Enlightening Mathematics Revision Book Volume 1

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Preface

The book was written in unison with the objectives of the secondary system of education. Secondary Mathematics Revision Book One has been developed to meet the objectives indicated in the new syllabus. We wrote this book with students in mind; Mathematics is pertinent to all aspects of life since it makes our life orderly and prevents chaos. Eminent qualities that are nurtured by mathematics include the Power of reasoning, creativity, abstract or spatial thinking, critical thinking, problem-solving ability, and even effective communication skills. The book is organized in an accessible style; we have introduced each topic with well-solved examples. Each topic ends with questions, and problems to solve that are related to it with their answers at the back of the book. The book is divided into four sections; the first section introduces each topic with its problems to solve, section two gives 10 model sample papers with the **K.C.S.E** format, section three articulates answers to questions in each topic while section four displays answers to model sample papers.

Introduction

Enlightening Mathematics Book 1 is a series that consists form one work only and is intended for use at form 1-4 levels for revision purposes. Although the book is written primarily for the secondary school mathematics syllabus, it can also be useful to students undertaking similar courses both within and outside Kenya. The book precisely introduces each topic in an understandable summarized manner; incorporated with elaborate examples anticipated to exhibit simple mathematical ideas that are offered as a precursor to a wide range of questions after every topic. It ensures that even weak students can simply understand the calculations and apply the skills to the problems provided after the topic. The Model Sample Papers are articulated in a **KCSE** mode

1 Chapter 1: Natural Numbers

Natural Numbers

Natural Numbers are also called **Counting numbers**. They consist of $0, 1, 2, 3, 4, 5, 6, 7, 8,$ *and* 9 digits. Place value is the position of a digit in a number. Total value is the product of the digit and its place value. A prime number is a number with only two factors that is, one and it's self. e.g $2, 3, 5, 7, 11$.

Odd numbers are numbers ending with the digits: $1, 3, 5, 7,$ *or* 9.

Even numbers are numbers ending with the digits: $0, 2, 4, 6,$ *or* 8.

1.0.1 Solved Examples

Example 1

Find the place value and the total value of digit 3 in the numbers below.

a) $47\,387\,645$ $(1mk)$

b) $2\,312\,464\,085$ $(1mk)$

c) $12\,594\,534$ $(1mk)$

Solution

a) The place value of 3 in the first number is a hundred thousand.\ Its total value is:$3 \times 100\,000 = 300\,000$

b) The place value of 3 in the second number is a hundred million. \Its total value is: $3 \times 100,000000 = 300\,000\,000$

c) The place value of 3 in the third number is tens.\ Its total value is: $3 \times 10 = 30$

Example 2

All prime numbers less than ten are arranged in descending order to form a number.

a) Write down the number formed $(1mk)$

b) What is the total value of the second digit? $(2mks)$

Solution

a) The number formed is: $7\,532$

b) Total value is as calculated below:

$$\text{Total value} = \text{place value} \times \text{the digit} \tag{1.1}$$
$$= 100 \times 5 \tag{1.2}$$
$$= 500 \tag{1.3}$$

Example 3

In a 3-digit number, the tens digit is thrice the unit digit and the hundreds digit is four times the unit digit. Also, the sum of its digits is 16. Find the number. (3mks)

Solution

Let the digits be xyz

$$y = 3z$$
$$x = 4z$$
$$x + y + z = 16$$
$$4z + 3z + z = 16$$
$$8z = 16$$

$$z = \frac{\cancel{16}^{2}}{\cancel{8}}$$
$$z = 2$$
$$y = 3(2)$$
$$= 6$$
$$x = 4(2)$$
$$= 8$$
$$\therefore Number = 862$$

Problems to solve

1. The prime numbers less than 10 are multiplied to form a number.
   a) Write down the number formed. ^space $(2mks)$
   b) State the total value of the first digit in the number formed in 2(a) above. $(1mk)$
2. All prime numbers between ten and twenty are arranged in descending order to form a number.
   a) Write down the number. $(2mk)$
   b) State the total value of the third digit of the number formed in (i) above $(1mk)$
3. All prime numbers less than 10 are arranged in a descending order to form a number which forms a quotient of 1 076 with a certain number. Calculate the number $(3mks)$
4. A two-digit number is such that the sum of the ones and the tens digit is 10. If the

digits are reversed, then the new number formed exceeds the original number by 54. Find the number. (4*mks*)

5. In a three-digit number, the hundreds digits is 4 more than the units digit and the tens digit is twice the hundreds digit. If the sum of the digits is 12, find the three digits. Write the number. (4*mks*)
6. A 3-digit number has a 4 in the hundreds place. It has a greater digit in the tens place than in the ones place. The sum of the digits is 6 which is my number. (3*mks*)
7. In a three-digit number, the hundreds digit is equal to the tens digit and is 2 more than the ones digit. The number formed by reversing the digits is 19 times the sum of the digits. Find the original number. (4*mks*)
8. The sum of the digits of a two-digit number is 15. When the number is subtracted from the number formed by reversing the digits, the difference is 27. Find the number. (4*mks*)
9. A certain two-digit number is equivalent to five times the sum of the digits. It is found to be 9 less than the number formed when the digits are interchanged. Find the number. (3*mks*)
10. The product of the digits in a two-digit number is 24. Four times the ten digit exceeds the unit digit by 10. Calculate the number. (3*mks*)

1.1 Rounding Off

The following examples explain in detail how to round off a whole number or a decimal number.

1.1.1 Solved Examples

Example 1

Round off the following numbers to the nearest number indicated in the brackets:
a) 246 852 (thousands) (1*mk*)
b) 3 442 (tens) (1*mk*)
c) 0.00897 (thousandths) (1*mk*)

Solution

a) 247 000
b) 3 440
c) 0.009

Problems to solve

1. Round off the following numbers to the nearest numbers indicated in the brackets:
   a) Thirty-seven million, six hundred and forty-seven thousand, three hundred and

forty-one. (100 000). $(1mk)$

b) 324 481 (ten thousands) $(1mk)$

c) 46.18702 (Hundredth) $(1mk)$

2. A firm was reported to have made a profit of $Ksh.\ 90,578,463$. Two daily newspaper gave the figure, one to the nearest $1,000,000$ and the other to the nearest $100,000$. Find the difference the rounded off figures? $(2mks)$
3. A number was rounded off to the nearest 1,000 and given as 150,000. Which of the following numbers was likely to have been rounded off? $(1mk)$

   a) 150,960

   b) 149,680

   c) 149,240
4. What is the difference between 14.643 rounded off to the nearest tenth and 21.247 rounded off the nearest hundredth? $(2mks)$
5. Kelvin, Grace, Ciru, and Njihia are playing a game. The winner is the person whose number is smallest when rounded to the nearest tenth. Kelvin's number is 0.355, Grace's number is 0.199, Ciru's number is 0.261, and Njihia's number is 0.959. Who is the winner? $(2mks)$

1.2 Operations

This includes addition, subtraction, multiplication and division of numbers.

1.2.1 Solved Examples

Example 1

Njoroge had $2,568$ bags of beans each weighing $90Kg.$, he sold $1,324$ of them.
a) How many kilograms of beans were left? $(2mks)$
b) If he added 632 more bags of beans, how many kilograms of beans did he end up being with? $(2mks)$

Solution

a) One bag of beans weighs 90 Kg.

$$\begin{aligned} 2,568\ bags\ weighs &= 90 \times 2,568 \\ &= 231,120\ Kg \\ 1,324\ bags\ weighs &= 90 \times 1,324 \\ &= 119,160\ Kg \\ Amount\ of\ beans\ left &= 231,120 - 119,160 \\ &= 111,960\ Kg \end{aligned}$$

Solution

b)

$$(632 \times 90) + 111,960\,Kg = 56,880 + 111,960$$
$$= 168,840\,Kg.$$

Example 2

Compute the quotient: $6120 \div 45$ $(3mks)$

Solution

$$\begin{array}{r|l} & 136 \\ \hline 45 & 6120 \\ & 4500 \\ \hline & 1620 \\ & 1350 \\ \hline & 270 \\ & 270 \\ \hline & 0 \end{array}$$

The answer is 136

Figure 1.1: Example 2

Example 3

The Amos family borrows $ 20,880 to purchase a new car at a special 0% interest rate. The car dealer allows them 5 years to pay back the amount they borrow and requires equal monthly payments. How much are their monthly payments? $(2mks)$

Solution

Since there are 12 months in each year, they must make a total of $5 \times 12 = 60$, payments on the loan. Dividing $ 20,880 by 60 will result in the monthly payment:

$$
\begin{array}{r}
348 \\
60\overline{)\,20880} \\
\underline{18000} \\
2880 \\
\underline{2400} \\
480 \\
\underline{480} \\
0
\end{array}
$$

Figure 1.2: Example 3

The Amos' monthly payment will be $ 348.

Problems to solve

1. A bus charges $Ksh.$ 150 as fare from Embu to Meru. It carries a capacity of 18 passengers. However, it can carry 5 more passengers but will have to pay a penalty of $Ksh.$ 100 at each of the 8 police checkpoints it passes through. The distance between the two towns is 91 km and the cost of petrol is $Ksh.$ 102 per litre. If the bus uses 1 litre for every 7 km, calculate;

   a) How much is gained if the bus does not overload? $(4mks)$

   b) How much is lost if the bus overloads? $(4mks)$

2. A vegetable vendor had 1,652 cabbages. He sold 835 cabbages on the first day and 326 cabbages on the second day. He added 413 cabbages to the remaining stock on the third day.

   a) How many cabbages did he have at the end? $(3mks)$

   b) If he sold all the cabbages at an average cost of $Ksh.$ 15, how much money did he collect? $(1mk)$

3. Perform the following divisions: $(6mks)$

   a) $2,668 \div 58$

   b) $867,594 \div 2,317$

   c) $0.0021 \div 14$

4. A bookshop had $29,424$ exercise books which were packed in cartons. each carton contained 24 exercise books. The mass of an empty carton was 2 Kg and 11 Kg when full.

   a) How many cartons were there? $(1mk)$

   b) What was the total mass of empty cartons? $(2mks)$

   c) What was the total mass of the books alone? $(2mks)$

5. The average mass of students in a class of 45 was 46 Kg at the beginning of the year. At the end of the that year, they had each gained 4 Kg. Calculate:

a) Their total mass of the students at the end of the year. (2mks)

b) The difference between their total mass at the beginning and at the end of the year. (2mks)

6. A matatu had 23 passengers at the beginning of the journey. Twelve passengers alighted at the first stop while 9 boarded. Six of those who boarded at the first stop alighted at the second stop and 12 got in. The matatu did not stop again up to the final destination. The charges from the starting point were Ksh. 50 up to the first stop, Ksh. 70 up to the second stop, and Ksh. 85 up to the final destination.

   a) How many passengers alighted at the final destination? (3mks)

   b) How many passengers were carried by the matatu through the journey? (2mks)

   c) How much money was collected during the trip? (5mks)

7. a) State the value of digit 7 after the operations below.

   i) 3.45×20.54 (2mks)

   ii) 0.345×2.054 (2mks)

   iii) 34.5×0.2054 (2mks)

   iv) 0.0345×2.054 (2mks)

   b) states the value of the second digit in the product 675×44.4. (2mks)

2 Chapter 2: Factors

Factors

Factors are all numbers that divide a given number without leaving a remainder.

Example of Factors

Number	Factor
12	1,2,3,4,6,12
18	1,2,3,6,9,18
32	1,2,4,8,16,32
49	1,7,49

Figure 2.1: Example

2.1 Solved examples

Example 1

Express the following numbers in terms of their prime factors
a) 150 $(2mks)$
b) 196 $(2mks)$

Solution

(a)

2	150
3	75
5	25
5	5
	1

$$150 = 2 \times 3 \times 5 \times 5$$

$$= 2 \times 3 \times 5^2$$

Figure 2.2: Example 1 (a)

(b)

2	196
2	98
7	49
7	7
	1

$$196 = 2 \times 2 \times 7 \times 7$$

$$= 2^2 \times 7^2$$

Figure 2.3: Example 1 (b)

Problems to solve

Express the following numbers in terms of their prime factors:

a) 1859 $(2mks)$
b) 105 $(2mks)$
c) 900 $(2mks)$
d) 700 $(2mks)$
e) 5929 $(2mks)$
f) 1078 $(2mks)$
g) 2057 $(2mks)$
h) 1386 $(2mks)$
i) 1573 $(2mks)$
j) 993 $(2mks)$

3 Chapter Three: Divisibility Test

Divisibility Test

3.1 Divisibility Test for 2, 3, 4, 5, 6, 8, 10, and 11

Divisibility test for 2

A number is divisible by **2** if its last digit is **even or zero** . e.g., 12, 10, and 72

Divisibility test for 3

A number is divisible by **3** if the sum of its digits is divisible by **3**.

> Example
>
> $1,275$ is divisible by 3 because the sum of the digit is a multiple of 3 that is:
> $(1+2+7+5=15)=\frac{15}{3}=5$

Divisibility test for 4

A number is divisible by **4** if its last two digits are both zero or form a number which is divisible by 4.

> Example
>
> $1,144$ is divisible by 4 because its last two digits are divisible by 4 to give 11

Divisibility test for 5

A number is divisible by 5 if its last digit is zero or 5. e.g 55, 60, 105

Divisibility test for 6

A number is divisible by 6 if it is divisible by both 2 and 3

Divisibility test for 8

A number is divisible by 8 if the number formed by its last 3 digits is divisible by 8.

Divisibility test for 9

A number is divisible by 9 if the sum of its digits is divisible by 9

Divisibility test for 10

A number is divisible by 10 if the last digit is zero.

Divisibility test for 11

A number is divisible by 11 if the sum of its **1st, 3rd, 5th, 7th, 9th**, etc. digits and the sum of the 2nd, 4th, 6th, 8th, etc. digits are equal or differ by 11 or a multiple of 11.

Problems to solve

1. In each of the following numbers without doing actual division, determine whether the first number is divisible by the second number: $(5mks)$

   a) 3409122; 6

   b) 17218; 6

   c) 11309634; ,8

   d) 515712; , 8

   e) 3501804; , 4

2. Which of the following numbers has 9 as a factor? $(2mks)$

   a) 394683

   b) 1872546

   c) 5172354

3. a) Which are the smallest numbers that can be added to the following numbers to make them divisible by 11? $(4mks)$

   i) 5,234

   ii) 36,541

   iii) 96,287

   iv) 27,992

   b) Which are the smallest numbers that can be subtracted from the following numbers to make them divisible by 11? $(2mks)$

   i) 96,287 ii) 24,535

4. Test whether 712,038 is divisible by: $(3mks)$

   i) 2

   ii) 3

   iii) 4

4 Chapter 4: G.C.D and L.C.M

Greatest Common Divisor and Least Common Divisor

4.1 Greatest Common Divisor (GCD)

GCD is also called the Highest Common Factor (HCF) or Greatest Common Factor (GCF). To find the GCF of two numbers you write down their prime factors, then select the common factors and obtain their product.

Solved Example

Find the HCF of 36 and 64. $(3mks)$

Solution

$$36 = 2 \times 2 \times 3 \times 3$$
$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$2, 2 = Common\ Factors$$
$$\therefore GCD = 4$$

A table can also be used to find GCD

	36	64
2	18	32
2	9	16

$$GCD = 2 \times 2 = 4$$

Figure 4.1: GCD Example

4.2 Least Common Multiple (LCM)

The least common multiple, or smallest common multiple, or lowest common multiple of two integers is the smallest positive integer that is divisible by the two integers.

4.3 Solved Examples

Example 1

What is the LCM of 18, 24 and 36? $(2mks)$