Logit price dynamics

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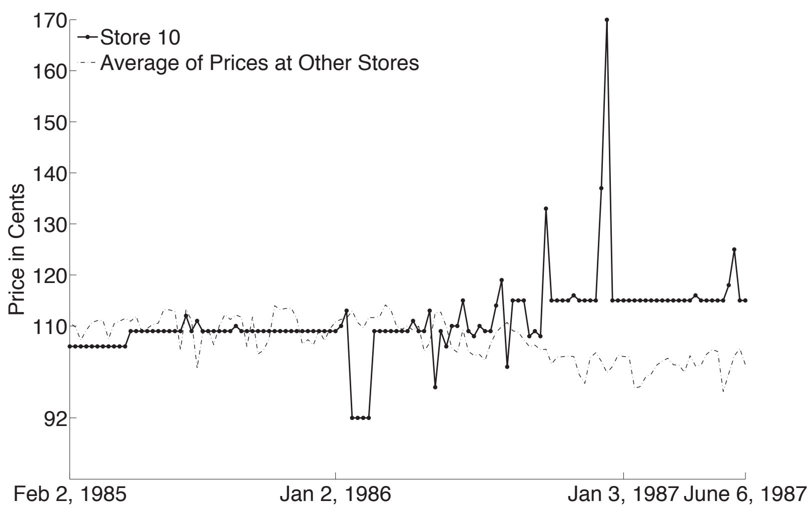
Many individual prices are "sticky"

Figure 1: The Price of Fleischmann's Margarine(i)



Source: Campbell and Eden (2010)

Motivating questions

1. What causes the "stickiness" of individual prices?
2. Does the rigidity of individual prices matter for aggregate business cycle fluctuations?

Implications for macroeconomic performance and for policy

Three approaches to price stickiness

1. **Arbitrary failures to adjust:**
   - Taylor (1979), Calvo (1983)
2. **"Menu costs":**
   - Barro (1972), Mankiw (1985), Caplin-Spulber (1987)
   - Dotsey et al (1999), Golosov-Lucas (2007), Midrigan (2011)
3. **Costly or imperfect information processing and decisions:**
   - Woodford (2009), Alvarez-Lippi-Paciello (2011)

- Case study evidence of Zbaracki et al (2004) points to **managerial costs**

This paper

1. **Main assumption: precise decisions are costly**
   Making exactly the right decision at all points in time is costly
2. Game theoretic approach: **“control costs”**
   1. Assume a **cost function** for **precision**
   2. Implies **mistakes** occur in equilibrium
   3. If precision is measured by **entropy**, then choices distributed as **logit**
3. Two margins for errors:
   1. **When** to adjust price (like Costain-Nakov JME 2011)
   2. **Which price** to set (like Costain-Nakov ECB WP 1375)
4. This paper puts the two margins together

Possible interpretations of our paper

1. Putting **“logit equilibrium”** or **“control costs”** in a macro model
2. Redefining **“menu costs”** so they can be interpreted as **costs of managerial decisions**
3. Showing that **near-rational** price adjustment, where errors can occur if they are not too costly, is **tractable and empirically successful**

Recent related papers

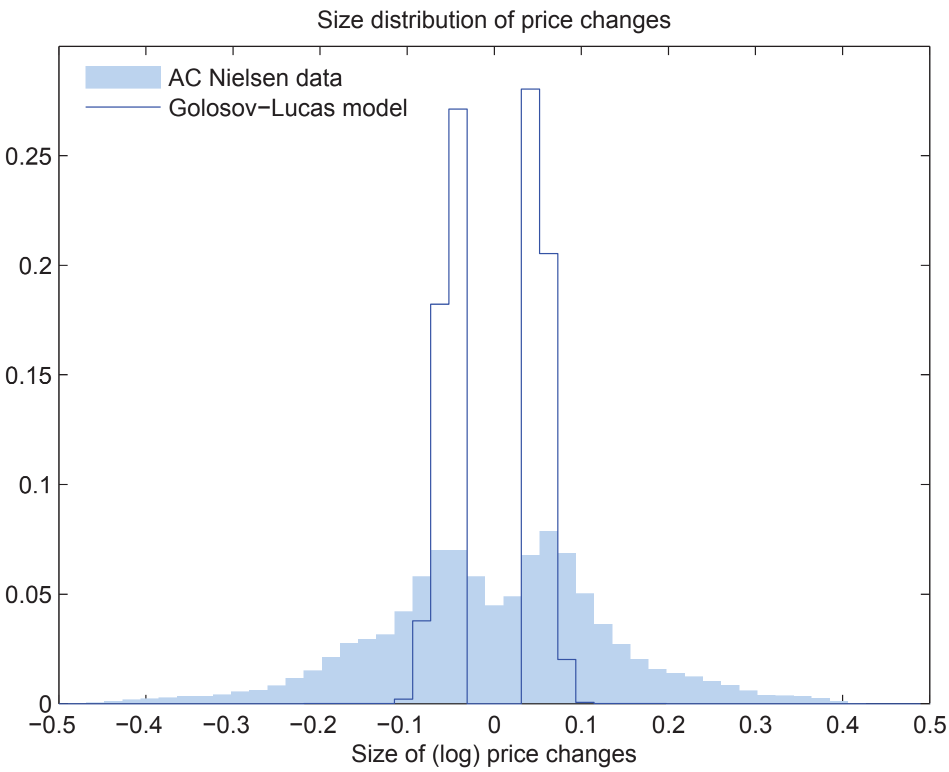
- Empirics of price adjustment
  - Klenow-Kryvtsov (2008), Nakamura-Steinsson (2008), Klenow-Malin (2010)
  - Document many stylized facts about micro price adjustment
- Menu cost models
  - Golosov-Lucas (2007), Midrigan (2011)
  - Feature aggregate and idiosyncratic shocks; fit to micro-data and then look for macro implications
  - In our model there is no menu cost but instead a “control cost”

Recent related papers

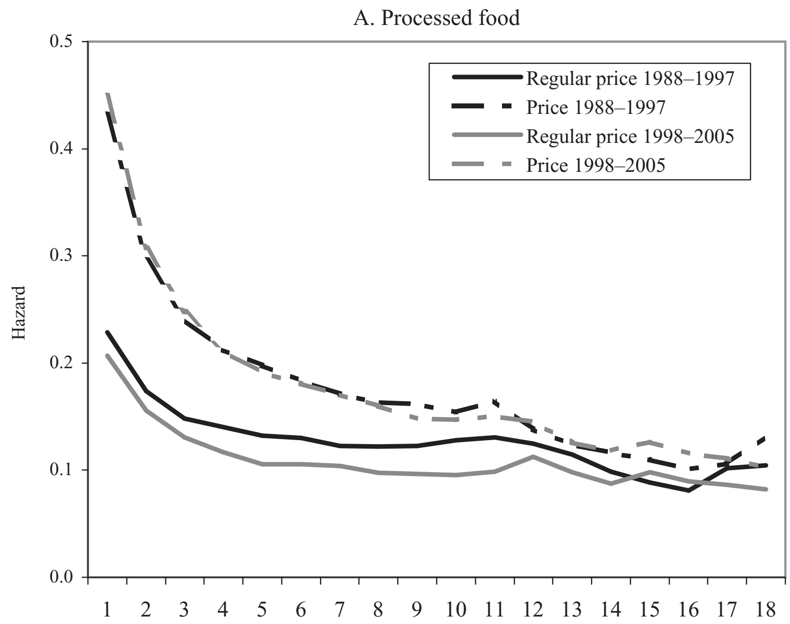
- Menu cost and observation cost
  - Alvarez-Lippi-Paciello (2011)
  - Highly successful in matching micro evidence in spite of relying only on two free parameters
  - But they don’t look at general equilibrium impulse-responses
- Rational inattention
  - Woodford (2009), Matejka (2011)
  - There is a constraint on the rate of information flow from the environment to the decisionmaker
  - In our model the decisionmaker has full information, yet decisions are subject to error due to control costs

SOME STYLIZED FACTS

Histogram of price changes: data vs. menu cost model



Declining price adjustment hazard

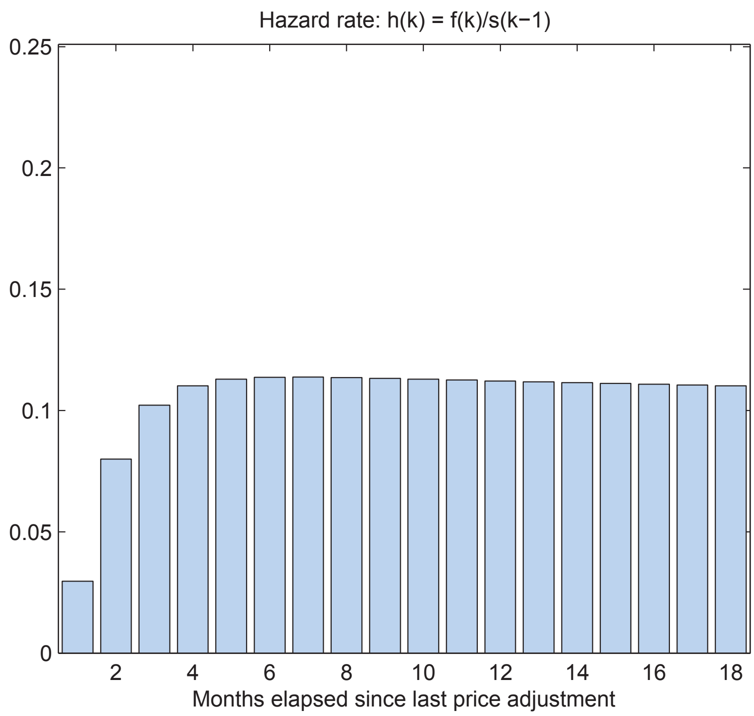


Months since the last price change

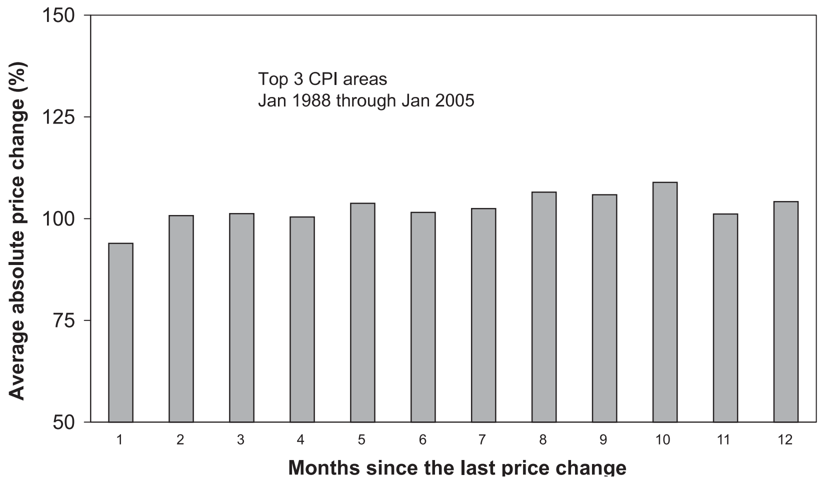
Source: Nakamura-Steinsson (2008)

Typical price adjustment hazard in the menu cost model

Idiosyncratic shocks with positive persistence

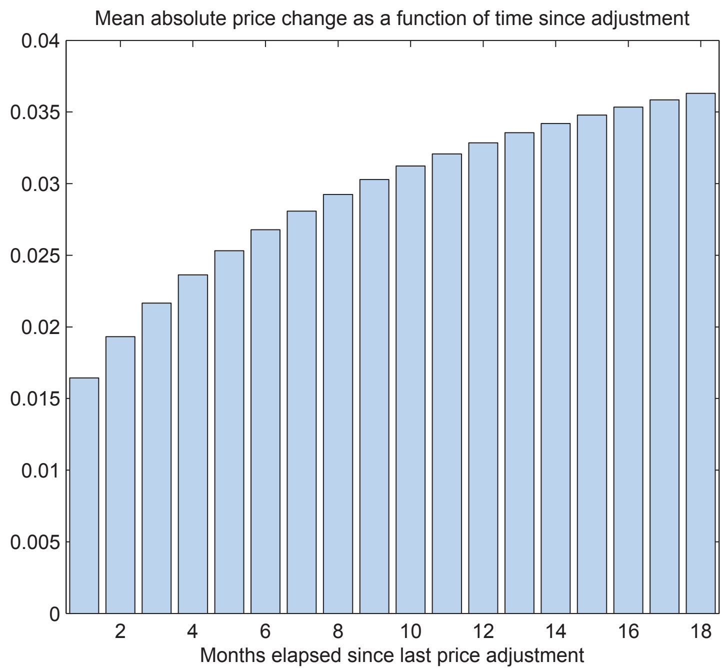


Average size of price changes as a function of price age



Source: Klenow-Kryvtsov (2008)

Average size of price changes in the Calvo model



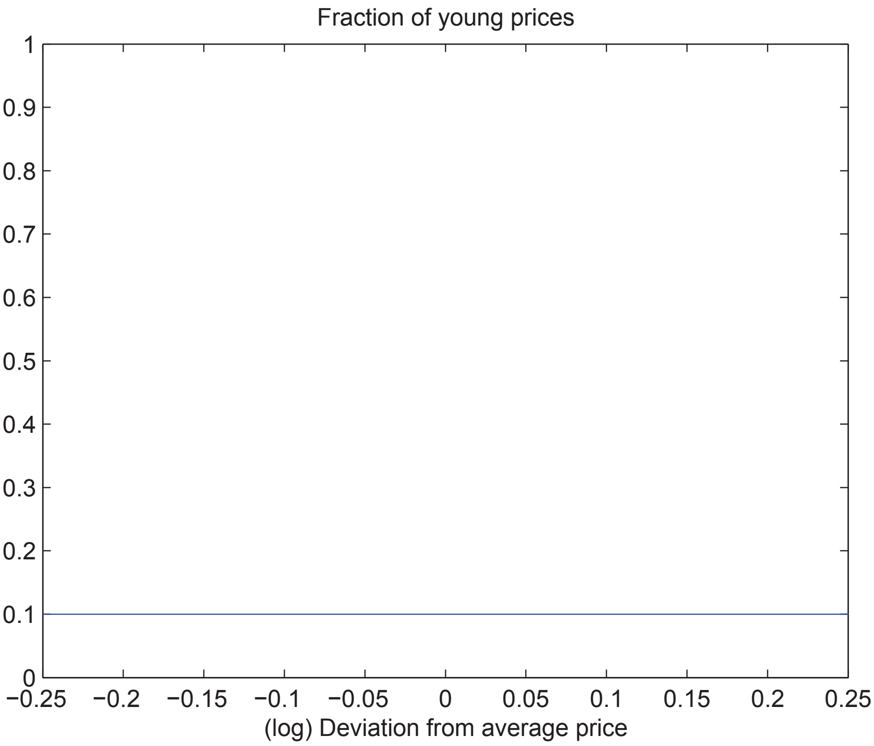
Extreme prices are young in the data

Figure 7: The Fraction of Young Prices by Relative Price(i)

Note: (i) Young prices are those with ages less than four weeks. The plotted fractions exclude prices one week old from both the numerator and denominator.

Source: Campbell-Eden (2010)

Extreme prices in the Calvo model



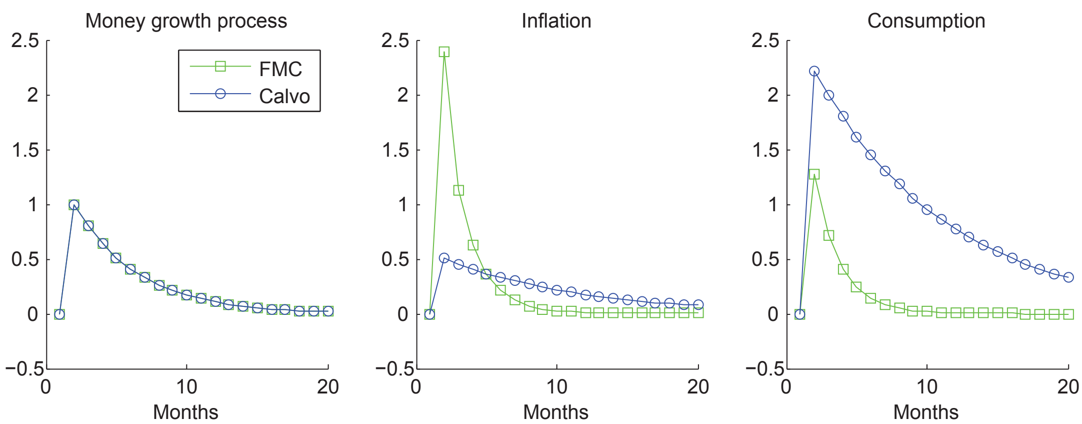
Prices are more volatile than costs

	Std(log(price))	Std(log(cost))	Ratio
*Eichenbaum et al**			
Weekly prices and costs	0.14	0.12	1.08
"Reference" prices and costs	0.08	0.07	1.15
*Menu cost simulation***	0.060	0.063	0.95
*Calvo simulation***	0.049	0.063	0.78

*Source: Eichenbaum-Jaimovich-Rebelo (2011)

**Source: Authors' simulations

Effects of money supply shocks



This paper: summary of results

1. This paper: allow **errors in timing and size** of adjustment.
2. **Entropy control costs** give rise to **logit equilibrium**
   - Calibrate two logit parameters to match size distribution and frequency of price changes
3. Microeconomic results (errors in choosing **which price** are helpful):
   - Large and small price adjustments coexist
   - Adjustment hazard is largely independent of age of price
   - Adjustment size is largely independent of age of price
   - Extreme prices are younger
   - Prices more volatile than costs
4. Macroeconomic results (errors in **when to adjust** are helpful):
   - Monetary nonneutrality, closer to the Calvo model

CONTROL COSTS AND LOGIT

Deriving multinomial logit from control costs

- Think of **decisions** as **probability distributions** over alternatives
- Suppose the **time cost** of decision π is:

$$\kappa\mathcal{D}(\pi|u) \equiv \kappa \sum_{j=1}^{n} \pi^j \log\left(\frac{\pi^j}{n^{-1}}\right) = \kappa\left(\log(n) + \sum_{j=1}^{n} \pi^j \log \pi^j\right)$$

  - This is the **relative entropy** of decision π, compared with perfectly uniform decision u
  - Also called Kullback-Leibler divergence
  - It means choice is more costly if more precise
  - Normalizes cost of uniform decision to zero

Deriving multinomial logit from control costs

- Maximize expected value net of costs:

$$\tilde{V} = \max_{\pi^j} \sum_{j=1}^{n} \pi^j V^j - \kappa W\left(\log(n) + \sum_j \pi^j \log \pi^j\right) \quad \text{s.t.} \sum_j \pi^j = 1$$

  - V^j is nominal value of alternative j
  - W is nominal value of time

- First-order condition:

$$V^j - \kappa W(1 + \log \pi^j) = \mu$$

- Rearranging, obtain

$$\pi^j = \frac{\exp(V^j/(\kappa W))}{\sum_k \exp(V^k/(\kappa W))}$$

Nesting frictionless choice

$$\pi^j = \frac{\exp(V^j/(\kappa W))}{\sum_k \exp(V^k/(\kappa W))}$$

- If $\kappa = 0$, then $\pi(P^*) = 1$
- If $\kappa = \infty$, then $\pi^j = 1/n$

Deriving logit timing from control costs

- Suppose **time cost** of the adjustment hazard λ_t is:

$$\kappa \mathcal{D}(\{\lambda_t, 1-\lambda_t\}||\{\bar{\lambda}, 1-\bar{\lambda}\}) \equiv \kappa \left(\lambda_t \log \frac{\lambda_t}{\bar{\lambda}} + (1-\lambda_t) \log \frac{1-\lambda_t}{1-\bar{\lambda}} \right)$$

  - This is the **relative entropy** of endogenous adjustment hazard λ_t, compared with exogenous adjustment hazard $\bar{\lambda}$.
  - It means costs are greater if adjustment probability deviates from $\bar{\lambda}$.
  - Normalizes cost of *some Calvo model* $\bar{\lambda}$ to zero.

Deriving logit timing from control costs

- Maximize expected gains net of costs

$$G_t = \max_{\lambda_t} \lambda_t D_t - \kappa W_t \left(\lambda_t \log \frac{\lambda_t}{\bar{\lambda}} + (1-\lambda_t) \log \frac{1-\lambda_t}{1-\bar{\lambda}} \right)$$

  - D_t is gain from adjustment at t
  - W_t is value of time at t

- First-order condition:

$$D_t = \kappa W_t \left(1 + \log \frac{\lambda_t}{\bar{\lambda}} - \left(1 + \log \frac{1-\lambda_t}{1-\bar{\lambda}} \right) \right)$$

- Rearranging,

$$\lambda_t = \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda}) \exp\left(-D_t/(\kappa W_t)\right)}$$

- Same as Woodford (2009)

Nesting Calvo and menu cost

$$\lambda_t = \frac{\bar{\lambda}}{\bar{\lambda} + (1-\bar{\lambda}) \exp\left(-D_t/(\kappa W_t)\right)}$$

- If $\kappa = 0$, menu cost: $\lambda_t = 1$ if $D_t > 0$ and $\lambda_t = 0$ if $D_t < 0$
- If $\kappa = \infty$, Calvo model: $\lambda_t = \bar{\lambda}$ regardless of D_t

New parameters

- Two free parameters: **noise** κ and **rate** $\bar{\lambda}$
- Interpretation of $\bar{\lambda}$: Adjustment probability when the firm is indifferent between adjusting or not
- Interpretation of κ: Level of noise in decision-making

MODEL

Model: monopolistic firms

- Firm's demand: $Y_{it} = (P_t/P_{it})^{\epsilon} Y_t$
- Firm's output: $Y_{it} = A_{it} N_{it}$
- Idiosyncratic productivity: $\log A_{it} = \rho \log A_{it-1} + \varepsilon^a_{it}$
- Profits: $U_{it} = P_{it} Y_{it} - W_t N_{it} = U_t(P_{it}, A_{it})$
- Frictionless optimal choice would imply:

$$V_t^*(A_{it}) = \max_P U_t(P, A_{it}) + E[Q_{t,t+1} V^*_{t+1}(A_{it+1})]$$

... but now there are mistakes and control costs.

Model: mistakes in price choice

- Instead of *optimal* price $P_t^*(A_{it})$...
- ... there is a **logit distribution** across possible prices:

$$\pi_t(P|A_{it}) = \frac{\exp(\kappa^{-1} W_t^{-1} V_t(P, A_{it}))}{\sum_{P'} \exp(\kappa^{-1} W_t^{-1} V_t(P', A_{it}))}$$

- The **value of adjusting** is:

$$\begin{aligned} \tilde{V}_t(A_{it}) &= \sum_P \pi_t(P|A_{it}) V_t(P, A_{it}) - W_t K_t^{\pi} \\ &= E^{\pi} V(P, A_{it}) - W_t K_t^{\pi} \end{aligned}$$

- ... which includes the adjustment cost:

$$W_t K_t^{\pi} = W_t \kappa \mathcal{D}(\pi_t | u)$$

Model: mistakes in timing

- Optimal timing is to adjust iff $E^{\pi} V_t(P, A_{it}) - W_t K_t^{\pi} > V_t(P_{it}, A_{it})$.
- But here, instead, adjustment hazard is a **weighted logit**:

$$\lambda(L) = \frac{\bar{\lambda}}{\bar{\lambda} + (1 - \bar{\lambda}) \exp(-L)},$$

- ... where L is **real loss from not adjusting:**

$$L = L_t(P_{it}, A_{it}) = \frac{E^{\pi} V_t(P, A_{it}) - W_t K_t^{\pi} - V_t(P_{it}, A_{it})}{\kappa W_t}$$

  - Noise parameter $\kappa \in [0, \infty)$ controls precision of timing.

- Each period, pay a cost to check whether it is a good time to adjust:

$$W_t K_t^{\lambda} = W_t \kappa \mathcal{D}\left(\{\lambda(L), 1 - \lambda(L)\} || \{\bar{\lambda}, 1 - \bar{\lambda}\}\right)$$

Bellman equation

- **Value of production now** at current firm-specific state (P, A):

$$\begin{aligned} V_t(P, A) &= U_t(P, A) \\ &+ E_t \left\{ Q_{t,t+1} \max_{\lambda} \left[(1 - \lambda) V_{t+1}(P, A') + \lambda \tilde{V}_{t+1}(A') \right.\right. \\ &- \left.\left. W_{t+1} \kappa \mathcal{D}\{(\lambda, 1 - \lambda) || (\bar{\lambda}, 1 - \bar{\lambda})\} \right] \middle| A \right\} \end{aligned}$$

  - Here $V_{t+1}(P, A')$ = value of continuing next period without adjusting
  - And $\tilde{V}_{t+1}(P, A')$ = expected value of continuing after adjustment:

$$\begin{aligned} \tilde{V}_{t+1}(A') &= \max_{\pi^j} \sum_j \pi^j V_{t+1}(P^j, A') - W_{t+1} \kappa \mathcal{D}(\pi || u) \\ & \text{s.t.} \quad \sum_j \pi^j = 1 \end{aligned}$$

Versions compared

In the paper we compare six versions of the model:

- **"Precautionary price stickiness"**: errors in price choice. Timing optimal.
  - PPS-logit
  - PPS-control
- **"Woodford"**: errors in timing. Set optimal price when adjustment occurs.
  - Woodford-logit
  - Woodford-control
- **"Nested"**: errors in price choice and timing.
  - Nested-logit
  - Nested-control

- Three versions just impose **logit**, without subtracting control costs
- The other three versions derive logit from **control costs**

Model: the rest is standard

- Household utility: $\frac{C^{1-\gamma}}{1-\gamma} - \chi N + \nu \log(M/P)$ with discount β
- Period budget constraint:

$$P_t C_t + M_t + R_t^{-1} B_t = W_t N_t + M_{t-1} + T_t + B_{t-1} + \Pi_t$$

- Consumption bundle:

$$C_t = \left[\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \text{ with price } P_t \equiv \left[\int_0^1 P_{it}^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

- Money supply: $M_t = \mu \exp(z_t) M_{t-1}$, where $z_t = \phi_z z_{t-1} + \epsilon_t^z$

Model: aggregate consistency and aggregate state variable

- Labor market clearing: $N_t = \Delta_t C_t + K_t^{\lambda} + K_t^{\pi}$
- Measure of price dispersion: $\Delta_t \equiv P_t^{\epsilon} \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di$
- Balanced budget: $M_t = M_{t-1} + T_t$
- Bond market clears: $B_t = 0$
- Aggregate state variable: $\Omega_t \equiv (z_t, M_{t-1}, \mathbf{\Psi_{t-1}})$ ...
  - ... where $\mathbf{\Psi_{t-1}}$ is the cross-sectional distribution of prices and productivities at time $t-1$

COMPUTATION

Computation

- Challenge: need to keep track of the *distribution* of firms
- Reiter's (2009) method of "projection & perturbation"
- Appropriate for price-setting by firms: idiosyncratic shocks are relatively large; aggregate shocks are relatively small
- Two-step procedure, computing:
  1. Aggregate steady-state by backwards induction on a finite grid
  2. Aggregate dynamics by linearization around each grid point

Finite grid approximation

- To keep track of value function and cross-sectional distribution, define them over finite grid.
- Grid of real firm-specific states: $\Gamma = \Gamma^a \times \Gamma^p$ ...
  - ... where $\Gamma^a \equiv \{a^1, a^2, ...a^{\#a}\}$, $\Gamma^p \equiv \{p^1, p^2, ...p^{\#p}\}$
- Exogenous Markov matrix describes productivity:

$$\mathbf{S}: \quad s^{jk} = prob(a^j | a^k)$$

- Endogenous, time-varying Markov matrix deflates real prices:

$$\mathbf{R}_t: \quad r^{jk} = prob(p^j | p^k, P_t / P_{t-1})$$

  - (If previous real price was p^k, $\mathbf{R}_t$ only allocates positive probability to the two grid points bounding $\frac{P_{t-1}}{P_t} p^k$.)

Computation: aggregate steady-state (projection)

Real prices converge to an ergodic distribution $\mathbf{\Psi}$.

1. Guess real wage: w
2. Consumption: $C = (\chi/w)^{1/\gamma}$
3. Payoff at grid points: $U^{jk} = \left(p^j - w/a^k\right) C(p^j)^{-\epsilon}$
4. Iterate on Bellman equation: $\mathbf{V} = \mathbf{U} + \beta \mathbf{R}' \left(\mathbf{V} + \mathbf{G}\right) \mathbf{S}$
5. Iterate on distribution matrices:
   - Beginning of period: $\tilde{\mathbf{\Psi}} = \mathbf{R}\mathbf{\Psi}\mathbf{S}'$
   - End of period: $\mathbf{\Psi} = (\mathbf{1}_{pa} - \mathbf{\Lambda}) .* \tilde{\mathbf{\Psi}} + \mathbf{\Pi} .* \left(\mathbf{1}_{pp} * (\mathbf{\Lambda} .* \tilde{\mathbf{\Psi}})\right)$
6. Check if $\sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} \Psi^{jk} (p^j)^{1-\epsilon} = 1$, and adjust w until it holds.

Computation: aggregate dynamics (perturbation)

- Dynamic Bellman equation:

$$\mathbf{V}_t = \mathbf{U}_t + \beta E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \mathbf{R}'_{t+1} \left(\mathbf{V}_{t+1} + \mathbf{G}_{t+1}\right) \mathbf{S} \right]$$

- Distributional dynamics:
  - $\tilde{\mathbf{\Psi}}_t = \mathbf{R}_t \mathbf{\Psi}_{t-1} \mathbf{S}'$
  - $\mathbf{\Psi}_t = (\mathbf{1}_{pa} - \mathbf{\Lambda}_t) .* \tilde{\mathbf{\Psi}}_t + \mathbf{\Pi}_t .* \left(\mathbf{1}_{pp} * (\mathbf{\Lambda}_t .* \tilde{\mathbf{\Psi}}_t)\right)$
- Collect variables in vector: $X_t = (vec(\mathbf{\Psi}_{t-1}), vec(\mathbf{V}_t), C_t, \pi_t, M_{t-1})$
- Model: $E_t \mathcal{F}\left(X_{t+1}, X_t, z_{t+1}, z_t\right) = 0$
- Linearization: $E_t \mathcal{A} \Delta X_{t+1} + \mathcal{B} \Delta X_t + E_t \mathcal{C} z_{t+1} + \mathcal{D} z_t = 0$
- Solve with Klein's QZ method for linear RE models

CALIBRATION

Common parameters (same in all specifications)

Discount factor	$\beta^{-12} = 1.04$	Golosov-Lucas (2007)
CRRA	$\gamma = 2$	Ibid.
Labor supply	$\chi = 6$	Ibid.
MIUF coeff.	$\nu = 1$	Ibid.
Elast. subst.	$\epsilon = 7$	Ibid.
Money growth	$\mu^{12} = 1.02$	Dominick's 2% annual inflation
Persistence prod.	$\rho = 0.95$	Blundell-Bond (2000)
Std. dev. prod.	$\sigma = 0.06$	Eichenbaum et. al. (2009)

Estimated parameters for each specification

Estimation criterion:

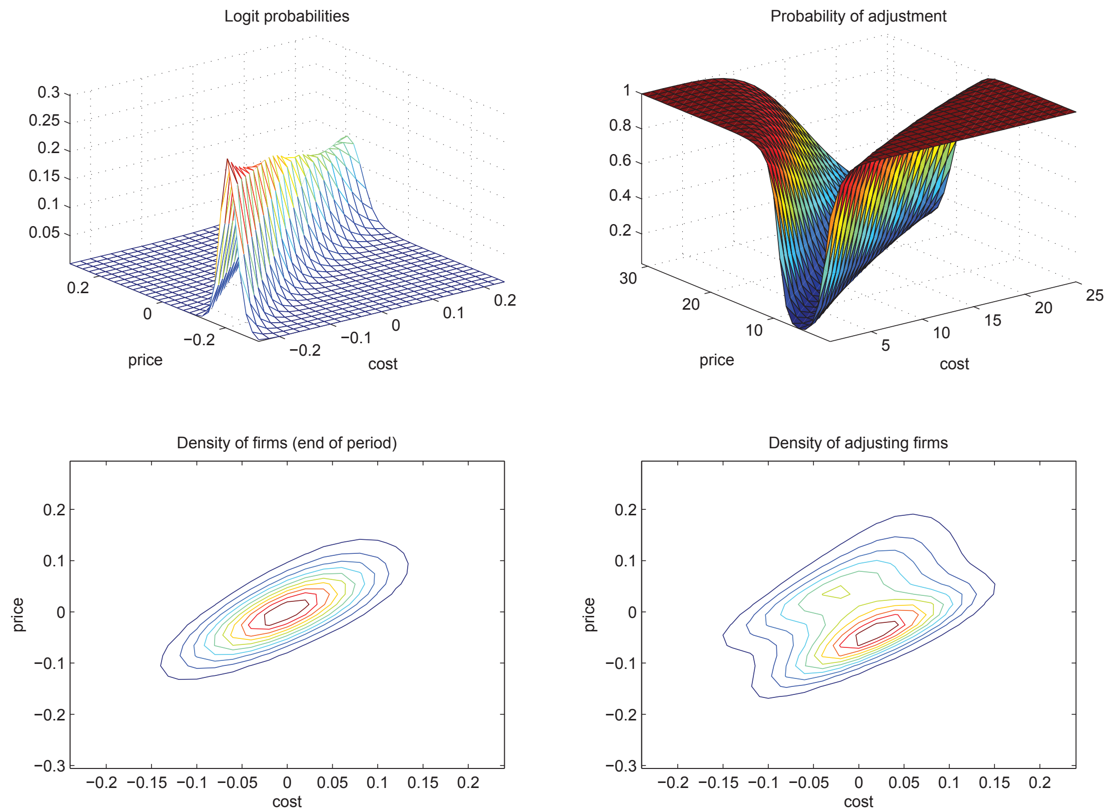
$$\text{distance} = \sqrt{n}||\lambda_{model} - \lambda_{data}|| + ||h_{model} - h_{data}||$$

where λ = frequency, h = histogram of changes, n = length(h).

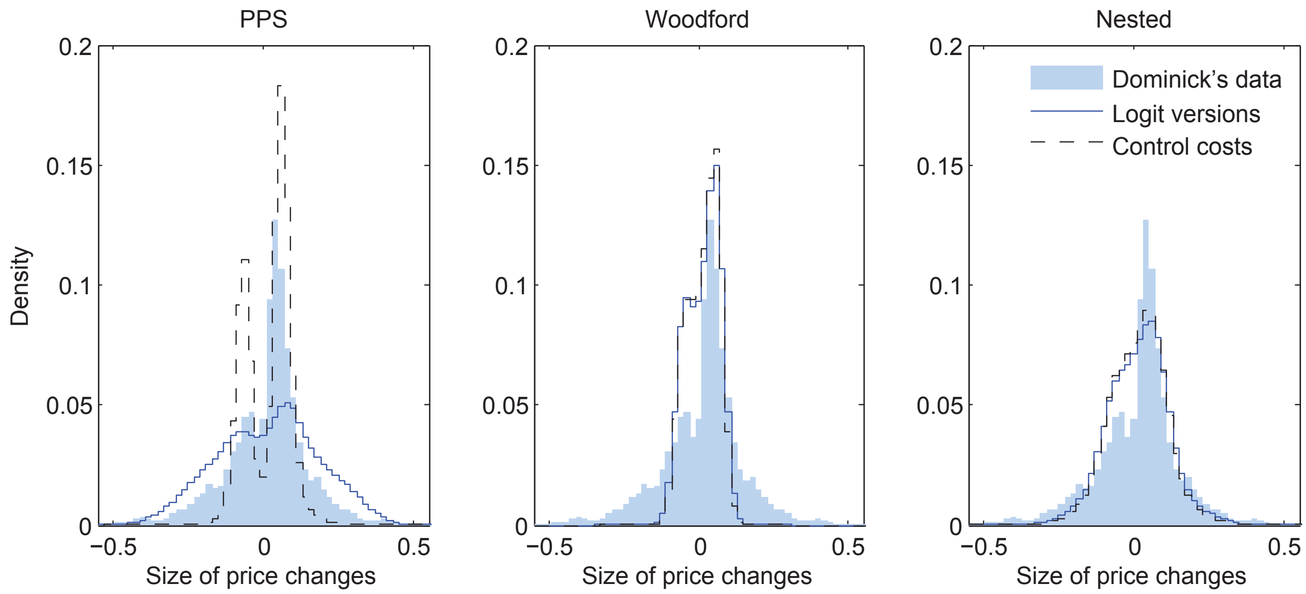
Specification	Rate: $\bar{\lambda}$	Noise: κ_π	Noise κ_λ
PPS-logit	–	0.049	–
PPS-control	–	0.0044	–
Woodford-logit	0.044	–	0.0051
Woodford-control	0.045	–	0.0080
Nested-logit	0.083	0.013	0.013
Nested-control	0.22	0.018	0.018

RESULTS

Equilibrium behavior (Nested control-cost model)



Histogram of nonzero price changes



Steady-state: statistics on price variability

	Wdfd logit	Wdfd cntrl	PPS logit	PPS cntrl	Nest logit	Nest cntrl	Data
$\text{Std}(p)/\text{Std}(a)$	0.95	0.91	1.13	0.98	1.09	1.04	1.15
Freq. Δp	10.2	10.2	10.2	10.2	10.2	10.2	10.2
Mean $\lvert\Delta p\rvert$	4.88	4.68	14.0	6.72	8.11	7.51	9.90
$\text{Std}(\Delta p)$	5.51	5.27	17.0	7.32	10.1	9.30	13.2
$\text{Kurt}(\Delta p)$	2.24	2.22	2.58	2.37	3.48	3.40	4.81
% $\Delta p > 0$	62.7	63.3	55.2	62.3	58.3	58.8	65.1
% $\lvert\Delta p\rvert \leq 0.05$	47.9	49.7	16.5	27.9	31.5	33.6	35.4

Note: Statistics in percent.

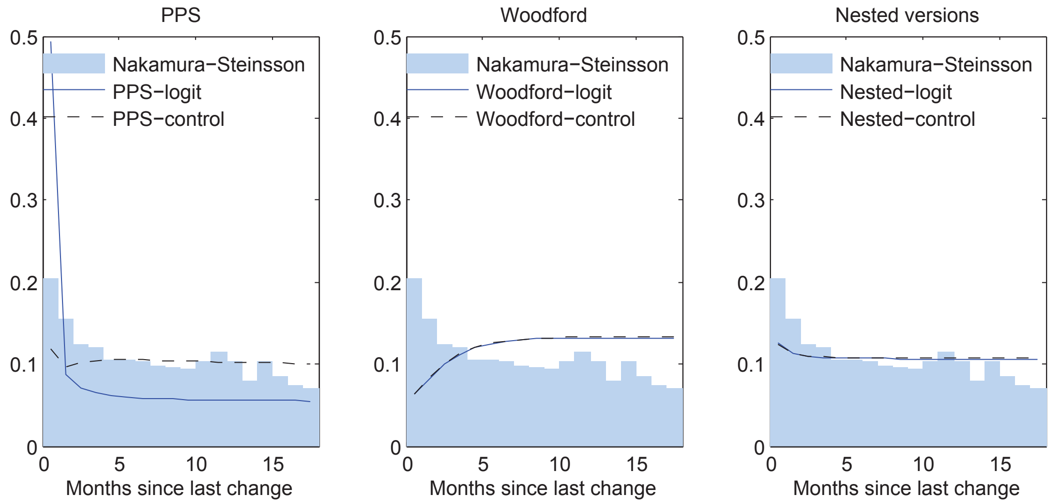
Dominick's data: "regular" price changes, excluding sales.

Steady-state: Costs of decision-making

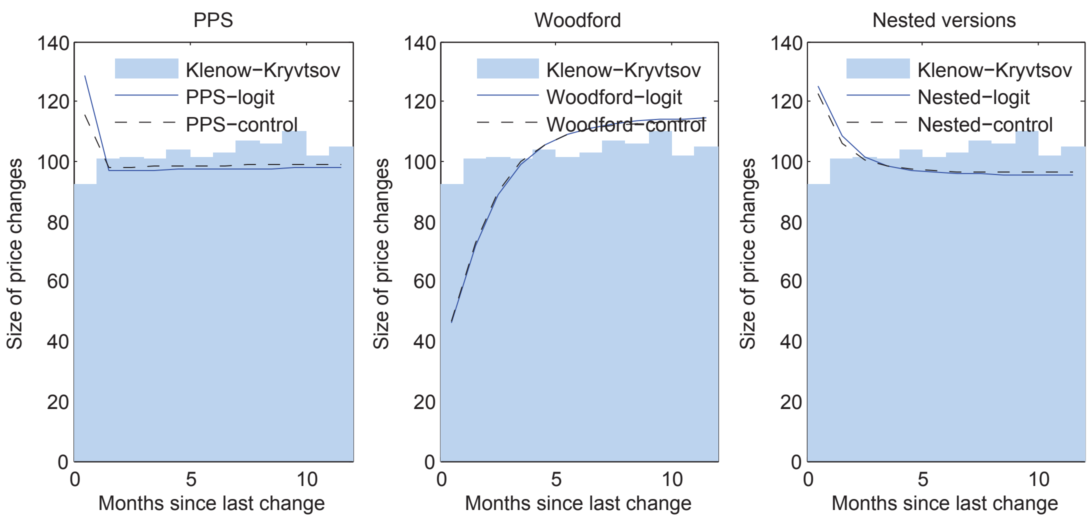
	Wdfd logit	Wdfd cntrl	PPS logit	PPS cntrl	Nested logit	Nested cntrl
Pricing costs	0	0	0	0.174	0	0.509
Timing costs	0	0.167	0	0	0	0.361
Gain if rational	0.258	0.416	0.665	0.365	0.582	1.41

Note: Costs and gains stated as percentage of average revenue.

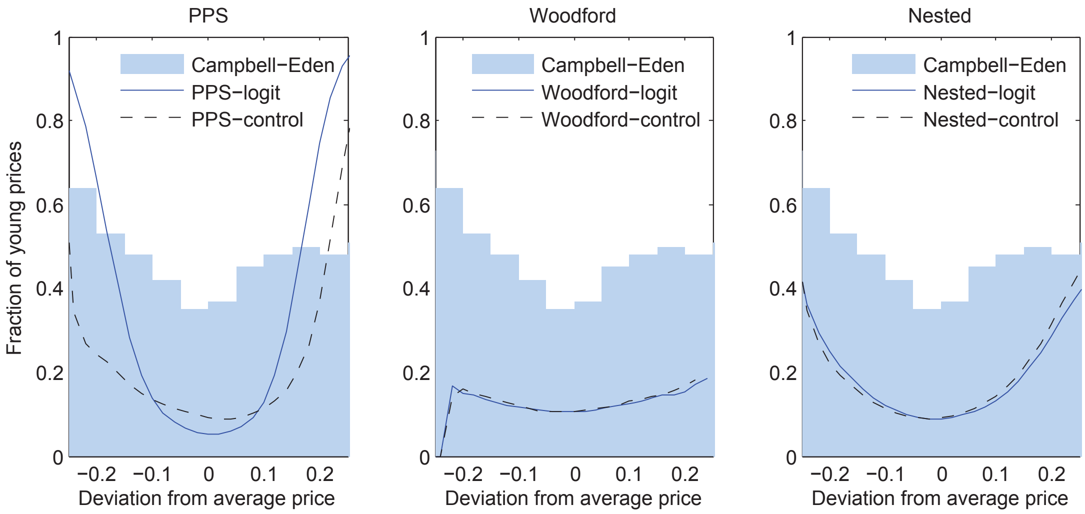
Price adjustment hazard



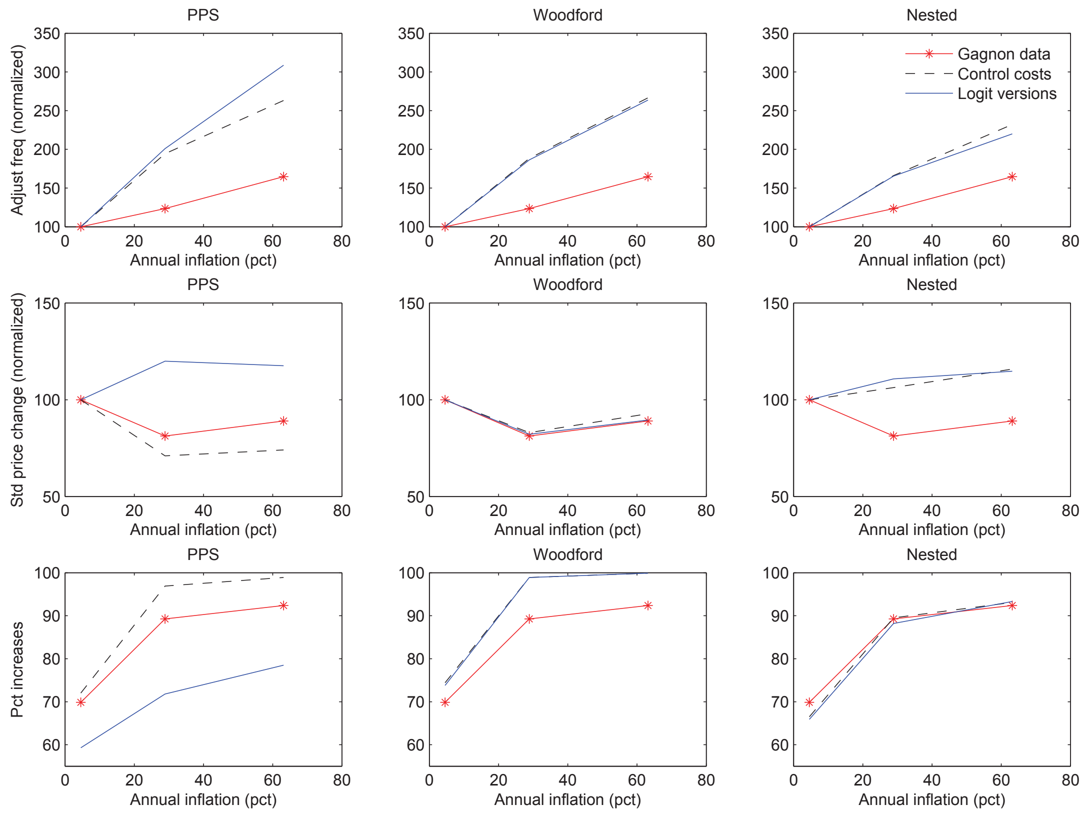
Size of price change as function of price age



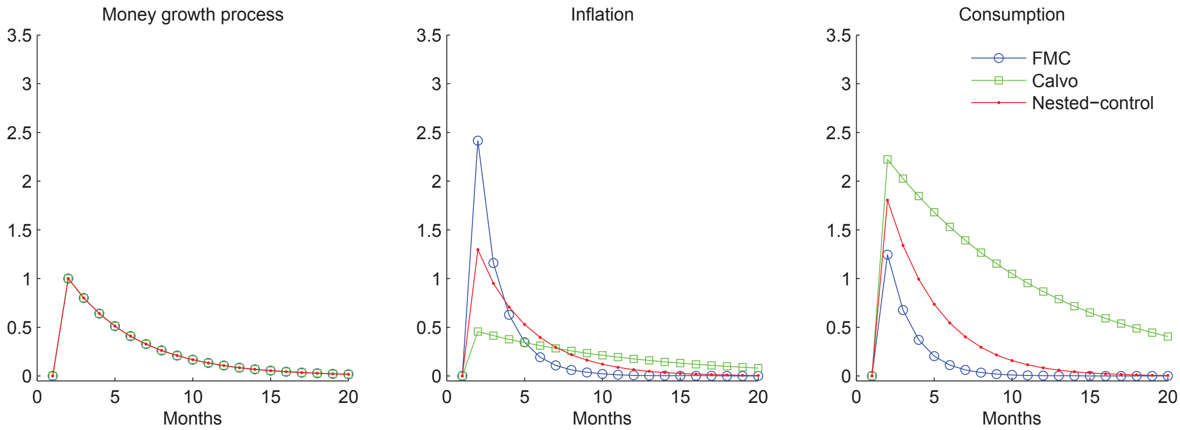
Fraction of young prices



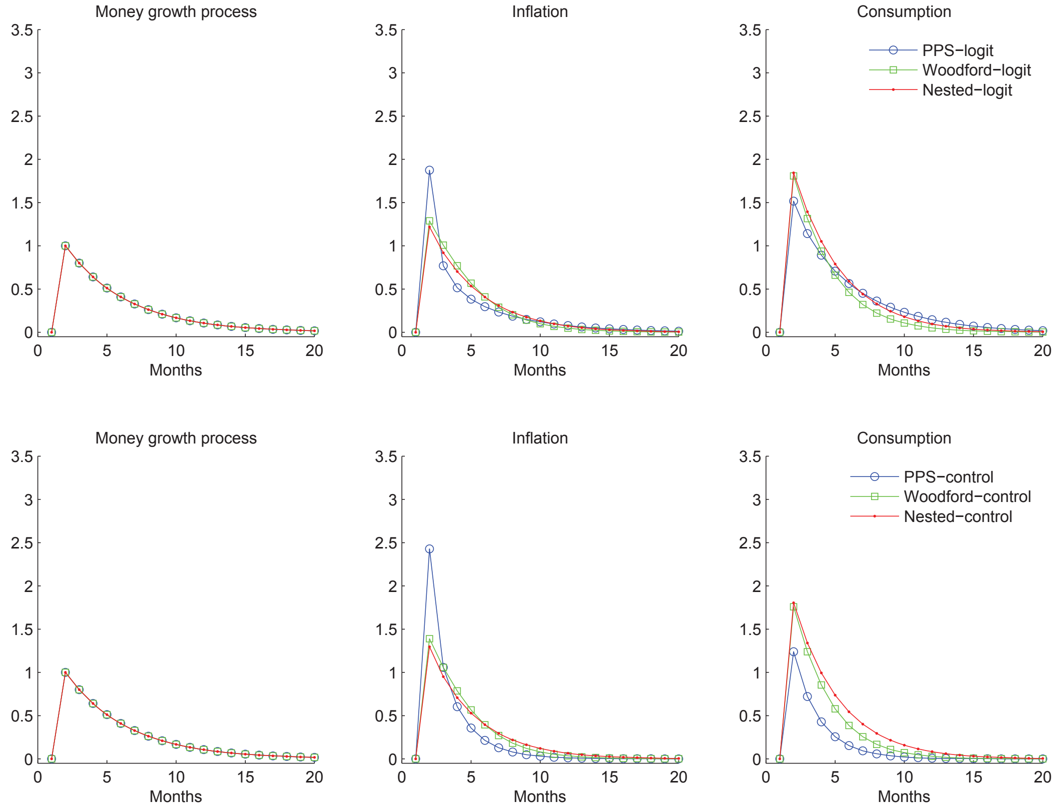
Effects of trend inflation



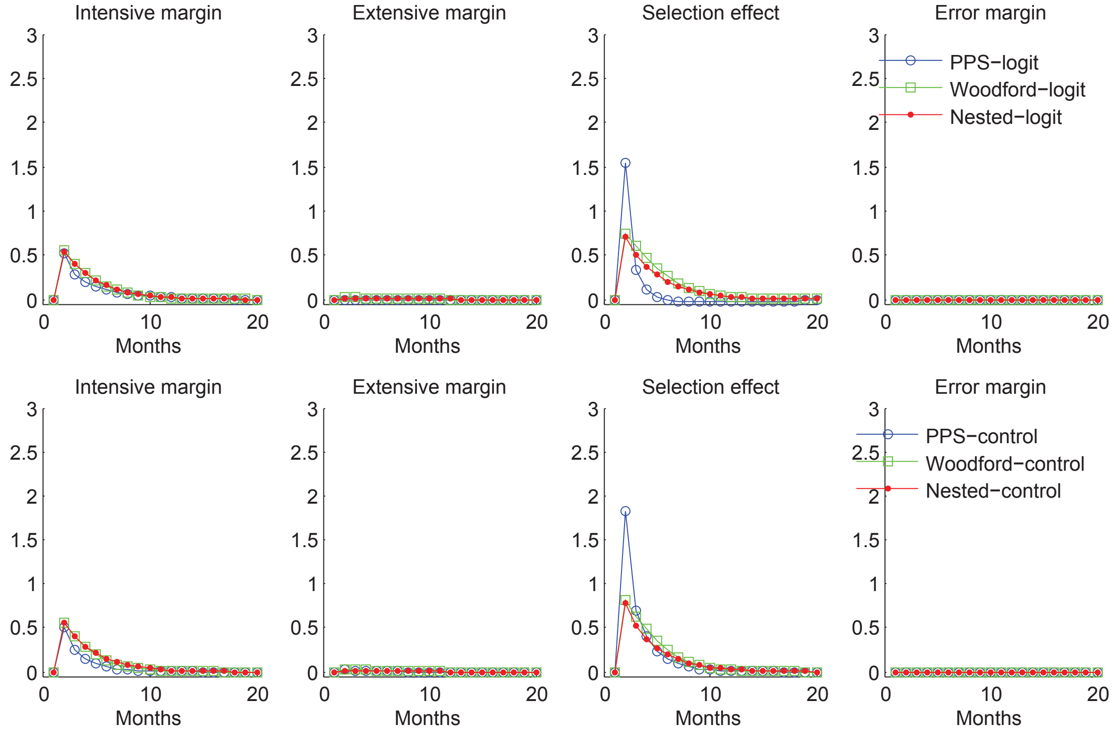
Effects of money growth shock



Responses to a money growth shock



Selection effect is dominant at low trend inflation rates



Estimated Phillips curve coefficients

Table 3. Variance decomposition and Phillips curves

Money shocks: ($\phi_z = 0.8$)	Wdfd logit	Wdfd cntrl	PPS logit	PPS cntrl	Nest logit	Nest cntrl	Data*
Std μ (%)	0.16	0.15	0.16	0.12	0.17	0.17	
Std inflation (%)	0.25	0.25	0.25	0.25	0.25	0.25	0.25
% explained by μ	100	100	100	100	100	100	
Std output (%)	0.41	0.37	0.34	0.20	0.45	0.43	0.51
% explained by μ	80	73	67	38	89	84	
Phillips slope*	0.32	0.29	0.31	0.15	0.38	0.35	

CONCLUSIONS

Conclusions

- Model: price stickiness as near-rational behavior
- Standard model of "mistakes": logit equilibrium
- Just two free parameters, but:
  - Matches micro facts well (due to price errors)
  - Generates monetary nonneutrality closer to the Calvo model (due to timing errors)
- Tractable enough to compute in DSGE

THANKS!