Enlightening Mathematics Revision Book Volume 1

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2023-10-01

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Preface

This book is aligned with the objectives of the secondary education system. **Secondary Mathematics Revision Volume 1 Book** has been developed to meet the goals outlined in the new syllabus, keeping students' needs in mind. Mathematics plays a critical role in everyday life, helping create order and prevent chaos. It nurtures key qualities such as reasoning skills, creativity, abstract and spatial thinking, critical thinking, problem-solving ability, and effective communication.

The book is organized in a clear and accessible style. Each topic begins with well-explained examples and ends with related questions and problems to solve, with answers provided at the back. The book is divided into four sections: the first introduces each topic along with practice problems; the second presents 10 model sample papers following the **K.C.S.E** format; the third provides answers to the topic-specific questions; and the fourth section includes answers to the model sample papers.

Introduction

Enlightening Mathematics Volume 1 Book is designed primarily for **Form 1** students, but is suitable for revision across **Form 1-4** levels. While it aligns with the secondary school mathematics syllabus, it can also benefit students pursuing similar courses both within and outside Kenya. Each topic is introduced in a concise and easily understandable format, accompanied by clear examples that simplify key mathematical concepts. These examples serve as a foundation for a variety of practice questions at the end of each topic. The book is crafted to ensure even students with weaker skills can grasp the calculations and apply the learned techniques to solve the provided problems. Model Sample Papers are presented in the **K.C.S.E** format for effective exam preparation

1 Chapter 1: Natural Numbers

Natural Numbers

Natural Numbers are also called **Counting numbers**. They consist of $0, 1, 2, 3, 4, 5, 6, 7, 8,$ $and\, 9$ digits. Place value is the position of a digit in a number. Total value is the product of the digit and its place value. A prime number is a number with only two factors that is, one and it's self. e.g $2, 3, 5, 7, 11$.

Odd numbers are numbers ending with the digits: $1, 3, 5, 7, or\, 9$.

Even numbers are numbers ending with the digits: $0, 2, 4, 6, or\, 8$.

1.0.1 Solved Examples

> **i Example 1**
>
> Find the place value and the total value of digit 3 in the numbers below.
>
> a) $47\,387\,645$ $(1mk)$
>
> b) $2\,312\,464\,085$ $(1mk)$
>
> c) $12\,594\,534$ $(1mk)$

> **◊ Solution**
>
> a) The place value of 3 in the first number is a hundred thousand.\ Its total value is:$3 \times 100\,000 = 300\,000$
>
> b) The place value of 3 in the second number is a hundred million. \Its total value is: $3 \times 100,000000 = 300\,000\,000$
>
> c) The place value of 3 in the third number is tens.\ Its total value is: $3 \times 10 = 30$

> **i Example 2**
>
> All prime numbers less than ten are arranged in descending order to form a number.
>
> a) Write down the number formed $(1mk)$
>
> b) What is the total value of the second digit? $(2mks)$

> **◊ Solution**
>
> a) The number formed is: $7\,532$
>
> b) Total value is as calculated below:

$$\text{Total value} = \text{place value} \times \text{the digit} \tag{1.1}$$
$$= 100 \times 5 \tag{1.2}$$
$$= 500 \tag{1.3}$$

ℹ Example 3

In a 3-digit number, the tens digit is thrice the unit digit and the hundreds digit is four times the unit digit. Also, the sum of its digits is 16. Find the number. (3mks)

🔥 Solution

Let the digits be xyz

$$y = 3z$$
$$x = 4z$$
$$x + y + z = 16 \tag{1.4}$$
$$4z + 3z + z = 16$$
$$8z = 16$$

$$z = \frac{16}{8}$$
$$z = 2$$
$$y = 3(2)$$
$$= 6 \tag{1.5}$$
$$x = 4(2)$$
$$= 8$$
$$\therefore \text{Number} = 862$$

ℹ Problems to solve

1. The prime numbers less than 10 are multiplied to form a number.

   a) Write down the number formed. $\widehat{\text{space}}$ (*2mks*)

   b) State the total value of the first digit in the number formed in 2(a) above. (*1mk*)

2. All prime numbers between ten and twenty are arranged in descending order to form a number.

   a) Write down the number. (*2mk*)

   b) State the total value of the third digit of the number formed in (i) above (*1mk*)

3. All prime numbers less than 10 are arranged in a descending order to form a number which forms a quotient of 1 076 with a certain number. Calculate the number (*3mks*)

4. A two-digit number is such that the sum of the ones and the tens digit is 10. If the

digits are reversed, then the new number formed exceeds the original number by 54. Find the number. (*4mks*)

5. In a three-digit number, the hundreds digits is 4 more than the units digit and the tens digit is twice the hundreds digit. If the sum of the digits is 12, find the three digits. Write the number. (*4mks*)

6. A 3-digit number has a 4 in the hundreds place. It has a greater digit in the tens place than in the ones place. The sum of the digits is 6 which is my number. (*3mks*)

7. In a three-digit number, the hundreds digit is equal to the tens digit and is 2 more than the ones digit. The number formed by reversing the digits is 19 times the sum of the digits. Find the original number. (*4mks*)

8. The sum of the digits of a two-digit number is 15. When the number is subtracted from the number formed by reversing the digits, the difference is 27. Find the number. (*4mks*)

9. A certain two-digit number is equivalent to five times the sum of the digits. It is found to be 9 less than the number formed when the digits are interchanged. Find the number. (*3mks*)

10. The product of the digits in a two-digit number is 24. Four times the ten digit exceeds the unit digit by 10. Calculate the number. (*3mks*)

1.1 Rounding Off

The following examples explain in detail how to round off a whole number or a decimal number.

1.1.1 Solved Examples

i Example 1

Round off the following numbers to the nearest number indicated in the brackets:
a) 246 852 (thousands) (*1mk*)
b) 3 442 (tens) (*1mk*)
c) 0.00897 (thousandths) (*1mk*)

◊ Solution

a) 247 000
b) 3 440
c) 0.009

i Problems to solve

1. Round off the following numbers to the nearest numbers indicated in the brackets:

   a) Thirty-seven million, six hundred and forty-seven thousand, three hundred and

forty-one. (100 000). (*1mk*)

b) 324 481 (ten thousands) (*1mk*)

c) 46.18702 (Hundredth)   (*1mk*)

2. A firm was reported to have made a profit of $Ksh. 90,578,463$. Two daily newspaper gave the figure, one to the nearest $1,000,000$ and the other to the nearest $100,000$. Find the difference the rounded off figures? (*2mks*)

3. A number was rounded off to the nearest 1,000 and given as 150,000. Which of the following numbers was likely to have been rounded off? (*1mk*)

   a) 150,960

   b) 149,680

   c) 149,240

4. What is the difference between 14.643 rounded off to the nearest tenth and 21.247 rounded off the nearest hundredth? (*2mks*)

5. Kelvin, Grace, Ciru, and Njihia are playing a game. The winner is the person whose number is smallest when rounded to the nearest tenth. Kelvin's number is 0.355, Grace's number is 0.199, Ciru's number is 0.261, and Njihia's number is 0.959. Who is the winner? (*2mks*)

1.2 Operations

This includes addition, subtraction, multiplication and division of numbers.

1.2.1 Solved Examples

ℹ Example 1

Njoroge had $2,568$ bags of beans each weighing $90Kg.$, he sold $1,324$ of them.
a) How many kilograms of beans were left? (*2mks*)
b) If he added 632 more bags of beans, how many kilograms of beans did he end up being with? (*2mks*)

🔥 Solution

a) One bag of beans weighs 90 Kg.

$$
\begin{aligned}
2,568 \ bags \ weighs &= 90 \times 2,568 \\
&= 231,120 \ Kg \\
1,324 \ bags \ weighs &= 90 \times 1,324 \\
&= 119,160 \ Kg \\
Amount \ of \ beans \ left &= 231,120 - 119,160 \\
&= 111,960 \ Kg
\end{aligned}
$$

> ♦ **Solution**
>
> b)
>
> $$(632 \times 90) + 111,960 \, Kg = 56,880 + 111,960$$
> $$= 168,840 \, Kg.$$

> **i Example 2**
>
> Compute the quotient: $6120 \div 45$ (*3mks*)

> ♦ **Solution**
>
>
>
> Figure 1.1: Example 2

> **i Example 3**
>
> The Amos family borrows $ 20,880 to purchase a new car at a special 0% interest rate. The car dealer allows them 5 years to pay back the amount they borrow and requires equal monthly payments. How much are their monthly payments? (*2mks*)

> ♦ **Solution**
>
> Since there are 12 months in each year, they must make a total of $5 \times 12 = 60$, payments on the loan. Dividing $ 20,880 by 60 will result in the monthly payment:

```
                  348
         60 ) 20880
             18000
             ──────
              2880
              2400
             ──────
               480
               480
              ────
                 0
```

Figure 1.2: Example 3

The Amos' monthly payment will be $ 348.

ℹ **Problems to solve**

1. A bus charges $Ksh.$ 150 as fare from Embu to Meru. It carries a capacity of 18 passengers. However, it can carry 5 more passengers but will have to pay a penalty of $Ksh.$ 100 at each of the 8 police checkpoints it passes through. The distance between the two towns is 91 km and the cost of petrol is $Ksh.$ 102 per litre. If the bus uses 1 litre for every 7 km, calculate;

   a) How much is gained if the bus does not overload? ($4mks$)

   b) How much is lost if the bus overloads? ($4mks$)

2. A vegetable vendor had 1,652 cabbages. He sold 835 cabbages on the first day and 326 cabbages on the second day. He added 413 cabbages to the remaining stock on the third day.

   a) How many cabbages did he have at the end? ($3mks$)

   b) If he sold all the cabbages at an average cost of $Ksh.$ 15, how much money did he collect? ($1mk$)

3. Perform the following divisions: ($6mks$)

   a) $2,668 \div 58$

   b) $867,594 \div 2,317$

   c) $0.0021 \div 14$

4. A bookshop had $29,424$ exercise books which were packed in cartons. each carton contained 24 exercise books. The mass of an empty carton was 2 Kg and 11 Kg when full.

   a) How many cartons were there? ($1mk$)

   b) What was the total mass of empty cartons? ($2mks$)

   c) What was the total mass of the books alone? ($2mks$)

5. The average mass of students in a class of 45 was 46 Kg at the beginning of the year. At the end of the that year, they had each gained 4 Kg. Calculate:

a) Their total mass of the students at the end of the year. (*2mks*)

b) The difference between their total mass at the beginning and at the end of the year. (*2mks*)

6. A matatu had 23 passengers at the beginning of the journey. Twelve passengers alighted at the first stop while 9 boarded. Six of those who boarded at the first stop alighted at the second stop and 12 got in. The matatu did not stop again up to the final destination. The charges from the starting point were Ksh. 50 up to the first stop, Ksh. 70 up to the second stop, and Ksh. 85 up to the final destination.

   a) How many passengers alighted at the final destination? (3mks)

   b) How many passengers were carried by the matatu through the journey? (2mks)

   c) How much money was collected during the trip? (5mks)

7. a) State the value of digit 7 after the operations below.

   i) 3.45×20.54 (2mks)

   ii) 0.345×2.054 (2mks)

   iii) 34.5×0.2054 (2mks)

   iv) 0.0345×2.054 (2mks)

   b) states the value of the second digit in the product 675×44.4. (2mks)

2 Chapter 2: Factors

Factors

Factors are all numbers that divide a given number without leaving a remainder.

> **i Example of Factors**
>
> | Number | Factor |
> |--------|--------------|
> | 12 | 1,2,3,4,6,12 |
> | 18 | 1,2,3,6,9,18 |
> | 32 | 1,2,4,8,16,32 |
> | 49 | 1,7,49 |
>
> Figure 2.1: Example

2.1 Solved examples

> **i Example 1**
>
> Express the following numbers in terms of their prime factors
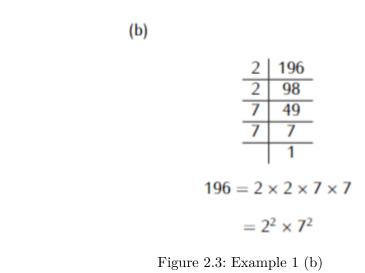> a) 150 ($2mks$)
> b) 196 ($2mks$)

> **Solution**
>
> (a)
>
> | 2 | 150 |
> |---|-----|
> | 3 | 75 |
> | 5 | 25 |
> | 5 | 5 |
> | | 1 |
>
> $$150 = 2 \times 3 \times 5 \times 5$$
>
> $$= 2 \times 3 \times 5^2$$
>
> Figure 2.2: Example 1 (a)

Factors

(b)

$$\begin{array}{c|c} 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$196 = 2 \times 2 \times 7 \times 7$$

$$= 2^2 \times 7^2$$

Figure 2.3: Example 1 (b)

i Problems to solve

Express the following numbers in terms of their prime factors:
a) 1859   (*2mks*)
b) 105   (*2mks*)
c) 900   (*2mks*)
d) 700   (*2mks*)
e) 5929   (*2mks*)
f) 1078   (*2mks*)
g) 2057   (*2mks*)
h) 1386   (*2mks*)
i) 1573   (*2mks*)
j) 993   (*2mks*)

3 Chapter Three: Divisibility Test

Divisibility Test

3.1 Divisibility Test for 2, 3, 4, 5, 6, 8, 10, and 11

Divisibility test for 2

A number is divisible by **2** if its last digit is **even or zero** . e.g., 12, 10, and 72

Divisibility test for 3

A number is divisible by **3** if the sum of its digits is divisible by **3**.

> **i** Example
>
> $1,275$ is divisible by 3 because the sum of the digit is a multiple of 3 that is:
> $(1 + 2 + 7 + 5 = 15) = \frac{15}{3} = 5$

Divisibility test for 4

A number is divisible by **4** if its last two digits are both zero or form a number which is divisible by 4.

> **i** Example
>
> $1,144$ is divisible by 4 because its last two digits are divisible by 4 to give 11

Divisibility test for 5

A number is divisible by 5 if its last digit is zero or 5. e.g 55, 60, 105

Divisibility test for 6

A number is divisible by 6 if it is divisible by both 2 and 3

Divisibility test for 8

A number is divisible by 8 if the number formed by its last 3 digits is divisible by 8.

Divisibility test for 9

A number is divisible by 9 if the sum of its digits is divisible by 9

Divisibility test for 10

A number is divisible by 10 if the last digit is zero.

Divisibility test for 11

A number is divisible by 11 if the sum of its **1st, 3rd, 5th, 7th, 9th**, etc. digits and the sum of the 2nd, 4th, 6th, 8th, etc. digits are equal or differ by 11 or a multiple of 11.

Divisibility Test

ℹ **Problems to solve**

1. In each of the following numbers without doing actual division, determine whether the first number is divisible by the second number: (*5mks*)

   a) 3409122; 6

   b) 17218; 6

   c) 11309634; ,8

   d) 515712; , 8

   e) 3501804; , 4

2. Which of the following numbers has 9 as a factor? (*2mks*)

   a) 394683

   b) 1872546

   c) 5172354

3. a) Which are the smallest numbers that can be added to the following numbers to make them divisible by 11? (*4mks*)

   i) 5,234

   ii) 36,541

   iii) 96,287

   iv) 27,992

   b) Which are the smallest numbers that can be subtracted from the following numbers to make them divisible by 11? (*2mks*)

   i) 96,287 ii) 24,535

4. Test whether 712,038 is divisible by: (*3mks*)

   i) 2

   ii) 3

   iii) 4

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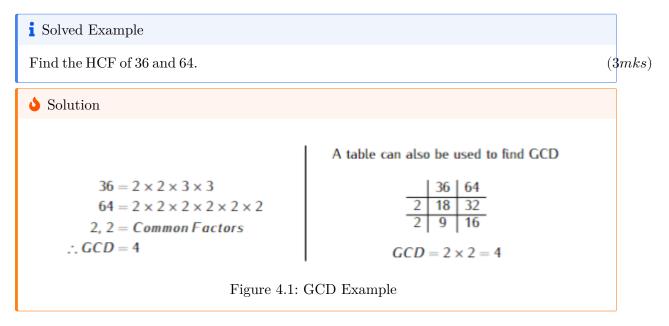
4 Chapter 4: G.C.D and L.C.M

Greatest Common Divisor and Least Common Divisor

4.1 Greatest Common Divisor (GCD)

GCD is also called the Highest Common Factor (HCF) or Greatest Common Factor (GCF). To find the GCF of two numbers you write down their prime factors, then select the common factors and obtain their product.

> **i Solved Example**
>
> Find the HCF of 36 and 64. $(3mks)$

> **🔥 Solution**
>
>
>
> $36 = 2 \times 2 \times 3 \times 3$
> $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
> $2, 2 = Common\,Factors$
> $\therefore GCD = 4$
>
> A table can also be used to find GCD
>
> | | 36 | 64 |
> |---|----|----|
> | 2 | 18 | 32 |
> | 2 | 9 | 16 |
>
> $GCD = 2 \times 2 = 4$
>
> Figure 4.1: GCD Example

4.2 Least Common Multiple (LCM)

The least common multiple, or smallest common multiple, or lowest common multiple of two integers is the smallest positive integer that is divisible by the two integers.

4.3 Solved Examples

> **i Example 1**
>
> What is the LCM of 18, 24 and 36? $(2mks)$

> **🔥 Solution**
>
>
>
> Figure 4.2: Example 1

> **ℹ Example 2**
>
> The G.C.D of two numbers is 12 and their L.C.M is 240. If one of the numbers is 60, find the other number (*3mks*)

> **🔥 Solution**
>
>
>
> Figure 4.3: Example 2

> **ℹ Problems to solve**
>
> 1. The GCD of three numbers is 30 and their LCM is 900. Two of the numbers are 60 and 150. Find the other possible numbers. (*3mks*)
>
> 2. the least common multiple of two numbers is 60 and one of the numbers is 7 less than the other. What are the numbers? (*3mks*)
>
> 3. The L.C.M of two numbers is 120 and their G.C.F is 6. One of the numbers is 30, what is the other number? (*3mks*)
>
> 4. Three bells rang at intervals of 9minutes, 15 minutes and 21minutes. The bells

will ring together at 11.00 p.m.Find the time the bells had last rang together. (*3mks*)

5. a) The difference between the GCD and the LCM of 36 and 54. (*2mks*)

   b) If three numbers 36, 54 and have a GCD of 6 and LCM of 216. Find the least value of the third number. (*2mks*)

6. The GCD and LCM of three numbers are 3 and 1,008 respectively. If two of the numbers are 48 and 72, find the least possible value of the third number. (*3mks*)

7. Three alarms ring at intervals of 40 minutes, 45 minutes, and 60 minutes. If they ring simultaneously at 6:30 a.m., at what time will they next ring together? (*3mks*)

8. Four traffic light signals are programmed at intervals of 40 seconds, 50 seconds, 60 seconds, and 75 seconds. What is the earliest they will give out light signals simultaneously if the last time they did this was at 8:15 a.m.? (*3mks*)

9. A number n is such that when it is divided by 27, 30, or 45, the remainder is 5. Find the smallest possible value of n. (*3mks*)

10. Find the greatest number which divides 181 and 170 leaving a remainder of 5. (*3mks*)

11. A square room is covered by a number of whole rectangular slabs of sides 60cm by 42 cm. Calculate the least possible area of the room in square meters. (*3mks*)

12. Three metal rods of lengths 234cm, 270cm, and 324cm were cut into shorter pieces all of the same length to make window grills. Calculate the length of the longest piece that can be cut from each of the rods and hence the total number of pieces that can be obtained from the rods. (*4mks*)

13. The GCD of two numbers is 7 and their LCM is 140. If one of the numbers is 20, find the other number. (*2mks*)

14. The GCD and LCM of three numbers are 84 and 7056 respectively. If two of the numbers are 168 and 336, find the least possible value of the third number. (*3mks*)

15. A fruit juice dealer sells the juice in a packet of 300ml, 500ml, and 750ml. Find the size of the smallest container that can fill each of the packets and leave a remainder of 200ml. (*3mks*)

16. Mr. Ombogo the principal of Chiga secondary would wish to cover the floor of the new administration block using the square tiles. The floor is a rectangle of sides 12.8m by 8.4m. Find the area of each

of the largest tiles which can be used to fit exactly without breaking. (*3mks*)

17. Three numbers, 1400, 1960, and n have a G.C.D and L.C.M of 70 and $2^2 \times 5^2 \times 7^2 \times 11$ respectively. Find the least possible value of n. (*3mks*)

18. a) Express 48 and 60 as a product of their prime factors. (*3mks*)

    b) A room of sides 48m and 60m is to be decorated using square tiles side XM. Find the greatest area of the tile. (*2mks*)

19. Three similar pieces of timber of length 240cm, 320cm, and 380cm are cut into equal pieces. Find the largest possible area of a square that can be made from any of the three pieces. (*3mks*)

5 Chapter 5: Integers

Integers

5.1 The Number Line

Integers are positive whole numbers, negative whole numbers, and zero. Integers are usually represented on the number line at equal intervals, as shown in the figure below, where each interval is equal to one unit.
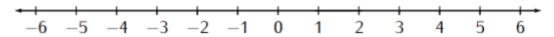

Figure 5.1: Number Line

Important properties to note while working with integers:

Multiplication and division properties of integers

- $(+) \times (-) = -$

- $(-) \times (+) = -$

- $(-) \times (-) = +$

- $(+) \times (+) = +$

- $(+) \div (-) = -$

- $(-) \div (+) = -$
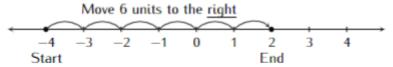
- $(-) \div (-) = +$

5.2 Solved Examples

> **i Example 1**
>
> Show how the following additions can be done using a number line and give the results: $(6mks)$
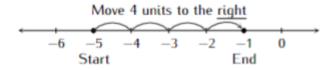> a) $(-4) + (+6)$
> b) $(-5) + (+4)$
> c) $(+2) + (-6)$

Integers

(a) The **answer is +2** as indicated by the number line below

Move 6 units to the right

(b) The **answer is -1** as shown on the number line below:

Move 4 units to the right

(c) The **answer is -4** as depicted on the number line below:

Move 6 units to the left
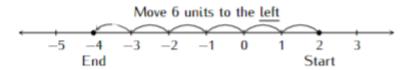
Figure 5.2: Example 1

ℹ **Example 2**

Fill in the boxes in numbers below: $(3mks)$
i) $(-3) + \square = +10$
ii) $\square + (-7) = -11$
iii) $(-4) + (-2) + \square = +3$

🔥 Solution

i) $\square = +10 + 3 = 13$
ii) $\square = -11 + 7 = -4$
iii) $\square - 4 - 2 = +3 \Rightarrow \square = 3 + 6 = +9$

ℹ **Example 3**

Without using a calculator evaluate, $(3mks)$

$$\frac{-2(4+3) - 12 \div 3 + 2}{-6 \times -3 + -2 \times 4}$$

🔥 Solution

Using BODMAS

$$\text{Numerator} = 2(4+3) - 12 \div 3 + 2$$
$$= 14 - 4 + 2$$
$$= 12$$
$$\text{Denominator} = -6 \times -3 + -2 \times 4$$
$$= 18 + -8$$
$$= 10$$
$$\therefore = \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$$

(5.1)

i Problem to Solve

1. Without using a calculator, evaluate *(3mks).*

$$\frac{(-8 + (-5) \times (-8) - (-6)}{-3 + (-8) \div 2 \times 4}$$

2. Orengo bought 1848 Mangoes on a Wednesday and sold 650 of them on the same day. On Thursday, he sold 180 more Mangoes than on Wednesday. On Friday he bought 460 more Mangoes. Later that day, he sold all the Mangoes he had at a price of Ksh. 10 each. How much money did he make? *(3mks)*

3. Evaluate: *(3mks)*

$$\frac{-12 \div (-3) \times 4 - (-20)}{-6 \times 6 \div + (-6)}$$

4. Without using tables or a calculator, evaluate *(3mks)*

$$\frac{(-2) \times 7 + (-4) \div (-3)}{3 \times (-2) + 5 \times (-4)}$$

5. Without using a calculator, evaluate (3

$$\frac{-5(-23 + 41) - (-10)}{-3 + (-8) \div 2 \times 4}$$

6. Show how the following additions can be done using a number line and give the results:

   a) $(-8) + (+5)$ *(2mks)*

   b) $(-7) + (+2)$ *(2mks)*

   c) $(-6) + (+4) + (+2)$ *(2mks)*

6 Chapter 6: Fractions

Fractions

A fraction is written in the form of $\frac{x}{y}$ where x and y are numbers and $y \neq 0$. The number on the upper side (x) is called numerator and the number on the lower side (y) is called Denominator.

There are three types of fractions:

- **Proper fractions:** These are fractions whose numerator is smaller than the denominator.

- **Improper fractions:** These are the fractions whose numerator is bigger than the denominator

- **Mixed fractions:** They are fractions written in the form of an integer and a proper fraction.

6.1 Solved Examples

> **ℹ Example 1**
>
> Evaluate: $\hspace{6cm}$ $(3mks)$
> $$\frac{8 \times \frac{1}{3} \, of \, 9 \div 2 - \frac{2}{3} \, of \, 144 \div 12 + 2 \times 3}{\frac{3}{4} \, of \, 36 \div 3 - 4 \div \frac{2}{5} \, of \, 10 + 3 \times (-2)}$$

> **🔥 Solution**
>
> **Using BODMAS**
>
> $$\begin{aligned}
Numerator :&= 8 \times \left(\frac{1}{3} \times 9\right) \div 2 - \left(\frac{2}{3} \times 144\right) \div 12 + 2 \times 3 \\
&= 8 \times (3 \div 2) - (96 \div 12) + 2 \times 3 \\
&= \left(8 \times \frac{3}{2}\right) - 8 + (2 \times 3) \\
&= 12 - 8 + 6 \\
&= 10
\end{aligned} \qquad (6.1)$$

$$Denominator := \left(\frac{3}{4} \times 36\right) \div 3 - 4 \div \left(\frac{2}{5} \times 10\right) + 3 \times (-2)$$
$$= (27 \div 3) - (4 \div 4) + 3 \times -2$$
$$= 9 - 1 + (3 \times -2)$$
$$= 9 - 1 - 6 \tag{6.2}$$
$$= 2$$
$$\therefore \frac{10}{2} = 5$$

ℹ Example 2

James withdrew some money from a bank. He spent $\frac{3}{8}$ of the money to pay for his son's school fees and $\frac{2}{5}$ to pay for his daughter's school fees. If he remained with $Ksh.12,330$, calculate the amount of money he paid for his daughter's school fees. (*3mks*)

🔥 Solution

$$Let\, his\, money\, be\, x$$
$$Son's\, school\, fees = \frac{3}{8}x$$
$$Daughter's\, school\, fees = \frac{2}{5}x \tag{6.3}$$
$$Remaining\, fraction = x - \left(\frac{3}{8}x + \frac{2}{5}x\right)$$
$$x - \left(\frac{31}{40}x\right) = \frac{9}{40}x$$

$$\frac{9}{40}x = 12330$$
$$\text{Multiply both sides by } \frac{40}{9}$$
$$\therefore x = 12330 \times \frac{40}{9} \tag{6.4}$$
$$x = 54800$$
$$\text{Daughter's school fees} = \left(\frac{2}{5}\right) \times 54800$$
$$= \text{Ksh. } 21,920$$

ℹ Example 3

In a certain church, there are 200 more women than men. One-third of the men and two-fifths of the women are elderly people. If there are 300 elderly people in the meeting, find out how many young people attend the church. (*3mks*)

> 🔥 Solution

$$Let\ men = x$$
$$women = x + 200$$
$$Elderly\ people = 300$$
$$\frac{1}{3}x + \frac{2}{5}(x + 200) = 300$$
$$\frac{1}{3}x + \frac{2}{5}x + 80 = 300$$

(6.5)

$$\frac{11}{15}x = 220$$
$$\frac{\cancel{15}}{\cancel{11}} \times \frac{\cancel{11}}{\cancel{15}}x = \cancel{220} \times \frac{\cancel{15}}{\cancel{11}}^{20}$$
$$x = 20 \times 15$$
$$= 300\,\text{men}$$

Total number of people:
$$\text{men} + \text{women} = 300 + 300 + 200$$
$$= 800$$
$$\text{young people} = 800 - 300$$
$$= 500$$

(6.6)

> ℹ️ Problems to solve

1. Three people Karimi, Omondi, and Ali contributed money to start a business. Karimi contributed a quarter of the total amount and Omondi two-fifths of the remainder. Ali's contribution was one and a half times that of Karimi. They borrowed the rest of the money from the bank which was $Ksh.\,60,000$ less than Ali's contribution, find the total amount required to start the business. (*4mks*)

2. Three people Gatungo, Martin, and Albert contributed money to purchase a flour mill. Gatungo contributed $\frac{1}{3}$ of the total amount, Martin contributed $\frac{3}{8}$ of the remaining amount and Albert contributed the rest of the money. The difference in contribution between Martin and Albert was $Ksh.\,40,000.$ Calculate the price of the flour mill. (*3mks*)

3. Agnes paid rent which was $\frac{1}{10}$ of her net salary. She used $\frac{1}{2}$ of the remaining amount to make a down payment for a plot. She gave her mother $Ksh.\,2,500$ and did shopping worth $Ksh.\,7,500$ for herself. She saved the remainder which was $Ksh.\,12,500.$ How much was the down payment that she made? (*4mks*)

4. King'oo spends one-third of his salary on food, one-quarter on rent, three–fifths of the remainder on transport, and saves the rest. If he spends $Ksh.\,1,800$ on transport, find how much money he saves. (*3mks*)

5. Without using a calculator or mathematical table evaluate: (*3mks*)

$$\frac{2\frac{1}{5} + \frac{2}{3}\,of\,3\frac{3}{4} - 4\frac{1}{6}}{1\frac{1}{4} + 2\frac{2}{5} \div 1\frac{1}{3} + 3\frac{3}{4}}$$

6. Without using a calculator, evaluate

$$\frac{\frac{3}{4} + 1\frac{5}{7} \div \frac{4}{7}\,of\,2\frac{1}{3}}{\left(1\frac{3}{7} - \frac{5}{8}\right) \times \frac{2}{3}}$$

7. Evaluate (*3mks*)

$$\frac{\frac{3}{5}\,of\,60 - 2\frac{2}{3} \times 1\frac{1}{2}}{5\frac{5}{8} \times 1\frac{7}{9} - \frac{5}{4}\,of\,4\frac{4}{5} + 2\frac{4}{5} \div \frac{7}{10}}$$

8. Two boys and a girl shared some money. The younger boy $\frac{5}{8}$ of it. The elder boy got $\frac{7}{12}$ of the remainder and the girl got the rest. Find the percentage share of the younger boy to the girl's share. (*2mks*)

9. Evaluate without using a calculator. (*3mks*)

$$\frac{\left(2\frac{3}{7} - 1\frac{5}{6}\right) \div \frac{5}{6}}{\frac{2}{3}\,of\,2\frac{1}{4} - 1\frac{1}{7}}$$

10. Evaluate: (*3mks*)

$$\frac{\sqrt{\frac{1}{4}\,of\,3\frac{1}{2}} + \frac{3}{2}\left(\frac{5}{2} - \frac{2}{3}\right)}{\frac{3}{4}\,of\,2\frac{1}{2} \div \frac{1}{4}}$$

11. Without using a calculator, evaluate: (*3mks*)

$$\frac{1\frac{4}{5}\,of\,\frac{25}{18} \div 1\frac{2}{3} \times 24}{2\frac{1}{3} - \frac{1}{4}\,of\,12 \div \frac{5}{3}}$$

12. Evaluate (*6mks*)

a)
$$\frac{5\frac{3}{5} \times 1\frac{3}{4} + 8\frac{1}{3} \div \frac{5}{9}}{5\frac{1}{6} \times 1\frac{1}{5}}$$

b)
$$\frac{8\frac{2}{5} - 3\frac{2}{3} \div 1\frac{5}{6}}{1\frac{1}{5} + 1\frac{1}{3} \times 1\frac{1}{2}}$$

13. A man spent $\frac{1}{9}$ his salary on food and $\frac{1}{4}$ the remainder on electricity and water bills. He paid fees with 20% of his salary and invested 16% of what was left on business. After taking a game drive on which he spent $Ksh.\,2,000$, he saved $Ksh.\,5,350$. Calculate his monthly earnings. (*3mks*)

14. In a mixed secondary school there are 60 more boys than girls. Half of the boys and $\frac{2}{3}$ of the girls are boarders. If there are 240 boarders, find the total number of students in the school. (*3mks*)

15. Five members of 'SILK', a self-supporting enterprise Jane, Jepchoge, Esther, Mama Charo, and Chepkoech were given a certain amount of money to share amongst themselves.Jane got $\frac{3}{8}$ of the total amount while Jepchoge got $\frac{2}{5}$ of the remainder.The remaining amount was shared equally among Esther, Mama Charo, and Chepkoech each of which received $Ksh.\,6,000$;

    a) How much was shared among the five businesswomen? (*3mks*)

    b) How much did Jepchoge get? (*2mks*)

    c) Jane, Jepchoge, and Chepkoech invested their money and earned a profit of $Ksh.\,12,000$. A third of the profit was left to maintain the business and the rest was shared according to their investments. Find how much each got. (*5mks*)

7 Chapter Seven: Decimals, Squares, and Square Roots

Decimals, Squares, and Square Roots

Decimal fraction (Decimal), is a fraction whose denominator can be written as a power of 10, e.g. $\frac{1}{10}$, $\frac{3}{100}$, and $\frac{50}{100}$.

Decimal fractions are usually written in a special way, e.g., $\frac{3}{10}$ written as 0.3. The dot in this notation is called the **decimal point**. A decimal fraction that represents the sum of a whole number and a proper fraction is called a **mixed decimal**. In division, a decimal fraction in which a digit or a group of digits repeat continuously without ending is called a **recurring decimal** e.g. $\frac{5}{11}$ = 0.4545..., in short, we place dots above the digits that recurs e.g. 0.4545...written as $0.\dot{4}\dot{5}$ A number is said to be in **standard form** if it is expressed in the form $A \times 10^n$ where $1 \leq A < 10$ and n is an integer e.g. 0.0065 is written in standard form as 6.5×10^{-3}.

Squares of numbers are tabulated and can be read from the table of squares which gives only approximate values of the square to four figures.

Square root of a number can be obtained using the factorization method or tables of square roots in a mathematical table.

7.1 Solved Exambles

> ℹ️ Example 1
>
> Without using table or calculators, evaluate:
>
> $$\sqrt{\frac{0.0032 + 0.0608}{1.44 \times 0.4}}$$
> $(3mks)$

> 🔥 Solution
>
> $$0.0032 + 0.0608 = 0.0640$$
> $$1.44 \times 0.4 = 0.576$$
> $$\Rightarrow \sqrt{\frac{0.064 \times 1000}{0.576 \times 1000}} = \sqrt{\frac{64}{576}}$$
> $$= \frac{\sqrt{2^6}}{\sqrt{2^6 \times 3^2}} \qquad (7.1)$$
> $$= \frac{2^{\cancel{3}}}{2^{\cancel{3}} \times 3}$$
> $$= \frac{1}{3}$$