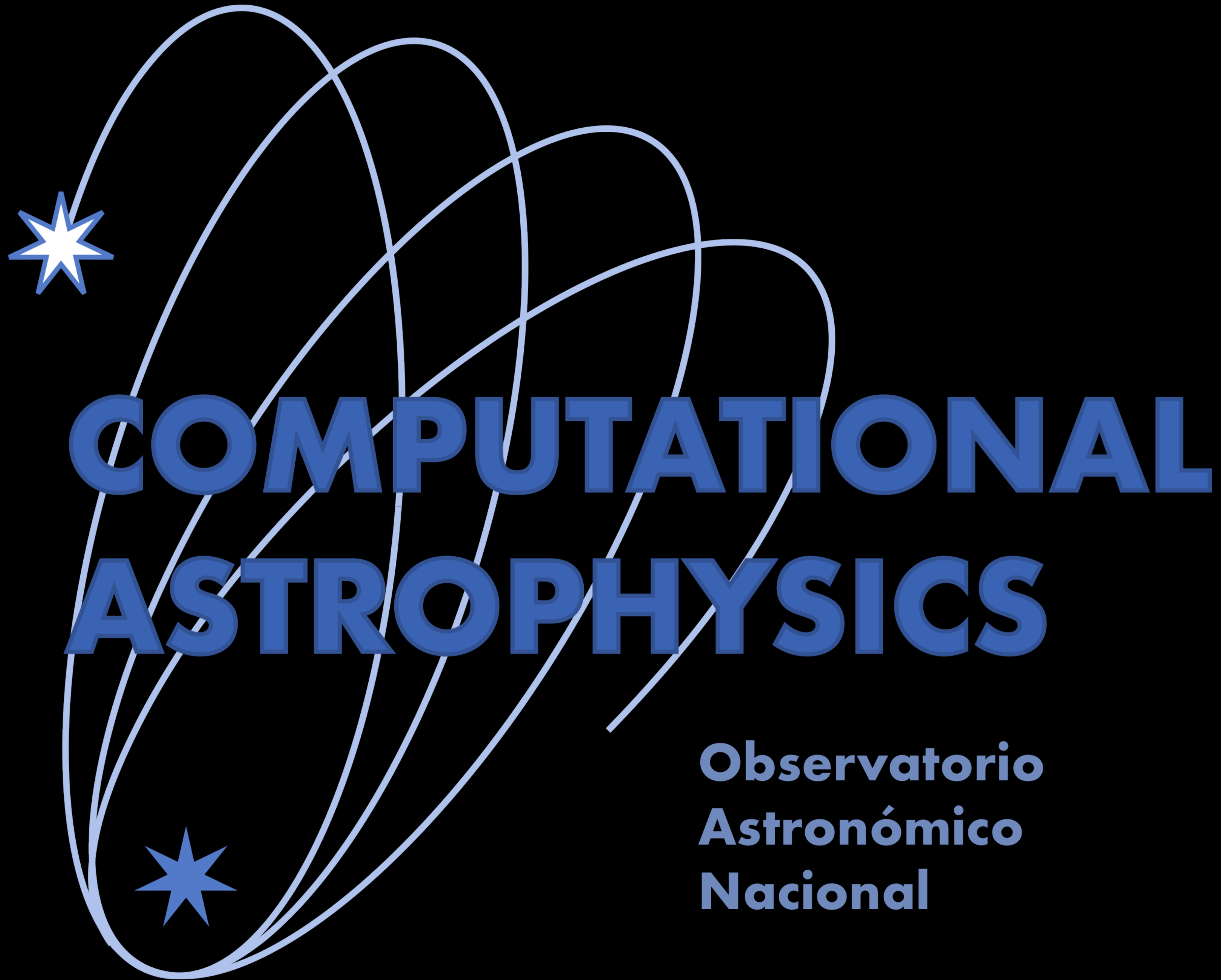




**ASTRONÓMICO**  

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**NACIONAL DE COLOMBIA**



# COMPUTATIONAL ASTROPHYSICS

Observatorio  
Astronómico  
Nacional



# Extended Seminar

## “A code to generate the image of a black hole”

**Eduard Larrañaga**  
**Observatorio Astronómico Nacional**

# Session 02

**The Image Plane and the Initial Conditions**

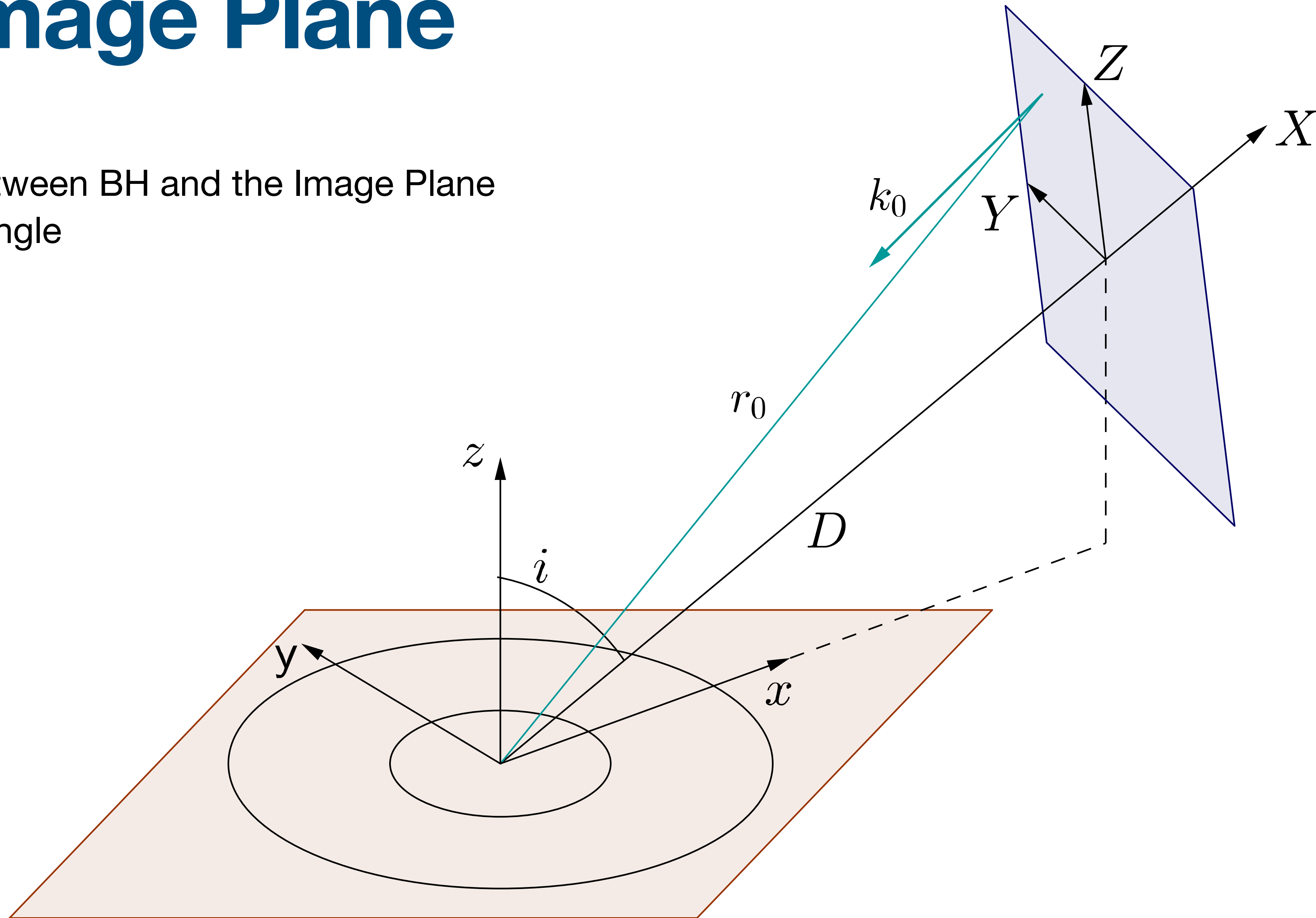


# The Image Plane

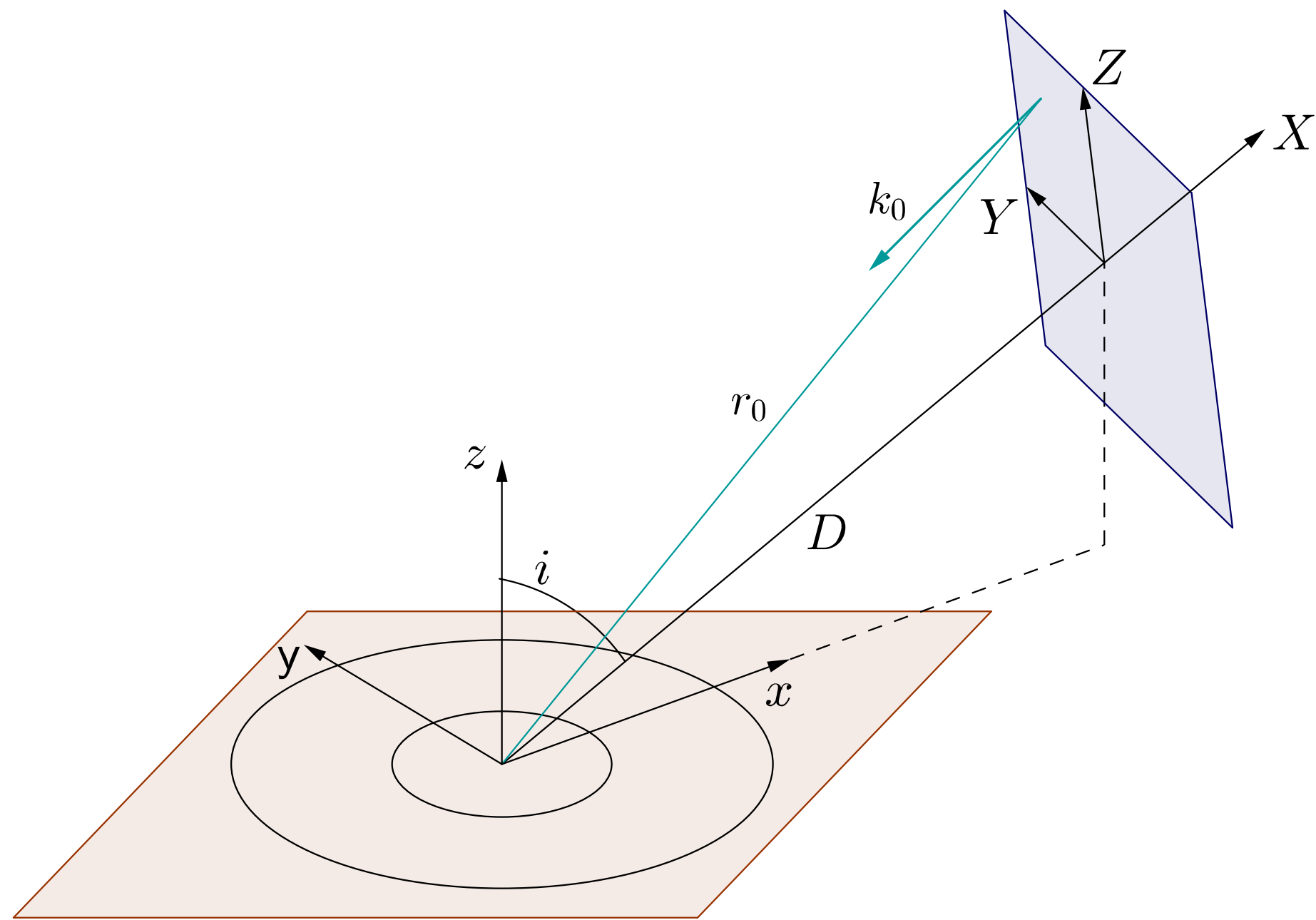
# The Image Plane

$D$  : Distance between BH and the Image Plane

$i$  : Inclination angle



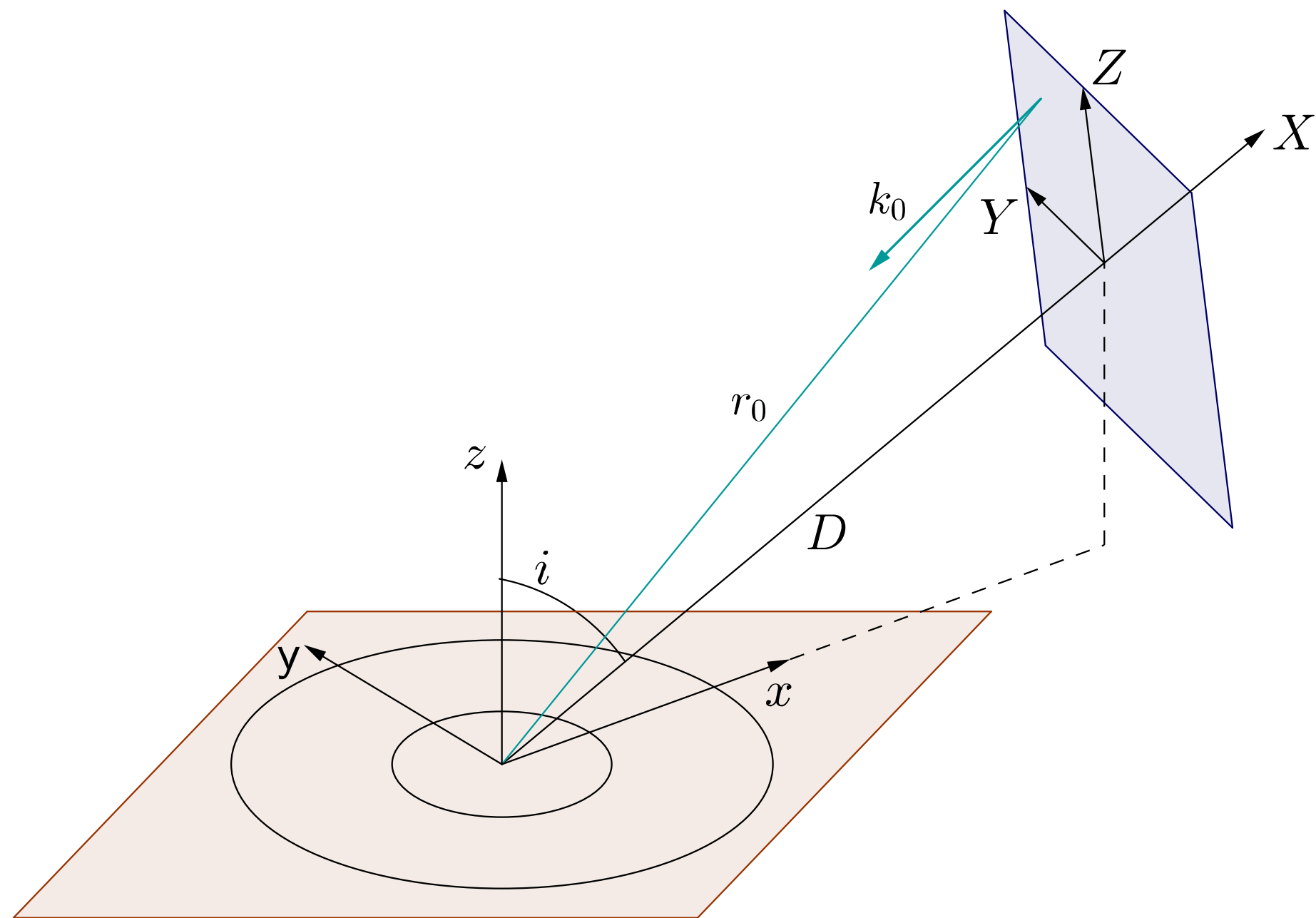
# Transformation between $(X, Y, Z)$ and $(x, y, z)$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_Y \left( \frac{\pi}{2} - \iota \right) \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} D \sin \iota \\ 0 \\ D \cos \iota \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (X + D) \sin \iota - Z \cos \iota \\ Y \\ (X + D) \cos \iota + Z \sin \iota \end{pmatrix}$$

# Transformation between $(x, y, z)$ and $(r, \theta, \phi)$



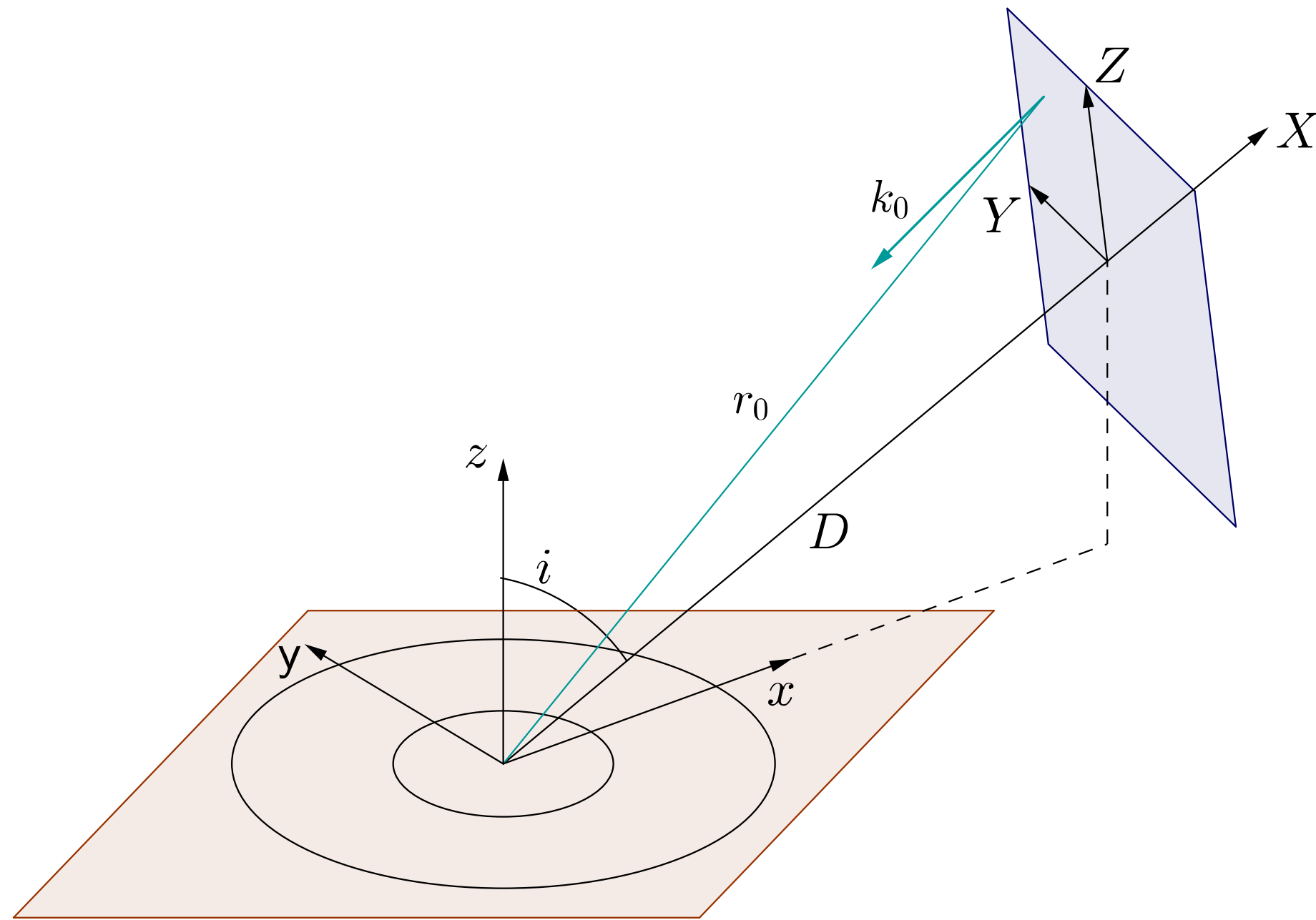
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arccos \left( \frac{z}{r} \right)$$

$$\phi = \arctan \left( \frac{y}{x} \right)$$



# Transformation between $(X, Y, Z)$ and $(r, \theta, \phi)$

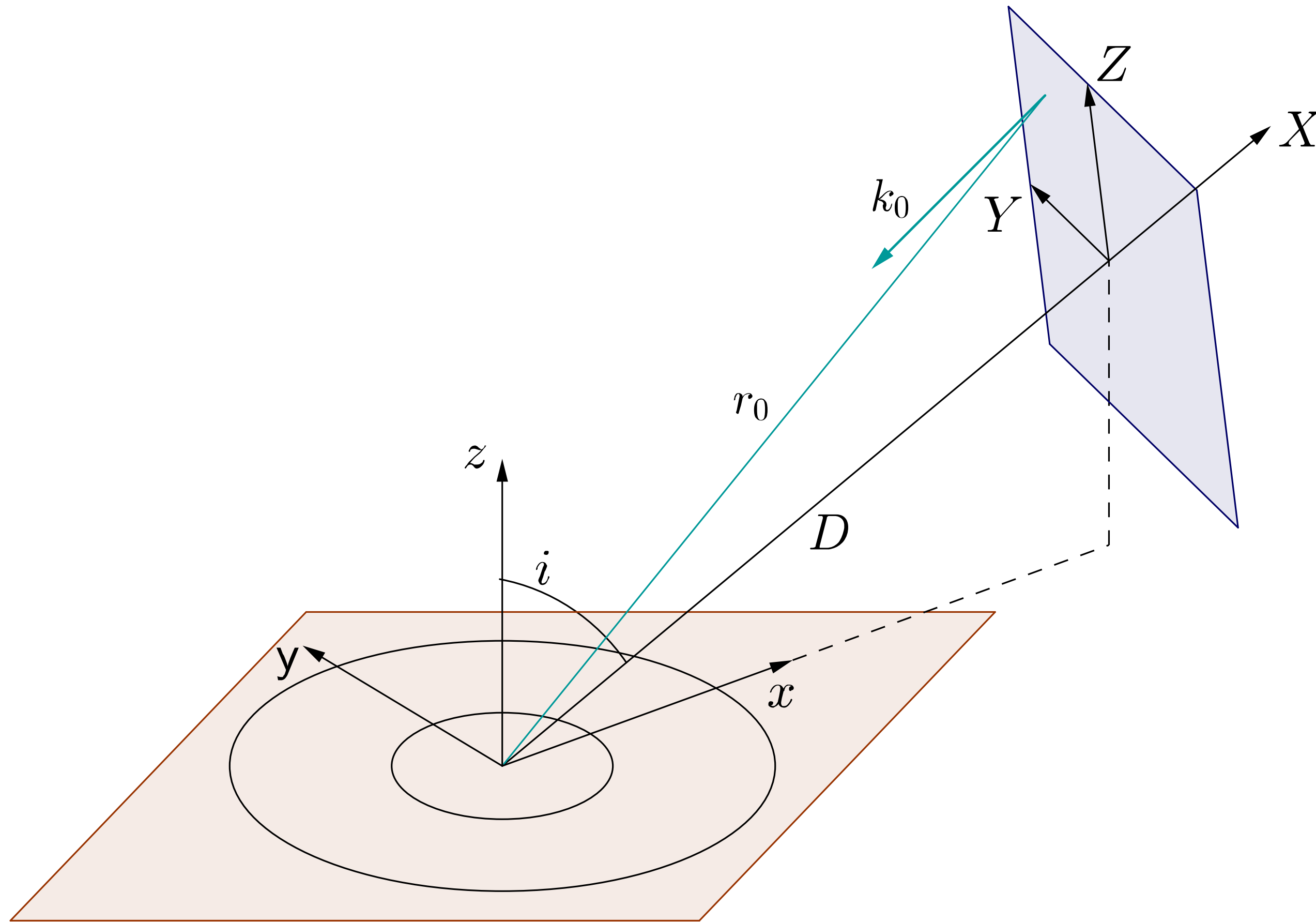


$$r = \sqrt{(X + D)^2 + Y^2 + Z^2}$$

$$\theta = \arccos \left( \frac{(X + D)\cos \iota + Z \sin \iota}{r} \right)$$

$$\phi = \arctan \left( \frac{Y}{(X + D)\sin \iota - Z \cos \iota} \right)$$

# A photon the Image Plane



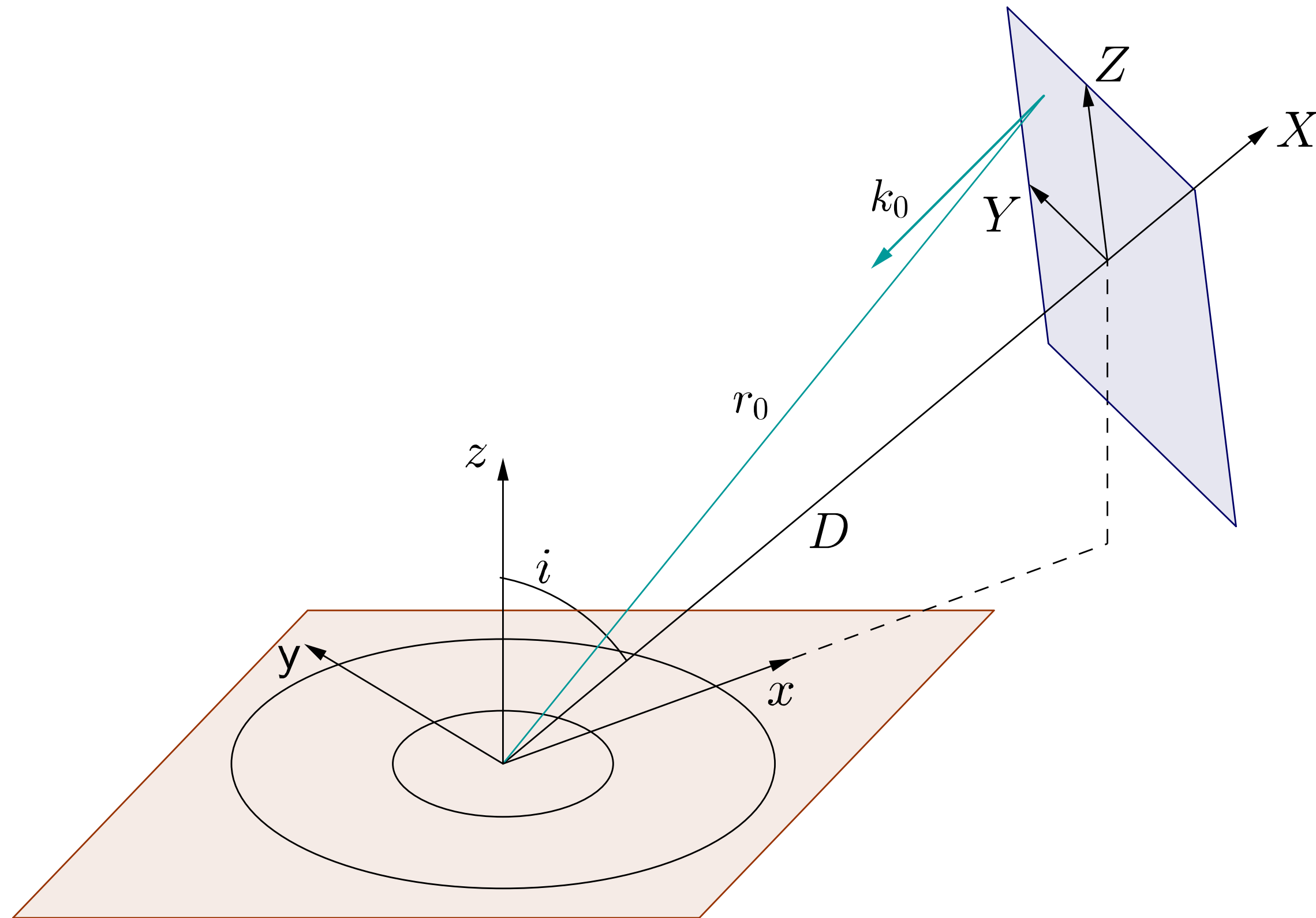
Initial coordinates of the photon  
at the image plane:

$$(0, Y_0, Z_0) = (0, \alpha, \beta)$$

Initial momentum of the photon  
at the image plane:

$$\vec{k} = (k_0, 0, 0)$$

# Initial Conditions for the photon



Initial coordinates of the photon  
at the image plane:

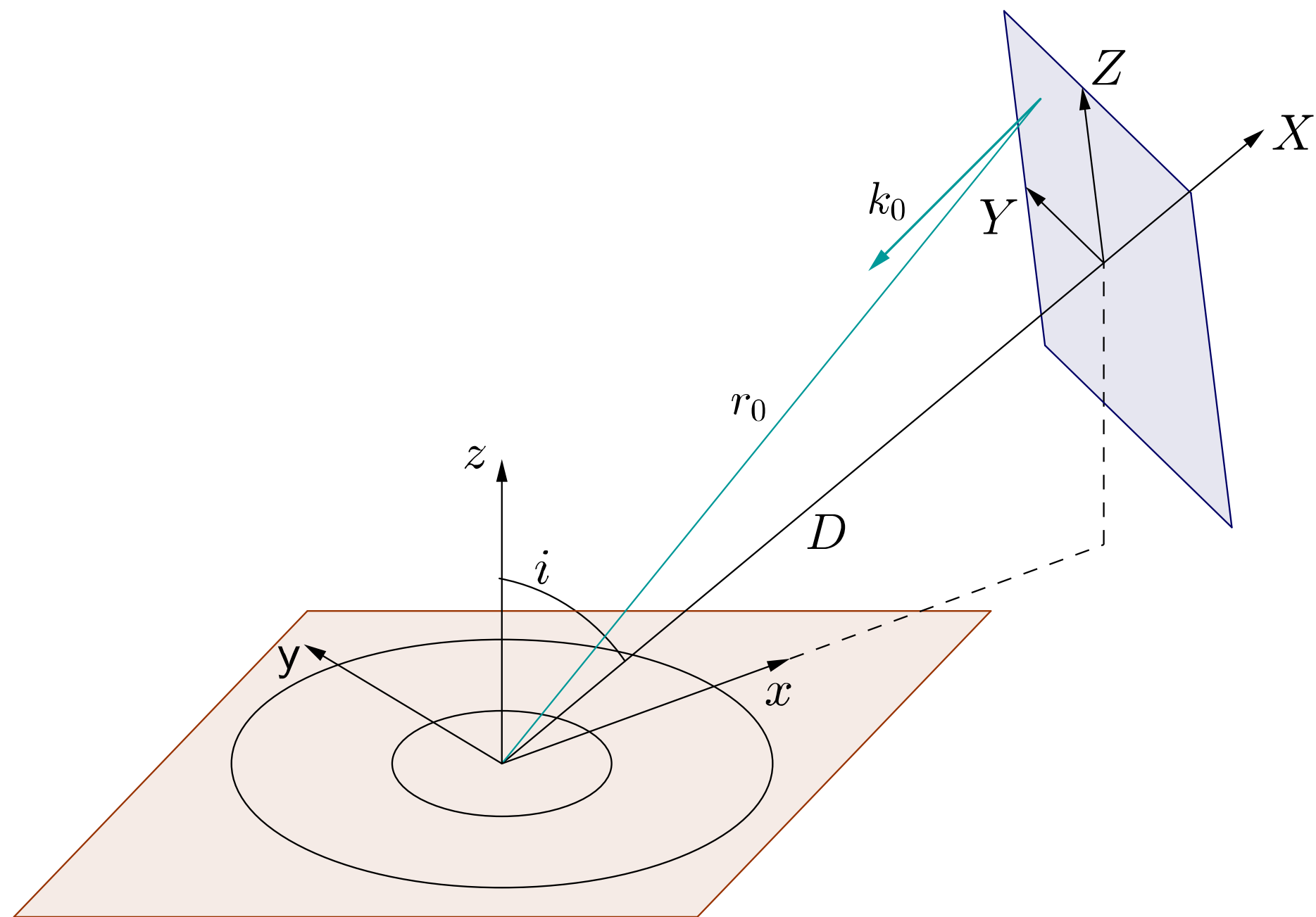
$$(0, Y_0, Z_0) = (0, \alpha, \beta)$$

$$r_0 = \sqrt{D^2 + \alpha^2 + \beta^2}$$

$$\theta_0 = \arccos \left( \frac{D \cos \iota + \beta \sin \iota}{\sqrt{D^2 + \alpha^2 + \beta^2}} \right)$$

$$\phi_0 = \arctan \left( \frac{\alpha}{D \sin \iota - \beta \cos \iota} \right)$$

# A photon the Image Plane



Initial momentum of the photon  
at the image plane:

$$\vec{k} = (k_0, 0, 0)$$

$$k_0^r = \frac{D}{r_0} k_0$$

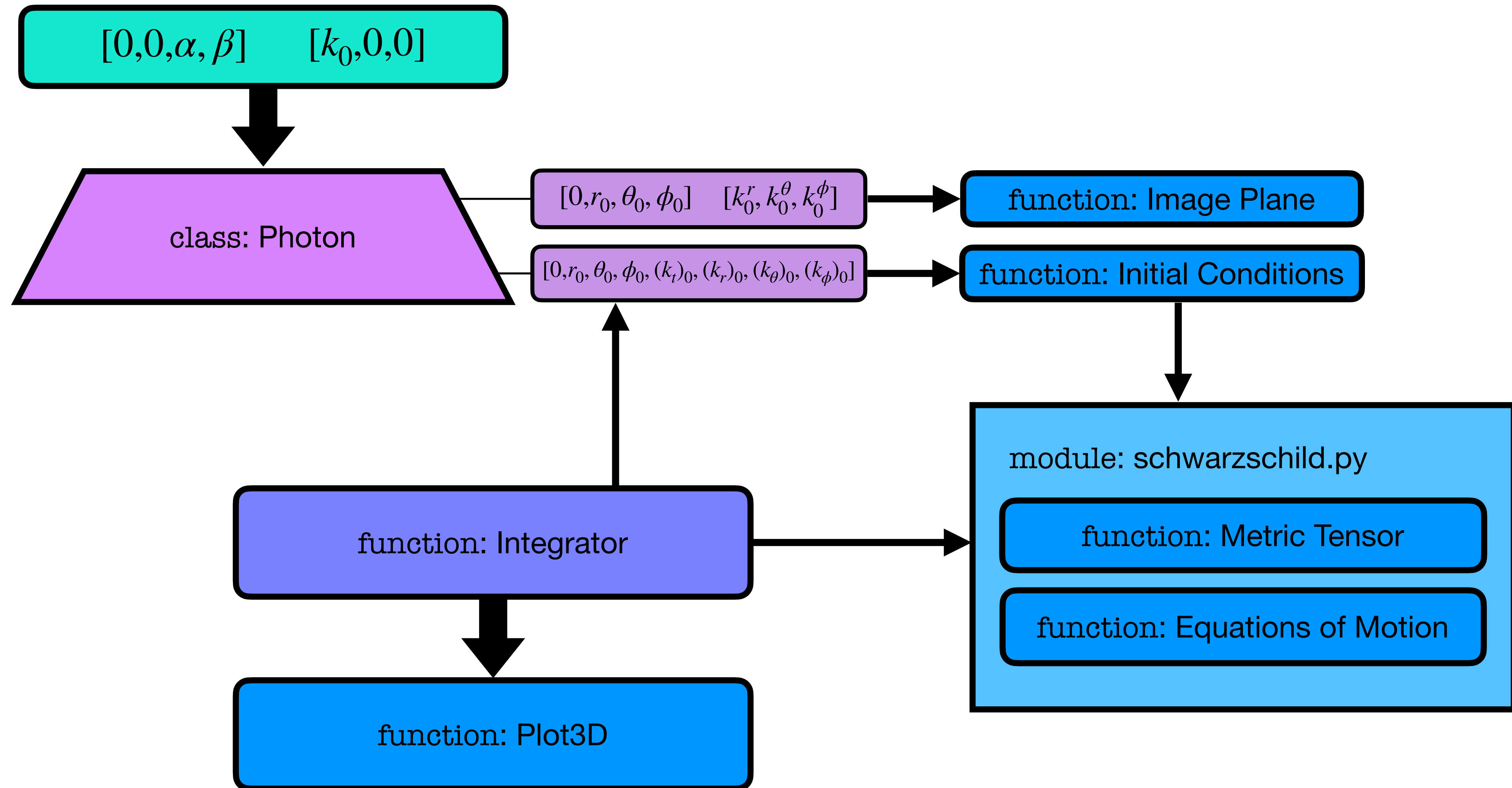
$$k_0^\theta = \frac{k_0}{\sqrt{r_0^2 - (D \cos \iota + \beta \sin \iota)^2}} \left( \cos \iota - \frac{D}{r_0^2} (D \cos \iota + \beta \sin \iota) \right)$$

$$k_0^\phi = \frac{\alpha \sin \iota k_0}{\alpha^2 + (D \cos \iota + \beta \sin \iota)^2}$$

$$k_0^t = \sqrt{(k_0^r)^2 + r_0^2 (k_0^\theta)^2 + r_0^2 \sin^2 \theta_0 (k_0^\phi)^2}$$

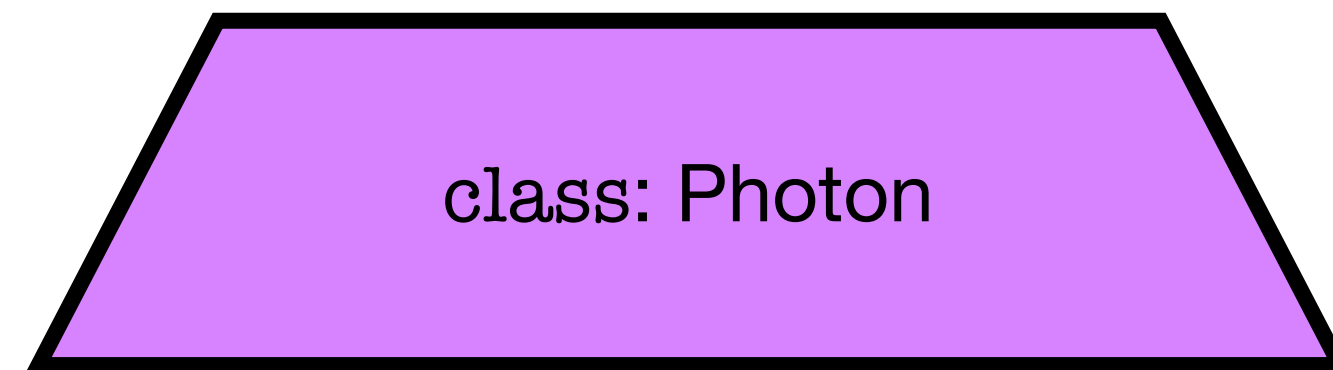
The Code [ver:0.1]

# Structure of the Code





# The Photon class



Attributes:

photon.alpha photon.beta

photon.xin photon.kin

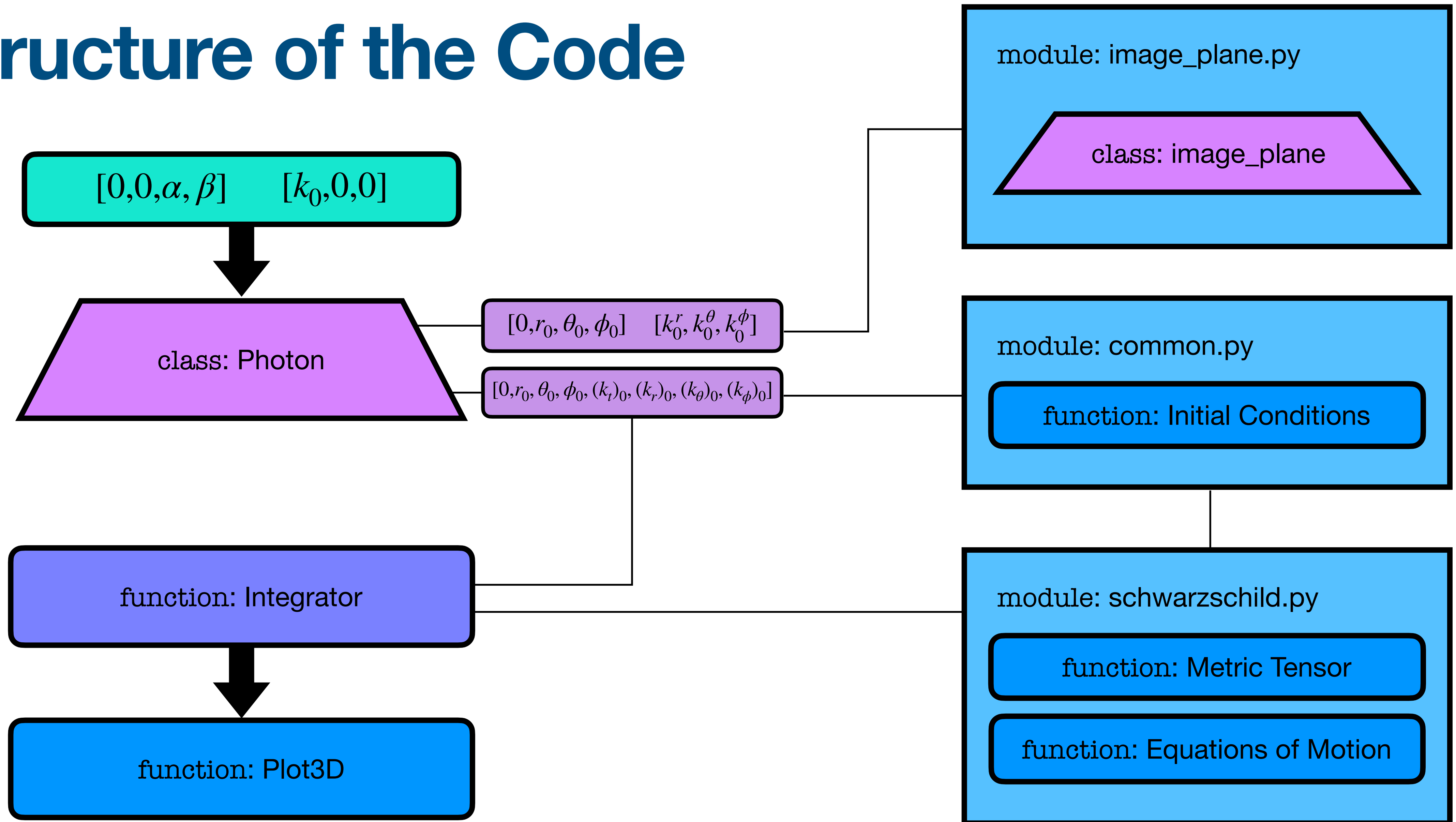
$[0, r_0, \theta_0, \phi_0]$   $[k_0^r, k_0^\theta, k_0^\phi]$

photon.iC

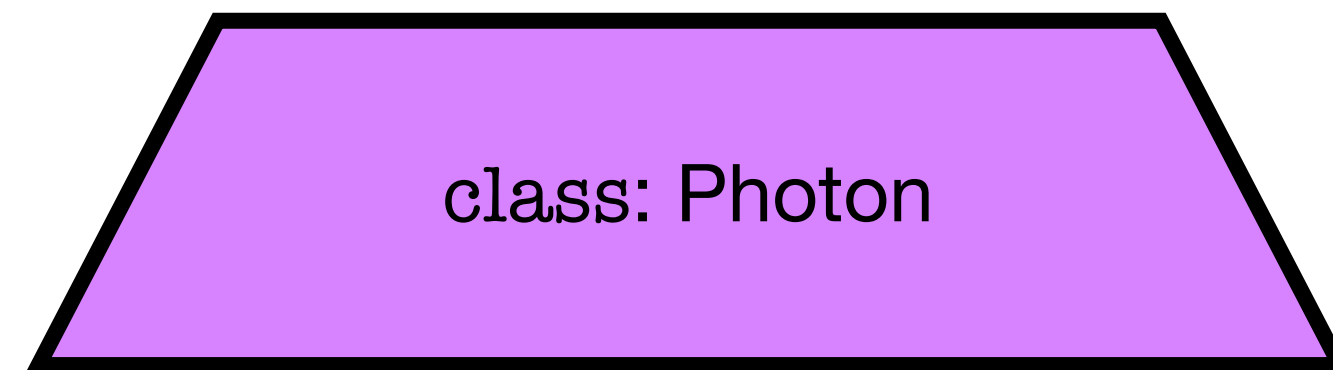
$[0, r_0, \theta_0, \phi_0, (k_t)_0, (k_r)_0, (k_\theta)_0, (k_\phi)_0]$

The Code [ver:0.2]

# Structure of the Code



# The Photon class



Attributes:

photon.alpha photon.beta

photon.xin photon.kin

$[0, r_0, \theta_0, \phi_0]$   $[k_0^r, k_0^\theta, k_0^\phi]$

photon.iC

$[0, r_0, \theta_0, \phi_0, (k_t)_0, (k_r)_0, (k_\theta)_0, (k_\phi)_0]$

# The image\_plane class



## Attributes:

image\_plane.D  
image\_plane.iota

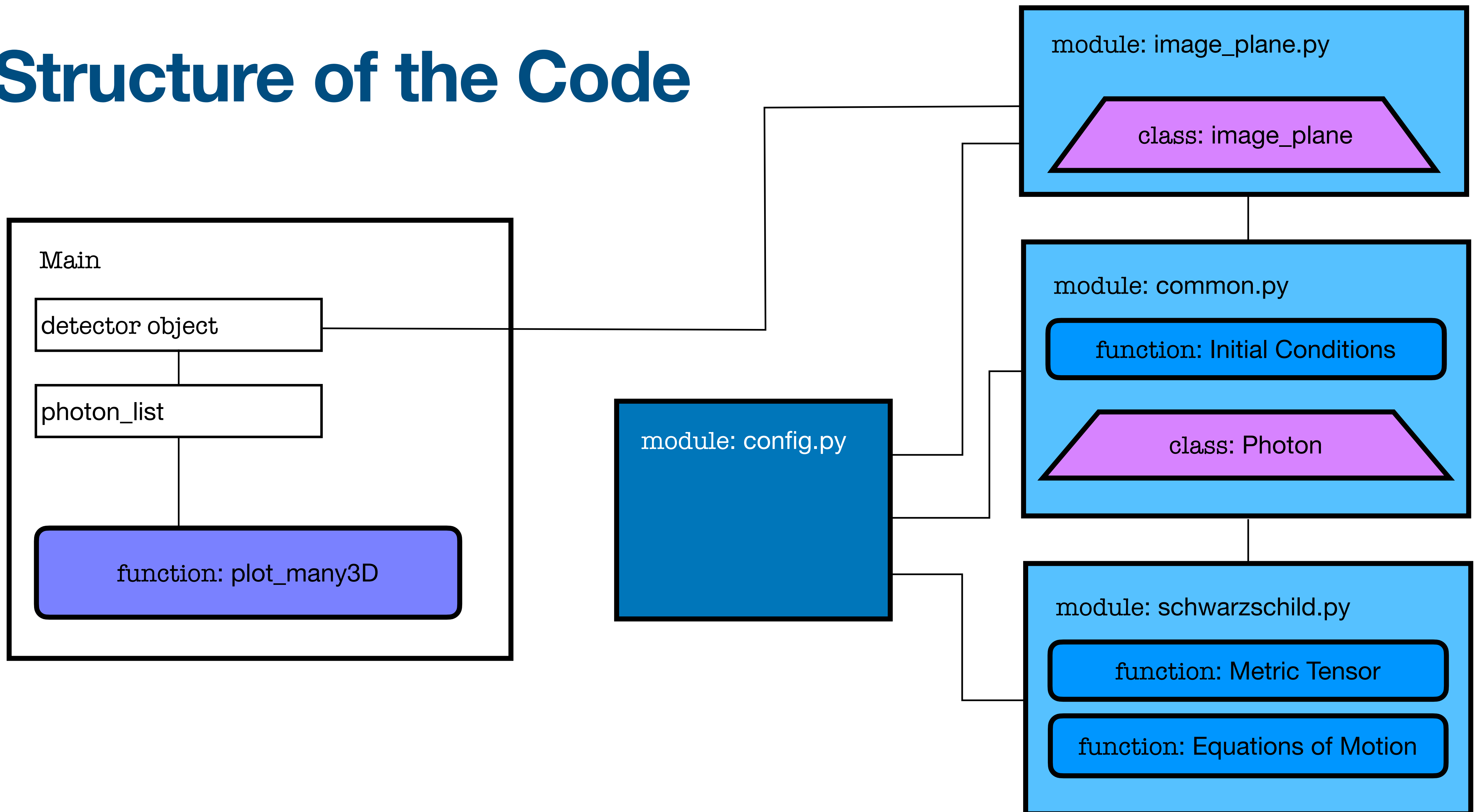
## Methods:

image\_plane.photon\_coords(alpha,beta,freq)

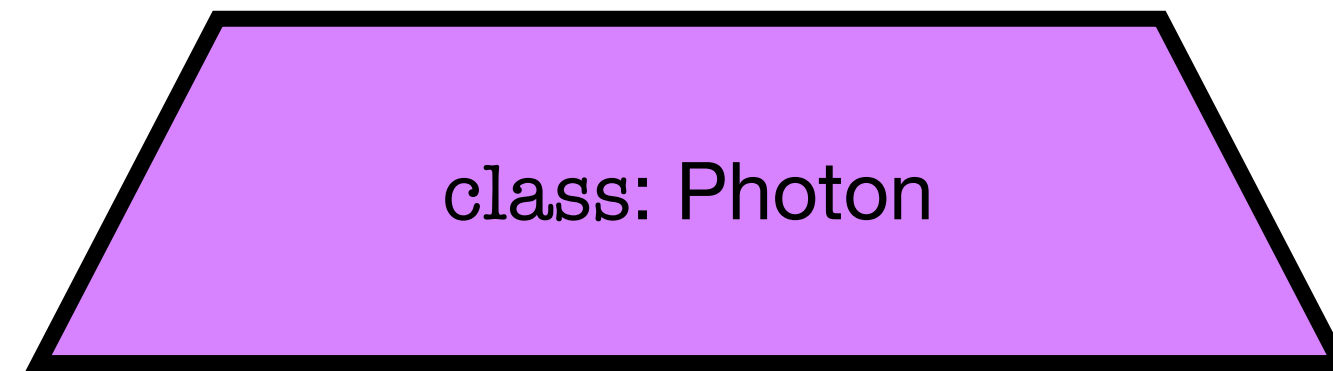
The Code [ver:0.3]



# Structure of the Code



# The Photon class



## Attributes:

`photon.alpha` `photon.beta`

`photon.xin` `photon.kin`

`photon.iC`

## Methods:

`photon.initial_conditions()`

# The image\_plane class



## Attributes:

image\_plane.D  
image\_plane.iota  
image\_plane.numPixels

## Methods:

image\_plane.photon\_coords(alpha,beta,freq)  
image\_plane.create\_photons()



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