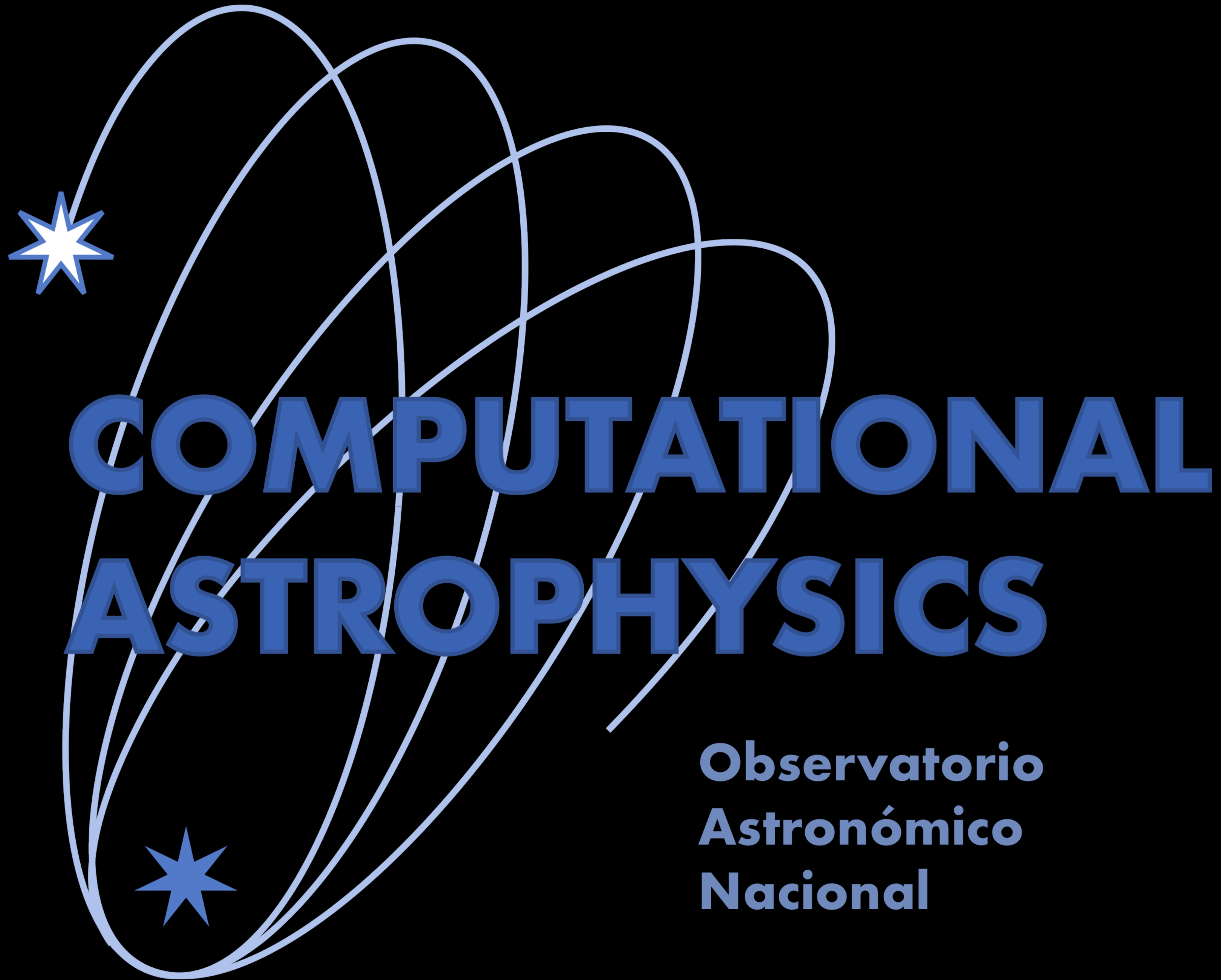




ASTRONÓMICO

NACIONAL DE COLOMBIA



COMPUTATIONAL ASTROPHYSICS

Observatorio
Astronómico
Nacional



Extended Seminar

“A code to generate the image of a black hole”

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Observatorio Astronómico Nacional



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Session 01: Motion of a single photon in a Schwarzschild background.

Session 02: The image plane and the initial conditions.

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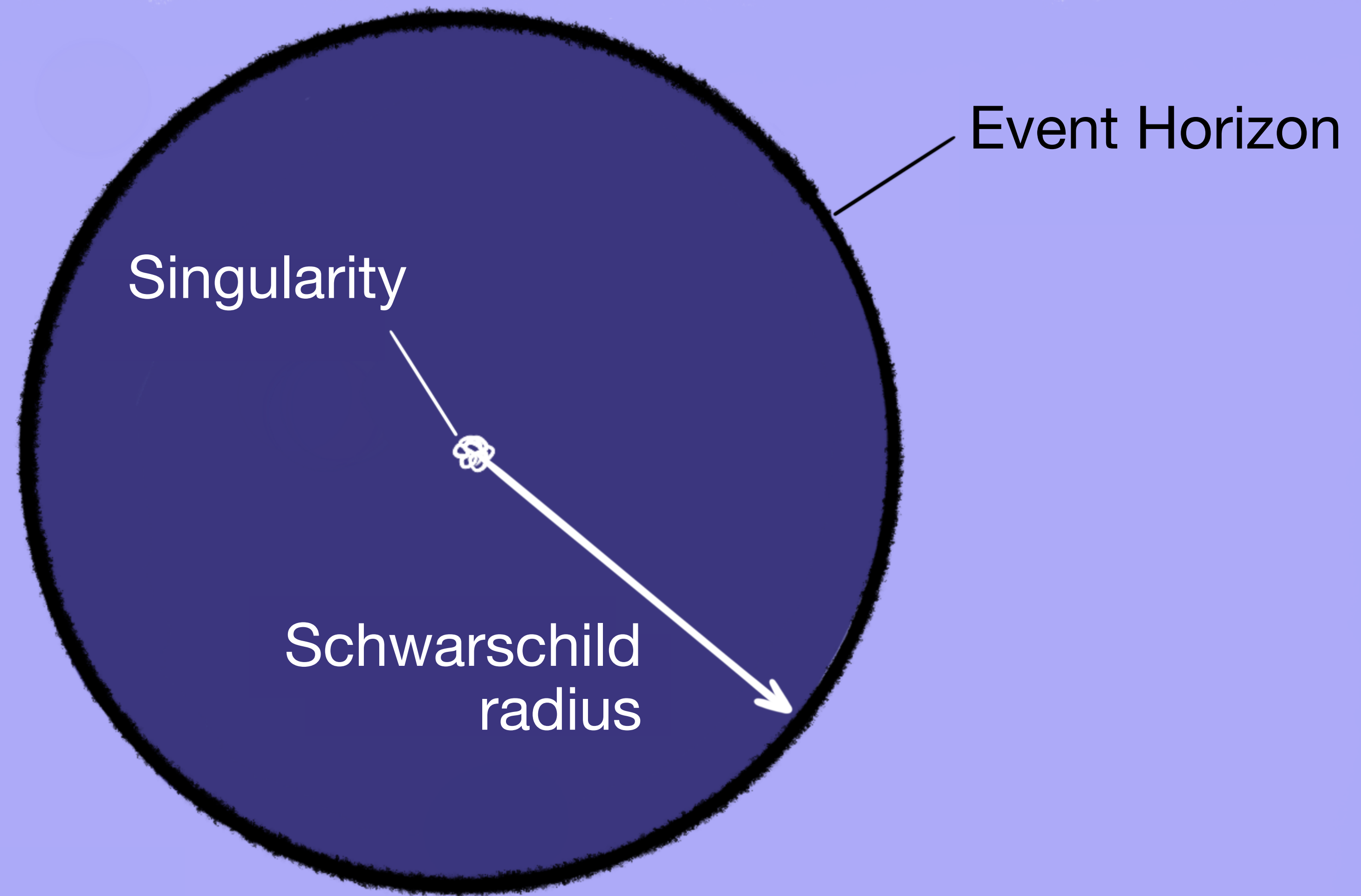
Session 01

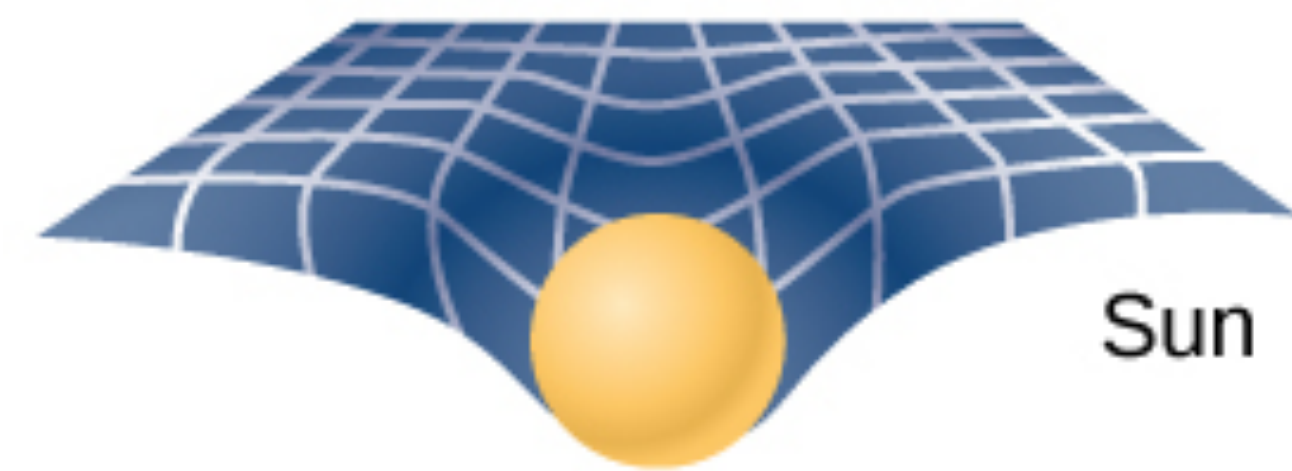
Motion of a Photon in a Schwarzschild Background

The Schwarzschild Spacetime

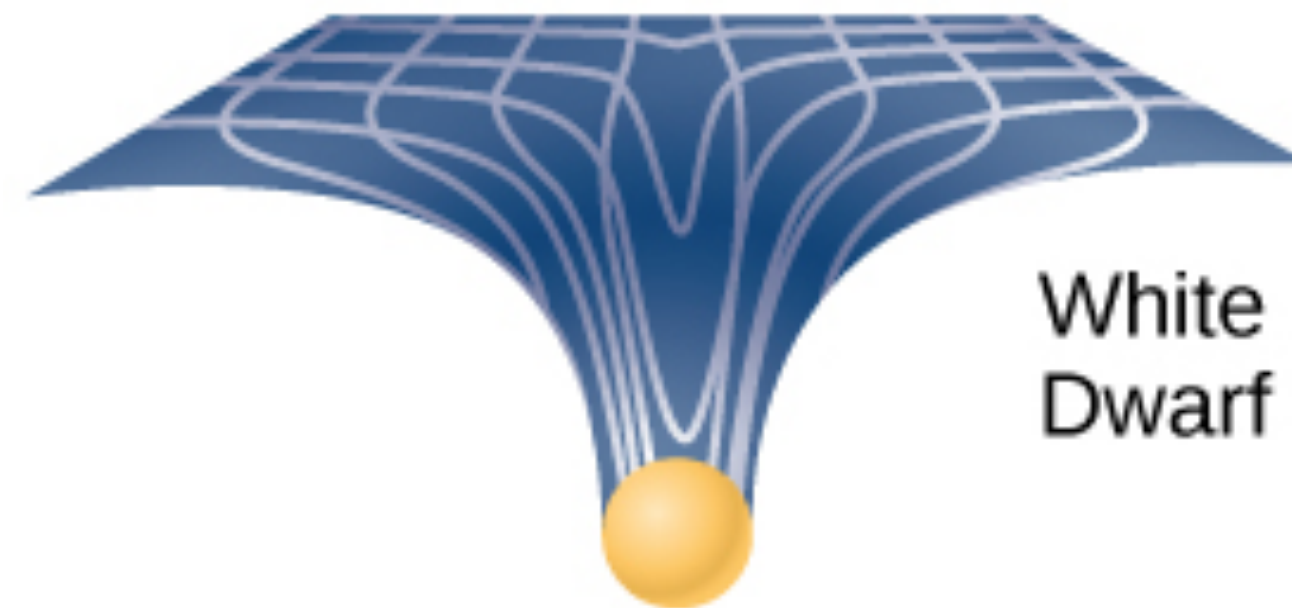
The Schwarzschild spacetime

$$ds^2 = - \left(1 - \frac{2GM}{c^2 r} \right) dt^2 + \left(1 - \frac{2GM}{c^2 r} \right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

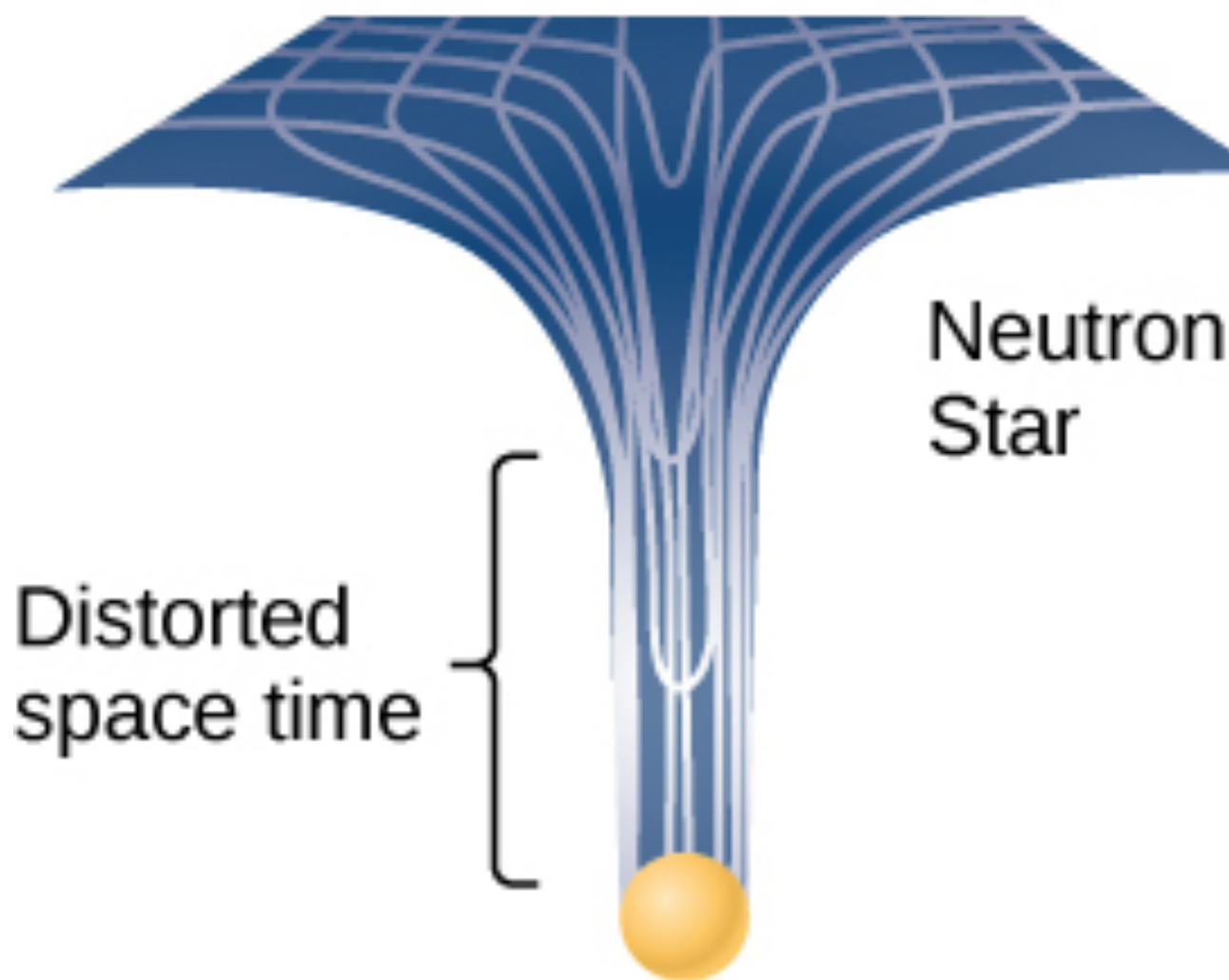




Sun

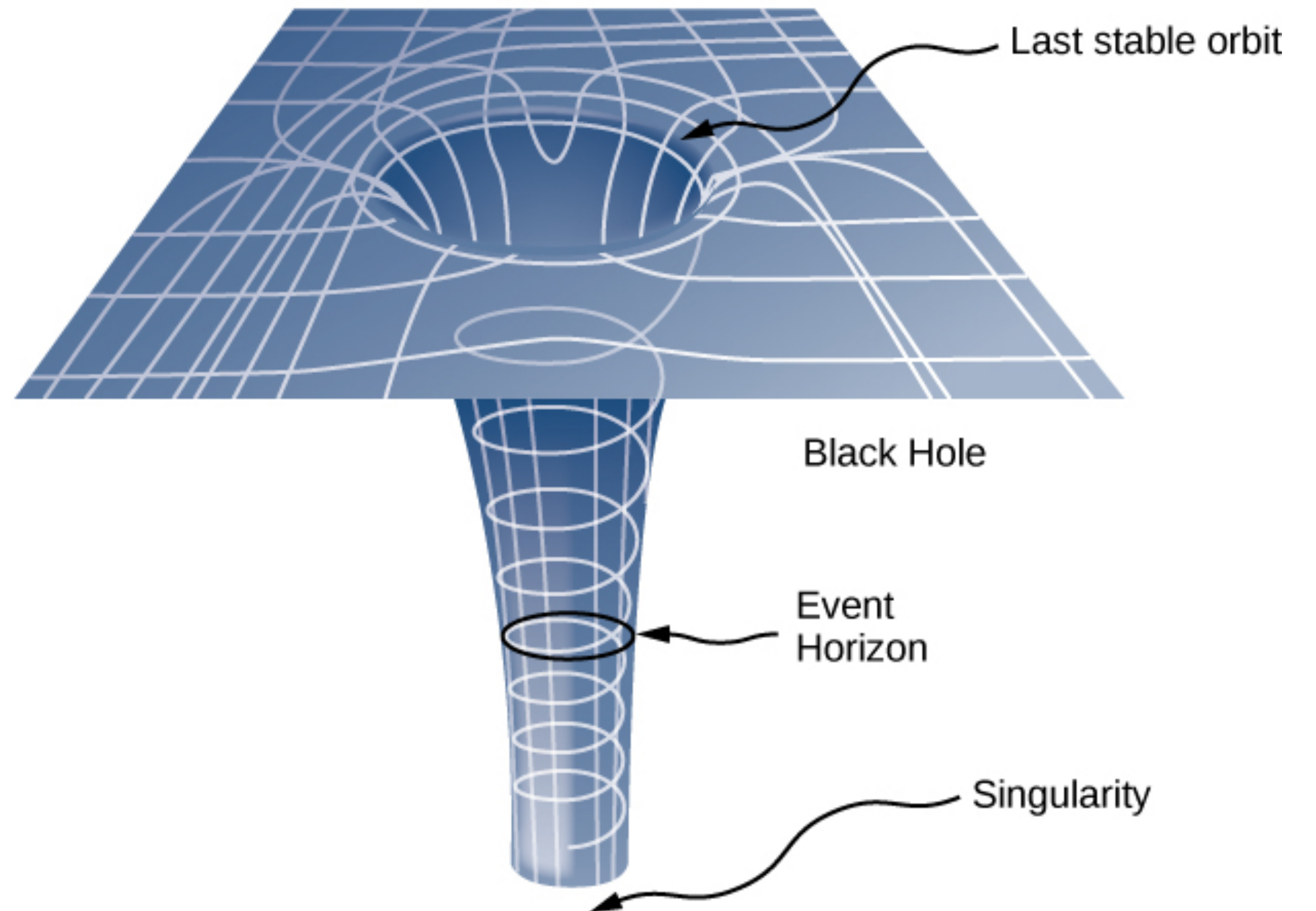


White Dwarf



Neutron Star

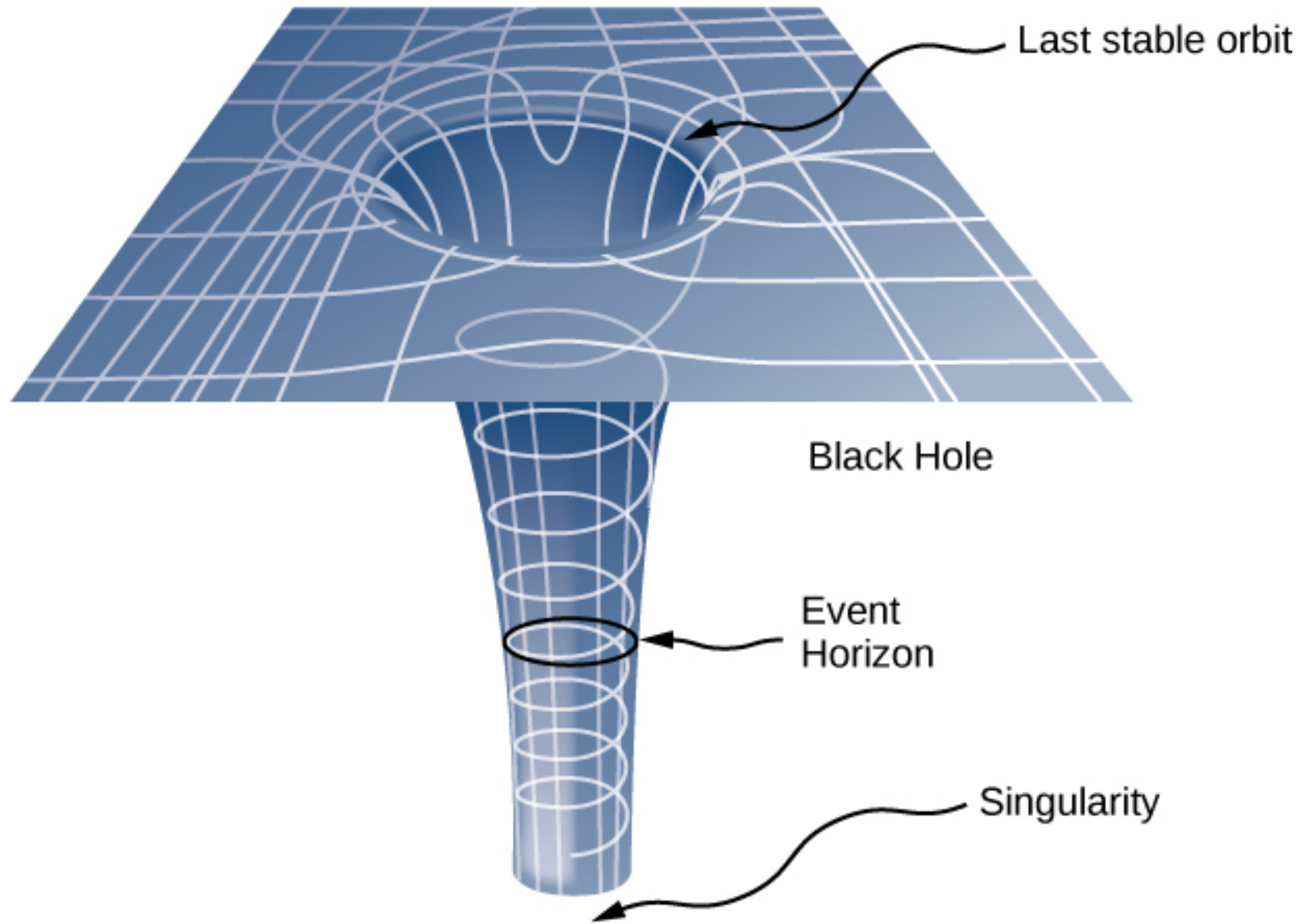
Distorted
space time



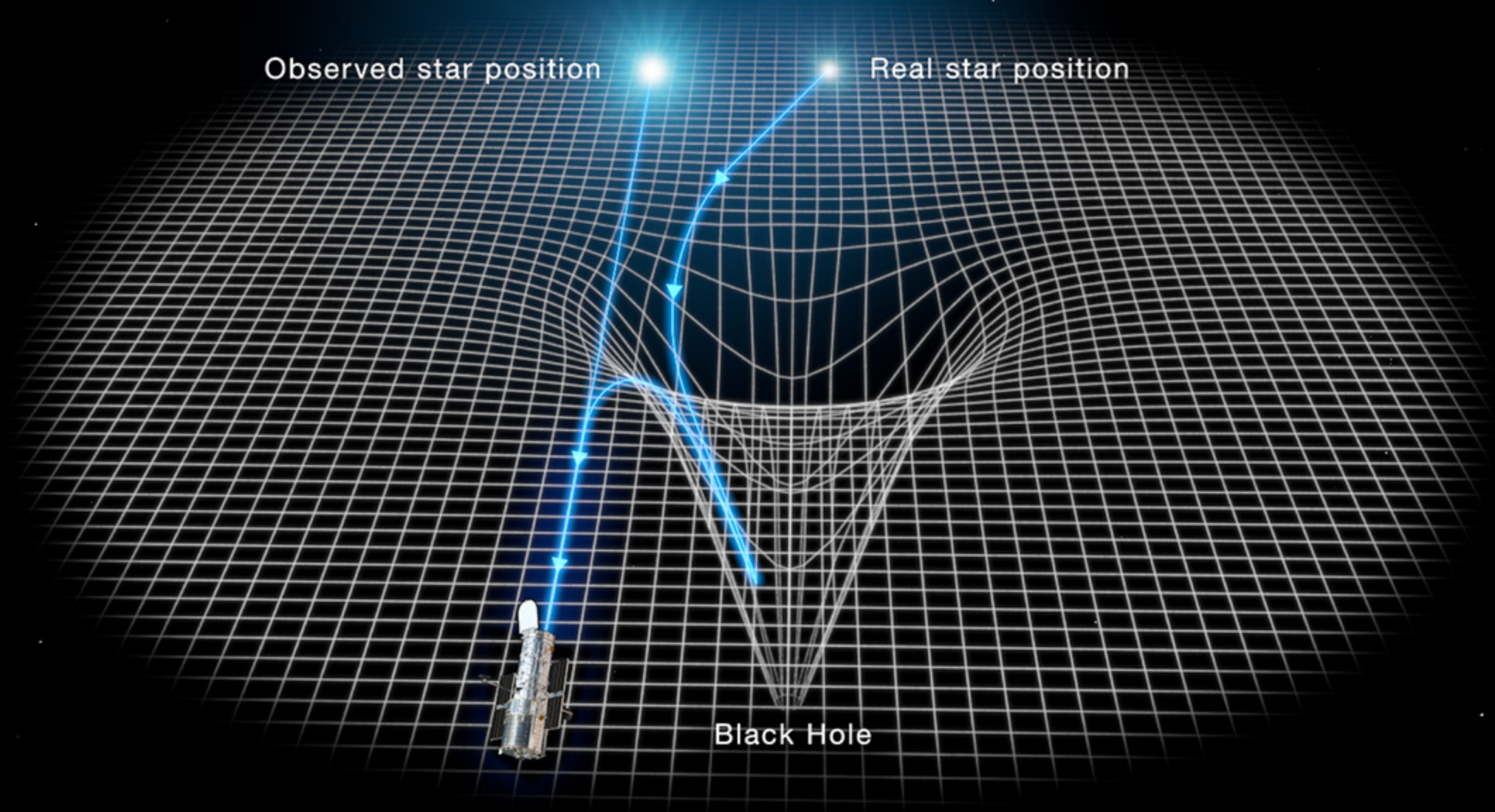
Black Hole

Event
Horizon

Singularity



Hubble Measures Deflection of Starlight by a Foreground Black Hole



The Schwarzschild spacetime

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \left(1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

$$g_{\mu\nu} = \begin{bmatrix} - \left(1 - \frac{2M}{r} \right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2M}{r} \right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{bmatrix}$$

Motion of a Photon in Schwarzschild Spacetime

Equations of Motion

$$x^\mu = [t, r, \theta, \phi]$$

$$k_\mu = [k_t, k_r, k_\theta, k_\phi]$$

Constants of Motion: $k_t = -\varepsilon$
 $k_\phi = \ell$

$$\dot{t} = \frac{\varepsilon}{\left(1 - \frac{2M}{r}\right)}$$
$$\dot{r} = \left(1 - \frac{2M}{r}\right) k_r$$
$$\dot{\theta} = \frac{k_\theta}{r^2}$$
$$\dot{\phi} = \frac{\ell}{r^2 \sin^2 \theta}$$

$$\dot{k}_t = \dot{\varepsilon} = 0$$

$$\dot{k}_r = -\frac{M}{(r-2M)^2} \varepsilon^2 - \frac{M}{r^2} k_r^2 + \frac{k_\theta^2}{r^3} + \frac{\ell^2}{r^3 \sin^2 \theta}$$

$$\dot{k}_\theta = \frac{\cos \theta}{r^2 \sin^3 \theta} \ell$$

$$\dot{k}_\phi = \dot{\ell} = 0$$

Equations of Motion

$$\dot{t} = \frac{\varepsilon}{\left(1 - \frac{2M}{r}\right)}$$

$$\dot{r} = \left(1 - \frac{2M}{r}\right) k_r$$

$$\dot{\theta} = \frac{k_\theta}{r^2}$$

$$\dot{\phi} = \frac{\ell}{r^2 \sin^2 \theta}$$

$$\dot{k}_t = \dot{\varepsilon} = 0$$

$$\dot{k}_r = -\frac{M}{(r - 2M)^2} \varepsilon^2 - \frac{M}{r^2} k_r^2 + \frac{k_\theta^2}{r^3} + \frac{\ell^2}{r^3 \sin^2 \theta}$$

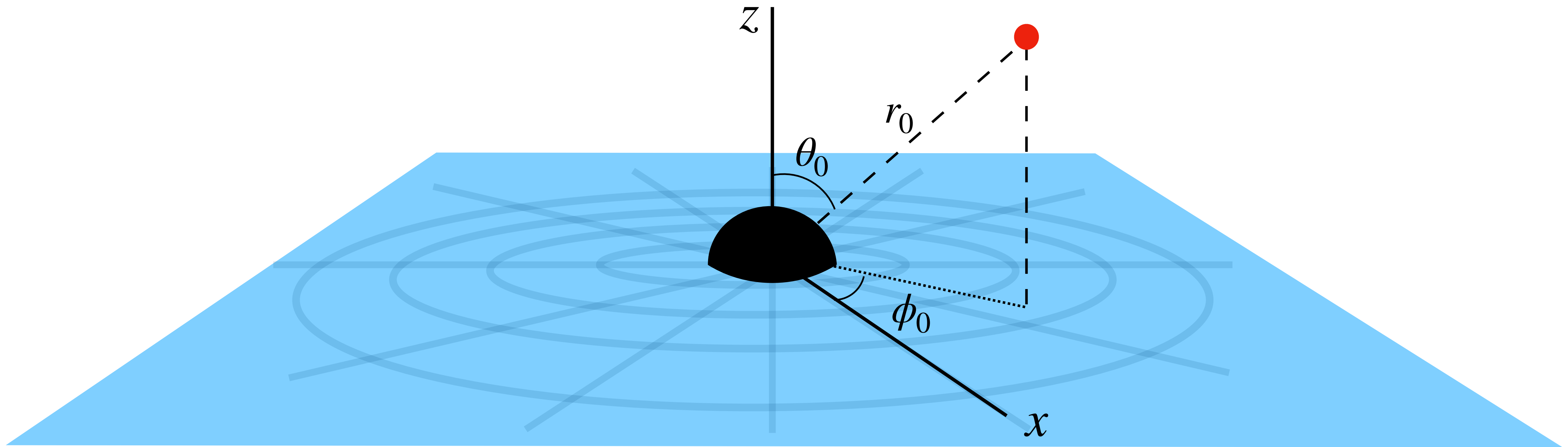
$$\dot{k}_\theta = \frac{\cos \theta}{r^2 \sin^3 \theta} \ell$$

$$\dot{k}_\phi = \dot{\ell} = 0$$

The Initial Conditions

Initial Conditions

$$[0, r_0, \theta_0, \phi_0]$$

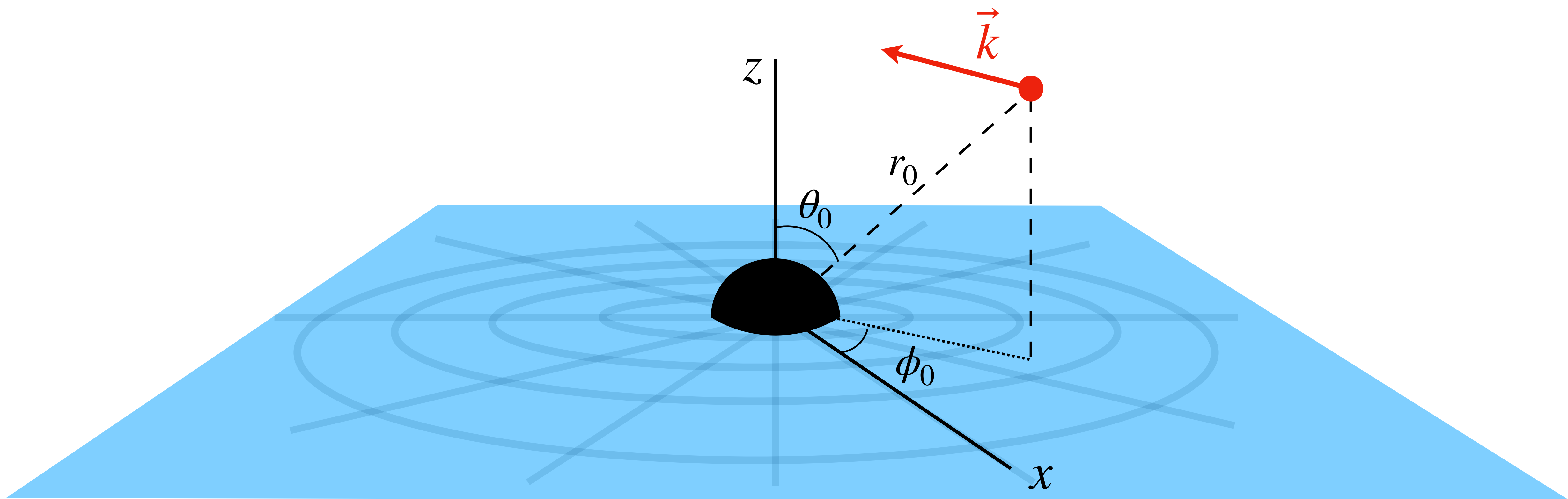


Initial Conditions

$$\left[(k_t)_0, (k_r)_0, (k_\theta)_0, (k_\phi)_0 \right]$$



$$[k_0^t, k_0^r, k_0^\theta, k_0^\phi]$$



Initial Conditions

$$\left[(k_t)_0, (k_r)_0, (k_\theta)_0, (k_\phi)_0 \right] \longleftarrow [k_0^t, k_0^r, k_0^\theta, k_0^\phi]$$

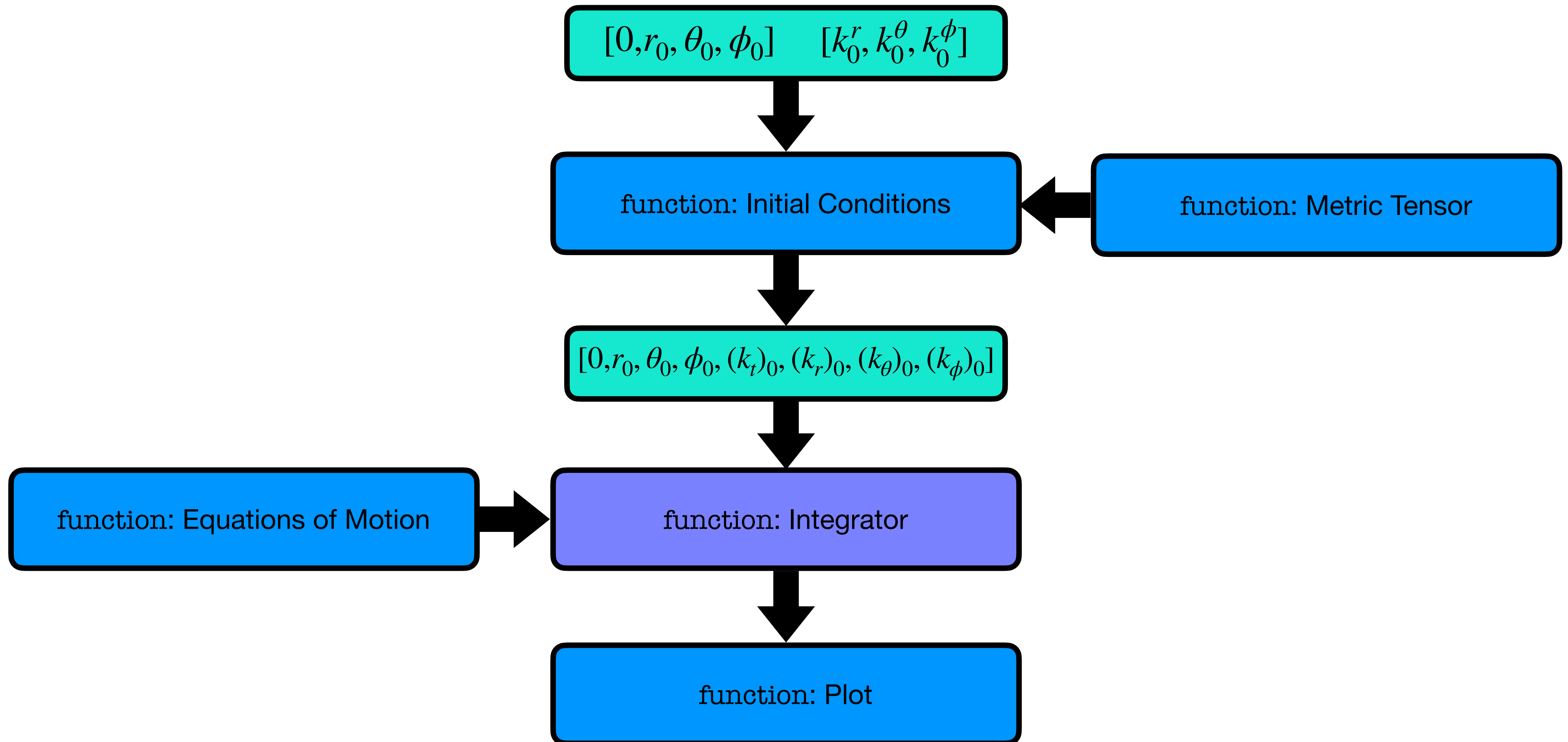
$$k^2 = g_{\mu\nu} k^\mu k^\nu = 0$$

$$k^t = \sqrt{\frac{g_{rr}(k^r)^2 + g_{\theta\theta}(k^\theta)^2 + g_{\phi\phi}(k^\phi)^2}{g_{tt}}}$$

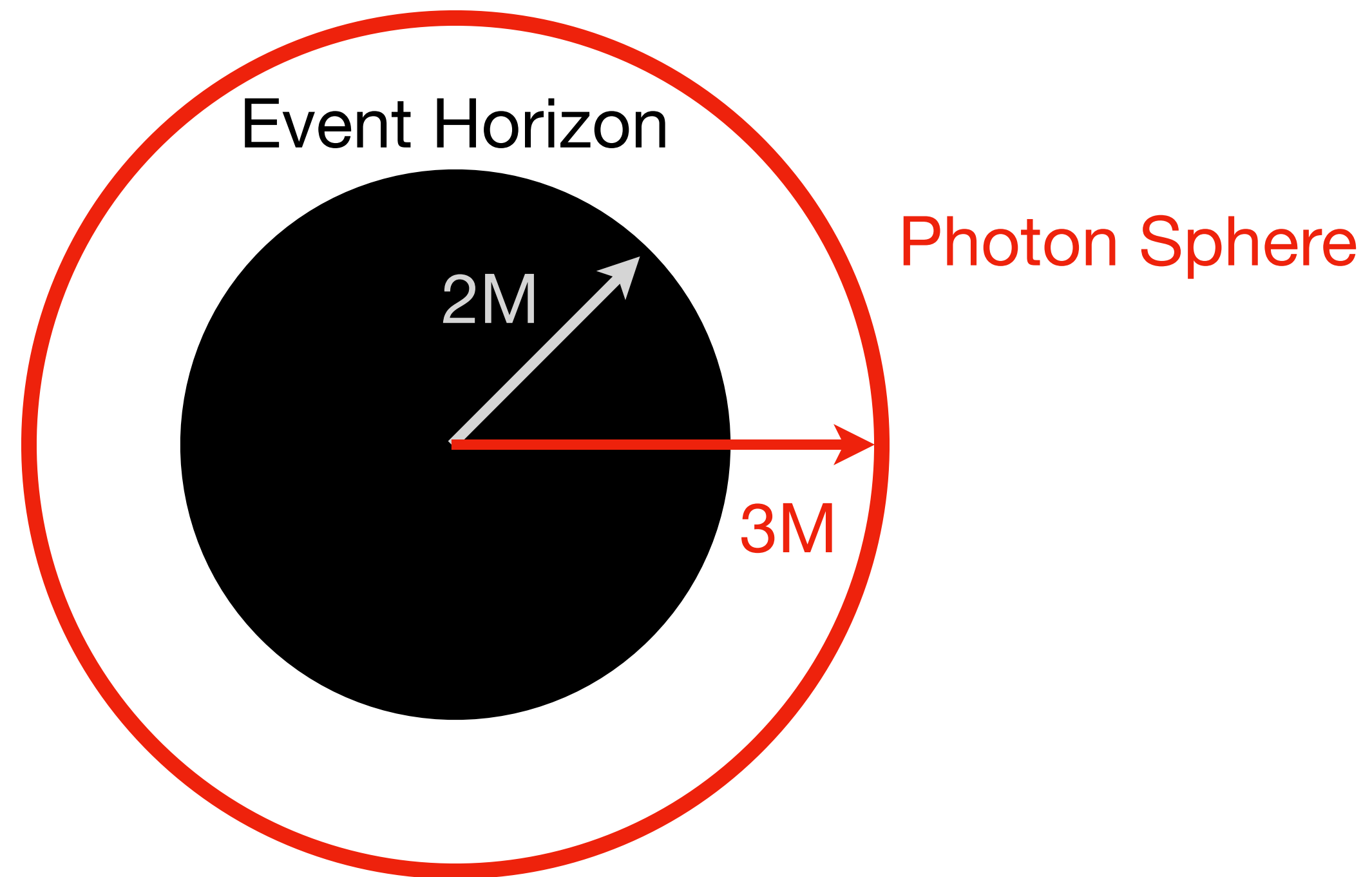
$$k_\mu = g_{\mu\nu} k^\nu$$

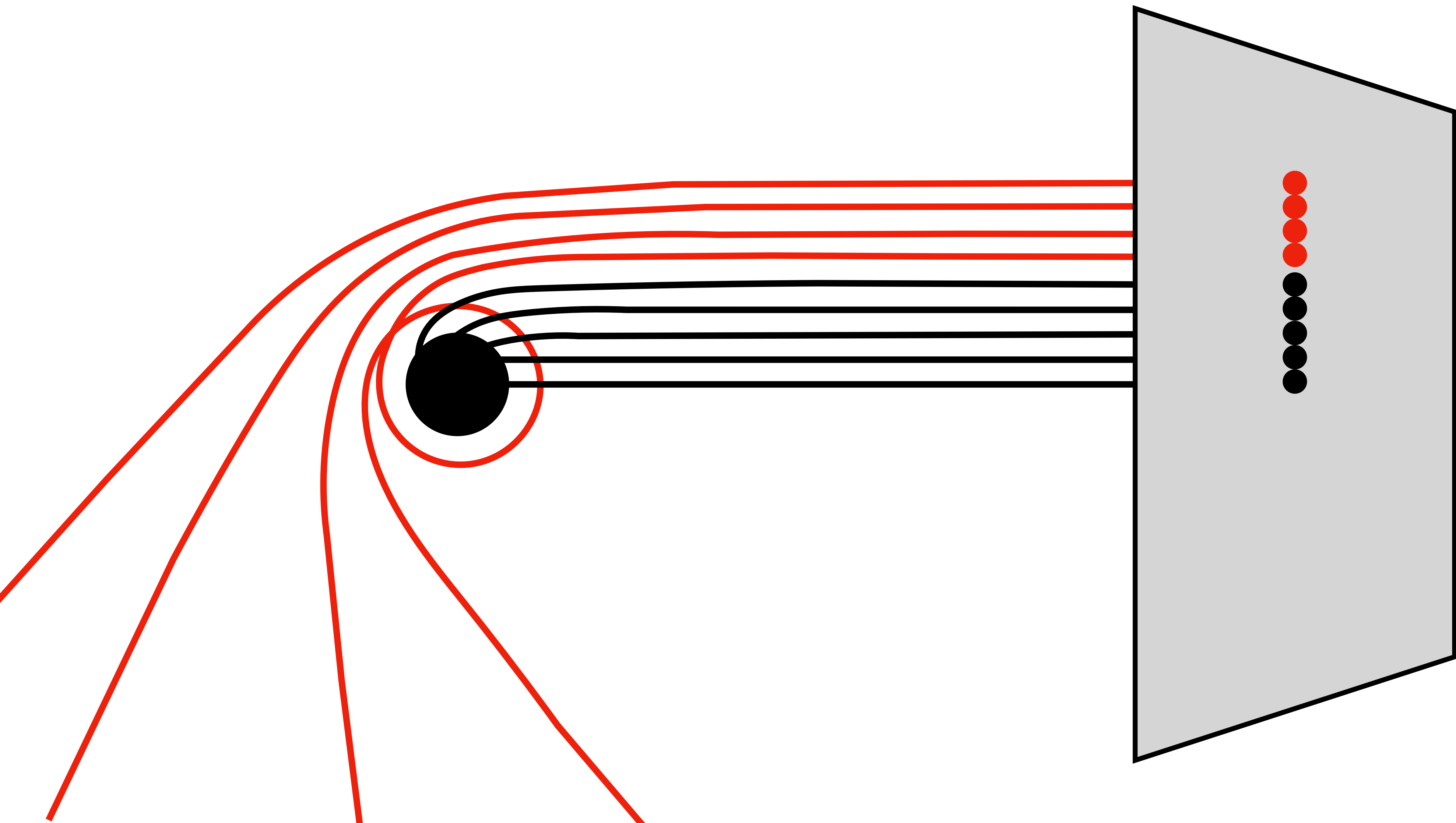
The Code

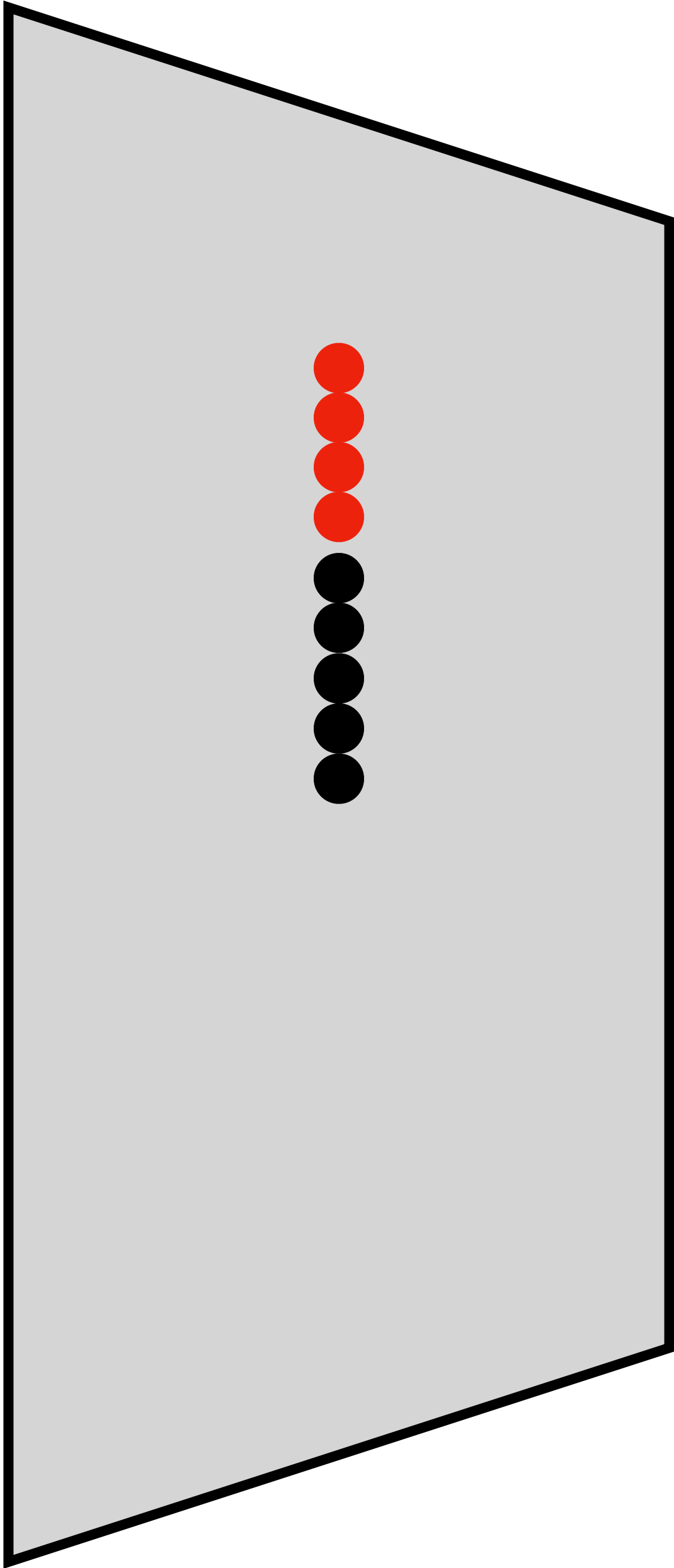
Structure of the Code

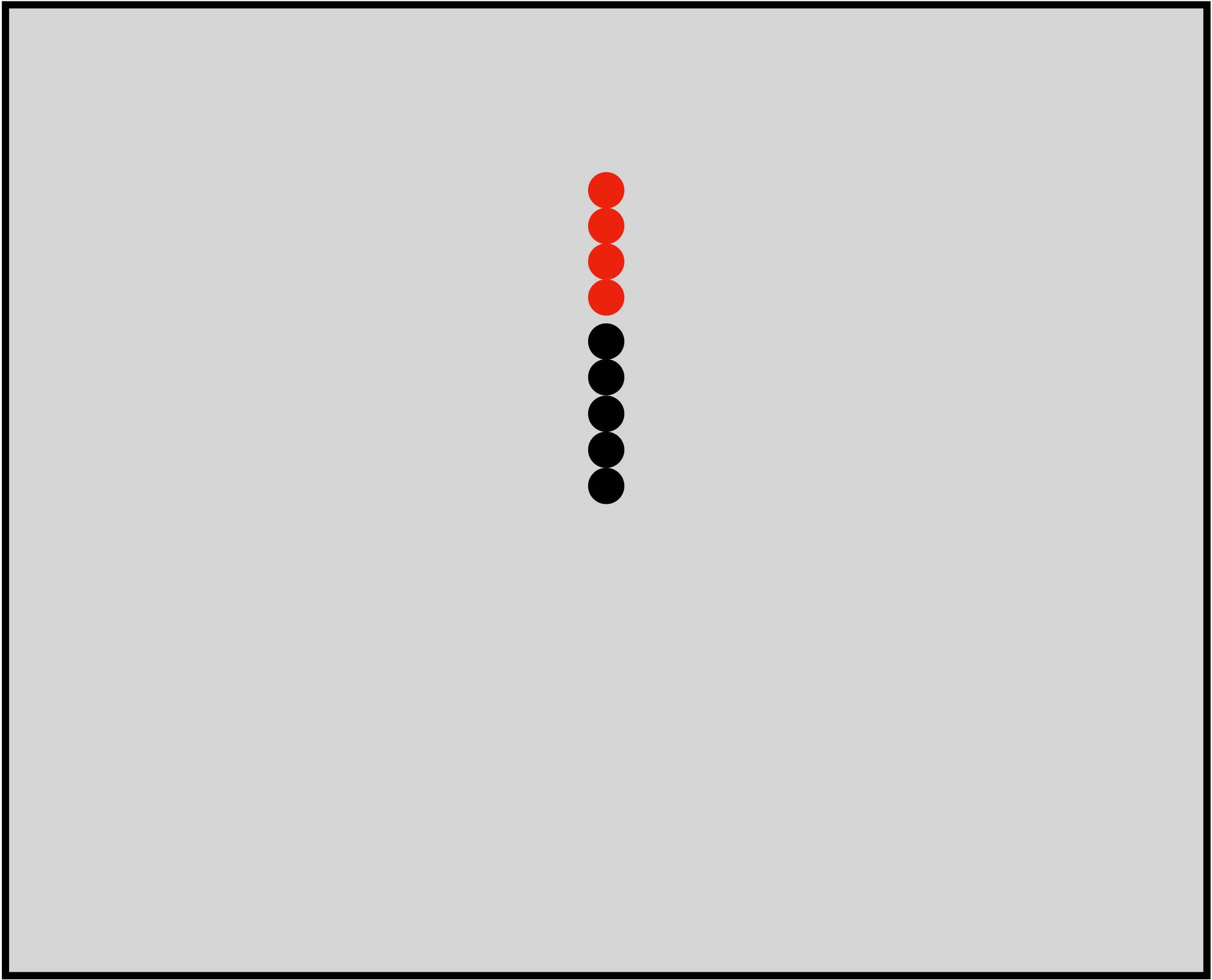


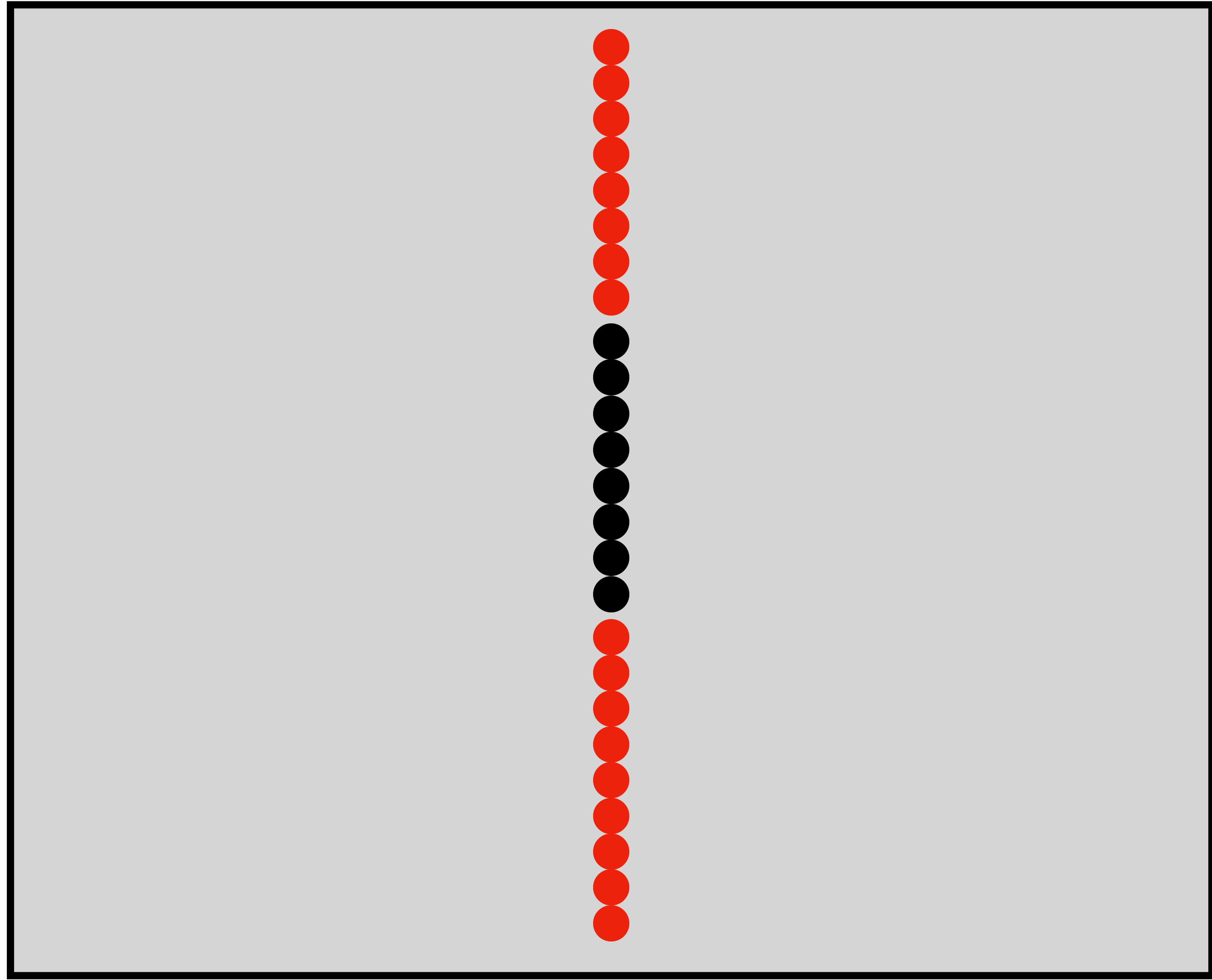
The Photon Sphere and the Shadow of a Black Hole



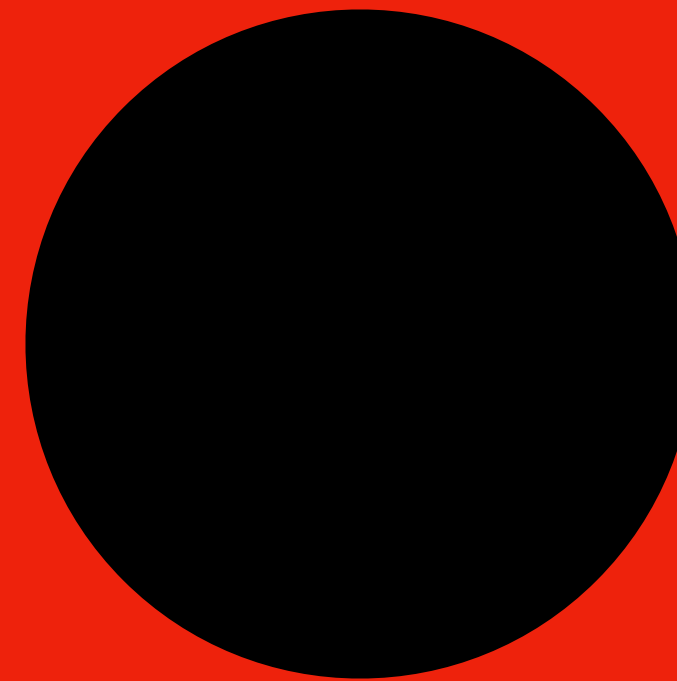








Shadow of the Black Hole





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