Strike while the Iron is Hot:

Optimal Monetary Policy with Nonlinear Inflation Dynamics

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Motivation

▶ The recent inflation surge featured

    ▶ Increase in the frequency of price changes (Montag and Villar, 2023) US

    ▶ Increase in Phillips curve slope (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023) US

▶ Optimal monetary policy is mainly studied in models, in which the Phillips curve is linear and the frequency is held constant (Galí, 2008; Woodford, 2003)

▶ What does optimal monetary policy look like with a nonlinear Phillips curve and endogenous variation in frequency? How should CBs respond to a large inflation surge?

What do we do?

▶ We use the standard state-dependent pricing model of Golosov and Lucas (2007)

▶ Solve it nonlinearly over the sequence space

▶ Positive analysis under a Taylor rule

    ▶ Trace the responses to shocks of different sizes

    ▶ Assess the nonlinearity of the Phillips curve

▶ Normative analysis: Ramsey optimal policy

    ▶ Optimal long-run inflation

    ▶ Trace the optimal responses to shocks

    ▶ Characterize the (nonlinear) targeting rule after *large* cost-push shocks

What do we find?

▶ In this model the Phillips curve is nonlinear: it gets steeper as frequency increases

▶ In response to small shocks, optimal monetary policy is similar to the one under Calvo

▶ In response to efficiency shocks, there is divine coincidence, as in Calvo

▶ Different responses to small and large cost-push shocks. Optimal policy leans forcefully against inflation when frequency rises: "it strikes while the iron is hot"

Literature

▶ Nonlinear Phillips curve (Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023)

  ▶ Microfounded by state-dependent price setting

    (Golosov and Lucas, 2007; Gertler and Leahy, 2008; Auclert et al., 2022)

  ▶ In the presence of large aggregate shocks (Karadi and Reiff, 2019; Alvarez and Neumeyer, 2020;

    Costain et al., 2022; Alexandrov, 2020; Blanco et al., 2024)

▶ Optimal policy in a menu cost economy

  ▶ Optimal inflation target (Burstein and Hellwig, 2008; Adam and Weber, 2019; Blanco, 2021)

  ▶ Small shocks, large shocks, optimal nonlinear target rule (comp. Galí, 2008; Woodford, 2003)

  ▶ Focus on aggregate shocks (unlike Caratelli and Halperin, 2023, who study *sectoral* shocks)

Overview of the model

▶ Heterogeneous-firm DSGE model with fixed costs of price-adjustment

▶ Households: consume a Dixit-Stiglitz basket of goods, and work

▶ Firms: produce differentiated goods using labor only and are subject to aggregate TFP shocks and idiosyncratic "quality" shocks. They have market power and set prices optimally subject to a fixed cost (Golosov and Lucas, 2007)

▶ Monetary policy: either follows Taylor rule or set optimally to maximize household welfare under commitment

Households

▶ A representative household consumes (C_t), supplies labor hours (N_t) and saves in one-period nominal bonds (B_t).

▶ The household's problem is:

$$\max_{C_t, N_t, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log (C)_t - \nu N_t$$

$$\text{s.t. } P_t C_t + B_t + T_t = R_{t-1} B_{t-1} + W_t N_t + D_t,$$

where P_t is the price level, R_t is the gross nominal interest rate, W_t is the nominal wage, T_t are lump sum transfers and D_t are profits

Consumption and labor

▶ Aggregate consumption C_t and the price level are defined as:

$$C_t = \left\{ \int \left[A_t(i) C_t(i) \right]^{\frac{\epsilon - 1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon - 1}}, \quad P_t = \left[\int_0^1 \left(\frac{P_t(i)}{A_t(i)} \right)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

where $A_t(i)$ is product quality, ϵ is the elasticity of substitution.

▶ Labor supply condition and Euler equation are given by:

$$W_t = \upsilon P_t C_t, \quad 1 = \mathbb{E}_t \left[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{R_t}{\Pi_{t+1}} \right]$$

Monopolistic producers

▶ Production of good i is given by $Y_t(i) = A_t \frac{N_t(i)}{A_t(i)}$, where quality follows a random walk

$$log(A_t(i)) = log(A_{t-1}(i)) + \varepsilon_t(i), \quad \varepsilon_t(i) \sim N(0, \sigma_t^2)$$

▶ Firms real profits

$$\frac{D_t(i)}{P_t} = \frac{P_t(i)}{P_t} Y_t(i) - (1 - \tau_t) \frac{W_t}{P_t} N_t(i)$$

▶ Firms face a fixed cost η to update prices

Quality-adjusted relative prices

▶ Let $p_t(i) \equiv \log \left(P_t(i)/(A_t(i)P_t) \right)$ be the quality-adjusted log relative price

▶ Real profit then is

$$\Pi(p_t(i), w_t, A_t) \equiv \frac{D_t(i)}{P_t} = C_t e^{p_t(i)(1-\epsilon)} - C_t(1-\tau_t)\frac{w_t}{A_t}e^{p_t(i)(-\epsilon)}$$

where w_t is the real wage.

▶ When nominal price $P_t(i)$ stays constant, $p_t(i)$ evolves: $p_t(i) = p_{t-1}(i) - \sigma_t \varepsilon_t(i) - \pi_t$

Pricing decision

► Let $\lambda_t(p)$ be the endogenous price-adjustment probability

► Value function is

$$
\begin{aligned}
V_t(p) &= \Pi(p, w_t, A_t) \\
&+ \mathbb{E}_t \left[(1 - \lambda_{t+1} (p - \sigma_{t+1}\varepsilon_{t+1} - \pi_{t+1})) \Lambda_{t,t+1} V_{t+1}(p - \sigma_{t+1}\varepsilon_{t+1} - \pi_{t+1}) \right] \\
&+ \mathbb{E}_t \left[\lambda_{t+1} (p - \sigma_{t+1}\varepsilon_{t+1} - \pi_{t+1}) \Lambda_{t,t+1} \left(\max_{p'} V_{t+1} (p') - \eta w_{t+1} \right) \right].
\end{aligned}
$$

► The adjustment probability is

$$
\lambda_t(p) = I[\max_{p'} V_t (p') - \eta w_t > V_t(p)]
$$

where $I[\cdot]$ is the indicator function.

Monetary Policy

▶ The central bank either sets policy optimally, or follows a Taylor rule:

$$\log\left(R_t\right) = \rho_r \log\left(R_{t-1}\right) + (1 - \rho_r)\left[\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y_t^e)\right] + \varepsilon_{r,t} \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2)$$

▶ Shocks: employment subsidy (τ_t), TFP (A_t), volatility (σ_t)

$$\log\left(A_t\right) = \rho_A \log\left(A_{t-1}\right) + \varepsilon_{A,t} \quad \varepsilon_{A,t} \sim N(0, \sigma_A^2)$$

$$\tau_t - \tau = \rho_\tau(\tau_{t-1} - \tau) + \varepsilon_{\tau,t} \quad \varepsilon_{\tau,t} \sim N(0, \sigma_\tau^2),$$

$$\log\left(\sigma_t/\sigma\right) = \rho_\sigma \log\left(\sigma_{t-1}/\sigma\right) + \varepsilon_{\sigma,t} \quad \varepsilon_{\sigma,t} \sim N(0, \sigma_\sigma^2)$$

Aggregation and market clearing

▶ Aggregate price index

$$1 = \int e^{p(1-\epsilon)} g_t(p)\, dp,$$

▶ Labor market equilibrium

$$N_t = \frac{C_t}{A_t} \underbrace{\int e^{p(-\epsilon)} g_t(p)\, dp}_{\text{dispersion}} + \eta \underbrace{\int \lambda_t(p - \sigma_t \varepsilon - \pi_t) g_{t-1}(p) dp}_{\text{frequency}},$$

Law of motion of the price density

$$
g_t(p) = \begin{cases}
(1 - \lambda_t(p)) \int g_{t-1}(p + \sigma\varepsilon + \pi_t)d\xi(\varepsilon) & \text{if } p \neq p_t^*, \\[2ex]
(1 - \lambda_t(p_t^*)) \int g_{t-1}(p_t^* + \sigma\varepsilon + \pi_t)d\xi(\varepsilon) + \\
\quad \int_{\underline{p}}^{\overline{p}} \lambda_t(\tilde{p}) \left(\int g_{t-1}(\tilde{p} + \sigma\varepsilon + \pi_t)d\xi(\varepsilon) \right) d\tilde{p} & \text{if } p = p_t^*.
\end{cases}
$$

Calibration

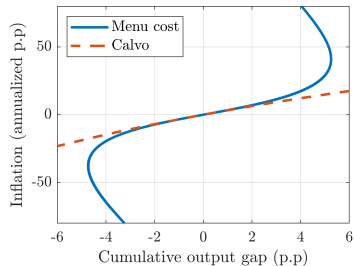| Households			
β	$0.96^{1/12}$	Discount rate	Golosov and Lucas (2007)
ϵ	7	Elasticity of substitution	Golosov and Lucas (2007)
γ	1	Risk aversion parameter	Midrigan (2011)
υ	1	Utility weight on labor	Set to yield $w = C$
Price setting			
η	3.6%	Menu cost	Set to match 8.7% of frequency
σ	2.4%	Std dev of quality shocks	and 8.5% size in Nakamura and Steinsson (2008)
Monetary policy			
ϕ_π	1.5	Inflation coefficient in Taylor rule	Taylor (1993)
ϕ_y	0.5/12	Output gap coefficient in Taylor rule	Taylor (1993)
ρ_i	$0.75^{1/3}$	Smoothing coefficient	
Shocks			
ρ_A	$0.95^{1/3}$	Persistence of the TFP shock	Smets and Wouters (2007)
ρ_τ	$0.9^{1/3}$	Persistence of the cost-push shock	Smets and Wouters (2007)

Large shocks: illustration

- ▶ Let $x_t(j) \equiv p_t(j) - p_t^*(j)$ be the price gap

- ▶ Large aggregate shock: shifts the distribution of price gaps for all firms

- ▶ Limited impact on the (s,S) bands

- ▶ Pushes a large fraction of firms outside of the inaction region

- ▶ Large increase in the frequency of price changes and hence additional flexibility of the aggregate price level (different from "selection")
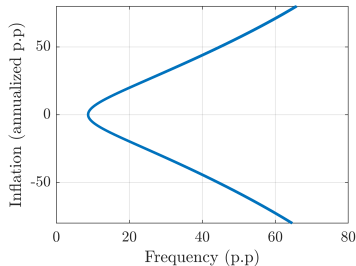

Distribution of Price Gaps in Steady State
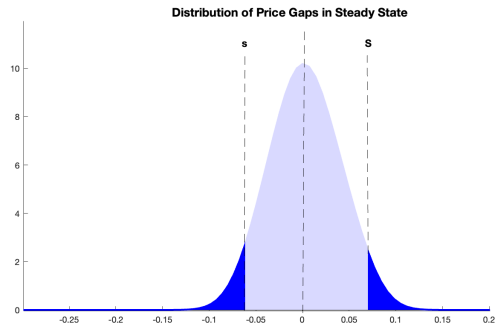
Large shocks: illustration, cont.

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Distribution of Price Gaps after Large Shock
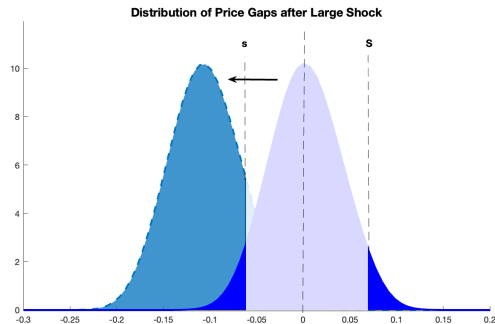
Nonlinearity of the Phillips Curve at realistic frequency (20%) US

Consider the model under a Taylor rule

Nonlinear Phillips Curve (PC)   Frequency   PC slope
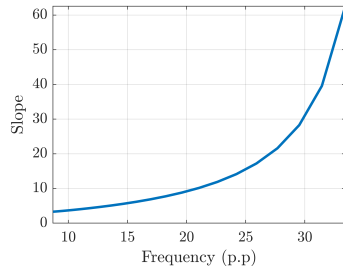
Nonlinearity of the Phillips Curve at realistic frequency (20%)
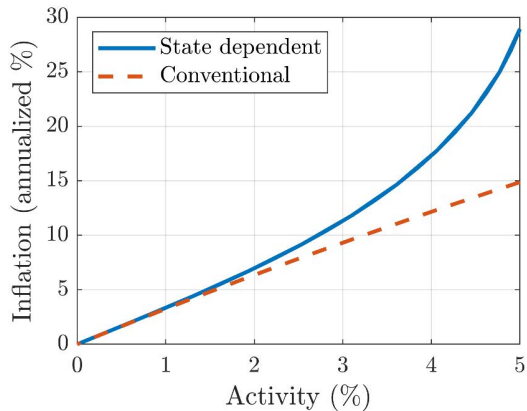
Normative results: The Ramsey problem $\left(x_t \equiv p_t - p_t^*\right)$

$$\max_{\left\{g_t^c(\cdot),g_t^0,V_t(\cdot),C_t,w_t,p_t^*,s_t,S_t,\pi_t^*\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left(C_t, \frac{C_t}{A_t}\left(\int e^{(x+p_t^*)(-\epsilon_t)} g_t^c(p)\,dx + g_t^0 e^{(p_t^*)(-\epsilon)}\right) + \eta g_t^0\right)$$

subject to

$$1 = \int e^{(x+p_t^*)(1-\epsilon)} g_t^c(x)\,dx + g_t^0 e^{(p_t^*)(1-\epsilon)},$$

$$V_t'(0) = \Pi_t'(x) + \frac{1}{\sigma}\Lambda_{t+1}\int_{s_{t+1}}^{S_{t+1}} V_{t+1}(x') \frac{\partial\phi\left(\frac{x-x'-\pi_t^*}{\sigma}\right)}{\partial x}\,dx' + \Lambda_{t+1}\left(\phi\left(\frac{S_{t+1}-\pi_t^*}{\sigma}\right) - \phi\left(\frac{s_{t+1}-\pi_t^*}{\sigma}\right)\right)(V_{t+1}(0)-\eta w_{t+1}),$$

$$V_t(s_t) = V_t(0) - \eta w_t,$$

$$V_t(S_t) = V_t(0) - \eta w_t,$$

$$w_t = \upsilon C_t^\gamma,$$

$$V_t(x) = \Pi(x,p_t^*,w_t,A_t) + \Lambda_{t,t+1}\frac{1}{\sigma}\int_{s_t}^{S_t}\left[V_{t+1}(x')\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right]dx' + \Lambda_{t,t+1}\left(1-\frac{1}{\sigma}\int_{s_t}^{S_t}\left[\phi\left(\frac{(x-x')-\pi_{t+1}^*}{\sigma}\right)\right]dx'\right)\left[(V_{t+1}(0)-\eta w_{t+1})\right],$$

$$g_t^c(x) = \frac{1}{\sigma}\int_{s_{t-1}}^{S_{t-1}} g_{t-1}^c(x_{-1})\phi\left(\frac{x_{-1}-x-\pi_t^*}{\sigma}\right)dx_{-1} + g_{t-1}^0\phi\left(\frac{-x-\pi_t^*}{\sigma}\right),$$

$$g_t^0 = 1 - \int_{s_t}^{S_t} g_t^c(x)dx.$$

Normative results: Computation

- ▶ Challenges
  - ▶ Price change distribution and firms' value function are infinite-dimensional objects
 - ▶ In the Ramsey problem, we need derivatives w.r.t. both

- ▶ Algorithm following González et al. (2024)
  - ▶ Approximate distribution and value functions by piece-wise linear interpolation on grid
  - ▶ Endogenous grid points: (s,S) bands and the optimal reset price
  - ▶ Solve in the sequence space using Dynare (Adjemian et al. (2023))

Optimal long-run inflation rate

▶ The Ramsey steady-state inflation rate is slightly above zero: $\pi^* = 0.25\%$

  ▶ Close to the inflation rate that minimizes the steady-state frequency of price changes

▶ Why not exactly zero as in Calvo (1983)?

  ▶ Asymmetry of the profit function leads to asymmetric (s,S) bands: a negative price gap is less desirable than a positive price gap of the same size

  ▶ At zero inflation, more mass around the lower (s) band than around the higher (S) band

  ▶ Slightly positive inflation raises p^* and pushes the mass of firms to the right inside (s,S)

  ▶ This leads to lower frequency and lower price-adjustment costs

Steady-state price distribution (at zero inflation)
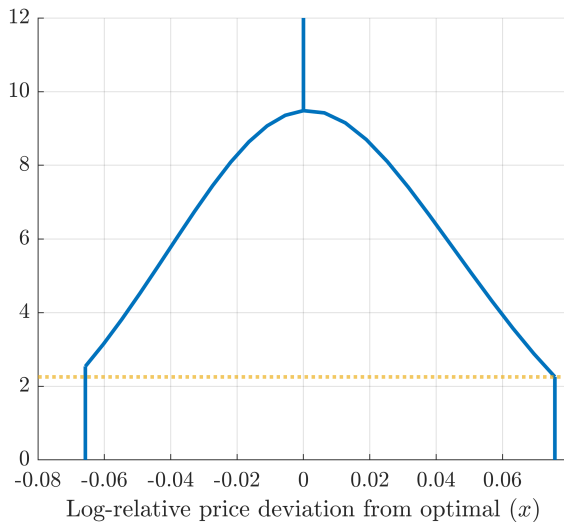
Optimal response to cost-push shocks

▶ In linearized Calvo (1983), optimal policy is a flexible inflation targeting rule

$$\pi_t = -\frac{1}{\epsilon}\Delta\tilde{y}_t$$

▶ Slope $-1/\epsilon$ is independent of the frequency of repricing or the slope of the PC

▶ For small cost-push shocks, the slope of the targeting rule in Golosov and Lucas (2007) is also $-1/\epsilon$!

Nonlinear targeting rule

▶ Globally, the target rule is nonlinear

▶ After large shocks, the planner stabilizes inflation more relative to the output gap

▶ Why? Stabilizing inflation is cheaper due to the lower sacrifice ratio (higher freq.)

  ▶ Similar results with quadratic objective

  ▶ The nonlinearity of the targeting rule is due to the nonlinear Phillips curve
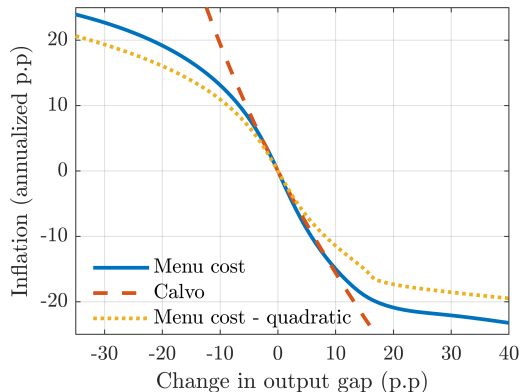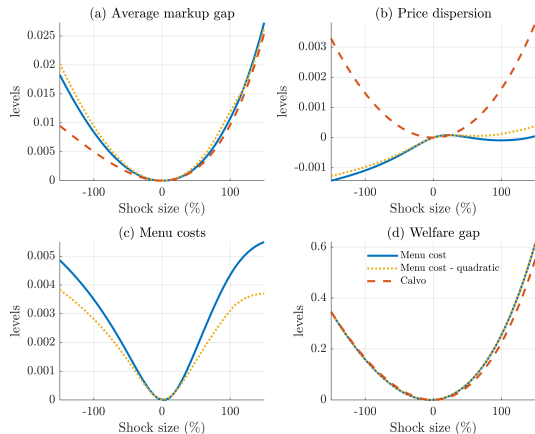
Nonlinear targeting rule
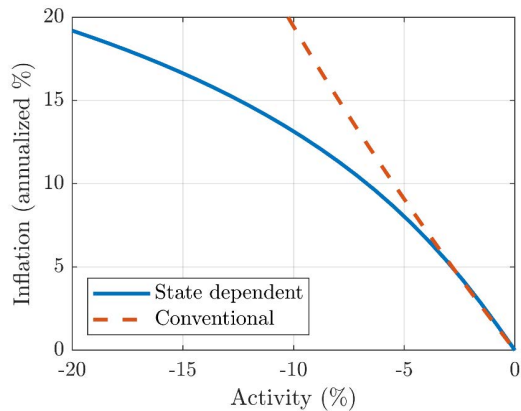
Nonlinear targeting rule for the real interest rate
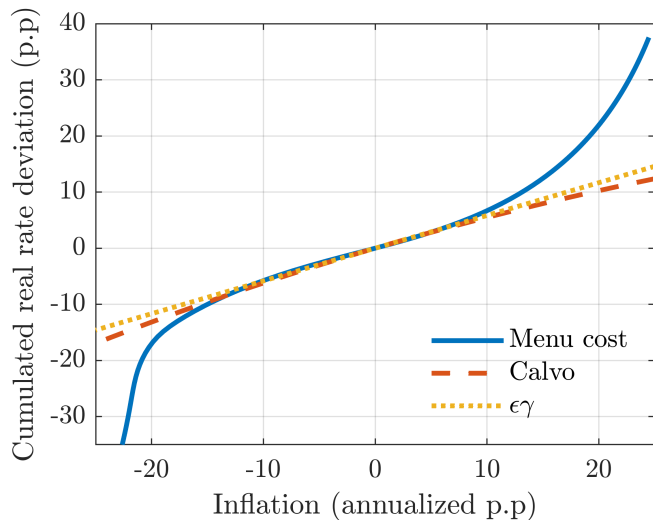
Distortions

- ▶ Monopolistic competition and nominal frictions imply three distortions:
  - ▶ Inefficient markup fluctuations
  - ▶ Price dispersion
  - ▶ Price adjustment (menu) costs
- ▶ Inflation is costly
  - ▶ In GL primarily due to menu costs
  - ▶ In Calvo due to price dispersion
- ▶ A quadratic welfare in inflation and output gap is a good approximation

(a) Average markup gap   (b) Price dispersion   (c) Menu costs   (d) Welfare gap

Legend: Menu cost; Menu cost - quadratic; Calvo

Optimal responses to efficiency shocks: "divine coincidence"

▶ In the standard NK model with Calvo pricing: divine coincidence holds after TFP and other shocks affecting the efficient allocation

▶ Optimal policy fully stabilizes inflation and closes the output gap

▶ We show analytically, that, after a TFP shock, divine coincidence holds also in the menu cost model: inflation is fully stabilized at steady state and the output gap is closed
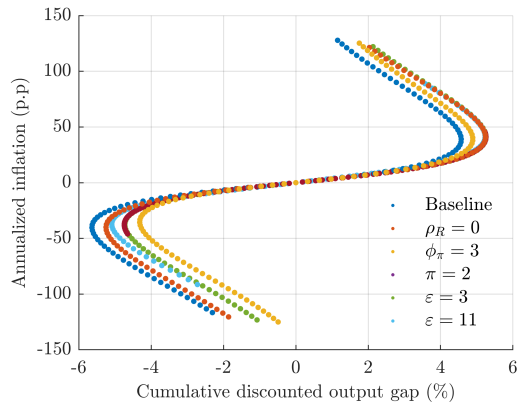
Time inconsistency

▶ Optimal policy without precommitment (time-0)

▶ Zero labor subsidy: steady state output is inefficiently low

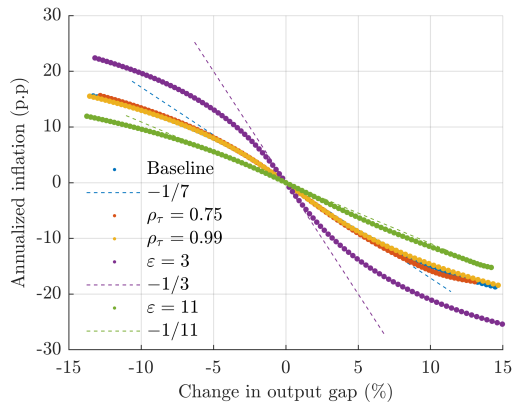▶ Weaker time inconsistency in GL than in Calvo: costlier to push up the output gap

(a) Output gap   (b) Inflation   (c) Frequency
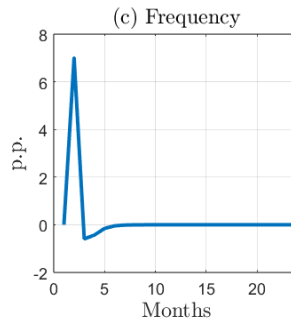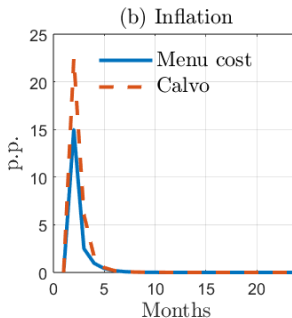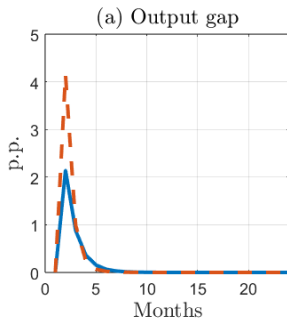
Menu cost
Calvo

Robustness: alternative parametrization

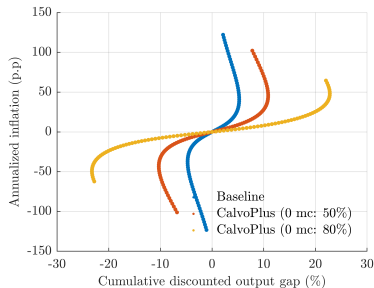Nonlinear Phillips Curve

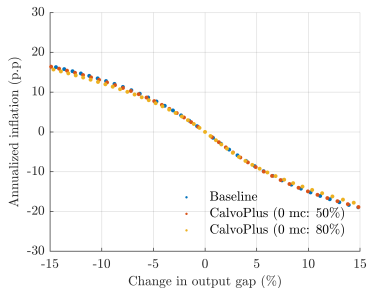Target rule

Robustness: CalvoPlus model

Menu cost is a random variable (Nakamura and Steinsson, 2010)

$$\widetilde{\eta} = \begin{cases} \eta & \text{with prob } \alpha \\ 0 & \text{with prob } 1 - \alpha \end{cases}$$

Nonlinear Phillips Curve   Target rule

Conclusion

We study optimal policy in a state-dependent framework with a nonlinear Phillips curve

▶ Optimal long-run inflation is near zero (slightly positive)

▶ Divine coincidence holds for efficiency shocks

▶ For small cost shocks the optimal response is similar to Calvo (1983)

▶ For large cost shocks, CB leans aggressively against inflation: strike while the iron is hot!

▶ Results robust in CalvoPlus framework

CPI and frequency of price changes in the US, Montag and Villar (2023)
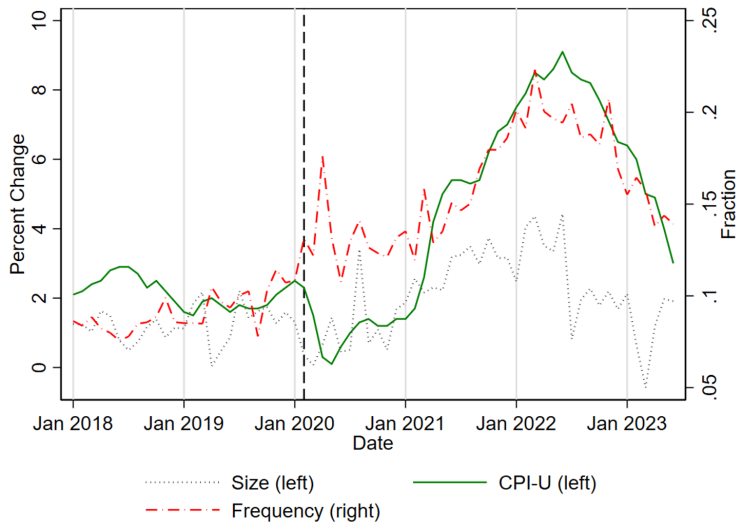
Phillips correlation across US cities, Cerrato and Gitti (2023)
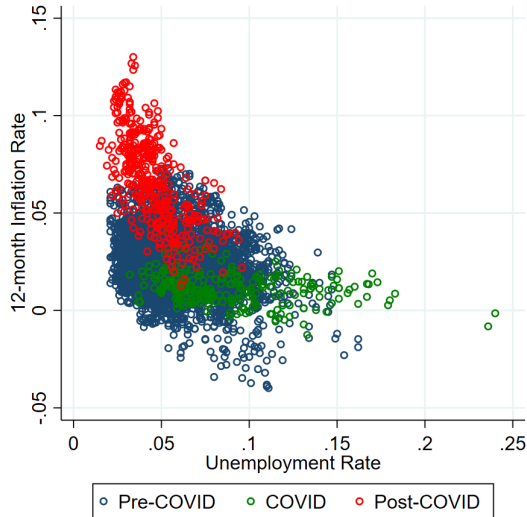
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