the economy without networks, as documented in Figure 4(a).

Figure 6: **Fully non-linear "Phillips Curves"**



Notes: the figure shows the fully non-linear "Phillips Curves", obtained by tracing out the impact response of CPI inflation and the cumulative GDP response ("output gap") following monetary shocks of different sizes; the "Phillips Curves" are constructed for three economies: the baseline economy with fixed menu costs and networks, as well as the otherwise identical economies without networks and with time-dependent pricing.

5.2 Disaggregated dynamics

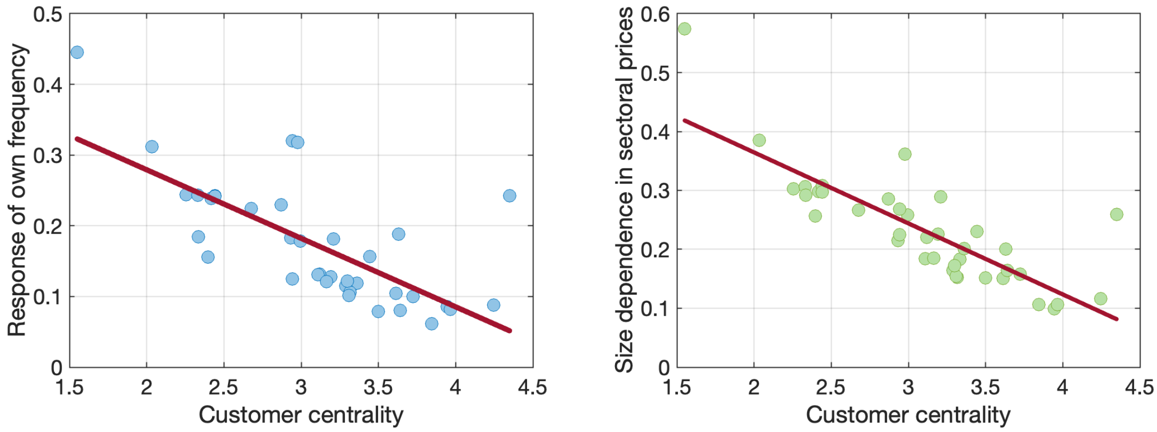
Having analyzed the behavior of macroeconomic aggregates, we now move to studying sector-level behavior following a large monetary shock. In Figure E.2 we report changes in sectoral adjustment frequencies, as well as scaled impact responses of sectoral price indices, following a monetary shock of $\varepsilon_0^M = 10\%$. A few important patterns emerge. First, all sectoral frequencies increase, with the natural exception of the fully price-flexible sectors. The largest rise in the fraction of adjusters is in "Education" (+0.65), whereas the smallest (non-zero) increase is in "Financial services" (+0.08). Second, removing production networks leads to a larger increase in adjustment frequency in every sector, so that cascades dampening also holds at the sector-by-sector level. Third, cascades dampening translates into smaller increases in sectoral price indices in the economy with networks.

To better understand the role production networks play in shaping the frequency responses, Figure 7(a) plots an estimated linear relationship between the sectoral frequency responses to the 10% monetary shock and the sectoral customer centrality measure, which we introduced in (39).[9] As can be seen, *ceteris paribus*, a one unit increase in customer centrality is associated with an approximately 0.05 smaller increase in the fraction of adjusters. This is because, all

[9]Formally, we regress the change in sectoral adjustment frequency on the measure of customer centrality, further controlling for the sectoral steady-state frequency of adjustment and the sectoral standard deviation of price changes.

Figure 7: **Sectoral responses to a monetary shock vs. Customer centrality**

(a) Sectoral frequency responses (b) Size-dependence of sectoral prices



Notes: Panels (a) and (b) plot fitted linear relationships between the sectoral Customer centrality and, respectively, the responses of sectoral frequencies and the degree of size dependence in sectoral prices following a 10% monetary shock. The vertical axes show the fitted response to Customer centrality, to which we add the fitted response to the control variables (frequency, size of of price changes) evaluated at their sample means.

else constant, cascades dampening is stronger for sectors that are more exposed to intermediate inputs, directly or indirectly, since their desired price changes move less with the monetary shock.

Similarly, we also investigate the association between customer centrality and the degree of size dependence in sectoral price responses to small vs. large monetary shocks. We measure size dependence in sectoral price indices by the difference in *scaled* impact responses to 10% and 0.1% monetary shocks. In Figure 7(b) we plot an estimated linear relationship between the measure of size dependence and the sectoral customer centrality.[10] As one would expect, larger customer centrality is associated with a smaller degree of size dependence, which in turn is driven by the smaller movement in the fraction of adjusters following the large monetary shock.

6 Quantitative results: TFP shocks

For our second set of quantitative results, we turn to the general equilibrium dynamics following aggregate and sector-specific total factor productivity (TFP) shocks. First, we show that following large aggregate TFP shocks, the economy with networks features much stronger

[10] As before, we regress the sectoral measure of size dependence on the measure of customer centrality, further controlling for the sectoral steady-state frequency of adjustment and the sectoral standard deviation of price changes.

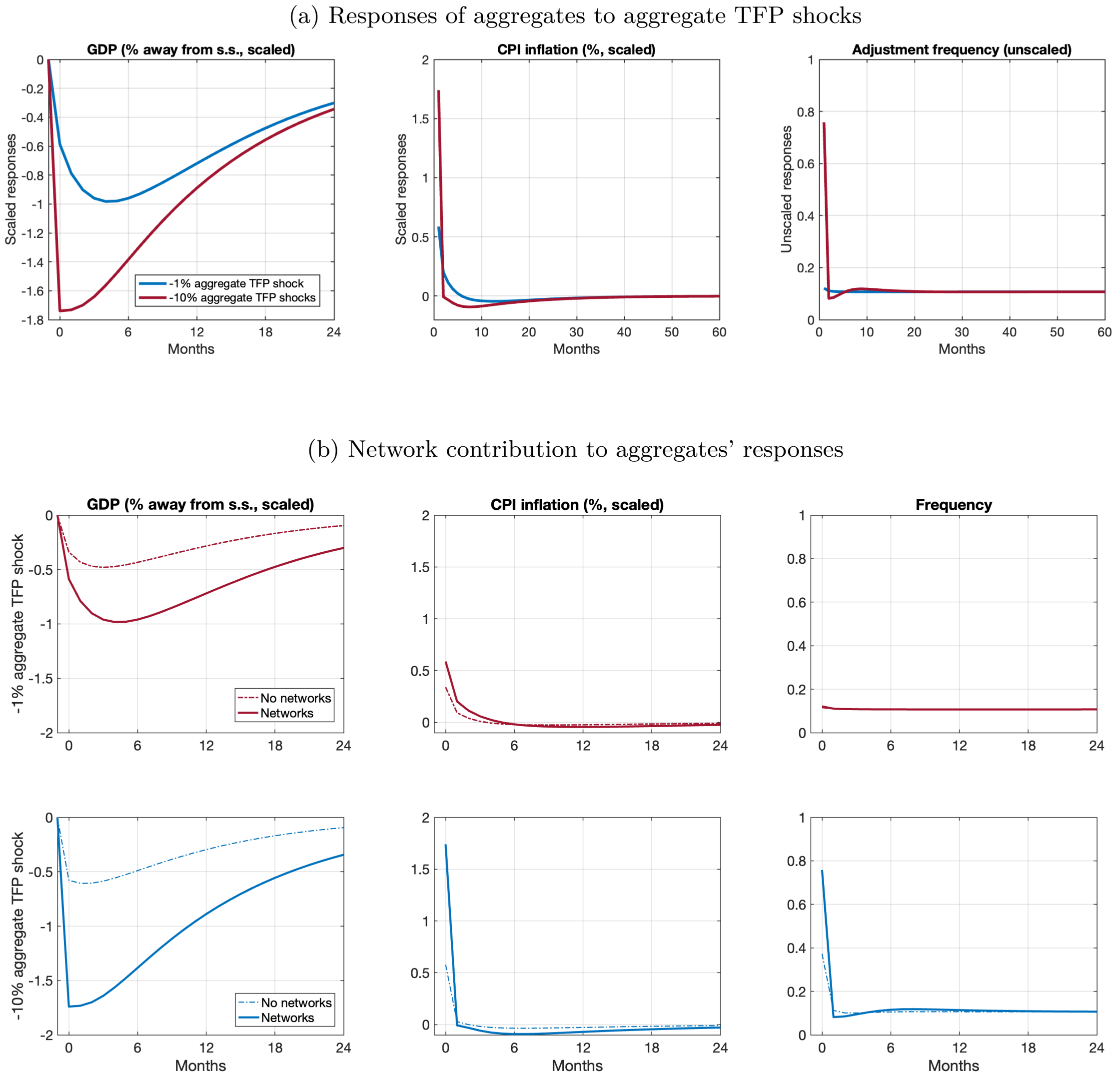
response of the repricing frequency, implying that the cascades amplification channel is indeed quantitatively important. The amplification of cascades in turn generates much stronger response of aggregate inflation for a given shock, relative to the otherwise identical economy with time-dependent pricing. We also show that sectors with a larger customer centrality exhibit stronger responses of the fraction of adjusters and more size dependence in sectoral price responses following large aggregate TFP shocks. Second, our results suggest that TFP shocks to sectors with a large supplier centrality lead to more sizable movements in the aggregate repricing frequency, and can in turn generate non-linearities in aggregate inflation.

6.1 Aggregate TFP shocks

Aggregate dynamics In Figure 8(a), we report scaled (per % shock) responses of aggregate GDP and CPI inflation, as well as the unscaled responses of the aggregate fraction of adjusters following two negative aggregate TFP shocks: $\varepsilon_0^A = -1\%$ and $\varepsilon_0^A = -10\%$. Just as with monetary shocks in the previous section, there is substantial size dependence: for the -1% shock the impact scaled response of GDP is -0.6%, whereas it is -1.7% for the -10% shock. At the same time, the scaled response of CPI inflation increases in the magnitude of the aggregate TFP shock, implying that the aggregate price changes rise more than proportionally in the size of the TFP innovation. Quantitatively, the -1% shock generates a scaled impact response of CPI inflation of 0.6%, whereas the -10% corresponds to a normalized response of almost 1.7% on impact. Key to the observed size dependence is the endogenous response of the fraction of adjusters: for the -1% shock it remains unchanged, whereas the larger -10% shock brings the fraction of adjusters to almost 80%.

In order to understand the contribution of networks to the observed size dependence, in Figure 8(b) we additionally document the responses to the same aggregate TFP shocks in an otherwise identical economy without networks. For the -1% shock, networks amplify the response of aggregate GDP by a factor of two, while the scaled response of CPI inflation is nearly 0.3% without networks versus 0.6% under networks. Importantly, for the larger shock of -10%, the network amplification of both aggregate GDP and CPI inflation is greater than under the small -1% shock. When it comes to the response of inflation, this is the opposite of our findings under monetary shocks, where the amplification of inflation response weakens for larger innovations. To understand the difference, it is instructive to look at the response of the adjustment frequency. One can see that for the -10% shock, the fraction of adjusters increases

Figure 8: **Aggregate responses and network contribution under aggregate TFP shocks**



Notes: Panel (a) shows the the responses of aggregate GDP, CPI inflation and aggregate adjustment frequency following -1% and -10% one-time aggregate TFP shocks in the baseline economy with fixed menu costs and networks; Panel (b) additionally shows the corresponding responses in the otherwise identical economy without networks.

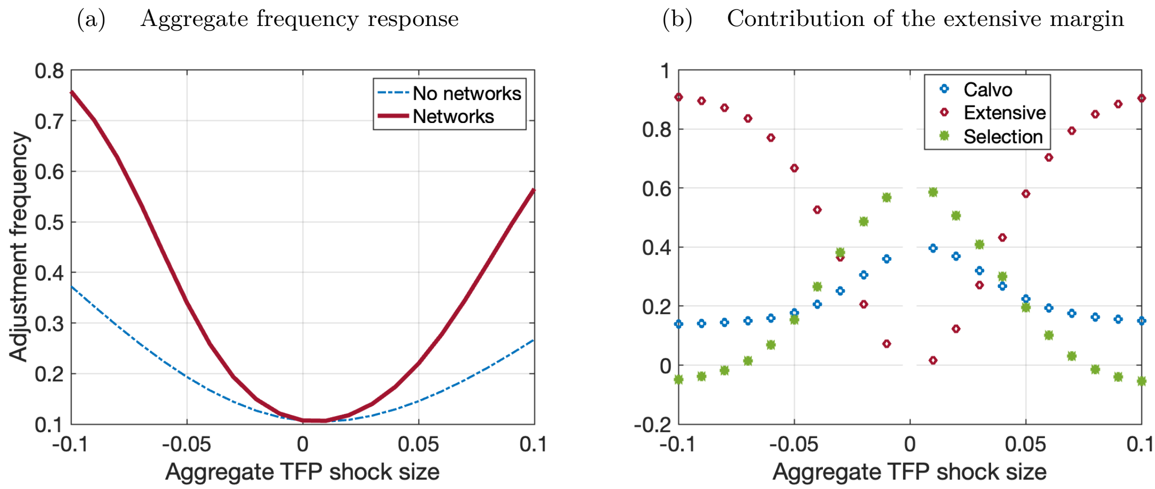
substantially more in the economy with networks. This is the exact opposite of what happens under monetary shocks, where networks dampen the response of the adjustment frequency. Instead, under TFP shocks networks make the response of the adjustment frequency stronger, representing cascades amplification.

In Figure 9(a), we further investigate the interaction between the aggregate repricing frequency and the size of the aggregate TFP shock. Once again, contrary to the findings under monetary shocks, networks consistently and substantially amplify the response of the aggregate fraction of adjusters to aggregate TFP innovations. For example, following a -10% aggregate TFP shock, the economy with networks features a rise in the fraction of adjusters to 75%, while without networks the aggregate adjustment frequency rises to just below 40%. In this sense, the cascades amplification effect is quantitatively sizable. In order to understand the contribution of cascades amplification to aggregate CPI movements, we once again rely on the decomposition in (41). Figure 9(b) decomposes the difference in impact responses of aggregate CPI in economies with and without networks for different shock sizes. It follows that for aggregate TFP shocks below 3% in absolute value, the difference between the network and no-network cases is explained mainly by the selection effect, whereas for larger shocks the extensive margin effect is dominant. Therefore, for large aggregate TFP shocks the cascades amplification mechanism, working through the extensive margin of price changes, is the main channel through which networks amplify the aggregate CPI response.

The cascades amplification channels is important for generating non-linearity in aggregate CPI inflation response. In Figure 10 (a), we plot the impact response of aggregate CPI inflation to aggregate TFP shocks of different signs and sizes. The inflation response rises much faster in the economy with networks relative to the no-network benchmark, being almost three and a half times higher after a -10% shock. We also report the inflation responses in an economy with networks, but with time-dependent (Calvo, 1983) pricing, matching the same sectoral frequencies of adjustment in steady state. One can see that the economy with time-dependent pricing predicts much smaller inflation responses. In this sense, the otherwise identical economy with Calvo (1983) pricing requires substantially larger TFP shocks in order to generate the same amount of aggregate inflation.

Disaggregated dynamics We now turn to analyzing the responses of individual sectors to an aggregate TFP shock. In particular, Figure E.3 reports the impact responses of sector-specific fractions of adjusters and scaled sectoral price indices to a large aggregate TFP shock

Figure 9: **Extensive margin response to aggregate TFP shocks**



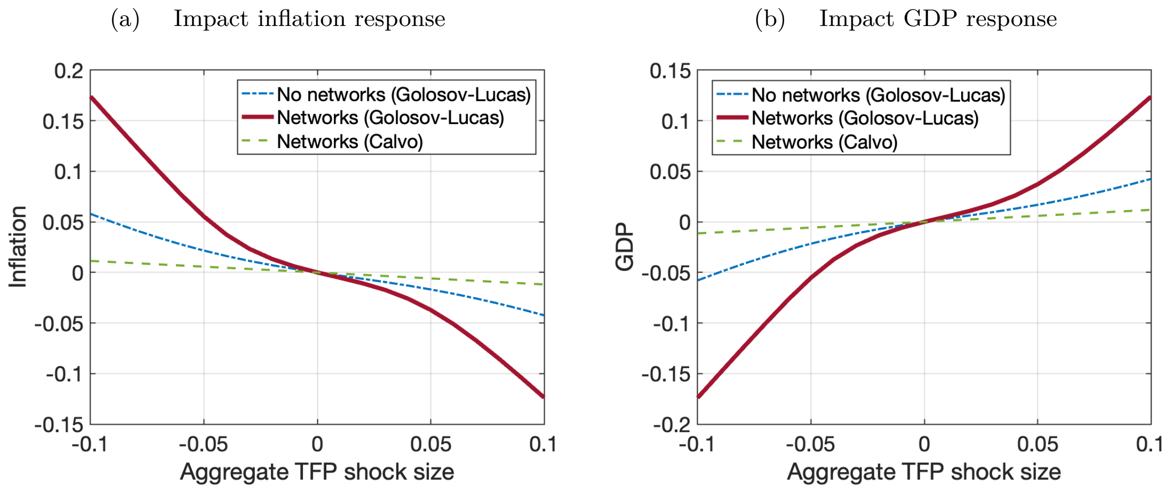
Notes: Panel (a) shows the impact responses of the aggregate adjustment frequency following aggregate TFP shocks of different sizes in the baseline economy with fixed menu costs and networks, as well as in the otherwise identical economy without networks; Panel (b) uses the decomposition in (41) to show the proportional contributions of the Calvo, Extensive and Selection components to the effect of networks on the impact response of CPI inflation to aggregate TFP shocks of different sizes.

($\varepsilon_0^A = -10\%$). Some key results are of note. First, with the natural exception of the two fully flexible sectors, frequency of adjustment rises in all the sectors. The largest rise is in ”Publishing” (+0.95), whereas the smallest positive change is in ”Legal, accounting, management” (+0.18). Second, the cascades amplification effect holds sector-by-sector: in an otherwise identical economy without networks, frequency increases by less in every sector. Third, the cascades amplification leads to larger sectoral price increases in every sector.

In order to pin down the role played by network characteristics in shaping sectoral frequency responses, Figure 11(a) plots an estimated linear relationship between the change in the fraction of adjusters in a given sector and its customer centrality, introduced in (39). There is a clear positive relationship, with a unit increase in customer centrality being associated to 0.11 larger increase in the fraction of adjusters, *ceteris paribus*. This is the cascades amplification mechanism in action: higher customer centrality means the sector has a larger total exposure to intermediate inputs; as a result, an aggregate TFP shock leads to a larger desired price change, making the adjustment decision more likely.

The established relationship between frequency responses and customer centrality also has an implication for the degree of size dependence in sectoral price dynamics. To see that, in Figure 11(b) we plot the estimated linear relationship between a measure of sectoral price size dependence, given by the difference between normalized responses to -10% and -0.1% aggregate TFP shocks, and the sectoral customer centrality. The estimated relationship is positive, im-

Figure 10: **Inflation and GDP responses to aggregate TFP shocks**



Notes: Panels (a) and (b) show the impact responses of, respectively, CPI inflation and aggregate GDP to aggregate TFP shocks of different sizes in three economies: the baseline economy with fixed menu costs and networks, as well as the otherwise identical economies without networks and with time-dependent pricing.

plying that a higher total exposure to intermediate inputs is associated with a greater degree of size dependence in the sectoral price response to a large aggregate TFP shock.

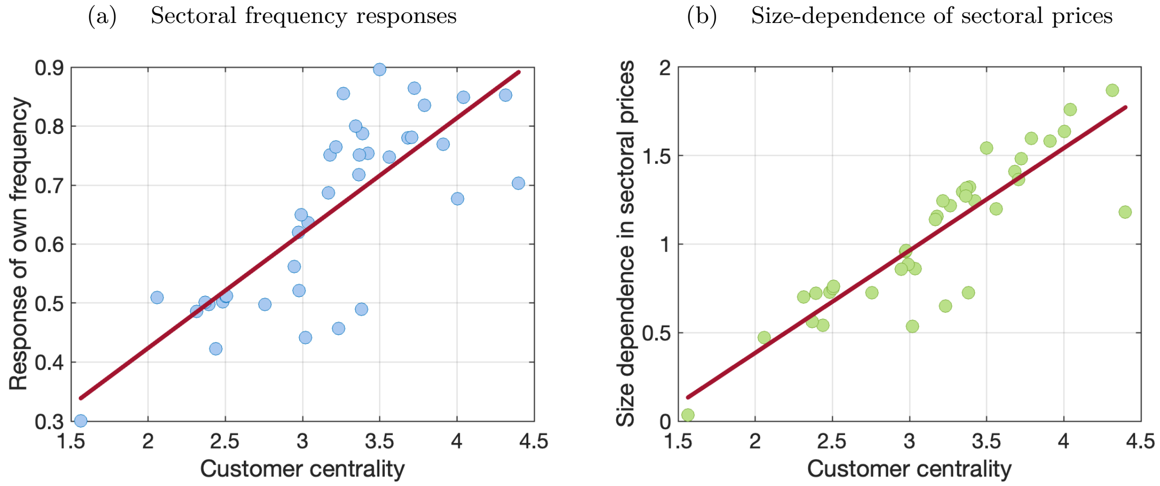
6.2 Sectoral TFP shocks

For our next set of results, we study the transmission of sector-specific TFP shocks. Our particular interest is in how very large shocks originating in specific parts of the economy can affect macroeconomic aggregates. To do that, we subject each production sector of our economy to a large negative TFP shock ($\varepsilon^A_{i,0} = -20\%, \forall i$), and study how each individual shock affects aggregates.

In Figure 12(a), we show the responses of the aggregate fraction of adjusting firms to the large sector-specific TFP shocks. First, for the majority of sectors, the effect of their own shock on aggregate frequency is relatively modest. However, there are notable exceptions: individual shocks to ”Food and beverages”, ”Crop and animal production”, as well as ”Chemicals and chemical products” generate an aggregate frequency increase of over 4%. Second, for all sectors, networks amplify the aggregate frequency response, to the sector-specific TFP shock. Moreover, the amplification is particularly strong for the three aforementioned sectors that are particularly important for aggregate frequency. In this sense, the disproportionate effect of certain sectors on the aggregate fraction of adjusting firms is potentially driven by their position in the networks.

We also study the response of aggregate CPI inflation to sectoral TFP shocks. Specifically, in

Figure 11: **Sectoral responses to an aggregate TFP shock vs Customer centrality**

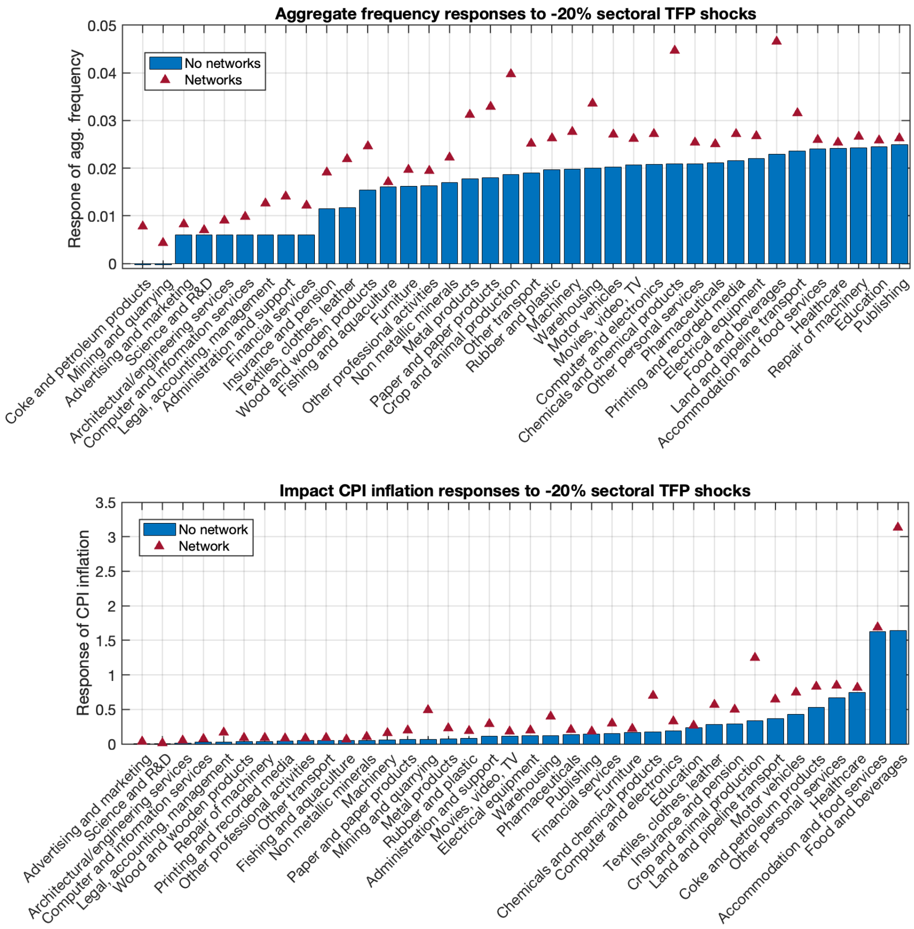


Notes: Panels (a) and (b) plot fitted linear relationships between the sectoral Customer centrality and, respectively, the responses of sectoral frequencies and the degree of size dependence in sectoral prices following a -10% aggregate TFP shock. The vertical axes show the fitted response to Customer centrality, to which we add the fitted response to the control variables (frequency, size of of price changes) evaluated at their sample means.

Figure 12(b) we depict scaled impact responses of aggregate CPI inflation to the large negative sectoral TFP shocks. First, networks amplify the aggregate CPI inflation responses for all sectoral TFP shocks. Second, just as with aggregate frequencies, for the majority sectors, the network amplification is relatively modest. Third, some sectors pose an exception: shocks to ”Food and Beverages”, ”Crop and animal production”, ”Mining and Quarrying”, as well as “Chemicals and Chemical Products” have an effect on aggregate CPI that is amplified be networks to a large degree than for other sectors.

An important pattern emerges: large shocks to certain sectors have the capacity to disproportionally affect the aggregate adjustment frequency. Moreover, this disproportionate importance stems from network amplification. In order to shed light on how network characteristics may affect the systemic importance of a sector for aggregate frequency, in Figure 13(a) we plot an estimated linear relationship between the aggregate frequency change (net of movements in the own sectoral frequency) and the sectoral supplier centrality, introduced in (40). The relationship is clearly positive: a unit increase in supplier centrality is associated with a 0.3% additional increase in aggregate frequency. Once again, this is a manifestation of the cascades amplification mechanism: higher supplier centrality implies that the sector is, directly or indirectly, a more important provider of intermediate inputs to the rest of the economy; as a result, a shock to that sector strongly affects marginal costs, and hence desired price changes, of firms

Figure 12: **Aggregate responses to sectoral TFP shocks**



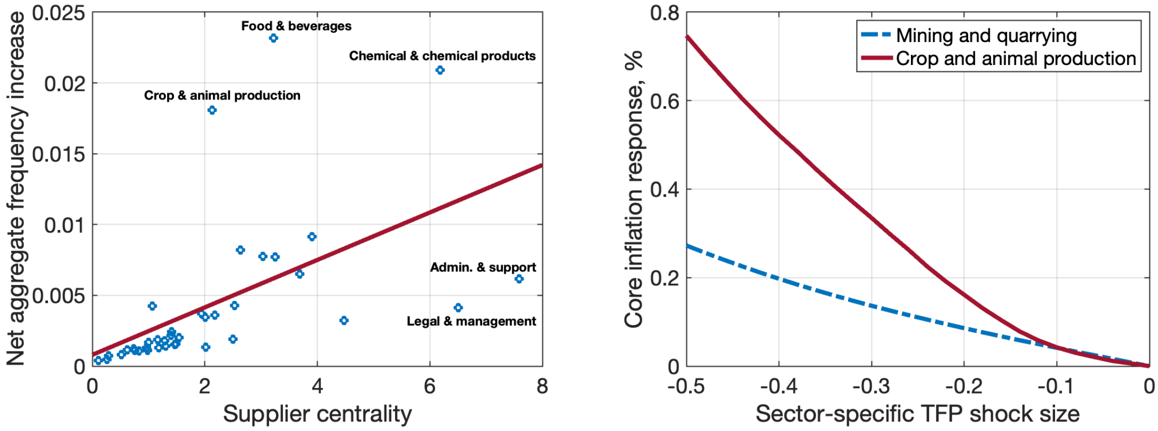
Notes: the top Panel shows the impact responses of aggregate adjustment frequency to a -20% TFP shocks to a specific production sector of our economy; the bottom Panel similarly shows the impact responses of CPI inflation to a -20% sectoral TFP shock.

in other sectors, making the latter more likely to adjust prices.

The cascades amplification channel also has important implications for the non-linearity of aggregate inflation in response to sector-specific shocks of different sizes. In order to see that, in Figure 13(b) we consider how progressively larger shocks to the two commodity sectors, "Mining and quarrying" and "Crop and animal production", affect aggregate *core* inflation, which excludes movements in the commodity prices themselves. First, one can see that as the negative TFP shocks to the "Crop and animal production" sector become larger in magnitude, core inflation responds more than proportionally on impact, thus rising in a fast non-linear fashion. This is because, as documented in Figure 13(a), shocks to "Crop and animal production" create large increases in the (net) aggregate adjustment frequency. Second, much less non-linearity in impact core inflation response occurs under shocks to "Mining and quarrying". This is because shocks to the latter sector do not induce substantial movements in aggregate adjustment

Figure 13: **Sectoral TFP shocks, aggregate responses and Supplier centrality**

(a) (Net) aggregate frequency response (b) Core inflation response



Notes: Panel (a) shows the fitted linear relationship between the sectoral Supplier centrality and the response of net aggregate adjustment frequency (excluding the shocked sector) to a -20% TFP shock to that sector; Panel (b) shows effect of TFP shocks to "Mining and quarrying" and "Crop and animal production" sectors on core inflation (excludes the two shocks commodity sectors).

frequency, which is ultimately driven by the close-to-average value of supplier centrality.

7 Extensions and robustness checks

In this section present three extensions to our baseline model. First, we consider a version of our economy in the cashless limit, where the central bank conducts monetary policy by setting the nominal interest rate, which endogenously responds to aggregate inflation and output according to a Taylor rule. Second, we relax the assumption of fixed menu costs, and consider a version of our economy with random menu costs instead. Third, we extend our baseline model with a constant elasticity of substitution (CES) aggregation across sectors, which allows for the consumption and input-output shares to vary endogenously along the intensive margin, capturing potentially low substitutability across inputs.

7.1 Endogenous monetary policy

In our baseline results, the central bank conducts policy by setting an exogenous path of money supply. We now consider an extension that adds realism to the monetary policy conduct. In particular, we use the cashless limit setup of Woodford (2004) and Galí (2015), where the central bank conducts policy by setting the level of the nominal interest rate, which also responds endogenously to movements in macroeconomic aggregates. In particular, we assume that the nominal interest rate follows the following Taylor-type rule, capturing the empirically-realistic policy persistence (Coibion and Gorodnichenko, 2012):

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_t^C + \phi_c c_t\right] + \varepsilon_t^i, \tag{42}$$

where $\hat{\imath}_t \equiv \log \frac{1+i_t}{\overline{\Pi}/\beta}$ is the log-deviation of the nominal interest rate from its steady-state value, $\pi_t^C \equiv \log \Pi_t^C/\overline{\Pi}$ is deviation of CPI inflation from target and $c_t \equiv \log C_t/\overline{C}$ is aggregate GDP in deviation from steady state. In the rule, $\rho_i \in [0,1)$ determines the degree of policy persistence, $\phi_\pi > 0$ and $\phi_c > 0$ pin down how aggressively the central bank responds to deviations of inflation and GDP from their steady-state values, and ε_t^i is the mean-zero i.i.d. monetary policy shock. For our monthly calibration, we set $\rho_i = 0.9, \phi_\pi = 1.5$ and $\phi_c = 0.5/12$.

In Appendix D we detail the full alternative version of our model in the cashless limit with the Taylor-type rule for the nominal interest rate. Here we present an overview of the key results. In Figure D.1 we report impulse responses of the aggregate adjustment frequency, GDP, inflation and the nominal interest rate to an annualized one-time monetary policy shock of -500 basis points. It follows that in the baseline economy with networks, the aggregate frequency rises up to 0.13, as opposed to only 0.11 without networks. Therefore, cascades dampening carries through even under endogenous monetary policy. Moreover, one can also see that the cumulative GDP response is larger in the economy with networks, so that cascades dampening contributes towards additional monetary non-neutrality.

As for supply shocks, in Figure D.2 we report the responses of aggregates to a one-time transitory ($\rho = 0$) aggregate TFP shock of -5%. Find find that the response of frequency is stronger in the economy with networks, so that cascades amplification also holds under endogenous monetary policy. One can also see that the response of aggregate CPI inflation is larger under networks, marking the contribution of cascades amplification.

7.2 Random menu costs

In our baseline results, we work under the assumption that nominal price re-setting is subject to a fixed sector-specific menu cost as in Golosov and Lucas (2007). In order to illustrate that our novel channel of interaction between networks and pricing cascades is not limited to the fixed menu cost setup, as an extension, we consider a random menu cost setup instead.[11] More

[11] We study the random menu cost setup in the context of the cash economy. However, we can also feasibly study random menu costs in the chasless limit.

specifically, we use the CalvoPlus setup of Nakamura and Steinsson (2010), which assumes that each period a randomly selected fraction of firms within each sector draws a menu cost of zero, whereas the complementary fraction is still subject to the fixed menu cost.

Formally, the CalvoPlus setup corresponds to the following functional form of the probability of adjustment function $\eta_{i.t}(.)$:

Assumption 4′ (CalvoPlus pricing). *Consider a firm in sector i with the quality adjusted log relative price p at time t. Then the probability that this firm adjusts its nominal price is given by:*

$$\eta_{i,t}(p) = \ell_i + (1 - \ell_i) \times \mathbf{1}(L_{i,t}(p) > 0) \tag{43}$$

where ℓ_i is the sectoral probability of drawing a zero menu cost, $\mathbf{1}(\cdot)$ is the indicator function, and

$$L_{i,t}(p) = \max_{p'} V_{i,t}(p') - V_{i,t}(p) - \overline{\kappa}_i \tag{44}$$

is the gain from adjustment (or loss from inaction), net of the menu cost.

Crucially, as the non-zero menu cost tends to infinity ($\overline{\kappa}_i \to \infty$), the pricing problem collapses to the time-dependent model of Calvo (1983), as only the randomly selected fraction ℓ_i in each sector gets to adjust. At the same time, setting the probability of drawing a zero menu cost to zero ($\ell_i = 0$) collapses the pricing problem in that sector to the fixed menu cost setup of Golosov and Lucas (2007).

In order to quantitatively discipline the probabilities of free adjustment, we estimate them so that, in steady state around 75% of all price adjustments are free in each sector, following Nakamura and Steinsson (2010) and Blanco et al. (2024b). As before, the non-zero menu costs and standard deviation of idiosyncratic shocks are estimated to jointly match the sector-specific frequencies and standard deviations of price changes in the Euro Area.

In Figure E.4 we study the responses of aggregate repricing frequency and GDP to monetary shocks of different sizes under CalvoPlus pricing. In panel (a) one can see that the response of aggregate repricing frequency, both with and without networks, is dampened relative to otherwise identical economies with fixed menu costs. This is because the presence of free adjustment opportunities implies that much larger shocks are needed for firms to get pushed out of their inaction region. At the same time, just like in the economy with fixed menu costs, the economy with networks features smaller frequency movements, which is the effect of dampening pricing cascades. As for the GDP responses in panel (b), the economy with networks and random menu costs features much stronger non-neutrality than an otherwise identical economy without networks.

As for the propagation of supply shocks, in Figure E.5 we report the responses of aggregate repricing frequency and CPI inflation to aggregate TFP shocks of different sizes. Panel (a) show that, as with monetary shocks, the introduction of random menu costs dampens the responses of frequency to aggregate TFP shocks, both with and without networks. At the same time, one can see that conditional on CalvoPlus pricing, the economy with networks features stronger movements in aggregate frequency, implying that networks amplify cascades, just as in the economy with fixed menu costs. The amplification of pricing cascades creates a strong nonlinearity in aggregate CPI dynamics, as can be seen in panel (b). For a -10% aggregate TFP shock, networks amplify the aggregate CPI response from 0.03 to 0.08 on impact.

7.3 Alternative elasticity of substitution across sectors

In our baseline analysis, we use Cobb-Douglas aggregation across sectors, as well as a Cobb-Douglas production technology. In this subsection we relax this assumption, and consider more general constant elasticity of substitution (CES) aggregation across sectoral consumptions, as well as across productive inputs.

First, we consider the following CES final consumption aggregator:

Assumption 2′ (CES consumption aggregation)**.** *The consumption aggregator $\mathcal{D}(\cdot)$ is given by:*

$$\mathcal{D}(C_{1,t},...,C_{N,t}) = \left(\sum_{i=1}^{N} \overline{\omega}_i^{C\,\frac{1}{\theta_c}} C_{i,t}^{\frac{\theta_c-1}{\theta_c}} \right)^{\frac{\theta_c}{\theta_c-1}}, \tag{45}$$

where $\theta_c > 0$ is the elasticity of substitution across sectoral varieties and $\sum_i \overline{\omega}_i^C = 1$, $\overline{\omega}_i^C \geq 0, \forall i$.

Under this assumption, the equilibrium final consumption shares are given by:

$$\omega_{i,t}^C = \overline{\omega}_i^C \times \frac{\tilde{P}_{i,t}^{1-\theta_c}}{\sum_{k=1}^{N} \overline{\omega}_k^C \tilde{P}_{k,t}^{1-\theta_c}} \tag{46}$$

which is constant in the special case when the sectoral consumption aggregator is Cobb-Douglas ($\theta_c = 1$). It follows that the final consumption shares are time-varying and depend on relative movements in (real) sectoral price indices. Whenever final sectoral varieties are complements, $\theta_c \in (0, 1)$, a relative increase in a sectoral price index leads to a rise in that sector's final consumption share, and *vice versa* whenever the varieties are substitutes, $\theta_c > 1$.

Similarly, we also assume the following CES production technology:

Assumption 3′ (CES production technology)**.** *The production technology $\mathcal{F}_i(\cdot)$ for a firm j in sector i is given by:*

$$\mathcal{F}_i[L_{i,t}(j), X_{i,1,t}(j), ..., X_{i,N,t}(j)] = \frac{1}{\zeta_{i,t}(j)} \times A_{i,t} \times \left(\overline{\alpha}_i^{\frac{1}{\theta_i}} N_{i,t}^{\frac{\theta_i - 1}{\theta_i}}(j) + \sum_{k=1}^{N} \overline{\omega}_{ik}^{\frac{1}{\theta_i}} X_{i,k,t}^{\frac{\theta_i-1}{\theta_i}}(j) \right)^{\frac{\theta_i}{\theta_i - 1}}, \quad (47)$$

where $\theta_i > 0$ is the elasticity of substitution across inputs and $\overline{\alpha}_i + \sum_i \overline{\omega}_{ik} = 1$, $\overline{\alpha}_i, \overline{\omega}_{ik} \geq 0, \forall i$.

Such a production technology delivers the following equilibrium cost shares of labor and intermediate inputs:

$$\alpha_{i,t} = \overline{\alpha}_i \times \frac{1}{\overline{\alpha}_i + \sum_{k'=1}^{N} \overline{\omega}_{ik'} \tilde{P}_{k',t}^{1-\theta_i}}, \qquad \omega_{ik,t} = \overline{\omega}_{ik} \times \frac{\tilde{P}_{k,t}^{1-\theta_i}}{\overline{\alpha}_i + \sum_{k'=1}^{N} \overline{\omega}_{ik'} \tilde{P}_{k',t}^{1-\theta_i}} \quad (48)$$

which are constant in the special case when the production function is Cobb-Douglas ($\theta_i = 1$). As with consumption aggregation, time variation in the input cost shares is pinned down by relative movements in (real) input prices. As before, whenever inputs are complements, a relative increase in the price of an input leads to an increase in the cost share of that input, and *vice versa* whenever inputs are substitutes.

We now revisit our key quantitative exercises in an economy with fixed menu costs and CES aggregation. We calibrate $\theta_c = \theta_i = 0.001, \forall i$, to consider an economy where goods are almost perfect complements, capturing the potential difficulty of substituting both consumption and production varieties. This may represent the supply chain disruptions that we observed during and after the Covid pandemic across the globe.

In Figure E.6, we study the propagation of monetary shocks in our economy with CES aggregation. First, once can see that, just like under Cobb-Douglas, networks dampen the response of frequency to monetary shocks. In other words, our key mechanism of interaction of networks and the extensive margin continues to hold under CES aggregation. Quantitatively, conditional on the presence of networks, moving from Cobb-Douglas to CES with $\theta_c = \theta_i = 0.001, \forall i$ delivers slightly larger frequency movements for expansions and slightly smaller frequency movements under monetary contractions. This is because under complements, sectors with rising prices see their input and consumption shares rise, thus creating a pro-inflation asymmetry.

As for supply disturbances, in Figure E.7 we study the propagation of aggregate TFP shocks. Just as in the economy with Cobb-Douglas, networks amplify the response of aggregate repricing frequency to aggregate TFP shocks. Therefore, our key mechanism that networks amplify pricing cascades continues to hold under CES aggregation. Also, as with monetary shocks, the fact that sectoral varieties are complements creates a pro-inflation asymmetry: conditional on networks, CES aggregation amplify frequency movements after negative TFP shocks, and dampens frequency movements following positive TFP shocks.

8 Application: (post-)COVID inflation in the Euro Area

We now assess whether the novel interaction between networks and pricing cascades is important for quantitatively explaining macroeconomic dynamics in the Euro Area in the (post-)Covid era. To do that, we feed four structurally interpretable shock series into our model, corresponding to the widely perceived major drivers of business cycles: money supply, energy price movements, food price movements and the labor market conditions. We show that when subjected to those four series, our model successfully captures the rise in the aggregate repricing frequency *and* the surge in consumer price inflation in the Euro Area. At the same time, removing either state-dependent pricing or networks dramatically worsens the quantitative performance of the model. This stresses the quantitative relevance of our novel interaction between networks and pricing cascades.

8.1 Four exogenous shock series

In our exercise, we consider four exogenous monthly shock series, spanning the period between January 2019 and June 2024.

First, we feed in the Euro Area nominal GDP in order to approximate the aggregate money supply series $\{M_t\}_{2019:1}^{2024:6}$. We treat this series as an amalgamation of monetary and fiscal stance in the (post-)Covid era, capturing the overall aggregate demand conditions.[12] Second, we are fitting an exogenous TFP process in the labor union sector $\{A_t^{LU}\}_{2019:1}^{2024:6}$ to make sure the nominal cost of labor faced by firms exactly matches the observed Euro Area nominal hourly earnings series in equilibrium. Equivalently, this amounts to fitting an exogenous process for the aggregate labor wedge. We believe this is important, since the labor market in our model is much too parsimonious to reconcile the observed wage dynamics, which is in turn crucial for

[12]We use the observed nominal GDP, as opposed to money supply, since the latter series is heavily affected by time variation in velocity of money, which our model assumes to be constant, in line with most of the theoretical literature.

price setting.[13]

Third and fourth, we fit exogenous TFP processes in the ”Mining and Quarrying” and ”Crop and Animal Production” sectors, $\{A_t^{\text{ENERGY}}\}_{2019:1}^{2024:6}$ and $\{A_t^{\text{FOOD}}\}_{2019:1}^{2024:6}$, in order to exactly match the real IMF Energy Price Index and the IMF Food Price Index as the respective sectoral price indices in equilibrium.[14] In this way, we subject the model to empirically-realistic commodity price shocks, which represent a supply-side influence on aggregate inflation. Since the global commodity prices are largely orthogonal to the Euro Area economic conditions, we believe it is plausible to assume those are driven purely by exogenous sector-specific shocks.

8.2 Explaining the surge in frequency and inflation

Figure 14(a) shows the actual observed changes in aggregate adjustment frequency and aggregate inflation in the Euro Area, as well as the variation generated by four shocks in our baseline non-linear model with menu costs and production networks. In panel (a), one can see that the baseline model successfully reproduces almost the entire surge in the aggregate adjustment frequency, as observed in the Euro Area microdata. In addition, the baseline model can also generate the magnitude of the empirically-observed increase in aggregate CPI inflation.

In order to discern the relative contribution of supply shocks, in Figure 14(b) we compare the actual Euro Area data with the model-implied variation generated exclusively by the aggregate demand and the aggregate labor wedge shocks. It follows that the model produces essentially no surge in the the adjustment frequency, whereas the peak inflation response is just below 5%, as opposed to almost 11% in the data. Since it is the supply-side commodity shocks that generate pricing cascades that get amplified by networks, one can see that the novel mechanism specific to our model is quantitatively important for matching the observed pricing dynamics.

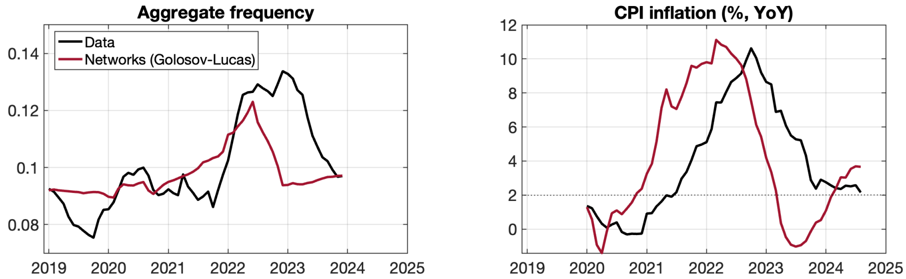
To highlight that our novel mechanism requires an *interaction* of large shocks, networks and state-dependent pricing, in Figure 14(c) we consider alternative modeling setups, which omit our model ingredients one-by-one. In particular, an otherwise identical model without networks, when subjected to the same four shock series, produces no major surge in adjustment frequency and less than half of observed inflation at the peak. At the same time, a model with networks

[13] A more realistic labor market setup would feature search-and-matching frictions with, for example, a bargaining process for the wage.

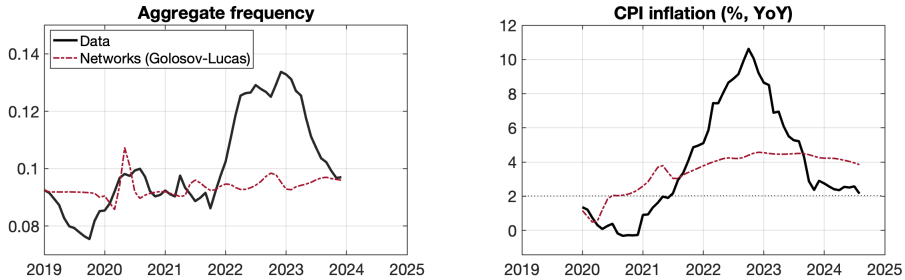
[14] The IMF Energy and Food Price Indices track the respective price movements in US dollars. In order to match them to the *real* sectoral price indices in the Euro Area, we apply two transformations to the IMF Indices. First, we adjust them by movements in the US dollar/Euro nominal exchange rate. Second, we deflate the exchange rate adjusted series by Euro Area nominal GDP in order to get model-consistent real price indices for the Euro Area.

Figure 14: **Explaining the observed surge in frequency and inflation (Euro Area)**

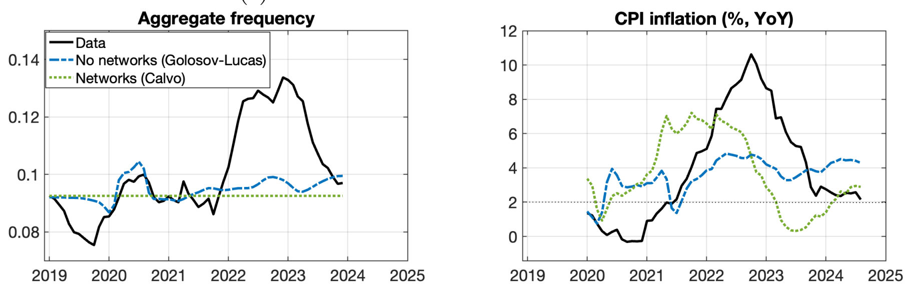
(a) Baseline model: all shocks



(b) Baseline model: no commodity shocks



(c) Alternative models: all shocks



Notes: the figure shows the model-implied changes in aggregate frequency of adjustment and CPI inflation versus the actual observed values in the Euro Area. Panel (a) considers the baseline model with fixed menu costs and networks, subjected to all four shocks; Panel (b) considers the baseline model, which is subjected to the aggregate demand and labor wedge shocks, but not the energy and food price shocks; Panel (c) considers the models with fixed menu costs and no networks, as well as the model with networks and time-dependent pricing, subjected to all four shocks.

and time-dependent Calvo (1983) pricing generates zero variation in frequency by construction, while generating only 7% inflation at the peak.

9 Conclusions

Recent business cycle episodes have shed light on novel aspects of macro fluctuations, such as inflationary swings driven by large sectoral shocks, as well as the crucial role of the extensive margin of price adjustment. Rationalizing such evidence requires broadening our modeling toolkit, which we do by developing a novel theoretical framework featuring an economy with production networks, state-dependent pricing and large shocks. The *interaction* of our three ingredients creates a novel theoretical channel, namely pricing *cascades*: large movements in aggregates triggering price adjustment decisions at the extensive margin. Beyond its conceptual novelty, we show that the interaction of networks with pricing cascades is quantitatively important for rationalizing the Euro Area inflationary experience in the (post-)Covid era.

Key to our novel theoretical mechanism is the differential interaction of networks and pricing cascades, depending on the type of shock driving the business cycle. In particular, under demand shocks, such as central bank interventions, networks *dampen* cascades, slowing down the movements of aggregate adjustment frequency and inflation, as well as strengthening monetary non-neutrality. On the other hand, networks *amplify* cascades following aggregate or sector-specific supply shocks, leading to strong inflationary spiral led by rising fraction of adjusting firms. Quantitatively, we find the network amplification of cascades set off by large movements in energy and food prices to be a crucial contributor towards the surges of Euro Area inflation and adjustment frequency between 2020 and 2024.

We believe that our novel framework creates ample opportunities for future research. First, while our current work is purely positive in character, a natural next step is to move towards a normative analysis of business cycles with pricing cascades, including solving for optimal monetary and fiscal policies. Second, while our framework features a rich supply-side structure, it remains quite parsimonious when it comes to the side of households. Extending the model to feature realistic heterogeneity on the demand side, especially when it comes to marginal propensities to consume and the sectoral composition of purchases, would allow to study the propagation of a broader class of shocks and interventions. In particular, the inflationary consequences of fiscal stimulus through checks and transfers, is one question of utmost importance that could be analyzed.

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Appendix

Business Cycles with Pricing Cascades

Mishel Ghassibe, *CREi, UPF & BSE*

Anton Nakov, *European Central Bank*

A Proofs

Proof of Lemma 1. We want to find a second-order approximation of the firm-level profit function $\tilde{D}_{i,t}(j)$ in the log quality-adjusted real price of that firm $\log \tilde{P}_{i,t}(j)$ near the optimum $\log \tilde{P}^*_{i,t}$. By definition of the optimal reset price, $\frac{\partial \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}} = 0$. As for the second derivative, one can show that:

$$\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2} = \left[(1-\tau_{i,t})e^{(1-\epsilon)\log \tilde{P}_{i,t}(j)}(1-\epsilon)^2 - \epsilon^2 \tilde{\mathcal{Q}}_{i,t}e^{-\epsilon \log \tilde{P}_{i,t}(j)}\right] \times \tilde{P}^{\epsilon}_{i,t}Y_{i,t}. \tag{A.1}$$

Evaluating the second derivative at $\log \tilde{P}^*_{i,t}$, and after some algebra one obtains:

$$\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}(j)} = -(\epsilon-1)(1-\tau_{i,t})\left[\tilde{P}_{i,t}/P^*_{i,t}\right]^{\epsilon-1}\lambda_{i,t}. \tag{A.2}$$

Therefore, one can write the second-order approximation as:

$$\tilde{D}_{i,t} = \tilde{D}^*_{i,t} + \frac{1}{2}\frac{\partial^2 \tilde{D}_{i,t}(j)}{\partial \log \tilde{P}_{i,t}(j)^2}|_{\tilde{P}_{i,t}(j)=\tilde{P}^*_{i,t}(j)} \times [\tilde{p}_{i,t}(j)]^2 + \mathcal{O}[\tilde{p}_{i,t}(j)^3], \tag{A.3}$$

where $\tilde{p}_{i,t}(j) \equiv [\log \tilde{P}_{i,t}(j) - \log \tilde{P}^*_{i,t}]$ is the firm-level price gap. Inserting the expression for the second derivative, one obtains:

$$\tilde{D}^*_{i,t} - \tilde{D}_{i,t} = \frac{1}{2}(\epsilon-1)(1-\tau_{i,t})\left[\tilde{P}_{i,t}/\tilde{P}^*_{i,t}\right]^{\epsilon-1}\lambda_{i,t} \times [\tilde{p}_{i,t}(j)]^2 + \mathcal{O}[\tilde{p}_{i,t}(j)^3]. \tag{A.4}$$

□

Proof of Lemma 2. Focusing on period $t = 0$, a firm adjusts its price if the profit gain from adjustment exceeds the menu cost:

$$\tilde{D}_{i,0}(j)^* - \tilde{D}_{i,0}(j) \geq \kappa_{i,0} \tag{A.5}$$

Using the approximation for the profit gain in Lemma 1, as well as the menu cost form in Assumption 6, one can further rewrite the adjustment condition as:

$$\frac{1}{2}(\epsilon-1)(1-\tau_{i,0})\left[\tilde{P}_{i,0}/\tilde{P}^*_{i,0}\right]^{\epsilon-1}\lambda_{i,0} \times [\tilde{p}_{i,0}(j)]^2 \geq \overline{\kappa}_i(1-\tau_{i,0})[\tilde{P}_{i,0}/\tilde{P}^*_{i,0}]^{\epsilon-1}\lambda_{i,0}, \tag{A.6}$$

$$\implies \quad [\tilde{p}_{i,0}(j)]^2 \geq \frac{2\overline{\kappa}_i}{\epsilon-1}. \tag{A.7}$$

Using the expression for the price gap in (28), as well as the normalization $p_{i,-1}(j) = \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$, the adjustment condition becomes:

$$\left|-\sigma_i \varepsilon_{i,0}(j) - m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0}\right| \geq \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}. \tag{A.8}$$

Therefore, the inaction region is given by:

$$[\sigma_i \underline{\varepsilon}_{i,0}, \quad \sigma_i \overline{\varepsilon}_{i,0}] \quad = \quad -m_0 - \gamma_{i,0} + a_{i,0} - \sum_{k=1}^{N} \overline{\omega}_{ik} \log \tilde{P}_{k,0} \quad \pm \quad \sqrt{\frac{2\overline{\kappa}_i}{\epsilon - 1}}, \tag{A.9}$$

where $m_0 \equiv \log(M_0/M_{-1})$, $\gamma_{i,0} \equiv \log \Gamma_{i,0} - \overline{\gamma}_i$, $a_{i,0} \equiv \log A_{i,0}$ and $\overline{\gamma}_i \equiv \log \frac{\epsilon}{\epsilon-1}\frac{1}{1-\overline{\tau}_i}$ □

Proof of Proposition 1. Before providing a proof for Proposition 1, it is useful to formally establish an auxiliary technical result:

Lemma A1. *Define $f_+(x;c) \equiv \Phi(c+x) - \Phi(-c+x)$ and $f_-(x;c) \equiv \Phi(c-x) - \Phi(-c-x)$, where $c > 0$ is a parameter and $\Phi(.)$ is standard normal CDF. Then both $f_+(x;c)$ and $f_-(x;c)$ are decreasing in x for all $x > 0$.*

Proof. First, consider $f_+(x;c)$. Notice that $f_+'(x) = \Phi'(c+x) - \Phi'(-c+x) = \phi(c+x) - \phi(-c+x)$, where $\phi(.)$ is standard normal PDF. Hence, $f_+'(0) = \phi(c) - \phi(-c) = 0$. As for any $x \in (0, c]$, one can deduce that $f_+'(x) = \underbrace{\phi(c+x)}_{<\phi(c)} - \underbrace{\phi(-c+x)}_{>\phi(-c)} < 0$. Further, for any $x > c$ it follows that $f_+'(x) = \phi(c+x) - \phi(-c+x) < 0$, since standard normal PDF is decreasing in positive inputs. All in all, we conclude that $f_+'(x) < 0$ for all $x > 0$.

Similarly, $f_-'(x) = -\Phi'(c-x) + \Phi'(-c-x) = -\phi(c-x) + \phi(-c-x)$. As before, $f_-'(0) = -\phi(c) + \phi(-c) = 0$. For any $x \in (0, c]$, $f_-'(x) = -\underbrace{\phi(c-x)}_{>\phi(c)} + \underbrace{\phi(-c-x)}_{<\phi(-c)} < 0$. As for any $x > c$, $f_-'(x) = -\phi(c-x) + \phi(-c-x) < 0$, since standard normal PDF is increasing in negative inputs. In total, we conclude that $f_-'(x) < 0$ for all $x > 0$. □

Armed with the additional result in Lemma A1, we are now ready to prove Proposition 1. Consider a monetary expansion $m_0 > 0$. The probability that a firm draws an idiosyncratic innovation that lies in the inaction region following the monetary expansion is given by:

$$
\begin{aligned}
Pr(\underline{\varepsilon}_{i,0} \leq \varepsilon_{i,0}(j) \leq \overline{\varepsilon}_{i,0}) &= \Phi\left(\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} - \frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\
&\quad - \Phi\left(-\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} - \frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\
&= f_-\left(\frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\};\ \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right), \qquad \text{(A.10)}
\end{aligned}
$$

where $f_-(.)$ is defined in Lemma A1. Now, as long as the pass-through of the monetary expansion to sectoral prices is incomplete, $\log\tilde{P}_{k,0} < 0, \forall k$, it follows that $m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} < m_0$. Moreover, since $f_-(.)$ is falling in its positive inputs, it immediately follows that:

$$
f_-\left(\frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\};\ \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) > f_-\left(\frac{1}{\sigma_i}m_0;\ \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right). \qquad \text{(A.11)}
$$

Hence, *ceteris paribus*, the probability of drawing a shock in the inaction region following a monetary expansion is higher in the economy with networks.

Similarly, consider a monetary contraction $m_0 < 0$. The probability that a firm draws an idiosyncratic innovation that lies in the inaction region following the monetary contraction is given by:

$$
\begin{aligned}
Pr(\underline{\varepsilon}_{i,0} \leq \varepsilon_{i,0}(j) \leq \overline{\varepsilon}_{i,0}) &= \Phi\left(\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} + \frac{1}{\sigma_i}\left\{-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\
&\quad - \Phi\left(-\frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}} + \frac{1}{\sigma_i}\left\{-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\}\right) \\
&= f_+\left(\frac{1}{\sigma_i}\left\{-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\};\ \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right), \qquad \text{(A.12)}
\end{aligned}
$$

where $f_+(.)$ is defined in Lemma A1. Now, as long as the pass-through of the monetary contraction to sectoral prices is incomplete, $\log\tilde{P}_{k,0} > 0, \forall k$, it follows that $-m_0 - \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} < -m_0$. Moreover, since $f_+(.)$ is falling in its positive inputs, it immediately follows that:

$$
f_+\left(-\frac{1}{\sigma_i}\left\{m_0 + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0}\right\};\ \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right) > f_+\left(-\frac{1}{\sigma_i}m_0;\ \frac{1}{\sigma_i}\sqrt{\frac{2\overline{\kappa}_i}{\epsilon-1}}\right). \qquad \text{(A.13)}
$$

Hence, *ceteris paribus*, the probability of drawing a shock in the inaction region following a monetary contraction is higher in the economy with networks. □

Proof of Proposition 2. Consider a productivity deterioration $a_{i,0} < 0$ and/or a rise in desired markups $\gamma_{i,0} > 0$ in sector i. The probability that a firm in sector i' (which may or may not be the same as i) draws an idiosyncratic innovation that lies in the inaction region following productivity deterioration/markup increase in sector i is given by:

$$\begin{aligned} Pr(\underline{\varepsilon}_{i',0} \leq \varepsilon_{i',0}(j) \leq \overline{\varepsilon}_{i',0}) \quad &= \quad \Phi\left(\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon - 1}} - \frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &- \Phi\left(-\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon - 1}} - \frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &= \quad f_-\left(\frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon - 1}}\right), \end{aligned} \tag{A.14}$$

where $f_-(.)$ is defined in Lemma A1. Now, as long as the productivity deterioration/markup increase in sector i leads to a rise in sectoral prices of all other sectors, $\log \tilde{P}_{k,0} > 0, \forall k$, it follows that $-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} > -a_{i',0} + \gamma_{i',0}$, $\forall i'$. Moreover, since $f_-(.)$ is falling in its positive inputs, it immediately follows that:

$$f_-\left(\frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon - 1}}\right) < f_-\left(\frac{1}{\sigma_{i'}}\left\{-a_{i',0} + \gamma_{i',0}\right\}; \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon - 1}}\right). \tag{A.15}$$

Hence, *ceteris paribus*, the probability that a firm in sector i' draws shock in the inaction region following a productivity deterioration/markup increase in sector i is lower in the economy with networks.

Similarly, consider a productivity improvement $a_{i,0} > 0$ and/or a fall in desired markups $\gamma_{i,0} < 0$ in sector i. The probability that a firm in sector i' (which may or may not be the same as i) draws an idiosyncratic innovation that lies in the inaction region following productivity improvement/markup decrease in sector i is given by:

$$\begin{aligned} Pr(\underline{\varepsilon}_{i',0} \leq \varepsilon_{i',0}(j) \leq \overline{\varepsilon}_{i',0}) \quad &= \quad \Phi\left(\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon - 1}} + \frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &- \Phi\left(-\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon - 1}} + \frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}\right) \\ &= \quad f_+\left(\frac{1}{\sigma_{i'}}\left\{a_{i',0} - \gamma_{i',0} - \sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\}; \quad \frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon - 1}}\right), \end{aligned} \tag{A.16}$$

where $f_+(.)$ is defined in Lemma A1. Now, as long as the productivity improvement/markup decrease in sector i leads to a fall in sectoral prices of all other sectors, $\log \tilde{P}_{k,0} < 0, \forall k$, it follows that $-a_{i',0} + \gamma_{i',0} + \sum_{k=1}^{N}\overline{\omega}_{ik}\log\overline{P}_{k,0} < -a_{i',0} + \gamma_{i',0}$, $\forall i'$. Moreover, since $f_+(.)$ is falling in its positive inputs, it immediately follows that:

$$f_+\left(-\frac{1}{\sigma_{i'}}\left\{a_{i',0}-\gamma_{i',0}-\sum_{k=1}^{N}\overline{\omega}_{i'k}\log\overline{P}_{k,0}\right\};\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right)<f_+\left(-\frac{1}{\sigma_{i'}}\left\{a_{i',0}-\gamma_{i',0}\right\};\frac{1}{\sigma_{i'}}\sqrt{\frac{2\overline{\kappa}_{i'}}{\epsilon-1}}\right). \tag{A.17}$$

Hence, *ceteris paribus*, the probability that a firm in sector i' draws shock in the inaction region following a productivity deterioration/markup increase in sector i is lower in the economy with networks.

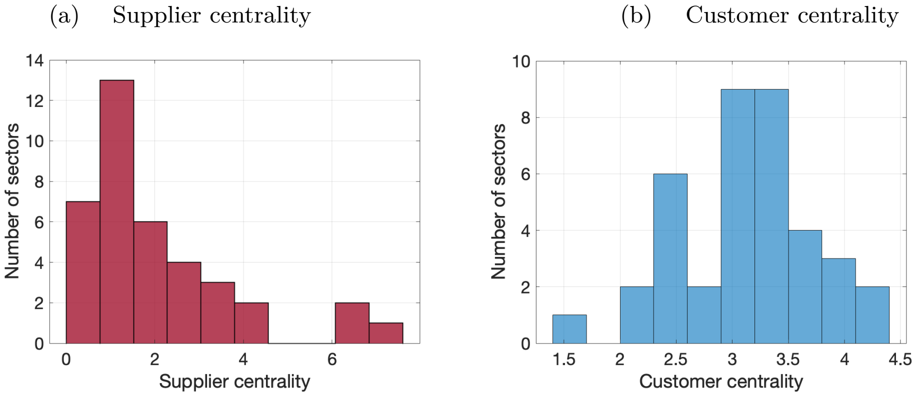
Note that while we provide a proof for sector-specific productivity/markup shocks, results are equivalent for aggregate productivity/markup shocks. This is the case since an aggregate productivity shock a is merely a combination of equally-sized sector-specific productivity shocks $a_i = a, \forall i$, and similarly for an aggregate markup shocks. $\square$

B Additional calibration details (Euro Area)

Table B.1: Consumption Shares, Supplier, Customer, and H Centrality

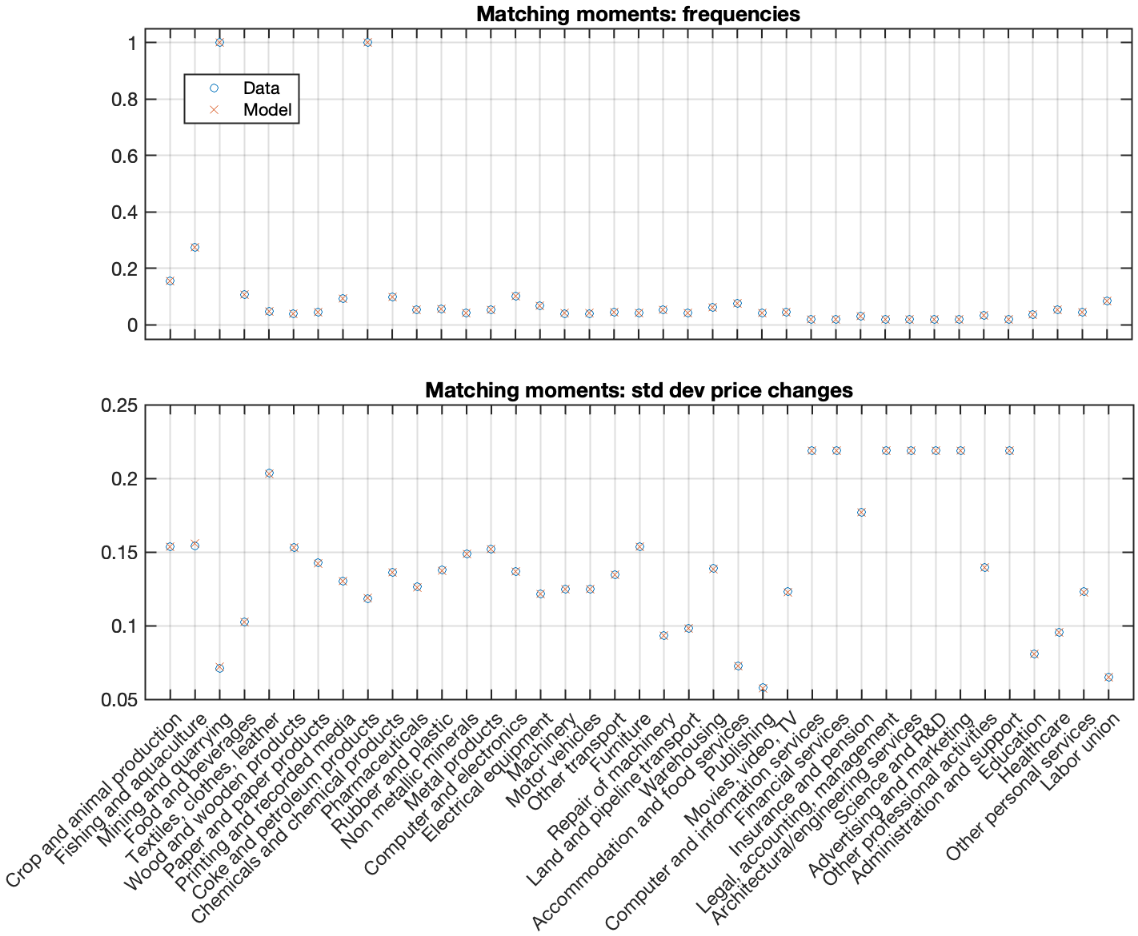
Sector Name	*Consumption Share*	*Customer Centrality*	*Supplier Centrality*	*Supplier Herfindahl*
Crop and animal production	0.0290	3.4452	0.0823	0.0575
Fishing and aquaculture	0.0039	3.2103	0.0291	0.0355
Mining and quarrying	0.0051	3.1919	0.0928	0.0401
Food and beverages	0.1430	3.9448	0.1112	0.0758
Textiles, clothes, leather	0.0403	3.6296	0.0653	0.0799
Wood and wooden products	0.0044	3.6121	0.0607	0.0708
Paper and paper products	0.0079	3.9664	0.0955	0.0794
Printing and recorded media	0.0035	3.3331	0.0544	0.0445
Coke and petroleum products	0.0398	4.3485	0.1061	0.0693
Chemicals and chemical products	0.0162	4.2462	0.1890	0.0929
Pharmaceuticals	0.0140	3.4977	0.0457	0.0493
Rubber and plastic	0.0088	3.7245	0.0791	0.0544
Non metallic minerals	0.0066	3.3615	0.0523	0.0482
Metal products	0.0081	3.1201	0.1118	0.0608
Computer and electronics	0.0175	3.2884	0.0773	0.0593
Electrical equipment	0.0115	3.3162	0.0633	0.0508
Machinery	0.0066	3.3086	0.0837	0.0530
Motor vehicles	0.0514	3.8439	0.0670	0.0726
Other transport	0.0057	3.6416	0.0481	0.0595
Furniture	0.0224	3.1077	0.0426	0.0383
Repair of machinery	0.0030	2.9322	0.0568	0.0373
Land and pipeline transport	0.0398	2.9952	0.1234	0.0483
Warehousing	0.0125	3.1653	0.1291	0.0704
Accommodation and food services	0.1475	2.9424	0.0527	0.0370
Publishing	0.0138	2.9781	0.0462	0.0351
Movies, video, TV	0.0131	2.9446	0.0600	0.0513
Computer and information services	0.0069	2.4193	0.0922	0.0464
Financial services	0.0391	2.6784	0.1442	0.0614
Insurance and pension	0.0502	3.2978	0.0648	0.0667
Legal, accounting, management	0.0079	2.2538	0.1976	0.0523
Architectural/engineering services	0.0037	2.3319	0.0793	0.0431
Science and R&D	0.0020	2.4415	0.0333	0.0319
Advertising and marketing	0.0020	2.8706	0.0574	0.0380
Other professional activities	0.0069	2.3978	0.0517	0.0336
Administration and support	0.0293	2.4434	0.2261	0.0472
Education	0.0261	1.5508	0.0400	0.0289
Healthcare	0.0743	2.0342	0.0340	0.0321
Other personal services	0.0765	2.3334	0.0631	0.0383

Figure B.2: **Distributions of Supplier and Customer centrality (Euro Area, 38 sectors)**



Notes: Panels (a) and (b) show the distributions of Supplier and Customer centrality across the 38 production sectors in the Euro Area.

Figure B.3: Matching pricing moments for each sector



Notes: the figure shows the sector-specific frequencies and sizes of price adjustment, as well as the corresponding model-based steady-state values under our estimated values of sectoral menu costs and standard deviations of idiosyncratic quality innovations.

C Details of the numerical algorithm

C.1 Steady state computation on a grid

For each sector, we solve the stationary Bellman equation and price distribution on an evenly spaced grid of log prices $\mathbf{\Gamma}$ with step size Δp, $p_j \in \left[\underline{p}, \underline{p} + \Delta p, ..., \overline{p}\right]$, $j = 1, .., J$ grid points, so that $V_j = V(p_j)$. The expectation $\mathbb{E}\left[V(p - \sigma\varepsilon_{t+1} - \pi)|p = p_j\right]$ is calculated as $\boldsymbol{TV}$ where we define transition matrix

$$\boldsymbol{T} = \begin{bmatrix} \mathcal{T}_{1,1} & \mathcal{T}_{1,2} & \cdots & \mathcal{T}_{1,J} \\ \mathcal{T}_{2,1} & \mathcal{T}_{2,2} & \cdots & \mathcal{T}_{2,J} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{T}_{J,1} & \mathcal{T}_{J,2} & \cdots & \mathcal{T}_{J,J} \end{bmatrix}.$$

with elements

$$\mathcal{T}_{j,k} = \int_{p=p_{k-1/2}}^{p_{k+1/2}} \psi\left(\frac{p-(p_j-\pi)}{\sigma}\right) dp = \Psi\left(\frac{p_{k+1/2}-(p_j-\pi)}{\sigma}\right) - \Psi\left(\frac{p_{k-1/2}-(p_j-\pi)}{\sigma}\right),$$

and where $p_{k-1/2} \equiv (p_{k-1} + p_k)/2$, $p_{k+1/2} \equiv (p_k + p_{k+1})/2$, $\psi(\cdot)$ is the standard normal probability density function, and $\Psi(\cdot)$ is the standard normal cumulative distribution function.

We also define the vectors

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_J \end{bmatrix}, \ \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_J \end{bmatrix}, \ \boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_J \end{bmatrix}, \ \boldsymbol{D} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_J \end{bmatrix}$$

The Bellman equation in matrix notation is then given by

$$\boldsymbol{V} \;=\; \boldsymbol{D} + \beta\left[\boldsymbol{T}\left((1-\eta)\cdot\boldsymbol{V}\right) + \boldsymbol{T}\left(\eta\cdot(\boldsymbol{\phi}'\boldsymbol{V} - \kappa w)\right)\right]$$

where $\cdot$ denotes element-by-element multiplication. Vector $\boldsymbol{\phi}$ distributes unit probability mass to grid points adjacent to p^* according to the logit formula

$$\boldsymbol{\phi} = \frac{\exp(\boldsymbol{V}/\chi)}{\sum_{\Gamma}\exp\left(\boldsymbol{V}/\chi\right)}$$

with $\chi = 0.0005$. Note that $\boldsymbol{\phi}'\boldsymbol{V}$ performs smooth maximization as in eq.(21).

To solve the problem for N sectors with input-output linkages, we use the following algorithm. Start with a guess for the vector of steady-state price dispersions Δ_k and sectoral taxes τ_k, then:[15]

1. Given $\pi = \overline{\pi}$, compute the transition matrix $\boldsymbol{T}$
2. Using $\frac{W}{M} \equiv w = 1$, compute[16] $\omega_{ik} = \overline{\omega}_{ik} \times \frac{(P_k/M)^{1-\theta_i}}{\overline{\alpha}_i + \sum_{k'=1}^{N} \overline{\omega}_{ik'}(P_{k'}/M)^{1-\theta_i}} = \overline{\omega}_{ik}$
3. λ is given by eq.(15) and η by eq.(22)
4. With that, construct the profit matrix $\boldsymbol{D}$ as in eq.(19)
5. Iterate backward on the value function $\boldsymbol{V}$ above to convergence
6. To compute the distribution, iterate forward on

$$\boldsymbol{g} = (1-\eta) \cdot (\boldsymbol{T}'\boldsymbol{g}) + \boldsymbol{\phi}\eta' \left(\boldsymbol{T}'\boldsymbol{g}\right). \tag{C.1}$$

   until convergence of $\boldsymbol{g}$.

7. Given the distribution, compute the residual vectors $resid_1$ and $resid_2$ as in

$$resid_1 = \Delta_k - (P_k/M)^{\epsilon} \int_0^1 \left(\frac{P_k(j')}{\zeta_k(j')M}\right)^{-\epsilon} dj', \tag{C.2}$$

$$resid_2 = P_k/M - \int_0^1 \left(\frac{P_k(j')}{\zeta_k(j')M}\right)^{1-\epsilon} dj' \tag{C.3}$$

8. Search for a vector of sectoral price dispersions and taxes such that $resid \to 10^{-14}$.

C.2 Solving for impulse-responses in sequence space

We compute fully non-linearly the responses to an MIT shock in the space of sequences, iterating backward in time on the value function and forward in time on the law of motion of the distribution, under the assumption of perfect foresight. The steps are similar to those for computing the steady state; only this time we keep track of the sequences over time. We start by guessing sequences for time t from 1 to $T = 500$ months, for sectoral prices and price dispersions (our starting guess simply equals the steady-state value for these variables). The key assumption is that all stationary variables must return to steady state by period T. Given

[15]We start with the guess $\Delta_k = 1$ and $\tau_k = -1/\epsilon$

[16]We are searching for taxes τ_k such that the steady state equilibrium is symmetric in sectoral prices, $P_k/M = 1$

this initial guess, we compute the price of the final good and consumption over time using their definitions. Given that, we calculate λ_t as in eq.(15). We compute the profits D_t as in eq.(19). Iterating backward in time from $t = T$ to $t = 0$, we solve for the value function as in eq.(21). Given the value function, we can compute the gain from adjustment L_t and the adjustment hazard η_t. Once the backward iteration on the value function reaches period 0, we start from the steady-state distribution and iterate forward in time on the law of motion of the price distribution from period 1 until period T. Given the distribution, we can compute via eq.(7) the sectoral price indices, and by

$$\Delta_{k,t} \equiv (P_{k,t}/M_t)^{\epsilon} \int_0^1 \left(\frac{P_{k,t}(j')}{\zeta_{k,t}(j')M_t} \right)^{-\epsilon} dj'$$

the sectoral price dispersions. This provides us with an updated guess, with which we repeat the previous steps until the change in the sequences (of sectoral prices and price dispersions) becomes near zero.

D Cashless limit

The representative household chooses a sequence of consumption, labor supply and one-period nominal bond holdings to maximize expected lifetime utility:

$$\max_{\{C_t,L_t,B_t\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{D.1}$$

subject to the period-by-period budget constraint

$$P_t^C C_t + B_t \quad = \quad (1+i_{t-1})B_{t-1} + W_t L_t + \sum_{i=1}^{N} \int_0^1 D_{i,t}(j)dj + T_t, \tag{D.2}$$

where C_t is consumption, L_t is labor supply, B_t is the level of nominal bond holdings, T_t is the level of lump-sum transfers from the government, $D_{i,t}(j)$ are the dividends received lump-sum from firm j in sector i at time t, $\Pi_t^C = \left(P_t^C / P_{t-1}^C\right)$ is the gross CPI inflation rate, W_t is the nominal wage and i_t is the nominal interest rate set by the central bank.

The nominal interest rate follows the following Taylor-type rule:

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_t^C + \phi_c c_t\right] + \varepsilon_t^i, \tag{D.3}$$

where $\hat{\imath}_t \equiv \log \frac{1+i_t}{\overline{\Pi}/\beta}$ is the log-deviation of the nominal interest rate from its steady-state value, $\pi_t^C \equiv \log \Pi_t^C/\overline{\Pi}$ is deviation of CPI inflation from target and $c_t \equiv \log C_t/\overline{C}$ is aggregate GDP in deviation from steady state. In the rule, $\rho_i \in [0,1)$ determines the degree of policy persistence, $\phi_\pi > 0$ and $\phi_c > 0$ pin down how aggressively the central bank responds to deviations of inflation and GDP from their steady-state values, and ε_t^i is the monetary policy shock.

We assume the following form of households' preferences:

$$u(C_t, L_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi}. \tag{D.4}$$

Note that the Golosov-Lucas log-linear preferences which we use in the main text arise as a special case when $\sigma \to 1$ and $\varphi = 0$.

Given the presence of possibly non-zero steady-state inflation and the non-stationarity of the quality processes, we appropriately normalize our variables. Unlike in the main text, where we normalize by money supply, in the current cashless setting, we instead normalize by the aggregate CPI price level P_{t-1}^C. In particular, we let $\tilde{P_{i,t}}(j) \equiv \frac{P_{i,t}(j)}{\zeta_{i,t}(j) P_{t-1}^C}$ be the quality-adjusted real price, $\tilde{P}_{i,t} \equiv \frac{P_{i,t}}{P^C_{t-1}}$ be the real sectoral price, and $\tilde{W}_t \equiv \frac{W_t}{P^C_{t-1}}$ be the real wage. Then the equilibrium conditions for the aggregate real variables are given by:

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left[\frac{1+i_t}{\Pi^C_{t+1}} C_{t+1}^{-\sigma} \right] \tag{D.5}$$

$$C_t^{\sigma} L_t^{\varphi} = \tilde{W}_t / \Pi^C_t \tag{D.6}$$

$$L_t = \Pi^C_t \frac{C_t}{\tilde{W}_t} \left[1 - \sum_{i=1}^{N} \lambda_{i,t} \left(1 - \frac{\Delta_{i,t}}{\mathcal{M}_{i,t}} \right) \right] + \sum_{i=1}^{N} \kappa_{i,t} \int_0^1 \eta_{i,t}(j) dj. \tag{D.7}$$

where $\lambda_{i,t}$ is the sectoral Domar weight (sales) share, $\Delta_{i,t}$ is the within-sector dispersion of real prices and $\mathcal{M}_{i,t}$ is the sectoral markup, which are given by:

$$\lambda_{i,t} = \omega^C_{i,t} + \sum_{k=1}^{N} \omega_{k,i,t} \lambda_{k,t} \frac{\Delta_{i,t}}{\mathcal{M}_{i,t}}, \qquad \Delta_{i,t} \equiv \tilde{P}^{\epsilon}_{i,t} \int_0^1 \tilde{P}_{i,t}(j)^{-\epsilon} dj, \qquad \mathcal{M}_{i,t} \equiv \frac{\tilde{P}_{i,t}}{\tilde{\mathcal{Q}}_{i,t}}. \tag{D.8}$$

The real sectoral price indices and marginal costs in turn satisfy:

$$\tilde{P}^{1-\epsilon}_{i,t} = \int_0^1 \tilde{P}_{i,t}(j)^{1-\epsilon} dj, \qquad \tilde{\mathcal{Q}}_{i,t} = \mathcal{Q}_i \left[\tilde{W}_t, \tilde{P}_{1,t}, ..., \tilde{P}_{N,t}; A_{i,t} \right], \qquad \Pi^C_t = \mathcal{P}^{\mathcal{C}} \left[\tilde{P}_{1,t}, ..., \tilde{P}_{N,t} \right]. \tag{D.9}$$

If the nominal price is not adjusted, then the quality-adjusted real price evolves according to:

$$p_{i,t}(j) = p_{i,t-1}(j) - \sigma_i \varepsilon_{i,t}(j) - \pi^C_{t-1}, \tag{D.10}$$

where $\pi^C_{t-1} \equiv \log \Pi^C_{t-1}$.

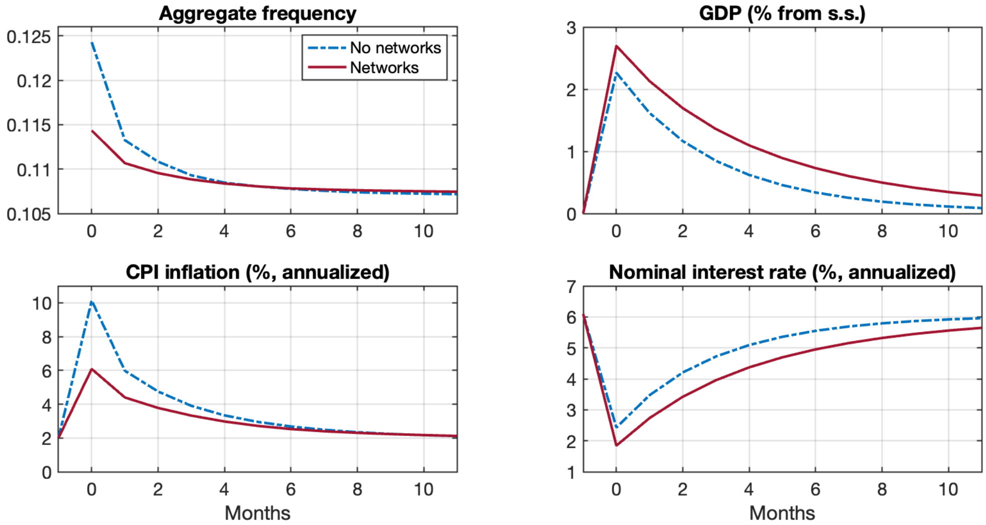
The per-period real profits of a firm are given by:

$$\tilde{D}_{i,t}(j) = \tilde{P}^{\epsilon-1}_{i,t} \left[(1-\tau_{i,t}) \tilde{P}_{i,t}(j) - \tilde{\mathcal{Q}}_{i,t} \right] \tilde{P}_{i,t}(j)^{-\epsilon} \times \lambda_{i,t} \times C_t \times \Pi^C_t. \tag{D.11}$$

Finally, consider a firm with real quality-adjusted price p at the end of period t, and let $p_+ \equiv (p - \sigma_i \varepsilon_{i,t+1}(j) - \pi^C_t)$, where $\pi^C_t \equiv \log \Pi^C_t$. Then this firm's real value at the end of period t is given by the following Bellman equation:

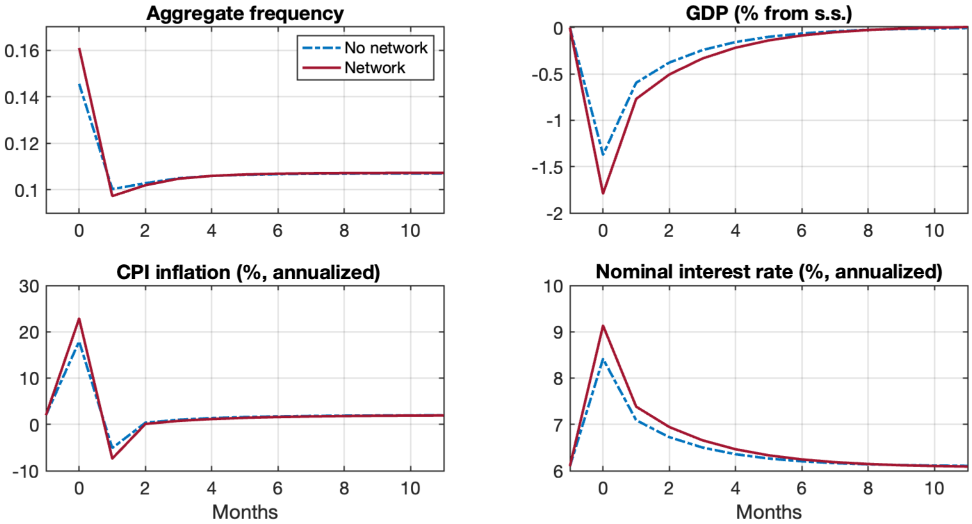
$$
\begin{aligned}
V_{i,t}(p) \quad &= \quad \tilde{D}_{i,t}(p) \quad + \\
&+ \beta \mathbb{E}_t \left[\left\{ 1 - \eta_{i,t+1}\left(p_+\right) \right\} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\Pi_t^C}{\Pi_{t+1}^C} V_{i,t+1}(p_+) \right] + \\
&+ \beta \mathbb{E}_t \left[\eta_{i,t+1}\left(p_+\right) \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\Pi_t^C}{\Pi_{t+1}^C} \left(\max_{p'} V_{i,t+1}\left(p'\right) - \kappa_{i,t+1} \tilde{W}_{t+1} \right) \right].
\end{aligned}
$$

Figure D.1: **Aggregate responses to a monetary shock under a Taylor rule**



Notes: the figure shows the responses of aggregate adjustment frequency, GDP, CPI inflation and the nominal interest rate to a one-time -500 basis points (annualized) shock to the Taylor rule ($\varepsilon_0^i = -0.05/12$).

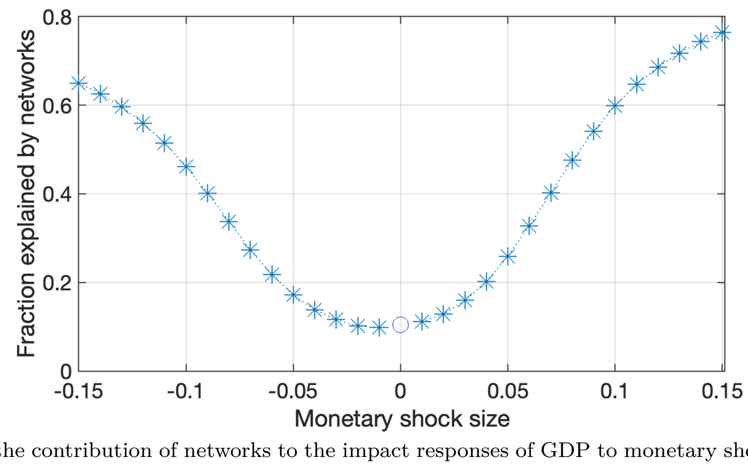
Figure D.2: **Aggregate responses to an aggregate TFP shock under a Taylor rule**



Notes: the figure shows the responses of aggregate adjustment frequency, GDP, CPI inflation and the nominal interest rate to a one-time transitory ($\rho = 0$) aggregate TFP shock of -5%.

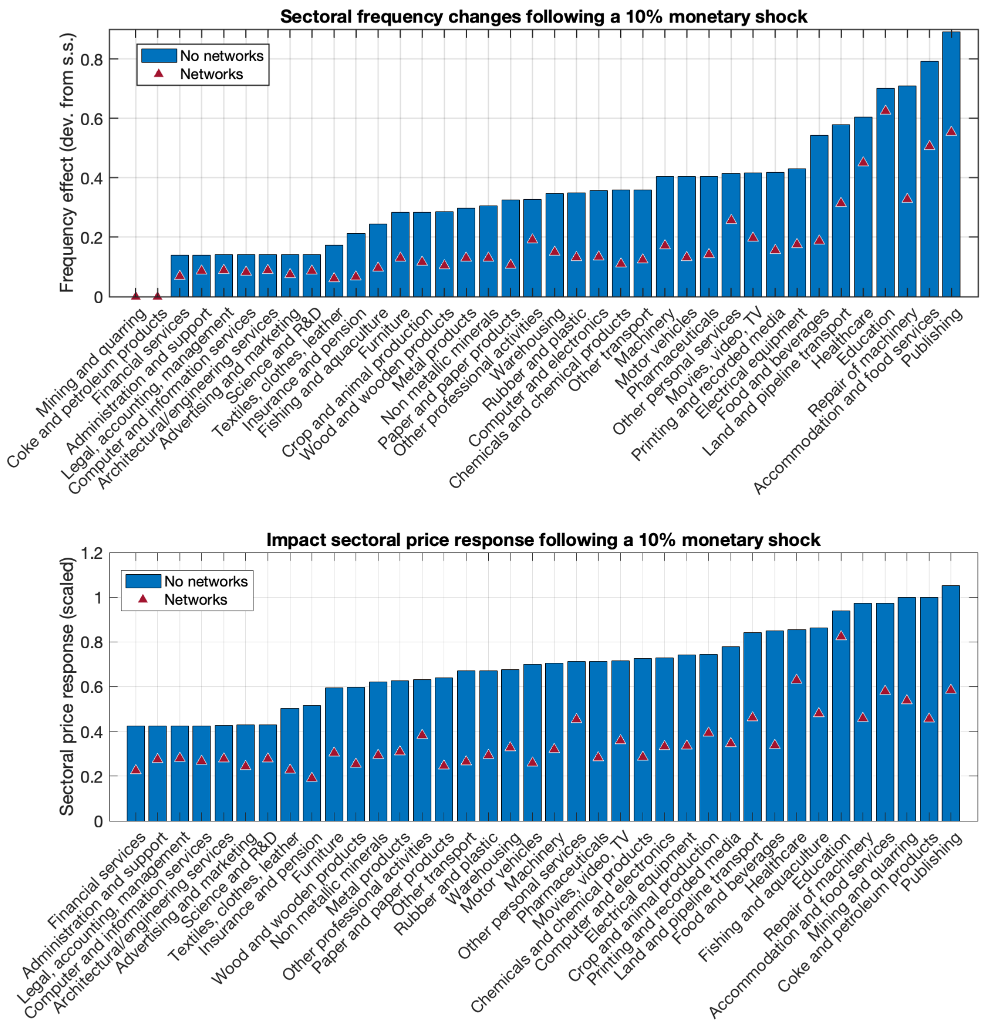
E Additional figures

Figure E.1: **Network amplification of GDP responses to monetary shocks**



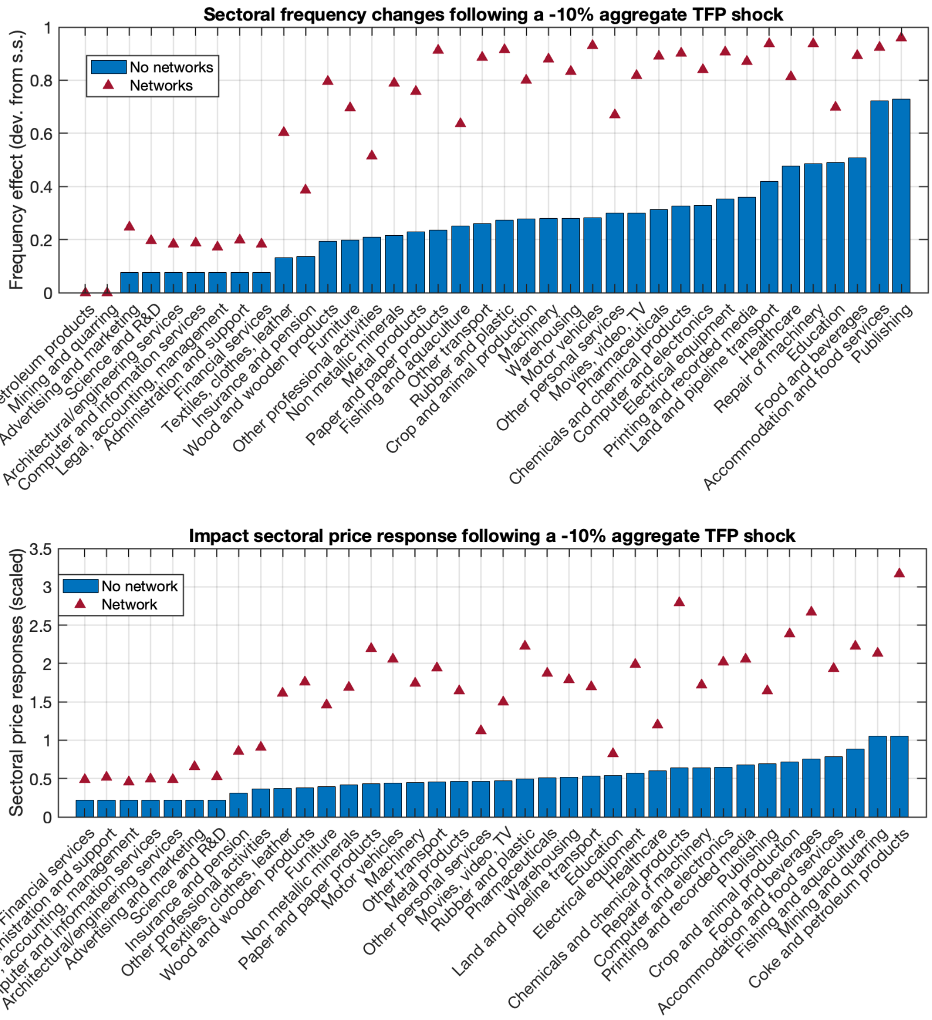
Notes: the figure shows the contribution of networks to the impact responses of GDP to monetary shocks of different sizes under fixed menu costs.

Figure E.2: **Sectoral responses to a monetary shock**



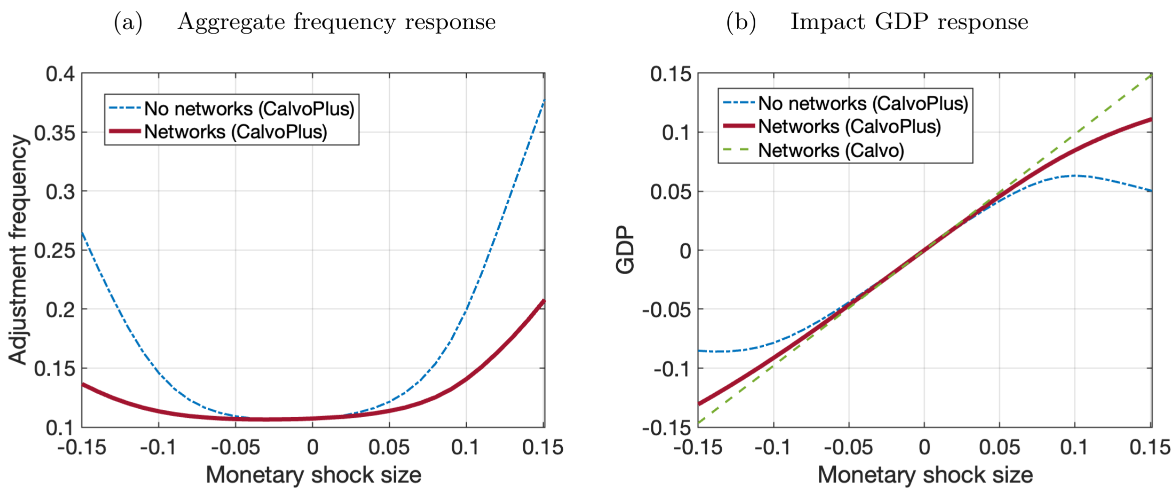
Notes: the top Panel shows the impact responses of sector-specific adjustment frequency to a one-time 10% monetary shock; the bottom Panel similarly shows the scaled impact responses of sectoral price indices one-time 10% monetary shock.

Figure E.3: **Sectoral responses to a aggregate TFP shock**



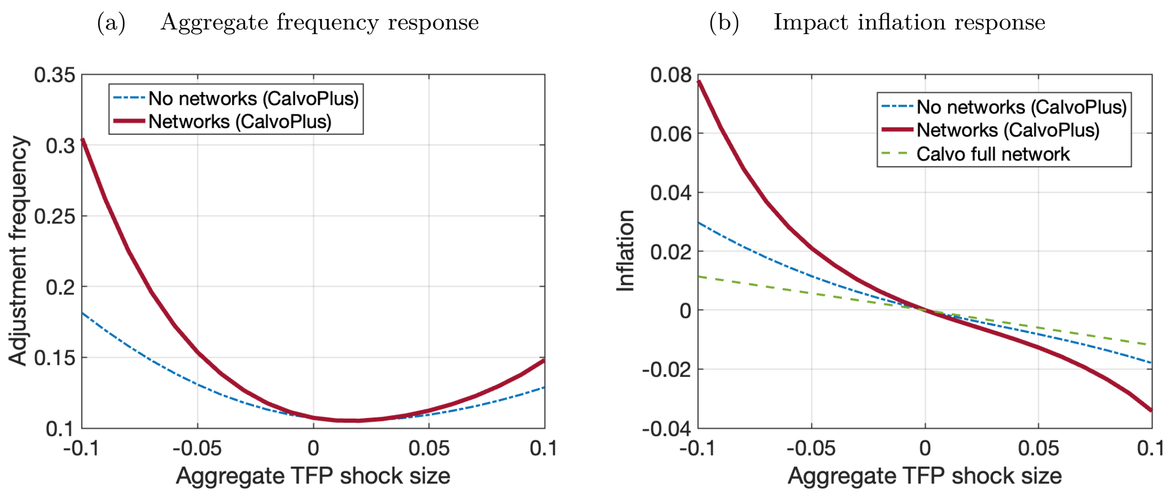
Notes: the top Panel shows the impact responses of sector-specific adjustment frequency to a one-time -10% aggregate TFP shock; the bottom Panel similarly shows the scaled impact responses of sectoral price indices one-time -10% aggregate TFP shock.

Figure E.4: **Frequency and GDP responses to monetary shocks: CalvoPlus**



Notes: Panel (a) shows the impact responses of the aggregate adjustment frequency following monetary shocks of different sizes in the economy with CalvoPlus pricing and networks, as well as in the otherwise identical economy without networks; Panel (b) shows the impact responses of GDP to monetary shocks of different sizes in three economies: the economy with CalvoPlus pricing and networks, as well as the otherwise identical economies without networks and with time-dependent pricing.

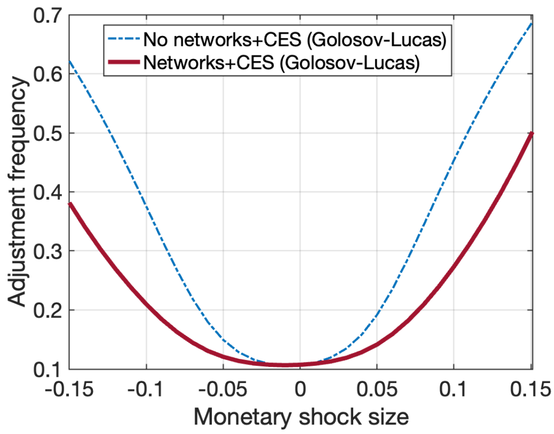
Figure E.5: **Frequency and inflation responses to agg. TFP shocks: CalvoPlus**



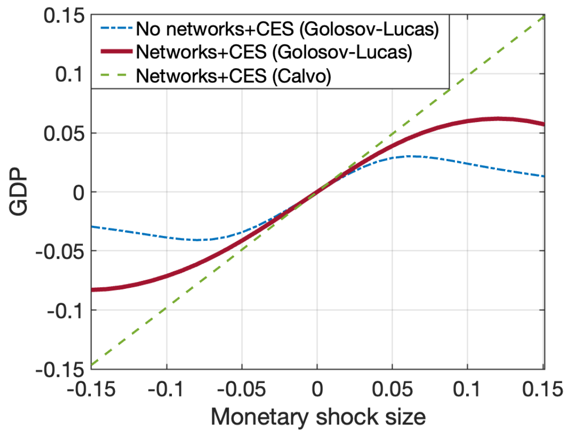
Notes: Panel (a) shows the impact responses of the aggregate adjustment frequency following aggregate TFP shocks of different sizes in the economy with CalvoPlus pricing and networks, as well as in the otherwise identical economy without networks; Panel (b) shows the impact responses of CPI inflation to aggregate TFP shocks of different sizes in three economies: the economy with CalvoPlus pricing and networks, as well as the otherwise identical economies without networks and with time-dependent pricing.

Figure E.6: **Frequency and GDP responses to monetary shocks: CES aggregation**

(a) Aggregate frequency response



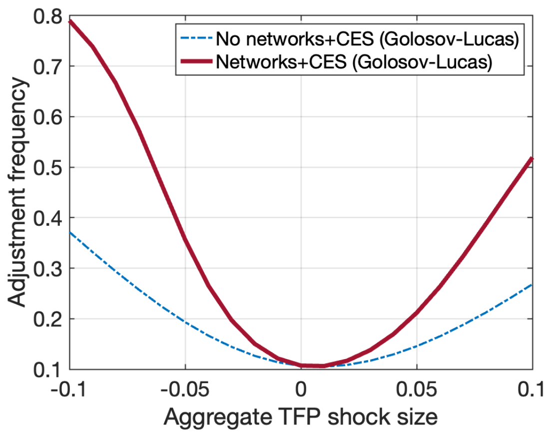
(b) Impact GDP response



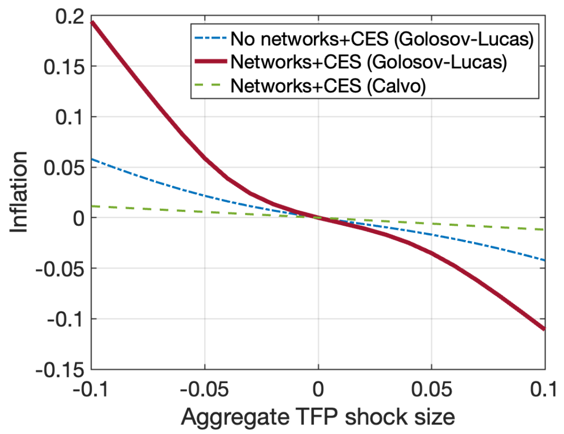
Notes: Panel (a) shows the impact responses of the aggregate adjustment frequency following monetary shocks of different sizes in the economy with CES aggregation, fixed menu costs and networks, as well as in the otherwise identical economy without networks; Panel (b) shows the impact responses of GDP to monetary shocks of different sizes in three economies: the economy with CES aggregation, fixed menu costs and networks, as well as the otherwise identical economies without networks and with time-dependent pricing.

Figure E.7: **Frequency and inflation responses to agg. TFP shocks: CES aggregation**

(a) Aggregate frequency response



(b) Impact inflation response



Notes: Panel (a) shows the impact responses of the aggregate adjustment frequency following aggregate TFP shocks of different sizes in the economy with CES aggregation, fixed menu costs and networks, as well as in the otherwise identical economy without networks; Panel (b) shows the impact responses of CPI inflation to aggregate TFP shocks of different sizes in three economies: the economy with CES aggregation, fixed menu costs and networks, as well as the otherwise identical economies without networks and with time-dependent pricing.