

# **Enlightening Mathematics Revision Book Volume 1**

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# Preface

This book, **Enlightening Mathematics Revision Book Volume 1**, serves as a comprehensive revision guide for students, particularly at the secondary school level. It covers a range of foundational topics in mathematics, structured to support students in mastering key mathematical concepts and problem-solving techniques. The book is carefully curated to make challenging concepts easier to understand, with detailed explanations, worked examples, and practice exercises.

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**Key features of the book include:**

- **Clear Explanations:** Each chapter provides step-by-step explanations of mathematical principles, making it ideal for both self-study and classroom use.
- **Worked Examples:** The book offers numerous worked examples that demonstrate how to apply mathematical concepts to solve problems.
- **Practice Exercises:** After each topic, students can practice their skills with well-structured exercises that reinforce learning.
- **Comprehensive Coverage:** Topics such as Natural Numbers, Factors, Decimals, Algebraic Expressions, Linear Equations, and Geometry are presented in a logical progression, ensuring that students build on their knowledge progressively.
- **Solutions:** The book includes answers to problems and model sample papers, allowing students to check their work and understand mistakes.
- **User-Friendly Design:** The layout is designed to be easy to follow, with a focus on clarity, making it accessible to a wide range of learners.

This first volume in the series covers the majority of Form 1 mathematics topics, providing a strong foundation in essential mathematical principles. Future volumes will build on this groundwork, extending students' knowledge into more advanced areas of mathematics. Authored by **Martin Nyamu and Ken Gatungo**, the book is designed to equip learners with the skills and confidence they need to excel in mathematics. It serves as a valuable resource for exam preparation and ongoing study, guiding students through key concepts while supporting their long-term academic success.

# Introduction

**Enlightening Mathematics Volume 1 Book** is designed primarily for **Form 1** students, but is suitable for revision across **Form 1-4** levels. While it aligns with the secondary school mathematics syllabus, it can also benefit students pursuing similar courses both within and outside Kenya. Each topic is introduced in a concise and easily understandable format, accompanied by clear examples that simplify key mathematical concepts. These examples serve as a foundation for a variety of practice questions at the end of each topic. The book is crafted to ensure even students with weaker skills can grasp the calculations and apply the learned techniques to solve the provided problems. Model Sample Papers are presented in the **K.C.S.E** format for effective exam preparation.

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# Chapter 1: Natural Numbers

## Natural Numbers

Natural Numbers are also called **Counting numbers**. They consist of  $0, 1, 2, 3, 4, 5, 6, 7, 8,$  and  $9$  digits. Place value is the position of a digit in a number. Total value is the product of the digit and its place value. A prime number is a number with only two factors that is, one and it's self. e.g  $2, 3, 5, 7, 11$ . Odd numbers are numbers ending with the digits:  $1, 3, 5, 7,$  or  $9$ . Even numbers are numbers ending with the digits:  $0, 2, 4, 6,$  or  $8$ .

## Solved Examples

### i Example 1

Find the place value and the total value of digit 3 in the numbers below.

- a) 47 387 645 (1mk)
- b) 2 312 464 085 (1mk)
- c) 12 594 534 (1mk)

### 🔥 Solution

- a) The place value of 3 in the first number is a hundred thousand. \ Its total value is:  $3 \times 100\,000 = 300\,000$
- b) The place value of 3 in the second number is a hundred million. \ Its total value is:  $3 \times 100,\,000\,000 = 300\,000\,000$
- c) The place value of 3 in the third number is tens. \ Its total value is:  $3 \times 10 = 30$

### i Example 2

All prime numbers less than ten are arranged in descending order to form a number.

- a) Write down the number formed (1mk)
- b) What is the total value of the second digit? (2mks)

 Solution

- a) The number formed is: 7532
- b) Total value is as calculated below:

$$\text{Total value} = \text{place value} \times \text{the digit} \quad (0.1)$$

$$= 100 \times 5 \quad (0.2)$$

$$= 500 \quad (0.3)$$

 Example 3

In a 3-digit number, the tens digit is thrice the unit digit and the hundreds digit is four times the unit digit. Also, the sum of its digits is 16. Find the number. (3mks)

 Solution

Let the digits be xyz

$$\begin{aligned} y &= 3z \\ x &= 4z \\ x + y + z &= 16 \\ 4z + 3z + z &= 16 \\ 8z &= 16 \end{aligned} \quad (0.4)$$

$$\begin{aligned} z &= \frac{16}{8} \\ z &= 2 \\ y &= 3(2) \\ &= 6 \\ x &= 4(2) \\ &= 8 \\ \therefore \text{Number} &= 862 \end{aligned} \quad (0.5)$$

 Problems to solve

1. The prime numbers less than 10 are multiplied to form a number.
  - a) Write down the number formed.  $\wedge$  space (2mks)
  - b) State the total value of the first digit in the number formed in 2(a) above. (1mk)

2. All prime numbers between ten and twenty are arranged in descending order to form a number.
  - a) Write down the number. (2mk)
  - b) State the total value of the third digit of the number formed in (i) above (1mk)
3. All prime numbers less than 10 are arranged in a descending order to form a number which forms a quotient of 1 076 with a certain number. Calculate the number (3mks)
4. A two-digit number is such that the sum of the ones and the tens digit is 10. If the digits are reversed, then the new number formed exceeds the original number by 54. Find the number. (4mks)
5. In a three-digit number, the hundreds digit is 4 more than the units digit and the tens digit is twice the hundreds digit. If the sum of the digits is 12, find the three digits. Write the number. (4mks)
6. A 3-digit number has a 4 in the hundreds place. It has a greater digit in the tens place than in the ones place. The sum of the digits is 6 which is my number. (3mks)
7. In a three-digit number, the hundreds digit is equal to the tens digit and is 2 more than the ones digit. The number formed by reversing the digits is 19 times the sum of the digits. Find the original number. (4mks)
8. The sum of the digits of a two-digit number is 15. When the number is subtracted from the number formed by reversing the digits, the difference is 27. Find the number. (4mks)
9. A certain two-digit number is equivalent to five times the sum of the digits. It is found to be 9 less than the number formed when the digits are interchanged. Find the number. (3mks)
10. The product of the digits in a two-digit number is 24. Four times the ten digit exceeds the unit digit by 10. Calculate the number. (3mks)

## Rounding Off

The following examples explain in detail how to round off a whole number or a decimal number.

### Solved Examples

#### i Example 1

Round off the following numbers to the nearest number indicated in the brackets:

- a) 246 852 (thousands) (1mk)
- b) 3 442 (tens) (1mk)
- c) 0.00897 (thousandths) (1mk)

#### 🔥 Solution

- a) 247 000
- b) 3 440
- c) 0.009

#### i Problems to solve

1. Round off the following numbers to the nearest numbers indicated in the brackets:
  - a) Thirty-seven million, six hundred and forty-seven thousand, three hundred and forty-one. (100 000). (1mk)
  - b) 324 481 (ten thousands) (1mk)
  - c) 46.18702 (Hundredth) (1mk)
2. A firm was reported to have made a profit of *Ksh.* 90,578,463. Two daily newspaper gave the figure, one to the nearest 1,000,000 and the other to the nearest 100,000. Find the difference the rounded off figures? (2mks)
3. A number was rounded off to the nearest 1,000 and given as 150,000. Which of the following numbers was likely to have been rounded off? (1mk)
  - a) 150,960
  - b) 149,680
  - c) 149,240
4. What is the difference between 14.643 rounded off to the nearest tenth and 21.247 rounded off the nearest hundredth? (2mks)
5. Kelvin, Grace, Ciru, and Njihia are playing a game. The winner is the person whose number is smallest when rounded to the nearest tenth. Kelvin's number is 0.355, Grace's number is 0.199, Ciru's number is 0.261, and Njihia's number is 0.959. Who is the winner? (2mks)

# Operations

This includes addition, subtraction, multiplication and division of numbers.

## Solved Examples

### Example 1

Njoroge had 2,568 bags of beans each weighing 90Kg., he sold 1,324 of them.

- How many kilograms of beans were left? (2mks)
- If he added 632 more bags of beans, how many kilograms of beans did he end up being with? (2mks)

### Solution

- a) One bag of beans weighs 90 Kg.

$$\begin{aligned} 2,568 \text{ bags weighs} &= 90 \times 2,568 \\ &= 231,120 \text{ Kg} \end{aligned}$$

$$\begin{aligned} 1,324 \text{ bags weighs} &= 90 \times 1,324 \\ &= 119,160 \text{ Kg} \end{aligned}$$

$$\begin{aligned} \text{Amount of beans left} &= 231,120 - 119,160 \\ &= 111,960 \text{ Kg} \end{aligned}$$

b)

$$\begin{aligned} (632 \times 90) + 111,960 \text{ Kg} &= 56,880 + 111,960 \\ &= 168,840 \text{ Kg.} \end{aligned}$$

### Example 2

Compute the quotient:  $6120 \div 45$  (3mks)

 Solution

$$\begin{array}{r} 136 \\ 45 ) 6120 \\ \underline{4500} \\ 1620 \\ \underline{1350} \\ 270 \\ \underline{270} \\ 0 \end{array}$$

The answer is 136

Figure 1: Example 2

 Example 3

The Amos family borrows \$ 20,880 to purchase a new car at a special 0% interest rate. The car dealer allows them 5 years to pay back the amount they borrow and requires equal monthly payments. How much are their monthly payments? (2mks)

 Solution

Since there are 12 months in each year, they must make a total of  $5 \times 12 = 60$ , payments on the loan. Dividing \$ 20,880 by 60 will result in the monthly payment:

$$\begin{array}{r} 348 \\ 60 ) 20880 \\ \underline{18000} \\ 2880 \\ \underline{2400} \\ 480 \\ \underline{480} \\ 0 \end{array}$$

Figure 2: Example 3

The Amos' monthly payment will be \$ 348.

**i** Problems to solve

1. A bus charges *Ksh.* 150 as fare from Embu to Meru. It carries a capacity of 18 passengers. However, it can carry 5 more passengers but will have to pay a penalty of *Ksh.* 100 at each of the 8 police checkpoints it passes through. The distance between the two towns is 91 km and the cost of petrol is *Ksh.* 102 per litre. If the bus uses 1 litre for every 7 km, calculate;
  - a) How much is gained if the bus does not overload? (4mks)
  - b) How much is lost if the bus overloads? (4mks)
2. A vegetable vendor had 1,652 cabbages. He sold 835 cabbages on the first day and 326 cabbages on the second day. He added 413 cabbages to the remaining stock on the third day.
  - a) How many cabbages did he have at the end? (3mks)
  - b) If he sold all the cabbages at an average cost of *Ksh.* 15, how much money did he collect? (1mk)
3. Perform the following divisions: (6mks)
  - a)  $2,668 \div 58$
  - b)  $867,594 \div 2,317$
  - c)  $0.0021 \div 14$
4. A bookshop had 29,424 exercise books which were packed in cartons. each carton contained 24 exercise books. The mass of an empty carton was 2 Kg and 11 Kg when full.
  - a) How many cartons were there? (1mk)
  - b) What was the total mass of empty cartons? (2mks)
  - c) What was the total mass of the books alone? (2mks)
5. The average mass of students in a class of 45 was 46 Kg at the beginning of the year. At the end of the that year, they had each gained 4 Kg. Calculate:
  - a) Their total mass of the students at the end of the year. (2mks)
  - b) The difference between their total mass at the beginning and at the end of the year. (2mks)
6. A matatu had 23 passengers at the beginning of the journey. Twelve passengers alighted at the first stop while 9 boarded. Six of those who boarded at the first stop alighted at the second stop and 12 got in. The matatu did not stop again up to the final destination. The charges from the starting point were *Ksh.* 50 up to the first stop, *Ksh.* 70 up to the second stop, and *Ksh.* 85 up to the final destination.
  - a) How many passengers alighted at the final destination? (3mks)
  - b) How many passengers were carried by the matatu through the journey? (2mks)
  - c) How much money was collected during the trip? (5mks)

7. a) State the value of digit 7 after the operations below.
- i)  $3.45 \times 20.54$  (2mks)
  - ii)  $0.345 \times 2.054$  (2mks)
  - iii)  $34.5 \times 0.2054$  (2mks)
  - iv)  $0.0345 \times 2.054$  (2mks)
- b) states the value of the second digit in the product:  $675 \times 44.4$  (2 mks)

# Chapter 2: Factors

## Factors

Factors are all numbers that divide a given number without leaving a remainder.

### **i** Example of Factors

Number	Factor
12	1,2,3,4,6,12
18	1,2,3,6,9,18
32	1,2,4,8,16,32
49	1,7,49

Figure 3: Example

## Solved examples

### **i** Example 1

Express the following numbers in terms of their prime factors

- 150 (2mks)
- 196 (2mks)

### **🔥** Solution

(a)

$$\begin{array}{c|c} 2 & 150 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$150 = 2 \times 3 \times 5 \times 5$$

$$= 2 \times 3 \times 5^2$$

Figure 4: Example 1 (a)

(b)

2	196
2	98
7	49
7	7
	1

$$196 = 2 \times 2 \times 7 \times 7$$

$$= 2^2 \times 7^2$$

Figure 5: Example 1 (b)

### i Problems to solve

Express the following numbers in terms of their prime factors:

- a) 1859 (2mks)
- b) 105 (2mks)
- c) 900 (2mks)
- d) 700 (2mks)
- e) 5929 (2mks)
- f) 1078 (2mks)
- g) 2057 (2mks)
- h) 1386 (2mks)
- i) 1573 (2mks)
- j) 993 (2mks)

# Chapter Three: Divisibility Test

## Divisibility Test

### Divisibility Test for 2, 3, 4, 5, 6, 8, 10, and 11

#### Divisibility test for 2

A number is divisible by **2** if its last digit is **even or zero**. e.g., 12, 10, and 72

#### Divisibility test for 3

A number is divisible by **3** if the sum of its digits is divisible by **3**.

#### Example

1,275 is divisible by 3 because the sum of the digit is a multiple of 3 that is:  
 $(1 + 2 + 7 + 5 = 15) = \frac{15}{3} = 5$

#### Divisibility test for 4

A number is divisible by **4** if its last two digits are both zero or form a number which is divisible by 4.

#### Example

1,144 is divisible by 4 because its last two digits are divisible by 4 to give 11

#### Divisibility test for 5

A number is divisible by 5 if its last digit is zero or 5. e.g 55, 60, 105

#### Divisibility test for 6

A number is divisible by 6 if it is divisible by both 2 and 3

#### Divisibility test for 8

A number is divisible by 8 if the number formed by its last 3 digits is divisible by 8.

#### Divisibility test for 9

A number is divisible by 9 if the sum of its digits is divisible by 9

#### Divisibility test for 10

A number is divisible by 10 if the last digit is zero.

#### Divisibility test for 11

A number is divisible by 11 if the sum of its **1st, 3rd, 5th, 7th, 9th**, etc. digits and the sum of the **2nd, 4th, 6th, 8th**, etc. digits are equal or differ by 11 or a multiple of 11.

#### Problems to solve

1. In each of the following numbers without doing actual division, determine whether the first number is divisible by the second number: (5mks)

*Chapter Three: Divisibility Test*

- a) 3409122; 6
  - b) 17218; 6
  - c) 11309634; ,8
  - d) 515712; , 8
  - e) 3501804; , 4
2. Which of the following numbers has 9 as a factor? (2mks)
- a) 394683
  - b) 1872546
  - c) 5172354
3. a) Which are the smallest numbers that can be added to the following numbers to make them divisible by 11? (4mks)
- i) 5,234
  - ii) 36,541
  - iii) 96,287
  - iv) 27,992
- b) Which are the smallest numbers that can be subtracted from the following numbers to make them divisible by 11? (2mks)
- i) 96,287 ii) 24,535
4. Test whether 712,038 is divisible by: (3mks)
- i) 2
  - ii) 3
  - iii) 4

# Chapter 4: G.C.D and L.C.M

## Greatest Common Divisor and Least Common Divisor

### Greatest Common Divisor (GCD)

GCD is also called the Highest Common Factor (HCF) or Greatest Common Factor (GCF). To find the GCF of two numbers you write down their prime factors, then select the common factors and obtain their product.

#### **i** Solved Example

Find the HCF of 36 and 64. (3mks)

#### **🔥** Solution

$$\begin{aligned}36 &= 2 \times 2 \times 3 \times 3 \\64 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\2, 2 &= \text{Common Factors} \\\therefore GCD &= 4\end{aligned}$$

A table can also be used to find GCD

$$\begin{array}{c|cc|cc} & 36 & 64 \\ \hline 2 & 18 & 32 \\ \hline 2 & 9 & 16 \\ \hline & & & \end{array}$$

$GCD = 2 \times 2 = 4$

Figure 6: GCD Example

### Least Common Multiple (LCM)

The least common multiple, or smallest common multiple, or lowest common multiple of two integers is the smallest positive integer that is divisible by the two integers.

### Solved Examples

#### **i** Example 1

What is the LCM of 18, 24 and 36? (2mks)

 Solution

	18	24	36
2	9	12	18
2	9	6	9
2	9	3	9
3	3	1	3
3	1	1	1

$$LCM = 2^3 \times 3^2$$

$$= 72$$

Figure 7: Example 1

 Example 2

The G.C.D of two numbers is 12 and their L.C.M is 240. If one of the numbers is 60, find the other number (3mks)

 Solution

*Method 1*

$$G.C.D \Rightarrow 12 = 2^2 \times (3)$$

$$L.C.M \Rightarrow 240 = (2^4) \times 3 \times 5$$

$$1^{st} No. \Rightarrow 60 = 2^2 \times 3 \times 5$$

$$2^{nd} No. \Rightarrow 2^4 \times 3 = 48$$

*Method 2*

$$2^{nd} No. = \frac{G.C.D \times L.C.M}{1^{st} No.}$$

$$= \frac{12 \times 240}{60}$$

$$= 48$$

Figure 8: Example 2

 Problems to solve

1. The GCD of three numbers is 30 and their LCM is 900. Two of the numbers are 60 and 150. Find the other possible numbers. (3mks)
2. the least common multiple of two numbers is 60 and one of the numbers is 7 less than the other. What are the numbers? (3mks)
3. The L.C.M of two numbers is 120 and their G.C.F is 6. One of the numbers is 30, what is the other number? (3mks)
4. Three bells rang at intervals of 9minutes, 15 minutes and 21minutes. The bells will ring together at 11.00 p.m. Find the time the bells had last rang together. (3mks)
5. a) The difference between the GCD and the LCM of 36 and 54. (2mks)

- b) If three numbers 36, 54 and have a GCD of 6 and LCM of 216. Find the least value of the third number. (2mks)
6. The GCD and LCM of three numbers are 3 and 1,008 respectively. If two of the numbers are 48 and 72, find the least possible value of the third number. (3mks)
7. Three alarms ring at intervals of 40 minutes, 45 minutes, and 60 minutes. If they ring simultaneously at 6:30 a.m., at what time will they next ring together? (3mks)
8. Four traffic light signals are programmed at intervals of 40 seconds, 50 seconds, 60 seconds, and 75 seconds. What is the earliest they will give out light signals simultaneously if the last time they did this was at 8:15 a.m.? (3mks)
9. A number  $n$  is such that when it is divided by 27, 30, or 45, the remainder is 5. Find the smallest possible value of  $n$ . (3mks)
10. Find the greatest number which divides 181 and 170 leaving a remainder of 5. (3mks)
11. A square room is covered by a number of whole rectangular slabs of sides 60cm by 42 cm. Calculate the least possible area of the room in square meters. (3mks)
12. Three metal rods of lengths 234cm, 270cm, and 324cm were cut into shorter pieces all of the same length to make window grills. Calculate the length of the longest piece that can be cut from each of the rods and hence the total number of pieces that can be obtained from the rods. (4mks)
13. The GCD of two numbers is 7 and their LCM is 140. If one of the numbers is 20, find the other number. (2mks)
14. The GCD and LCM of three numbers are 84 and 7056 respectively. If two of the numbers are 168 and 336, find the least possible value of the third number. (3mks)
15. A fruit juice dealer sells the juice in a packet of 300ml, 500ml, and 750ml. Find the size of the smallest container that can fill each of the packets and leave a remainder of 200ml. (3mks)
16. Mr. Ombogo the principal of Chiga secondary would wish to cover the floor of the new administration block using the square tiles. The floor is a rectangle of sides 12.8m by 8.4m. Find the area of each of the largest tiles which can be used to fit exactly without breaking. (3mks)
17. Three numbers, 1400, 1960, and  $n$  have a G.C.D and L.C.M of 70 and  $2^2 \times 5^2 \times 7^2 \times 11$  respectively. Find the least possible value of  $n$ . (3mks)
18. a) Express 48 and 60 as a product of their prime factors. (3mks)  
 b) A room of sides 48m and 60m is to be decorated using square tiles side XM. Find the greatest area of the tile. (2mks)

19. Three similar pieces of timber of length 240cm, 320cm, and 380cm are cut into equal pieces. Find the largest possible area of a square that can be made from any of the three pieces. (3mks)

# Chapter 5: Integers

## Integers

### The Number Line

Integers are positive whole numbers, negative whole numbers, and zero. Integers are usually represented on the number line at equal intervals, as shown in the figure below, where each interval is equal to one unit.

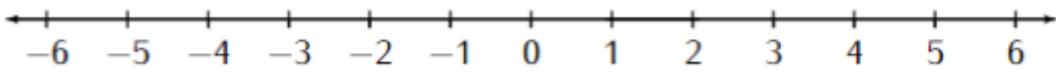


Figure 9: Number Line

Important properties to note while working with integers:

#### Multiplication and division properties of integers

- $(+) \times (-) = -$
- $(-) \times (+) = -$
- $(-) \times (-) = +$
- $(+) \times (+) = +$
- $(+) \div (-) = -$
- $(-) \div (+) = -$
- $(-) \div (-) = +$

## Solved Examples

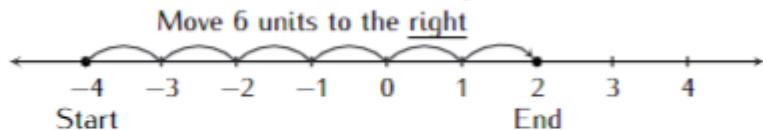
### i Example 1

Show how the following additions can be done using a number line and give the results: (6mks)

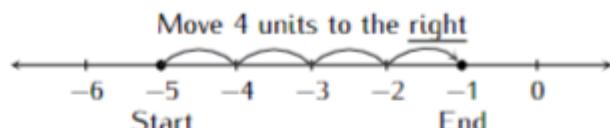
- $(-4) + (+6)$
- $(-5) + (+4)$
- $(+2) + (-6)$

### Solution

(a) The answer is +2 as indicated by the number line below



(b) The answer is -1 as shown on the number line below:



(c) The answer is -4 as depicted on the number line below:

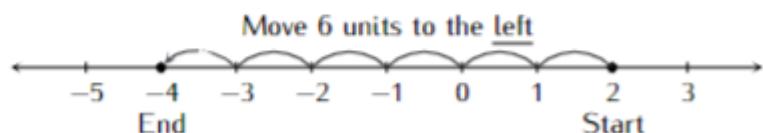


Figure 10: Example 1

### i Example 2

Fill in the boxes in numbers below: (3mks)

- i)  $(-3) + \square = +10$
- ii)  $\square + (-7) = -11$
- iii)  $(-4) + (-2) + \square = +3$

### Solution

- i)  $\square = +10 + 3 = 13$
- ii)  $\square = -11 + 7 = -4$
- iii)  $\square - 4 - 2 = +3 \Rightarrow \square = 3 + 6 = +9$

### i Example 3

Without using a calculator evaluate, (3mks)

$$\frac{-2(4+3)-12\div 3+2}{-6\times -3+-2\times 4}$$

### Solution

**Using BODMAS**

$$\begin{aligned}
 \text{Numerator} &= 2(4 + 3) - 12 \div 3 + 2 \\
 &= 14 - 4 + 2 \\
 &= 12
 \end{aligned}$$

$$\begin{aligned}
 \text{Denominator} &= -6 \times -3 + -2 \times 4 && (0.6) \\
 &= 18 + -8 \\
 &= 10 \\
 \therefore &= \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}
 \end{aligned}$$

### Problem to Solve

1. Without using a calculator, evaluate (3mks).

$$\frac{(-8 + (-5) \times (-8) - (-6))}{-3 + (-8) \div 2 \times 4}$$

2. Orengo bought 1848 Mangoes on a Wednesday and sold 650 of them on the same day. On Thursday, he sold 180 more Mangoes than on Wednesday. On Friday he bought 460 more Mangoes. Later that day, he sold all the Mangoes he had at a price of Ksh. 10 each. How much money did he make? (3mks)
3. Evaluate: (3mks)

$$\frac{-12 \div (-3) \times 4 - (-20)}{-6 \times 6 \div +(-6)}$$

4. Without using tables or a calculator, evaluate (3mks)

$$\frac{(-2) \times 7 + (-4) \div (-3)}{3 \times (-2) + 5 \times (-4)}$$

5. Without using a calculator, evaluate (3mks)

$$\frac{-5(-23 + 41) - (-10)}{-3 + (-8) \div 2 \times 4}$$

6. Show how the following additions can be done using a number line and give the results:

- a)  $(-8) + (+5)$  (2mks)
- b)  $(-7) + (+2)$  (2mks)
- c)  $(-6) + (+4) + (+2)$  (2mks)

# Chapter 6: Fractions

## Fractions

A fraction is written in the form of  $\frac{x}{y}$  where  $x$  and  $y$  are numbers and  $y \neq 0$ . The number on the upper side ( $x$ ) is called numerator and the number on the lower side ( $y$ ) is called Denominator.

There are three types of fractions:

- **Proper fractions:** These are fractions whose numerator is smaller than the denominator.
- **Improper fractions:** These are the fractions whose numerator is bigger than the denominator
- **Mixed fractions:** They are fractions written in the form of an integer and a proper fraction.

## Solved Examples

### Example 1

Evaluate: (3mks)

$$\frac{8 \times \frac{1}{3} \text{ of } 9 \div 2 - \frac{2}{3} \text{ of } 144 \div 12 + 2 \times 3}{\frac{3}{4} \text{ of } 36 \div 3 - 4 \div \frac{2}{5} \text{ of } 10 + 3 \times (-2)}$$

### Solution

#### Using BODMAS

$$\begin{aligned} \text{Numerator} &:= 8 \times \left(\frac{1}{3} \times 9\right) \div 2 - \left(\frac{2}{3} \times 144\right) \div 12 + 2 \times 3 \\ &= 8 \times (3 \div 2) - (96 \div 12) + 2 \times 3 \\ &= \left(8 \times \frac{3}{2}\right) - 8 + (2 \times 3) \\ &= 12 - 8 + 6 \\ &= 10 \end{aligned} \tag{0.7}$$

$$\begin{aligned}
 \text{Denominator} &:= \left(\frac{3}{4} \times 36\right) \div 3 - 4 \div \left(\frac{2}{5} \times 10\right) + 3 \times (-2) \\
 &= (27 \div 3) - (4 \div 4) + 3 \times -2 \\
 &= 9 - 1 + (3 \times -2) \\
 &= 9 - 1 - 6 \\
 &= 2 \\
 \therefore \frac{10}{2} &= 5
 \end{aligned} \tag{0.8}$$

**i Example 2**

James withdrew some money from a bank. He spent  $\frac{3}{8}$  of the money to pay for his son's school fees and  $\frac{2}{5}$  to pay for his daughter's school fees. If he remained with Ksh.12,330, calculate the amount of money he paid for his daughter's school fees. (3mks)

**🔥 Solution**

Let his money be  $x$

$$\begin{aligned}
 \text{Son's school fees} &= \frac{3}{8}x \\
 \text{Daughter's school fees} &= \frac{2}{5}x \\
 \text{Remaining fraction} &= x - \left(\frac{3}{8}x + \frac{2}{5}x\right) \\
 x - \left(\frac{31}{40}x\right) &= \frac{9}{40}x
 \end{aligned} \tag{0.9}$$

$$\begin{aligned}
 \frac{9}{40}x &= 12330 \\
 \text{Multiply both sides by } \frac{40}{9} & \\
 \therefore x &= 12330 \times \frac{40}{9} \\
 x &= 54800 \\
 \text{Daughter's school fees} &= \left(\frac{2}{5}\right) \times 54800 \\
 &= \text{Ksh. } 21,920
 \end{aligned} \tag{0.10}$$

**i Example 3**

In a certain church, there are 200 more women than men. One-third of the men and two-fifths of the women are elderly people. If there are 300 elderly people in the

meeting, find out how many young people attend the church. (3mks)

### Solution

$$\text{Let men} = x$$

$$\text{women} = x + 200$$

$$\text{Elderly people} = 300$$

$$\frac{1}{3}x + \frac{2}{5}(x + 200) = 300 \quad (0.11)$$

$$\frac{1}{3}x + \frac{2}{5}x + 80 = 300$$

$$\frac{11}{15}x = 220$$

$$\frac{15}{11} \times \frac{11}{15}x = 220 \times \frac{15}{11}$$

$$x = 20 \times 15$$

$$= 300 \text{ men}$$

(0.12)

Total number of people:

$$\text{men} + \text{women} = 300 + 300 + 200$$

$$= 800$$

$$\text{young people} = 800 - 300$$

$$= 500$$

### Problems to solve

- Three people Karimi, Omondi, and Ali contributed money to start a business. Karimi contributed a quarter of the total amount and Omondi two-fifths of the remainder. Ali's contribution was one and a half times that of Karimi. They borrowed the rest of the money from the bank which was *Ksh.* 60,000 less than Ali's contribution, find the total amount required to start the business. (4mks)
- Three people Gatungo, Martin, and Albert contributed money to purchase a flour mill. Gatungo contributed  $\frac{1}{3}$  of the total amount, Martin contributed  $\frac{3}{8}$  of the remaining amount and Albert contributed the rest of the money. The difference in contribution between Martin and Albert was *Ksh.* 40,000. Calculate the price of the flour mill. (3mks)
- Agnes paid rent which was  $\frac{1}{10}$  of her net salary. She used  $\frac{1}{2}$  of the remaining amount to make a down payment for a plot. She gave her mother *Ksh.* 2,500 and did shopping worth *Ksh.* 7,500 for herself. She saved the remainder which was *Ksh.* 12,500. How much was the down payment that she made? (4mks)

4. King'oo spends one-third of his salary on food, one-quarter on rent, three-fifths of the remainder on transport, and saves the rest. If he spends Ksh. 1,800 on transport, find how much money he saves. (3mks)
5. Without using a calculator or mathematical table evaluate: (3mks)

$$\frac{2\frac{1}{5} + \frac{2}{3} \text{ of } 3\frac{3}{4} - 4\frac{1}{6}}{1\frac{1}{4} + 2\frac{2}{5} \div 1\frac{1}{3} + 3\frac{3}{4}}$$

6. Without using a calculator, evaluate (3mks)

$$\frac{\frac{3}{4} + 1\frac{5}{7} \div \frac{4}{7} \text{ of } 2\frac{1}{3}}{(1\frac{3}{7} - \frac{5}{8}) \times \frac{2}{3}}$$

7. Evaluate (3mks)

$$\frac{\frac{3}{5} \text{ of } 60 - 2\frac{2}{3} \times 1\frac{1}{2}}{5\frac{5}{8} \times 1\frac{7}{9} - \frac{5}{4} \text{ of } 4\frac{4}{5} + 2\frac{4}{5} \div \frac{7}{10}}$$

8. Two boys and a girl shared some money. The younger boy  $\frac{5}{8}$  of it. The elder boy got  $\frac{7}{12}$  of the remainder and the girl got the rest. Find the percentage share of the younger boy to the girl's share. (2mks)
9. Evaluate without using a calculator. (3mks)

$$\frac{(2\frac{3}{7} - 1\frac{5}{6}) \div \frac{5}{6}}{\frac{2}{3} \text{ of } 2\frac{1}{4} - 1\frac{1}{7}}$$

10. Evaluate: (3mks)

$$\frac{\sqrt{\frac{1}{4}} \text{ of } 3\frac{1}{2} + \frac{3}{2}(\frac{5}{2} - \frac{2}{3})}{\frac{3}{4} \text{ of } 2\frac{1}{2} \div \frac{1}{4}}$$

11. Without using a calculator, evaluate: (3mks)

$$\frac{\frac{1}{5} \text{ of } \frac{25}{18} \div 1\frac{2}{3} \times 24}{2\frac{1}{3} - \frac{1}{4} \text{ of } 12 \div \frac{5}{3}}$$

12. Evaluate (6mks)

a)

$$\frac{5\frac{3}{5} \times 1\frac{3}{4} + 8\frac{1}{3} \div \frac{5}{9}}{5\frac{1}{6} \times 1\frac{1}{5}}$$

b)

$$\frac{8\frac{2}{5} - 3\frac{2}{3} \div 1\frac{5}{6}}{1\frac{1}{5} + 1\frac{1}{3} \times 1\frac{1}{2}}$$

13. A man spent  $\frac{1}{9}$  his salary on food and  $\frac{1}{4}$  the remainder on electricity and water bills. He paid fees with 20% of his salary and invested 16% of what was left on business.  
After taking a game drive on which he spent *Ksh.* 2,000, he saved *Ksh.* 5,350. Calculate his monthly earnings. (3mks)
14. In a mixed secondary school there are 60 more boys than girls. Half of the boys and  $\frac{2}{3}$  of the girls are boarders. If there are 240 boarders, find the total number of students in the school. (3mks)
15. Five members of ‘SILK’, a self-supporting enterprise Jane, Jepchoge, Esther, Mama Charo, and Chepkoech were given a certain amount of money to share amongst themselves. Jane got  $\frac{3}{8}$  of the total amount while Jepchoge got  $\frac{2}{5}$  of the remainder. The remaining amount was shared equally among Esther, Mama Charo, and Chepkoech each of which received *Ksh.* 6,000;
- How much was shared among the five businesswomen? (3mks)
  - How much did Jepchoge get? (2mks)
  - Jane, Jepchoge, and Chepkoech invested their money and earned a profit of *Ksh.* 12,000. A third of the profit was left to maintain the business and the rest was shared according to their investments. Find how much each got. (5mks)

# Chapter Seven: Decimals, Squares, and Square Roots

## Decimals, Squares, and Square Roots

**Decimal fraction (Decimal)**, is a fraction whose denominator can be written as a power of 10, e.g.  $\frac{1}{10}$ ,  $\frac{3}{100}$ , and  $\frac{50}{100}$ .

Decimal fractions are usually written in a special way, e.g.,  $\frac{3}{10}$  written as 0.3. The dot in this notation is called the **decimal point**. A decimal fraction that represents the sum of a whole number and a proper fraction is called a **mixed decimal**. In division, a decimal fraction in which a digit or a group of digits repeat continuously without ending is called a **recurring decimal** e.g.  $\frac{5}{11} = 0.\dot{4}\dot{5} = 0.4545\dots$ , in short, we place dots above the digits that recurs e.g. 0.4545...written as  $0.\dot{4}\dot{5}$ . A number is said to be in **standard form** if it is expressed in the form  $A \times 10^n$  where  $1 \leq A < 10$  and  $n$  is an integer e.g. 0.0065 is written in standard form as  $6.5 \times 10^{-3}$ .

**Squares** of numbers are tabulated and can be read from the table of squares which gives only approximate values of the square to four figures.

**Square root** of a number can be obtained using the factorization method or tables of square roots in a mathematical table.

## Solved Examples

### i Example 1

Without using table or calculators, evaluate: (3mks)

$$\sqrt{\frac{0.0032 + 0.0608}{1.44 \times 0.4}}$$

 Solution

$$\begin{aligned}
 0.0032 + 0.0608 &= 0.0640 \\
 1.44 \times 0.4 &= 0.576 \\
 \Rightarrow \sqrt{\frac{0.064 \times 1000}{0.576 \times 1000}} &= \sqrt{\frac{64}{576}} \\
 &= \frac{\sqrt{2^6}}{\sqrt{2^6 \times 3^2}} \\
 &= \frac{2^3}{2^3 \times 3} \\
 &= \frac{1}{3}
 \end{aligned} \tag{0.13}$$

 Example 2

Express as a fraction: (3mks)

$$3.2\dot{5}\dot{6}$$

 Solution

$$\begin{aligned}
 \text{Let } r &= 3.25656\dots & (i) \\
 10r &= 32.5656\dots & (ii) \\
 100r &= 325.656\dots & (iii) \\
 1000r &= 3256.56\dots & (iv)
 \end{aligned} \tag{0.14}$$

$$\begin{aligned}
 &\text{Subtracting (ii) from (iv)} \\
 990r &= 3224 \\
 r &= \frac{3224}{990} \\
 \therefore 3.2\dot{5}\dot{6} &= 3\frac{127}{495}
 \end{aligned} \tag{0.15}$$

 Example 3

Use tables of square and square roots to evaluate. (3mks)

$$\sqrt{337.5} - (3.375)^2$$

 Solution

$$\begin{aligned}
 \sqrt{337.5} &= \sqrt{3.375 \times 10^2} \\
 &= \sqrt{3.375} \times 10 \\
 &= 1.8358 \\
 &\quad + 14 \\
 \hline
 &= 1.8372 \\
 &= 1.8372 \times 10 \\
 &= 18.372
 \end{aligned}$$

$$\begin{aligned}
 (3.375)^2 &= \left[ \begin{array}{r} 11.357 \\ + 33 \\ \hline 11.390 \end{array} \right] \\
 \therefore \sqrt{337.5} - (3.375)^2 &= 18.372 - 11.390 \\
 &= 6.982
 \end{aligned}$$

Figure 11: Example 3

 Problem to Solve

1. Use the tables of squares and square roots only to find the value of: (2mks)

$$(0.0546)^{\frac{1}{2}}$$

2. Evaluate without using a calculator. (3mks)

$$\frac{23.4 - 2(5.2 + 5.3)}{3.2 \times 1.2}$$

3. Use square tables to evaluate, to 4 significant figures. (2mks)

$$(8.254)^2$$

4. Evaluate without using tables or calculator. (3mks)

$$\frac{2.5 \times \sqrt{324}}{3 \times \sqrt{729}}$$

5. Find the exact value of: (3mks)

$$2.\dot{4}\dot{1} - 0.\dot{3}\dot{2}$$

6. The number  $5.\dot{8}\dot{1}$  contains an integral part and a recurring decimal. Convert the number into an improper fraction and hence a mixed fraction. (3mks)
7. Without using a calculator or mathematical tables find the value of: (3mks)

$$\frac{0.0060 \times 2.4 \times 0.3^2}{0.9 \times 0.00015 \times 160}$$

8. Express the recurring decimal below as a fraction;  $0.\dot{3}\dot{7}\dot{2}$  leaving your answer in the form of  $\frac{a}{b}$  where  $a$  and  $b$  are integers. (3mks)

9. Use the square and square-root tables to evaluate to 4 significant figures the expression. (4mks)

$$(43.46)^2 - \sqrt{3785.4}$$

10. Using tables of square roots and reciprocals, evaluate: (3mks)

$$\sqrt{0.1964} + (0.478)^2$$

11. Convert  $0.63\dot{3}\dot{1}$  into a fraction without using a calculator. (3mks)

12. Use square and square root tables to calculate to 3 significant figures the value of: (4mks)

$$\sqrt{(0.04766)^2 - (0.00972)^2}$$

13. Use square roots and square tables to evaluate the expression: (3mks)

$$(0.005467)^2 + (0.04328)^2$$

14. The surface area of a sphere of radius,  $r$  is given by the formula  $A = 4\pi r^2$ . using square root tables, calculate the radius of the sphere whose surface area is  $120\text{cm}^2$  (3mks)

15. The periodic time for the swing of a pendulum in seconds is given by the formula,  $T = 2\pi\sqrt{\frac{l}{g}}$ . Using square root tables, calculate the value of  $T$  when  $l = 23.7\text{cm}$  and  $g = 1000\text{cms}^{-2}$ . (3mks)

# Chapter Eight: Algebraic Expressions

## Algebraic Expressions

**Algebraic expression**, is an expression built up from integer constants, variables, and algebraic operations (Addition, subtraction, division and multiplication).

## Solved problems

### i Example 1

Simplify the following:

### i Example 1 (a)

a)

$$\frac{2m - am - 2y + ay}{2m + 2y - am - ay} \quad (3\text{mks})$$

### 🔥 Solution

Factorizing the **Numerator**:

$$\begin{aligned} 2m - am - 2y + ay &= m(2 - a) - y(2 - a) \\ &= (m - y)(2 - a) \end{aligned} \quad (0.16)$$

Factorizing the **Denominator**:

$$\begin{aligned} 2m + 2y - am - ay &= 2(m + y) - a(m + y) \\ &= (2 - a)(m + y) \\ \Rightarrow \frac{(m - y)(2 - a)}{(2 - a)(m + y)} &= \frac{(m - y)}{(m + y)} \end{aligned} \quad (0.17)$$

### i Example 1 (b)

b)

$$\frac{x^2 - 4ax - 4a + x}{(x + 1)(4a^2 - ax)} \quad (3\text{mks})$$

 Solution

Factorizing the **Numerator**:

$$\begin{aligned}x^2 - 4ax - 4a + x &= x(x - 4a) + 1(x - 4a) \\&= (x + 1)(x - 4a)\end{aligned}\tag{0.18}$$

**Denominator:**

$$\begin{aligned}&(x + 1)(4a^2 - ax) \\&\Rightarrow \frac{(x+1)(x-4a)}{(x+1)(4a^2 - ax)} = \frac{x - 4a}{a(4a - x)} \\&= \frac{x - 4a}{-a(x - 4a)} \\&= \frac{-1}{a}\end{aligned}\tag{0.19}$$

 Example 1 (c)

c)

$$\frac{x - 2y}{12p} - \frac{x + 3y}{60p} \quad (3mks)$$

 Solution

$$\begin{aligned}\text{L.C.M} &= 60p \\ \frac{x - 2y}{12p} - \frac{x + 3y}{60p} &= \frac{5(x - 2y) - (x + 3y)}{60p} \\&= \frac{5x - 10y - x - 3y}{60p} \\&= \frac{4x - 13y}{60p}\end{aligned}\tag{0.20}$$

 Example 2

Given that p=4, q=-3, and r=-1 find the value of:

$$\frac{2pqr^4 + pqr - pr}{4pr - 2qr + 2pqr} \quad (3mks)$$

 Solution
**Numerator:**

$$\begin{aligned}
 2pqr^4 + pqr - pr &= 2(4)(-3)(-1)^4 + (4)(-3)(-1) - (4)(-1) \\
 &= 2(4)(-3)(1) + (4)(-3)(-1) - (4)(-1) \\
 &= -24 + 12 + 4 \\
 &= -8
 \end{aligned} \tag{0.21}$$

**Denominator:**

$$\begin{aligned}
 4pr - 2qr + 2pqr &= 4(4)(-1) - 2(-3)(-1) + 2(4)(-3)(-1) \\
 &= -16 - 6 + 24 \\
 &= 2
 \end{aligned} \tag{0.22}$$

$\therefore \frac{\cancel{8}}{2} = -4$

**i Example 3**

Simplify

$$\frac{(4a+b)^2 - (b-4a)^2}{(a+b)^2 - (b-a)^2} (4mks)$$

 Solution
**Numerator:**

$$\begin{aligned}
 (4a+b)^2 &= (4a+b)(4a+b) \\
 &= 4a(4a+b) + b(4a+b) \\
 &= 16a^2 + 4ab + 4ab + b^2 \\
 &= 16a^2 + 8ab + b^2
 \end{aligned}$$

$$\begin{aligned}
 (b-4a)^2 &= (b-4a)(b-4a) \\
 &= b(b-4a) - 4a(b-4a) \\
 &= b^2 - 4ab - 4ab + 16a^2
 \end{aligned} \tag{0.23}$$

$$\begin{aligned}
 (4a+b)^2 - (b-4a)^2 &= 16a^2 + 8ab + b^2 - (b^2 - 8ab + 16a^2) \\
 &= 16a^2 + 8ab + b^2 - b^2 + 8ab - 16a^2 \\
 &= 8ab + 8ab \\
 &= 16ab
 \end{aligned}$$

**Denominator:**

$$\begin{aligned}
 (a+b)^2 &= (a+b)(a+b) \\
 &= a(a+b) + b(a+b) \\
 &= a^2 + ab + ab + b^2 \\
 &= a^2 + 2ab + b^2
 \end{aligned}$$

$$\begin{aligned}
 (b-a)^2 &= (b-a)(b-a) \\
 &= b(b-a) - a(b-a) \\
 &= b^2 - ab - ab + a^2 \\
 &= b^2 - 2ab + a^2
 \end{aligned} \tag{0.24}$$

$$\begin{aligned}
 (a+b)^2 - (b-a)^2 &= a^2 + 2ab + b^2 - (b^2 - 2ab + a^2) \\
 &= a^2 + 2ab + b^2 - b^2 + 2ab - a^2 \\
 &= 2ab + 2ab \\
 &= 4ab
 \end{aligned}$$

$$\therefore \frac{16ab}{4ab} = 4$$

### **i** Example 4

Solve the equation

$$\frac{3}{2r} = \frac{5}{5r-1} \text{ (2mks)}$$

### **🔥** Solution

Multiply both sides by the L.C.M of the denominator

$$\begin{aligned}
 L.C.M &= 2r(5r-1) \\
 \frac{3(2r(5r-1))}{2r} &= \frac{5(2r(5r-1))}{5r-1} \\
 3(5r-1) &= 5(2r) \\
 15r - 3 &= 10r \\
 15r - 10r &= 3 \\
 \frac{5r}{5} &= \frac{3}{5} \\
 \therefore r &= \frac{3}{5}
 \end{aligned} \tag{0.25}$$

**i** Example 5

In fourteen years' time, a mother will be twice as old as her son. Four years ago, the sum of their ages was 30 years. Find how old the mother was when the son was born. (4mks)

**🔥 Solution**

14 yrs time :

$$\text{Mother} = 2x$$

$$\text{son} = x$$

Present age :

$$\text{Mother} = 2x - 14 \quad (0.26)$$

$$\text{Son} = x - 14$$

4 yrs ago :

$$\text{Mother} = 2x - 14 - 4$$

$$\text{Son} = x - 14 - 4$$

Sum of their age :

$$(2x - 18) + (x - 18) = 30$$

$$3x - 36 = 30$$

$$\begin{aligned} \frac{3x}{3} &= \frac{66}{3} \\ x &= 22, \text{years} \end{aligned} \quad (0.27)$$

$$\text{Mother} \Rightarrow (44 - 14) = 30 \text{ years now}$$

$$\text{Son} \Rightarrow (22 - 14) = 8 \text{ years now}$$

$$\begin{aligned} \text{Mother's age when giving birth} &= 30 - 8 \\ &= 22 \text{ years} \end{aligned}$$

∴ The mother was 22 years old when the son was born.

**i** Problems to Solve

1. Esther has 26 coins whose total value is Ksh. 205. There are thrice as many Ksh. 10 coins as there are Ksh. 20 coins. The rest are 50cts coins. Find the number of Ksh. 20 coins that Esther has. (3mks)
2. Joyce has 21 coins whose total value is Ksh. 72. There are twice as many five shillings coins as there are ten shillings coins. The rest are one shilling coin. Find the number of ten shillings coins that Joyce has. (3mks)

3. A rectangular plot is  $0.4m$  longer than it is wide. If its length is  $6m$  find its perimeter. When the breadth of the rectangle is reduced by  $0.5m$ , the length is increased such that the perimeter is increased by  $\frac{1}{4}$  of its original. What is the change in the length of the rectangle? (3mks)
4. Three fruit vendors Musa, Florence, and Josephine agreed to share *Ksh.* 1800 gained after a sale of the property. For every *Ksh.* 1 that Musa gets, Florence gets 50cts and for every *Ksh.* 2 that Florence gets Josephine gets *Ksh.* 3. Find Florence's share. (3mks)
5. Three-fifths of the work is done on the first day. On the second day,  $\frac{3}{4}$  of the remainder is completed. If on the third day  $\frac{7}{8}$  of what remained is done, what fraction of the work still remains to be done? (3mks)
6. Three years ago, Albina was three times as old as her son. In five years' time, the sum of their ages will be 76. Determine their present ages. (3mks)
7. Simplify (3mks)

$$\frac{(4p + 2q)^2 - (2q - 4p)^2}{(2p + q)^2 - (q - 2p)^2}$$

8. In a poultry farm, there are eight more hens than ducks, three times as many turkeys as hens, and three-quarters as many quails as turkeys.
  - a) If there are  $x$  ducks, write down a simplified expression in  $x$  for the total number of birds on the farm. (1mk)
  - b) Find the total number of birds given that there are 162 quails. (2mks)
9. Jane's mother is now four times as older than her. In eight years' time, she will be three times as old as her daughter. Find their present ages. (3mks)
10. Factorize completely 3mks)

$$(p - 2q)(3p + 2q) - (p - 2q)^2$$

11. Simplify the following expression by reducing it to a single fraction (3mks)

$$\frac{4p - 3}{3} - \frac{3p - 4}{4} = \frac{3 - p}{6}$$

12. After work a hawker had four times as many ten-shilling coins as twenty-shilling coins, eight times as many five-shilling coins as twenty-shilling coins and thrice as many one-shilling coins as ten-shilling coins. After counting his money he found that he had a total of *Ksh.* 560. Calculate the number of coins he had. (3mks)
13. Osinya is now two times as old as his younger brother and three times as old as his son. Four years from now Osinya's age will be twelve years more than the sum of the ages of his son and younger brother. Find Osinya and his son present age.(4mks)

14. Wathitha is now three times as old as Fatuma. Seven years ago Wathitha was exactly two-third times as old as Fatuma will be 14 years from now. Find Wathitha's present age. *3mks*)
15. Fadhili is now two and a half times as old as his son. Sixteen years ago his age was 7 years more than his son will be 7 years from now. Calculate Fadhili's present age. *3mks*)
16. In a church harambee meeting, there were 300 people present. Each man contributed *Ksh.* 500 and each woman *Ksh.* 300. The meeting raised a total of *Ksh.* 100,200. Calculate the number of men present in the meeting. *3mks*)
17. A group of 144 women and young girls in a certain church organized a trip to Kisumu. Each woman paid *Ksh.* 900 while each girl paid half as much. In this way the group raised a total of *Ksh.* 81,000 for the trip. Calculate the number of women in the group. *(3mks)*
18. The sum of age's three children of Awiti, Wafula, and Najala is 68 years. Wafula is three-quarter as old as Awiti and twice times as old as Najala. Determine their ages. *(3mks)*
19. Mwangi working on a coffee factory is paid *Ksh.* 40 for every normal working hour and *Ksh.* 60 for each hour worked overtime. During one week he worked for a total of 80 hours and was paid *Ksh.* 3840 in wages. Determine the number of hours he worked overtime. *(3mks)*
20. A retailer has bought a mixture of bags of beans and bags of peas from a wholesaler. One bag of beans has a mass of 90 Kg and one bag of peas has a mass of 75 Kg. the retailer bought 350 bags whose total mass is 28.5 tonnes. Find the number of bags of each type he bought. *(3mks)*
21. Given that  $x = 4$ ,  $y = -3$ , and  $z = -1$  evaluate. *(6mks)*
- a)
- $$\frac{2(x+z)^2 - (x-y)(y-z)}{3(x+y) - 2(y-z)}$$
- b)
- $$\frac{2xy^2 - 3xy^2z^2 - 3xy^2z}{x^2yz - 2xyz^2 - 4xyz}$$
22. In a certain year, Waithera had 5 times as many goats as cows and three-quarter as many sheep as goats. In the next year the number of goats increased by 24%, the number of cows decreased by 50% while the number of sheep decreased by 30%. At the end of the preceding year Waithera had a total of 1865 animals. Calculate to three significant figures the percentage decrease in the total number of animals during the preceding year. *(10mks)*
23. A family has two children whose age difference is 9. Twice the sum of their ages is equal to the age of their father.
- a) By letting the age of the younger child be  $x$ , write an expression of the:

- i) Age of the elder child. (1mk)
  - ii) Age of their father. (1mk)
  - iii) If in 19 years' time, the product of the ages of the two children is equal to 14 times the age of their father; form an equation in  $x$  and hence determine the present possible age of the younger child. (4mks)
  - iv) Determine the possible age of the elder child in 19 years' time. (2mks)
  - v) Find the possible age of the father. (2mks)
24. A group of women decided to raise *Ksh.* 480,000 to buy a piece of plot costing *Ksh.* 80,000 per hectare. Before the actual payment was made, four of the women pulled out and each of those remaining had to pay an additional *Ksh.* 20,000.
- a) If the original number of the women in the group was  $x$ , write down;
  - i) An expression of how much each was to contribute originally. (1mks)
  - ii) An expression of how the remaining number of women were to contribute after the four pulled out. (1mks)
  - b) Determine the number of women who actually contributed towards the purchase of the plot. (5mks)
  - c) Calculate the ratio of the supposed original contribution to the new contribution. (1mks)
  - d) If the plot was sub-divided equally, find the size of the plot each woman got. (2mks)

# Chapter Nine: Rate, Ratio, Proportion, and Percentage

## Rate, Ratio, Proportion, and Percentage

**Rate**, is a comparison of one quantity with another of a different kind. If a van takes three hours to travel a distance of 180 Km, then the van is traveling at an average rate of 60 Km/hr.

**Ratio**, is a way of comparing two similar quantities. If John is 7 years old and his sister Jane is 16 years old, then John's age is  $\frac{7}{16}$  of Jane's age and the ratio of their ages is 7:16. In stating the ratio the unit must be the same.

**Proportion**, is a comparison of two or more ratios. For example, if a, b, and c are three numbers such that \$ a:b:c = 3:5:7\$ then a, b, and c are said to be proportional to 3, 5, and 7 and the relationship should be integrated to mean  $\frac{a}{3} = \frac{b}{5} = \frac{c}{7}$ . In the same way,\$ a:b = 3:5\$, \$b:c = 5:7\$, and \$a:c = 3:7\$.

## Solved Examples

### Example 1

If , $a : b = 3 : 5$  and  $b : c = 7 : 9$ , Find the ratio:  $a : c$ . (3mks)

### Solution

$$a : b = 3 : 5$$

$$b : c = 7 : 9$$

$$\begin{array}{rcl} a & : & b & : & c \\ 7(3 & : & 5) & & & & \\ & & (7 & : & 9)5 \end{array}$$

$$\begin{array}{rcl} a & : & b & : & c \\ 21 & : & 35 & : & 45 \end{array}$$

$$\therefore a : c = 21 : 45$$

### Example 2

Twelve laborers each working six hours a day, take twelve days to plough a piece of land. How long would it take 24 laborers each working 9 hours a day to plough the same piece of land? (3mks)

### Solution

<i>men</i>	<i>hours</i>	<i>days</i>
12	6	12
24	9	?

Men increase in ratio 24 : 12

Days will decrease in ratio 12 : 24

Hours increase in ratio 9 : 6

Days will decrease in ratio 6 : 9

$$\text{Days needed} : \frac{12}{24} \times \frac{\frac{1}{2}}{\frac{2}{3}} \times 12 = 4$$

∴ The laborers would take 4 days

### i Example 3

A blend of juice is made from mango and passion. The cost of four limes of mango is Ksh. 180 and two limes of passion is Ksh. 160. In what ratio should the juice be mixed such that by selling the mixture at Ksh. 90 per lime, a profit of 20% is realized? (3mks)

### Solution

Juice	Mango	Passion	Blend
Ratio	1	n	1 + n
cost per litre	45	80	
Total cost	45	80n	45+80n

$$\text{Buying price}(B.P) \Rightarrow \frac{45 + 80n}{1 + n} = 100\%$$

$$\text{Selling Price}(S.P) \Rightarrow 90 = 120\%$$

$$B.P \Rightarrow \frac{90 \times 100}{120} = 75$$

$$\frac{45 + 80n}{1 + n} = 75$$

$$45 + 80n = 75 + 75n$$

$$5n = 30$$

$$\begin{aligned}\frac{5n}{5} &= \frac{30}{5}^6 \\ \frac{n}{1} &= \frac{6}{1} \\ \Rightarrow \frac{1}{n} &= \frac{1}{6}\end{aligned}$$

$$\therefore 1 : n = 1 : 6$$

### **i** Example 4

A cold water tap can fill a bath in 15 minutes while a hot water tap can fill it in 10 minutes. The drainage pipe can empty it in 8 minutes. The cold water and hot water taps are opened for 2 minutes. After two minutes all three taps are opened. Find the total time taken to fill the bath. (3mks)

### **🔥** Solution

**The 3 taps rate of filling the bath in 1 min:**

$$\begin{aligned}\text{Cold water tap} &= \frac{1}{15} \\ \text{Hot water tap} &= \frac{1}{10} \\ \text{Drainage tap} &= \frac{1}{8}\end{aligned}\quad (0.28)$$

**Fraction by hot and cold tap in 2 min:**

$$\left( \frac{1}{15} + \frac{1}{10} \right) \cdot 2 = \frac{1}{3}$$

**Fraction of water unfilled:**

$$\Rightarrow 1 - \frac{1}{3} = \frac{2}{3}$$

**Fraction of water by 3 taps in 1 min:**

$$\frac{1}{15} + \frac{1}{10} - \frac{1}{8} = \frac{1}{24}$$

**Time to fill the remaining water:**

$$\frac{2}{3} \div \frac{1}{24} \Rightarrow \frac{2}{3} \times 24^8 = 16$$

∴ Total time  $\Rightarrow 16 + 2 = 18$  min.

### Example 5

A plastic container manufacturer increased the radius of a cylindrical can by 22.5% but, decreased its height by 30%. Calculate in two decimal places the percentage increase in the volume of the can. (3mks)

### Solution

$$\text{Volume} = \pi r^2 h$$

$$\text{Old volume} = \pi r^2 h$$

$$\text{New radius} = 1.225r$$

$$\text{New height} = 0.7h \quad (0.29)$$

$$\begin{aligned}\text{New volume} &= \pi(1.225r)^2(0.7h) \\ &= 1.225^2 \times 0.7 \times r^2 \times h \\ &\approx 1.05044\pi r^2 h\end{aligned}$$

$$\begin{aligned}\text{Volume increase} &= 1.05044\pi r^2 h - \pi r^2 h \\ &= 0.05044\pi r^2 h\end{aligned}$$

$$\begin{aligned}\text{Percentage increase} &= \frac{\text{Increase}}{\text{Old volume}} \times 100 \\ &= \frac{0.05044\cancel{\pi r^2 h}}{\cancel{\pi r^2 h}} \times 100 \\ &\approx 5.044\% \\ \therefore \% \text{ increase} &= 5.04\%\end{aligned} \quad (0.30)$$

### Problems to Solve

1. A photograph is reduced in the ratio 2 : 7 for a newspaper, and further reduced in the ratio 5 : 7 for an exercise book. Find the ratio of the newspaper size to textbook size. (3mks)
2. Five plows working 8 hours daily complete a piece of work in 6 days. How long will it take 12 plows working 5 hours a day to complete the same work? (2mks)
3. There are two grades of beans, grade A and grade B. Grade A costs Ksh. 100 per kg and grade B costs Ksh. 80 per kg. In what ratio must the two grades be mixed in order to produce a blend worth Ksh. 95 per kg? (3mks)

4. A tradesman blends 340kg of tea costing *Ksh.*80 per kg with 160kg of tea costing *Ksh.*100 per kg. At what price must he sell the mixture, to make a 25% profit? (3mks)
5. Oil flows through a pipe whose cross-sectional radius is 7cm at a rate of 2m/min. Calculate how long it will take the pipe to fill a 28,000 litres tank. (3mks)
6. Nyamu and Gatungo working together can do a piece of work in 6 days. Gatungo working alone would take 10 days to complete the work. They start working together but, after 4 days Gatungo leaves and the remaining work is done by Nyamu. Find how long Nyamu takes to complete the remaining work. (4mks)
7. Five constructors can build a 25-meter-long wall in 10 days. What length of wall can 10 constructors working at the same rate build in 8 days? (3mks)
8. A businesswoman bought 160 mangoes at *Ksh.*50 for every four mangoes. She sold some of them at *Ksh.*30 for every three and the rest at *Ksh.*30 for every four. If she made a  $33\frac{1}{2}\%$  loss, calculate the number of mangoes sold at *Ksh.*30 for every four. (3mks)
9. One hundred and twenty examiners each marking 90 papers per day are needed to mark an examination in 2 weeks. How many days would 180 examiners each marking 35 papers per day take to mark the same examination? (3mks)
10. A group of 15 soldiers set off with enough food to last 6 days. After 6 soldiers evacuated. How many more days will the food last for the remaining soldiers? (3mks)
11. In the Moi University Christian Union choir, the ratio of male to female is 2 : 3. On one Sunday service, 10 male members were absent and six new female members joined the choir as guests for that day. If on this day the ratio of males to females was \$ 1:3\$, how many regular members does the choir have? (3mks)
12. The ratio of men to women in Njega Boys High School B.O.G which consists of 45 members is 7 : 2. Find the number of women required to join the existing members so that the ratio of men to women changes to 5 : 4. (3mks)
13. A coffee trader mixes two brands of coffee, A and B to obtain 40kg of the mixture worth *Ksh.*2,600. If brand A is valued at *Ksh.*70 per kg and brand B is valued at *Ksh.*55 per kg. Calculate the ratio in its simplest form in which brands A and B are mixed. (4mks)
14. Nyamu bought sorghum and millet at *Ksh.*65 per kg and *Ksh.*40 per kg respectively. He then mixed them and sold the mixture at *Ksh.*60 per kg making a profit of 20%. Determine the ratio of sorghum to millet in the mixture. (3mks)

15. The ratio of a spherical balloon increases by 4% as it rises up in the air. Find the percentage increase in its;
  - a) Surface area. (2mks)
  - b) Volume (2mks)
16. A dealer has two types of grades of tea, x and y. Grade x costs *Ksh.* 150 per kg and grade y costs *Ksh.* 170 per kg. If he mixes  $x$  and  $y$  in the ratio 2 : 3 to make a brand of tea which he sells at *Ksh.* 180 per kg, calculate the percentage profit that he makes. (3mks)
17. In what ratio will coffee grade A cost *Ksh.* 85 per kg to be mixed with grade B costing *Ksh.* 55 per kg so that a profit of 25% is realized by selling the mixture at *ksh.* 80 per kg? (3mks)
18. A mixture contains two powders A and B with masses in the ration 4 : 10. If the mixture costs *Ksh.* 650 per kg and powder A costs *Ksh.* 550 per kg, find the cost of a kg of powder B. (3mks)
19. It would take 20 workers 12 days to spray a piece of land. If they work for 8 hours a day, how long will it take 24 workers if they work 12 hours a day to three-quarters of the same land? (3mks)
20. A farmer has enough feed to last 35 pigs for 24 days. If he buys 5 more pigs, how long will the feed last? (3mks)
21. Mukami, a juice blender mixes two brands of Juice A and B to obtain 90ml of the mixture worth *Ksh.* 165 per litre. If brand A is valued at *Ksh.* 175 per 1 litre bottle and brand B at *Ksh.* 150 per 1-litre bottle, calculate the ratio in which the bands A and B are mixed. (2mks)
22. Twelve men can build 6 huts in 21 days. Find the number of men working at the same rate that will build 9 similar huts in 14 days. (3mks)
23. A rectangular dam with a surface area of 24 ares has a uniform depth of 5 m is to be drained for renovation. A pipe drains it at the rate of 250 litres per second. How long does it take to empty the dam? (2mks)
24. Tap A can fill a tank in 12 minutes while tap B can fill the same tank in 15 minutes. Another tap C can empty the tank when full in 20 minutes. Starting with an empty tank, the three taps are left open for 4 minutes after which tap A is closed. How much longer does it take to fill the tank? (3mks)
25. The radius of a cylindrical tin is increased by 20% while its height is decreased by 10%. If the capacity of the old tin is  $250 \text{ cm}^3$ , find the capacity of the new tin. (3mks)
26. The radius of a cylindrical container is increased by 28% while its height is reduced by 15%. In 4 significant figures find the percentage increase in the volume of the juice in the container. (3mks)

27. Pipe X and Y can fill a tank in 15 minutes and 30 minutes respectively. Pipe Z can empty the full tank in 25 minutes. Starting with an empty tank, how long does it take to fill the tank if:
- All the three pipes are open? (1mk)
  - Pipe Y is closed after 10 minutes? (3mks)
28.  $2,280 \text{ cm}^3$ , of milk was shared by three children, Josephine, Florence, and Moses in the ratio  $\frac{1}{4} : \frac{1}{2} : \frac{1}{5}$ , What volume did Moses get: (2mks)
29. Sewage is flowing through a cylindrical pipe at the speed of  $0.95 \text{ m/s}$ . If the pipe has an internal radius of 14cm, Calculate:
- The volume of sewage delivered by the pipe per second in  $\text{cm}^3$  (Take =  $\frac{22}{7}$ ) (2mks)
  - The depth to which the pipe fills a rectangular tank of base dimensions  $6.5\text{m} \times 5.2\text{m}$  in one hour to the nearest 0.1 metres. (3mks)
  - The time is taken, to the nearest second for the pipe to fill a 50,000-litre tank tub (initially empty) which has a hole at the base that drains the tub at the rate of 524 litres per minute. (5mks)
30. Three potters; A, B, and C work together to make a certain number of pots. If Potter C was to work alone he would take  $4\frac{4}{9}$ , hours to complete the job. If all working together they will take 1hr 40min to complete the job. They all started working together however, potter B left after the first 40 minutes, while Potter C left 20min later. Potter A took a further 1hr 46min. Calculate how long it would take if all the potters were made by:
- Potter A alone? (6mks)
  - Potter B alone? (2mks)
  - Potter A and C alone? (2mks)
31. Boniface purchased 3 brands of coffee A, B, and C. The cost prices of the brands were Ksh. 50, Ksh. 68 and Ksh. 75 per kilogram respectively. He mixed the brands in the ratio of  $7 : 5 : 3$  respectively. After selling the mixture, he made a profit of 32%.
- How much profit did he make per kilogram of the mixture? (4mks)
  - After one year, the cost price of each brand was increased by 15%.
  - i) For how much did he sell one kilogram of the mixture to make 20% Profit. (3mks)
  - ii) What would have been his percentage profit if he sold one kilogram of the mixture at Ksh. 85.60? (3mks)
32. A solution whose volume is 160 litres is made up of 75% milk and the rest water. When  $x$  litres of milk is added the percentage of water drops to 20%
- Find the value of  $x$  (4mks)

- b) The new solution is diluted further by the addition of 120 litres of water. Calculate the percentage of milk in the resulting solution. (2mks)
- c) A blend is made by mixing 10 litres of the solution in (b) above with 20 liters of the original solution. Calculate in the simplest form, the ratio of water to that of milk in the blend. (4mks)
33. Four hundred and twenty litres of homogeneous paint is made by mixing three paints P, Q, and R. The ratio by volume of paint P to point Q is 3 : 4 and paint Q to paint R is 1 : 2. Paint P costs *Ksh.* 150 per litre, paint Q *Ksh.* 180 per litre and paint R *Ksh.* 120.50 per litre. Determine:
- The volume of each type of paint in the mixture. (5mks)
  - The amount of money spent in making one litre of the mixture. (3mks)
  - The percentage profit made by selling the mixture at *Ksh.* 205 per litre. (2mks)
34. Olemapenzi's cows decreased by 16% in 2014 to stand at 2100 cows at the beginning of 2015. The number of cows increased by 24% in 2015 and also increased by 20% in 2016.
- Determine the number of cows Olemapenzi had at the beginning of:
    - The year 2014 (2mks)
    - The year 2016 (2mks)
    - The year 2017 (2mks)
  - Determine the percentage increase in Olemapenzi's cows between:
    - 2014 and 2016 (2mks)
    - 2014 and 2017 (2mks)

# Chapter Ten: Length, Area, Volume, and Capacity

## Length, Area, Volume, and Capacity

**Length**, is the distance between two points. Its SI Unit is the meter (m). The perimeter of a rectangle and a square is given by the formula:

$$\text{Perimeter of a rectangle} = 2(\text{length} + \text{width})$$

$$\text{Perimeter of a square} = 4 \times \text{length}$$

The circumference of a circle is given by the formula:

$$\text{Volume} = \pi r^2 h \quad (0.31)$$

$$\text{Old volume} = \pi r^2 h \quad (0.32)$$

$$\text{New radius} = 1.225r \quad (0.33)$$

$$\text{New height} = 0.7h \quad (0.34)$$

$$\text{New volume} = \pi(1.225r)^2(0.7h) \quad (0.35)$$

$$= 1.225^2 \times 0.7 \times r^2 \times h \quad (0.36)$$

$$\approx 1.05044\pi r^2 h \quad (0.37)$$

The length of an arc ( $l$ ) of a circle subtended by an angle,  $\theta$ , at the center of the circle is given by the formula:

$$l = \frac{\theta}{360} \times 2\pi r$$

**Area**, is the amount of surface enclosed within the boundaries of a plane shape. Its SI unit is a square meter ( $m^2$ ).

**Area of a rectangle (A):**

$$A = \text{length}(l) \times \text{width}(w) = lw$$

**Area of square (A):**

$$A = \text{length} \times \text{length} = l^2$$

**Area of a triangle (A):**

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

**Area of a parallelogram (A):**

$$A = \text{base} \times \text{height}$$

**Area of a rhombus (A):**

$$A = \text{base} \times \text{height}$$

OR

$$A = \frac{1\text{st diagonal} \times 2\text{nd diagonal}}{2}$$

**Area of a trapezium (A):**

$$A = \frac{1}{2} \times \text{height} \times (\text{sum of the two parallel sides}) = \frac{1}{2} \times h \times (a + b)$$

**Area of a circle (A):**

$$A = \pi r^2$$

**Area of a sector subtended by an angle  $\theta$  at the center of a circle:**

$$A = \frac{\theta}{360} \times \pi r^2$$

**Surface area of solids (S.A)**

**Surface area of a cuboid:**

$$\begin{aligned} S.A &= 2(\text{length} \times \text{width}) + 2(\text{width} \times \text{height}) + 2(\text{length} \times \text{height}) \\ &= 2(lw) + 2(wh) + 2(lh) \\ &= 2(lw + wh + lh) \end{aligned}$$

**Surface area of a cylinder:**

$$\begin{aligned} S.A &= 2\pi rh + 2\pi r^2 \\ &= 2\pi r(h + r) \end{aligned}$$

**Surface area of a sphere:**

$$S.A = 4\pi r^2$$

**Volume**, is the amount of space occupied by a matter. Its SI unit is a cubic meter ( $m^3$ ).

**Volume of a cuboid:**

$$\begin{aligned} v &= \text{length} \times \text{width} \times \text{height} \\ &= lwh \end{aligned}$$

**Volume of a cube:**

$$\begin{aligned} v &= \text{length} \times \text{length} \times \text{length} \\ &= l^3 \end{aligned}$$

**Volume of a cylinder:**

$$v = \pi r^2 h$$

**Volume of a sphere:**

$$v = \frac{4}{3} \pi r^3$$

**Volume of a prism:**

$$\begin{aligned} v &= \text{cross-sectional area} \times \text{length} \\ &= \frac{1}{2} \times \text{base} \times \text{height} \times \text{length} \\ &= \frac{1}{2} \times bhl \end{aligned}$$

**Capacity**, is the ability of the container to hold fluids. Its SI unit is the litre (l).

### i Conversion of Units

$$\begin{aligned} 1m &= 100cm \\ 1cm &= 10mm \\ 1000cm^3 &= 1 \text{ litre} = 1m^3 \\ 1cm^3 &= 1 ml \\ 10000m^2 &= 1 \text{ hectare}(ha) \\ 100m^2 &= 1 acre \end{aligned}$$

## Solved Examples

### i Example 1

An angle of 0.9 radians at the Centre of the circle subtends an arc of length 28.8cm. Find in 4 significant figures ( $\pi \text{radians} = 180^\circ$ ):

- The radius of the circle (3mks)
- The area of the sector enclosed by the arc and radii. (2mks)

 Solution

The angle of the arc =  $0.9\pi$  radians

Given that:  $\pi \text{ rad} = 180^\circ$

The length of the arc (l) is given by:

a)

$$\begin{aligned}\frac{\theta}{360} \times 2\pi r &= \text{length of the arc} \\ 28.8\text{cm} &= \frac{162^\circ}{\pi} \times \frac{1}{360} \cancel{\times 180^\circ} 2\pi r \\ \frac{162^\circ}{180^\circ} r &= 28.8\text{cm} \\ r &= 28.8\text{cm} \times \frac{180^\circ}{162^\circ} \\ \therefore r &= 32\end{aligned}\tag{0.38}$$

b)

$$\begin{aligned}\text{Area} &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{162^\circ}{360^\circ \pi} \times \pi \times 32 \times 32 \\ &= \frac{162^\circ}{360^\circ} \times 32 \times 32 \\ &= 460.8\text{cm}^2\end{aligned}\tag{0.39}$$

 Example 2

A solid metal cylinder with a radius of 7cm and height of 5cm is melted down and recast into a spherical ball. Calculate to 1 decimal place the surface area of this ball.  
(4mks)

 Solution

Radius (R) of the cylinder:  $R = 7\text{cm}$

Height (h) of the cylinder:  $h = 5\text{cm}$

Radius (r) of the sphere:  $r = ?$

Surface Area (S.A) of the Sphere:  $S.A = 4\pi r^2$

*Volume of Cylinder = volume of Sphere*

$$\pi R^2 h = \frac{4}{3} \pi r^3$$

$$7^2 \times 5 = \frac{4}{3} r^3 \quad (0.40)$$

$$r^3 = 7 \times 7 \times 5 \times \frac{3}{4}$$

$$r = \sqrt[3]{183.75}$$

$$\therefore r \approx 5.685 \text{ cm}$$

*S.A of the sphere =  $4\pi r^2$*

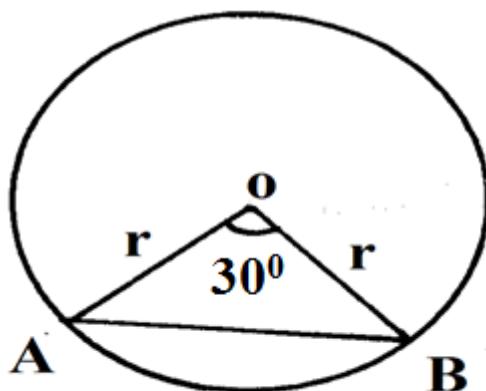
$$= 4 \times \frac{22}{7} \times 5.685^2 \quad (0.41)$$

$$= 406.299$$

$$\approx 406.3 \text{ cm}^2$$

### i Problems to Solve

1. An arc of length 'x' cm subtends an angle of,  $(\frac{p}{\pi})^0$ , at the center of the circle. Find an expression for the radius, r, of the circle in terms of x and p. (3mks)
2. Find the length of the minute hand of a wall clock if the tip of the minute hand traces a length of  $6p$  cm between 11:25 am and 11:45 am (Give your answer in terms of  $\pi$ ). (3mks)
3. The diagonal of a rectangular flower garden is 20m. If the width of this garden is 12m, calculate its length and perimeter to 3 significant figures. (3mks)
4. The figure below shows a circle Centre O. Chord AB subtends  $30^0$  at the Centre. If the area of the minor segment is  $5\frac{16}{21} \text{ cm}^2$ , find the radius of the circle. (hint area of  $\triangle OAB = \frac{1}{2}r^2 \sin(\theta)$ ) (3mks)



5. The curved surface area of a cylindrical container is  $2,112\text{ cm}^2$ . If the radius of the container is 21cm, calculate to one decimal place the capacity of the container in litres (3mks)
6. The base of a triangle is 3cm longer than its height. Given that the area of the triangle is  $35\text{ cm}^2$ , determine the base of the triangle. (3mks)
7. A carpenter constructed a closed wooden box with internal measurements 1.8 m long, 0.8 m wide, and 0.6 m high. The wood used in constructing the box was 1.0cm thick and had a density of  $0.75\text{ g/cm}^3$ . Determine the:
  - a) Volume in  $\text{cm}^3$  of the wood used in constructing the box. (3mks)
  - b) Mass of the box in kilograms correct to 4 significant figures. (1mk)
8. A solid hemisphere of radius 7cm has the same volume as a cube. Find the length of the cube to 3 significant figures. (3mks)
9. Mango juice in a factory is stored in a rectangular tank whose internal dimensions are 1.8m by 1.4m by 2.5m one day the tank was 80% full of Mango juice. Calculate the volume of Mango juice in the tank in litres. (3mks)
10. An open box has an external breadth of 12 cm, a height 10 cm, and a length of 15 cm. If the thickness of the material of the box is 1 cm, find the total surface area of the box. (5mks)
11. A triangular plot PQR is such that the length of the side PQ is two-thirds that of QR. The ratio of the lengths  $PQ : PR = 4 : 9$  and the angle at Q is obtuse. If the perimeter of the plot is 38m calculate the length of the 3 sides of  $\triangle PQR$  (4mks)
12. A metallic cuboid 10cm by 12cm by 15cm is melted. Half of it is used to make a cylinder of radius 4.2cm, the remaining is used to make a sphere. Determine in 4 significant figures using  $\pi = \frac{22}{7}$ 
  - a) The height and surface area of the cylinder to 2 decimal places. (4mks)
  - b) The radius and surface area of the sphere are correct to 3 significant figures. (4mks)
  - c) The difference between the surface area of the sphere and the cylinder. (2mks)

# Chapter Eleven: Mass, Weight and Density

## Mass, Weight and Density

**Mass**, is the quantity of matter in an object? The Mass of an object is constant everywhere. Its SI unit is Kilogram (K.g).

**Weight**, is the pull of the earth on an object. The weight of an object varies from one place to another on the earth's surface. Its SI unit is Newton (N). It is given by mass multiplied by the gravitational force (g).

$$Weight(N) = mass(kg) \times gravity(N/kg)$$

**Density**, is mass per unit volume. Its SI unit is Kilogram per Cubic meter ( $kgm^{-3}$ )

$$Density = \frac{mass}{volume}$$

$$Density\ of\ mixture = \frac{mass\ of\ mixture}{volume\ of\ mixture}$$

### i Conversion of Units

$$1\ tonne = 1,000\ Kilograms$$

$$1\ kilogram = 1,000\ grams \quad (0.42)$$

$$1g/cm^3 = 1,000Kg/m^3$$

## Solved Examples

### i Example 1

2.5 litres of water of density  $1g/cm^3$  is added to 4 litres of alcohol of density  $0.8g/cm^3$ . Calculate in 3 significant figures, the density of the mixture in its SI unit. (3mks)

### 🔥 Solution

$$Density\ of\ Mixture(D.M) = \frac{Mass\ of\ mixture}{Volume\ of\ mixture}$$

$$\text{Volume of water} = 2,500 \text{ cm}^3$$

$$\begin{aligned}\text{Mass of water} &= 2,500 \text{ cm}^3 \times 1 \text{ g/cm}^3 \\ &= 2,500 \text{ g}\end{aligned}$$

$$\text{Volume of alcohol} = 4,000 \text{ cm}^3$$

$$\begin{aligned}\text{Mass of alcohol} &= 4,000 \text{ cm}^3 \times 0.8 \text{ g/cm}^3 \\ &= 3,200 \text{ g}\end{aligned}$$

$$\begin{aligned}\text{Mass of Mixture} &= 2,500 \text{ g} + 3,200 \text{ g} \\ &= 5,700 \text{ g}\end{aligned}$$

$$\begin{aligned}\text{Volume of mixture} &= 2,500 \text{ cm}^3 + 4,000 \text{ cm}^3 \\ &= 6,500 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{Density (D.M.)} &= \frac{5,700}{6,500} \text{ g/cm}^3 \\ &= 0.8769 \text{ g/cm}^3 \\ &= 877 \text{ Kg/m}^3\end{aligned}$$

### Example 2

Two coils of the same mass are made by winding zinc of different gauges and lengths. The first coil is made by winding 648 m of wire with a cross-sectional diameter of 5.4mm and the second coil is made by winding 450 m with different cross-sections. Find the cross-sectional radius of the second coil. (3mks)

### Solution

**Note:** Since mass is proportional to volume for the same density, the volume of the first coil is equal to volume of the second coil.

**First coil volume:**

$$\frac{22}{7} \times \left(\frac{5.4}{2}\right)^2 \times 648,000 = 14,846,605.71 \text{ mm}^3$$

**Second coil volume:**

$$\frac{22}{7} \times r^2 \times 450,000 = 1,414,285.714r^2 \text{ mm}^3$$

**Equating second coil volume to first coil volume:**

$$1,414,285.714r^2 = 14,846,605.71 \text{ mm}^3$$

$$\frac{\cancel{1,414,285.714}r^2}{\cancel{1,414,285.714}} = \frac{\cancel{14,846,605.71}}{\cancel{1,414,285.714}}^{10.4976}$$

$$r^2 = 10.4976 \text{ mm}^2$$

$$\therefore r = 3.24 \text{ mm}$$

### i Example 3

The external measurements of a closed metal box are 1.6 m long, 0.7 m wide, and 0.4 m high. The metal used in making box is 1.0 cm thick and has a density of 0.85 g/cm<sup>3</sup>. If the box contains 40 packets of 12 similar tools each and each tool has a mass of 115 g, calculate:

- The volume of metal used in making the box. (4mks)
- The mass of the empty box in Kilograms to 3 significant figures. (3mks)
- The total mass of the box in kilograms to 3 significant figures. (3mks)

### 🔥 Solution

a)

**Volume of the Cuboid (v):**

$$v = \text{Length} \times \text{Width} \times \text{Height}$$

**External Measurement:**

$$\text{Length} = 160 \text{ cm}$$

$$\text{Width} = 70 \text{ cm}$$

$$\text{Height} = 40 \text{ cm}$$

**Internal Measurement:**

$$\text{Thickness} = 1 \text{ cm}$$

$$\text{Internal Length} = 158 \text{ cm}$$

$$\text{Internal Width} = 68 \text{ cm}$$

$$\text{Internal Height} = 38 \text{ cm}$$

$$\text{Width} = 68 \text{ cm}$$

$$\text{Height} = 38 \text{ cm}$$

**External Volume:**

$$160 \times 70 \times 40 = 448,000 \text{ cm}^3$$

**Internal Volume:**

$$158 \times 68 \times 38 = 408,272 \text{ cm}^3$$

$$448,000 \text{ cm}^3 - 408,272 \text{ cm}^3 = 39,728 \text{ cm}^3$$

$$\therefore \text{Volume of the metal} = 39,728 \text{ cm}^3$$

b)

$$\text{Mass} = \text{density} \times \text{volume}$$

$$= 0.85 \text{ g/cm}^3 \times 39,728 \text{ cm}^3$$

$$= 33,768.8 \text{ g}$$

$$\frac{33,768.8}{1000} = 33.7688 \text{ Kg}$$

$$\therefore \text{Mass of the empty box} = 33.8 \text{ Kg}$$

c)

**Mass of tools:**

$$12 \times 40 \times 115 = 55,200 \text{ g}$$

$$\frac{55,200}{1000} = 55.2 \text{ Kg}$$

$$\therefore \text{The total Mass} = 55.2 + 33.8$$

$$= 89.0 \text{ Kg}$$

**i Problems to Solve**

1. A solid block in the shape of a cylinder has a height of 14cm and weighs 26kg. If it is made of material of density  $0.45 \text{ g/cm}^3$ , find the radius of the cylinder

- to four significant figures. Take  $\pi = \frac{22}{7}$  (3mks)
2. 2000 cm<sup>3</sup> of a mixture consists of 2.5 kg of substance x and 7.5 kg of substance y. find the density of mixture in g/cm<sup>3</sup> (3mks)
  3. An artisan has 63kg of metal of density 7g/cm<sup>3</sup>. He intends to use it to make a rectangular pipe with external dimensions 12cm by 15cm and internal dimensions 10cm by 12 cm. Calculate the length of the pipe in metres. (3mks)
  4. A solid metal cuboid 2.1m long, 0.8m wide, and 0.75m high of material of density 0.75g/cm<sup>3</sup>. Calculate its mass in kilograms. (2mks)
  5. A metal R is an alloy of two metals X and Y. Metal X has a mass of 70g and a density of 16000kg/m<sup>3</sup>. Metal Y has a mass of 42g and a density of 4000kg/m<sup>3</sup>. In 4 significant figures, calculate the density of the metal R in its SI unit. (4mks)
  6. Two coils of the same mass are made by winding aluminum wire of different gauges and lengths. If the first coil is made by winding 540 m of the wire with diameter 1.96 mm cross-sectional diameter and the second coil is made by winding a certain length of the wire with a cross-sectional diameter of 2.94 mm, find the length of the second coil wire. (3mks)
  7. The external measurements of a wooden box are 1.2 m long, 65 cm wide, and 40 cm high. The wood used in making the box is 2 cm thick and has a density of 950Kg/m<sup>3</sup>. Given that the box contains 30 packets of tools and each packet holds a dozen tools each weighing 125 g, calculate:
    - a) The volume of wood used in making the box. (4mks)
    - b) The mass of the empty box in kilograms to four significant figures. (3mks)
    - c) The total mass of the box in kilograms to 3 significant figures. (3mks)

# Chapter Twelve: Time

## Time

**Time** can be given in either a 12-hour clock system or a 24-hour system. In a 12-hour system, time is counted from midnight (12:00 midnight). The time is written as am from midnight to midday and pm from midday to midnight. In the 24-hour clock system, time is expressed in hours and counted from midnight to midnight.

To convert a 12-hour clock system to a 24-hour system, add 12 to a pm time. To convert a 24-hour time to a 12-hour time, subtract 12 if it's 13 or more, then add the right suffix (am if the original value is less than 13, or pm if the original value is 13 or more).

### Problems to Solve

1. A car left town A for town B at 1000h and had a puncture after traveling for 2 h 30 min fixing a new tyre took 36 minutes. The car then traveled for another 1 hour 45 min before reaching town B. At what time did it arrive? (3mks)
2. An airplane left Nairobi at 2045h and arrived in London at 0320h. It stayed for  $1\frac{1}{2}$  hours for rest and refreshment of passengers and crew. It then headed for Washington D.C and took  $9\frac{1}{4}$  hours.
  - a) How long did the journey from Nairobi to London take in hours and minutes? (2mks)
  - b) At what time did it arrive in Washington D.C.? (2mks)
3. A watch that loses a half-minute every hour was set to read the correct time at 0445h on Monday. Determine the time, in the 12-hour system, the watch will show on the following Friday at 1845h. (3mks)
4. The average lap time for 3 athletes in a long-distance race is 36 seconds, 40 seconds, and 48 seconds respectively. If they all start the race at the same time, find the number of times the slowest runner will have been overlapped by the fastest at the time they all cross the starting point together again. (3mks)
5. The travel timetable below shows the departure and arrival time for a bus plying between two towns M and R, 450 kilometers apart.

TOWN	ARRIVAL	DEPARTURE
M		0830h
N	1000h	1020h

P	1310h	1340h
Q	1510h	1520h
R	1600h	

6. Calculate the average speed for the whole journey. (3mks)

# Chapter 15: Linear Equations

## Linear Equations

Linear equations are equations of straight lines involving one or two unknown variables.

A system of two linear equations forms simultaneous equations.

There are three methods used to solve simultaneous equations:

- Elimination method
- Substitution method
- Graphical method

## Solved examples

### Substitution method

#### i Example 1

$$\begin{aligned} 2x - 3y &= -13 & (i) \\ y &= 2x + 7 & (ii) \end{aligned}$$

#### 🔥 Solution

Substituting equation (ii) into (i);

$$\begin{aligned} 2x - 3(2x + 7) &= -13 \\ 2x - 6x - 21 &= -13 \\ -4x &= -13 + 21 \\ \frac{-4x}{-4} &= \frac{8}{-4} \\ x &= -2 \end{aligned}$$

but,  $y = 2x + 7$

Solve for  $y$ :

$$\begin{aligned}y &= 2(-2) + 7 \\&= 3\end{aligned}$$

$\therefore$  Solution set =  $(-2, 3)$

### Example 2

$$\begin{aligned}7x + 15y &= 22 \dots & (i) \\8x + 17y &= 25 \dots & (ii)\end{aligned}$$

### Solution

Solving for  $x$  in (i);

$$x = \frac{22 - 15y}{7}$$

substituting  $x = \frac{22 - 15y}{7}$  in (ii)

$$8 \left( \frac{22 - 15y}{7} \right) + 17y = 25$$

Multiplying both side by 7;

$$176 - 120y + 119y = 175$$

$$\cancel{y} = \cancel{1}$$

$$y = 1$$

$$\text{then, } x = \frac{22 - 15(1)}{7}$$

$$x = 1$$

$\therefore$  Solution set =  $(1, 1)$

## Elimination method

### Example 3

$$\begin{aligned}3x - 7y &= 13 \dots & (i) \\6x - 5y &= 8 \dots & (ii)\end{aligned}$$

### Solution

To eliminate x multiply equation (i) by 2 and (ii) by 1 and then subtract (ii) from (i);

$$2 \left( \begin{array}{l} 3x - 7y = 13 \\ 6x - 5y = 8 \end{array} \right)$$

Subtracting the two equations:

$$\begin{array}{r} 6x - 14y = 26 \\ 6x - 5y = 8 \\ \hline -9y = 18 \end{array}$$

$$\frac{-9y}{-9} = \frac{18}{-9}$$

$$y = -2$$

Plugging  $y = -2$  in (i) or (ii) to determine the value of x;

$$3x - 7(-2) = 13$$

$$3x + 14 = 13$$

$$3x = -1$$

$$\frac{3x}{3} = -\frac{1}{3}$$

$$x = -\frac{1}{3}$$

$$\text{Solution set} = \left( -\frac{1}{3}, -2 \right)$$

### i Example 4

Solve the following linear equations (4mks)

$$\frac{x+y}{6} - \frac{x+y}{4} = -\frac{1}{4}$$

$$\frac{x-y}{5} + \frac{x-y}{6} = 1\frac{1}{10}$$

### Solution

Simplifying the two equations;

Multiply the 1st equation by 12;

$$\left( \frac{x+y}{6} - \frac{x+y}{4} = -\frac{1}{4} \right) 12$$

$$2x + 2y - 3x - 3y = -3$$

$$-x - y = -3 \dots \dots \dots (i)$$

Multiply the 2nd equation by 30;

$$\left( \frac{x-y}{5} + \frac{x-y}{6} = \frac{11}{10} \right) 30$$

$$6x - 6y + 5x - 5y = 33$$

$$11x - 11y = 33 \dots \dots \dots (ii)$$

Multiply equation (i) by 11 and add to equation (ii) to eliminate x;

$$\begin{array}{r} 11 \\ \times 1 \\ \hline -x - y = -3 \\ 11x - 11y = 33 \\ \hline \end{array}$$

$$\begin{array}{r} -11x - 11y = -33 \\ 11x - 11y = 33 \\ \hline -22y = 0 \\ y = 0 \end{array}$$

Plugging  $y = 0$  in (i) or (ii) to determine the value of x;

$$\begin{aligned} -x - y(0) &= -3 \\ -x &= -3 \\ x &= 3 \\ \therefore \text{Solution set} &= (3, 0) \end{aligned}$$

### **i** Problems to Solve

1. Solve the following equations (4mks)

$$\begin{array}{l} \frac{3y - 2x}{15} + \frac{2x + 1}{3} = 2 \\ \frac{2x - 3y}{15} - \frac{1 - x}{3} = 1 \end{array} \quad (0.43)$$

2. Solve the simultaneous equations. (3mks)

$$\begin{aligned} 2x + 3y &= 4 \\ x + 4y &= 7 \end{aligned}$$

3. Solve the simultaneous equations: (3mks)

$$\begin{aligned} 2x - y + 2 &= 0 \\ -3y + x &= -6 \end{aligned}$$

4. Mercy a student at Mucagara mixed Secondary bought 5 pens and 3 exercise books from Magunas supermarket at Ksh. 135, at the same time Murugi her class mate also bought 4 pens and 5 exercise books and spent Ksh. 25 more than Mercy. Find the cost of each pen and exercise book. (4mks)
5. In July, Kiama donated  $\frac{1}{6}$ th of his salary to a children's home while Joshua donated  $\frac{1}{5}$ th of his salary to the same children's home. Their total donation for July was *Ksh.* 14,820. In August, Kiama donated  $\frac{1}{8}$ th of his salary to the children's home while Joshua donated  $\frac{1}{12}$ th of his salary to the children's home. Their total donation for August was *Ksh.* 8,675. Calculate Kiama's monthly salary. (4mks)
6. Three spoons and four plates cost *Ksh.* 87. Two spoons and five plates cost *Ksh.* 93. Find the cost of one spoon and one plate. (4mks)
7. Mwendia bought 8 pairs of trousers and six socks at *Ksh.* 4,160. Had he bought twice as many socks and half as many trousers, he would have saved *Ksh.* 100. Find the cost of each item. (3mks)
8. Esther bought 144 mangoes at *Ksh.* 100 for every six mangoes. She sold some of them at *Ksh.* 72 for every three and the rest at *Ksh.* 60 for every two. If she made a 45% profit, calculate the number of mangoes sold at *Ksh.* 72 for every three. (3mks)
9. Four men took their cows to the market. John had two more cows than Enoch. Alex had as many cows as John, whereas Jeff had 10 cows less than the sum of both John and Alex.
  - a) Write a simplified expression with one variable, representing the total number of cows. (1mk)
  - b) Three butchers bought all the cows and shared them equally. If each butcher got 17 cows, how many did Jeff sell to the butchers (3mks)
10. Wanjiru bought three cups and four plates for Ksh. 324. Moraa bought five cups and Anyango bought two plates of the same type as those bought by Wanjiru. Moraa paid *Ksh.* 228 more than Anyango. Find the price of each cup and spoon. (3mks)
11. Daniel and Sokoro bought the same types of pens and blades from the same shop. Daniel bought 2 pens and 3 blades for *Ksh.* 78. Sokoro bought 3 pens and 4 blades and spent *Ksh.* 36 more than Daniel. Calculate the cost of each item (3mks)
12. Karani bought 4 pencils and 6 blades for *Ksh.* 66 and Kanuni bought 2 pencils and 5 blades for *Ksh.* 51.
  - a) Find the price of each item. (3mks)
  - b) Naomi spent *Ksh.* 228 to buy the same type of pencils and blades. If the number of blades she bought was 4 more than the number of pencils, find the number of pencils bought. (3mks)

13. A retailer bought 50 plates and 30 spoons from a wholesaler P for *Ksh.* 4260. Had she bought 15 plates less and half spoons more, she would have paid *Ksh.* 990 less. Had the retailer bought from wholesaler Q, she would have paid 50% more for a plate and 25% less for a spoon. How much would she have lost if she had bought the 50 plates and the 30 spoons from wholesaler Q.? (10mks)

# Chapter Fourteen: Commercial Arithmetic

## Commercial Arithmetic

Currency is the medium of any business transaction. In Kenya, shillings are used as the basic currency unit. 1 Kenyan shilling (*Ksh*) is equal to 100 cents (*ct*).

### Solved Examples

#### i Example 1

Use the exchange rates below to answer this question. (3mks)

	<i>Buying</i>	<i>Selling</i>
1 US dollar	102.20	102.80
1 UK £	132.30	132.95

A European tourist arriving in Kenya from Britain had 12,600 UK Sterling pounds (£). He converted the pounds to Kenya shillings at a commission of 5%. While in Kenya, he spent  $\frac{4}{5}$  of this money. He changed the balance to US dollars after his stay. If he was not charged any commission for this last transaction, calculate to the nearest US dollars, the amount he received.

#### 🔥 Solution

To Convert foreign currency to Kenyan currency, the bank Buys from you (use the Buying column. But, from Kenyan currency to foreign currency, the bank sells to you (use the Selling column)

$$\begin{aligned} \text{He received} &= 12,500 \times 132.30 \\ &= \text{Ksh.}1,666,980 \end{aligned}$$

*After commission, he gets :*

$$\begin{aligned} &= \frac{95}{100} \times 1,666,980 \\ &= 1,583,631 \end{aligned}$$

Balance after spending  $\frac{4}{5}$  of his money:

$$\begin{aligned}
 &= \frac{1}{5} \times 1,583,631 \\
 &= Ksh. 237,667.20
 \end{aligned}$$

*US dollars he received :*

$$\begin{aligned}
 &= \frac{237,667.20}{102.80} \\
 &= 2,311.94
 \end{aligned}$$

*∴ He received : 2,312 US dollars*

### Example 2

A Kenyan bank buys and sells foreign currencies at the exchange rates shown below.

	<i>BUYING (KSH)</i>	<i>SELLING (KSH)</i>
1 Euro	147.56	148.00
1 U.S Dollar	102.22	102.50

A foreign woman arrived in Kenya with 25,000 Euros. He converted all the Euros into Kenyan Shillings at the bank. He spent *Ksh. 2,610,200* while in Kenya and converted the remaining Kenya shillings into U.S Dollars at the bank. Calculate to the nearest dollars the amount that she received. (3mks)

### Solution

$$\begin{aligned}
 \text{She received} &= 25000 \times 147.56 \\
 &= Ksh. 3,689,000
 \end{aligned}$$

*Balance after spending :*

$$\begin{aligned}
 &= 3,689,000 - 2,610,200 \\
 &= Ksh. 1,078,800
 \end{aligned}$$

*US dollars she received :*

$$\begin{aligned}
 &= \frac{1,078,800}{102.5} \\
 &= 10,524.88
 \end{aligned}$$

*∴ She received : 10,525 US dollars*

### Example 3

Mr. Albert who deals in electronics sells a radio to a customer at *Ksh. 1,350* after giving him a discount of 10% but finds that he still makes a 20% profit. Find the percentage profit Mr. Albert would make if he does not give a discount. (3mks)

### Solution

**Note:** The marked price and the Buying Price (B.P) are always 100% Marked Price (M.P) of the radio:

$$\frac{1350 \times 10\varnothing}{9\varnothing} = Ksh. 1,500$$

*Buying Price(B.P) :*

$$\frac{1,35\varnothing \times 100}{12\varnothing} = Ksh. 1,125$$

Profit gained if no discount was offered:

$$1,500 - 1,125 = Ksh. 375$$

Percentage profit (P.P):

$$\begin{aligned} &= \left( \frac{\text{profit}}{\text{B.P}} \right) \times 100 \\ P.P &= \frac{375}{1,125} \times 100 \\ &= 33.33\% \end{aligned}$$

∴ He would make  $33\frac{1}{3}\%$  profit

### Example 4

Judy bought some rice at the wholesale price of *Ksh.* 65 per kg. She packed three-fifths of the rice in 2 kg bags and sold each bag at *Ksh.* 160. She packed the remaining in 1 kg bags and sold each bag at *Ksh.* 85. After selling all the rice she found that she had made a profit of *Ksh.* 6460.

- a) Calculate the amount of rice she bought (6mks)
- b) In three significant figures, determine:
  - i) The percentage profit she made. (2mks)
  - ii) The percentage profit he would have made if sold all the rice in 2 kg bags. (2mks)

### Solution

a)

Let the amount of rice be  $x$

$$2Kg \text{ bags} = \frac{3}{5}x$$

Selling price (S.P) for 2kg bags:

$$= \left(\frac{3}{5}x\right) \frac{160}{2}$$

$$= 48x$$

$$1Kg\ bags = x - \frac{3}{5}x$$

$$= \frac{2}{5}x$$

Selling price for 1Kg bags:

$$= \frac{2}{5}x \times 85$$

$$= 34x$$

$$\begin{aligned} \text{Total selling price} &= 48x + 34x \\ &= 82x \end{aligned}$$

Buying Price (B.P) of the rice:

$$\begin{aligned} &= 65x \\ Profit &= S.P - B.P \\ 82x - 65x &= 6460 \\ \frac{17x}{17} &= \frac{6460}{17}^{380} \\ &= 380 \\ \therefore \text{The amount of Rice} &= 380\ kg \end{aligned}$$

b) i)

$$\begin{aligned} \text{Percentage profit(p.p)} &= \frac{\text{profit}}{\text{B.p}} \times 100 \\ \text{B.p} &= 380 \times 65 \\ &= 24700 \\ &= \left(\frac{6460}{24700}\right) 100 \\ &\approx 26.154\% \\ \therefore p.p &= 26.2\% \end{aligned}$$

ii)

$$\begin{aligned}
 S.p &= \left( \frac{380}{2} \right) 160 \\
 &= 30400 \\
 profit &= 30,400 - 24,700 \\
 &= 5700 \\
 p.p &= \left( \frac{5700}{24700} \right) 100 \\
 &\approx 23.077\% \\
 \therefore p.p &= 23.1\%
 \end{aligned}$$

### **i** Problems to solve

1. A Kenya bank buys and sells foreign currencies as shown

	<i>Buying (Ksh)</i>	<i>Selling (Ksh)</i>
1 Euro	116.26	116.80
100 Japanese Yen	91.36	91.45

A Japanese traveling from France to Kenya had 5,000 Euros. He converted all the 5,000 Euros to Kenya shillings at the bank. While in Kenya, he spent a total of Ksh. 389,850 and then converted the remaining Kenya shilling to Japanese Yens at the bank. Calculate the amount in Japanese Yen that he received. (3mks)

2. A Kenyan bank buys and sells foreign currency as shown below.

	<i>Buying (Ksh)</i>	<i>Selling (Ksh)</i>
1 Euro	116.15	116.26
1 US Dollar	100.43	100.80

A foreigner traveling from Britain arrives in Kenya with 6,500 Euros. He converts all the Euros to Kenya shillings at the bank. While in Kenya he spends a total of KSh. 459,650 and then converts the remaining Kenya shillings to US dollars at the bank. Calculate (to the nearest dollar) the amount he receives. (3mks)

3. A forex bureau buys and sells American dollars in Kenya shillings at the rate shown below.

<i>Buying</i>	<i>Selling</i>
102.40	102.81

An American woman at the end of her tour in Kenya had Ksh. 107,500 which he converted to the dollar through the Forex bureau. How many dollars did she get? (2mks)

4. A tourist arrived from Los Angeles and changed his US Dollar 3,650 to Ksh. He spent Ksh. 3,000 per night in a hotel for 20 nights and a further Ksh. 9,000 daily for the entire period. He left for South Africa having changed the balance to South African Rand. Calculate the amount of South African Rands he left with, if the bank buys and sells currencies using the table below. (3mks)

Currency	Buying	Selling
1 US Dollar (\$)	100.8133	100.8744
1 Sterling Pound (£)	116.1616	116.3043
1 South African Rand	7.4842	7.5141

5. Kavula sold a bag of potatoes for *Ksh.* 420 and made a profit. If she sold it at *Ksh.* 320, she could have made a loss. Given that the profit is thrice the loss, how much did she pay for the bag of potatoes? (3mks)
6. The marked price of a Toyota car in a dealer's shop was *Ksh.* 450,000. Tom bought the car at 7% discount. The dealer still made a profit of 13%. Calculate the amount of money the dealer had paid for the car. (3mks)
7. A 6kg gas cylinder was bought at *Ksh.* 2,250 and then later sold for *Ksh.* 2,790. Calculate:-
- The percentage profit. (2mks)
  - The price at which it should be sold to make a profit of 28%. (2mks)
8. Mwikali paid *Ksh.* 160 for a blouse after getting a discount of 20%. The vendor made a profit of 25% on the sale of this blouse. What percentage profit would the vendor have made if no discount was allowed? (3mks)
9. Mr. Nyamu who deals in computer accessories sells a laptop to a customer at *Ksh.* 21,500 after giving him a discount of 10% but finds that he still makes a 25% profit. Find the profit Mr.Nyamu would make if he does not give a discount. (3mks)
10. The marked price of a pro-box in a dealer's shop was *Ksh.* 850,000. Mucai bought the car at 8% discount. The dealer still made a profit of 15%. Calculate the amount of money the dealer had paid for the car. (3mks)
11. An electronic company imported into the country some speakers that cost *Ksh.* 25,750 each. The government imposed an import duty of 20% and a sales tax of 15%. If the company decides to make a 20% profit on sales, calculate to the nearest shillings the selling price of each speaker. (4mks)
12. A manufacturer sells an empty crate of soda to a trader at a profit of 50%. The trader sells it for *Ksh.* 360 at a profit of 20%. Find
- The trader's buying price. (2mks)
  - The cost of manufacture of an empty crate. (3mks)
13. A Kenyan tradesman owes US \$ 180,000 to a company in the United States of America. The Kenyan can either pay through his account in Kenya or through his account in the United Kingdom. Which method is cheaper and by how much? (Give your answer in Kenyan shillings given that: (4mks)

*1 US dollar = 102.74 Kenyan shillings.*

*1 Sterling pound = 1.79 US dollar*

*1 Sterling pound = 132.87 Kenyan shillings*

14. A Forex Bureau in Kenya buys and sells foreign currencies as shown below:

	<i>Buying (Ksh)</i>	<i>Selling Ksh)</i>
Chinese Yuan	15.34	15.58
South African Rand	11.28	11.45

A merchant from China converted 205,250 Chinese Yuan into Kenya Shillings.

- a) Calculate the amount of Money, in Kenya shillings, that she received. (1mk)
  - b) While in Kenya, the merchant spent *Ksh. 1,858,000* and then converted the balance to South African Rand. Calculate the amount of money, to the nearest Rand, that he received. (3mks)
15. A vehicle sales agent is paid a commission on all vehicles bought through him. During a certain month, he sold 2 cars at *Ksh.1.5 million* each, 5 probox at *Ksh. 650,000* each and 5 vans at *Ksh. 1.8 million* each. If he was paid a total commission of *Ksh. 720,000*, calculate the percentage rate of commission he was paid in 3 significant figures. (3mks)
16. Kang'ethe bought a pair of shoes for *Ksh. 1,600* and marked it at a price such that after allowing his customer a 20% discount, he would make a profit of 25%. Calculate the marked price of the shoes. (4mks)
17. Muringo bought a skirt at *Ksh. 600* and marked it at a price such that after allowing her customer a 5% discount she would make a profit of 33%. Find the marked price of the skirt. (4mks)
18. Noah sold a second-hand computer which was marked at *Ksh.24,000* to a customer at 19% discount. If he still made a 20% profit on the cost price, what was its cost price? (4mks)
19. A businessman sold a pair of shoes which was marked at *Ksh. 2,700* to a customer allowing a 15% discount. If he still made a 35% profit on the cost price, determine how much he had paid for the pair of shoes. (4mks)
20. The marked price of a second-hand car was *Ksh. 625,000*. Mwendia sold the car at a discount of 7.2% and received *Ksh. 49,300* as a commission of the sale. Calculate the percentage rate of commission he was paid. (3mks)
21. Three partners Ndirangu, Isa, and Mukami contributed *Ksh. 700,000*, *Ksh. 500,000* and *Ksh. 900,000* respectively to start a business of a mini-bus plying the Embu– Nairobi route. The mini-bus carries 25 passengers with each paying *Ksh. 250*. The mini-bus makes two round trips each day and ever full. Each day *Ksh. 8,000* is used to cover running costs and wages.
- a) Calculate their net profit per day. (2mks)

- b) The matatu works for 25 days per month and is serviced every month at a cost of *Ksh.* 12,000. Calculate their monthly in July. (1mk)
- c) The three partners agreed to save 42% of the profit, 75% of the remainder to be shared in the ratio of their contribution. Calculate Mukami's share in the month of July. (4mks)
- d) The mini-bus developed mechanical problems and they decided to sell it through an agent who charged a commission of 5% on the selling price. Each partner received *Ksh.* 520,000 from the agent after he had taken his commission. Determine the price at which the agent sold the matatu. (3mks)
22. Wangari bought some sugar at *Ksh.* 80 per kg. She packed five-eighths of the sugar in  $\frac{1}{2}$  kg packets which she sold at *Ksh.* 60 per packet. She packed the remaining sugar in 1 kg packets and sold them at *Ksh.* 110 per packet. She sold all the sugar in this way and made a profit of *Ksh.* 10,875.
- Determine the amount of sugar she bought. (4mks)
  - Calculate to one decimal place:
  - i) The percentage profit she made. (3mks)
  - ii) The percentage profit she would have made if she had sold all the sugar at  $\frac{1}{2}$  kg packets. (3mks)
23. A manufacturer made a mattress and sold it to a wholesaler at a profit of 25%. The wholesaler sold the mattress to a shopkeeper at a profit of 32%. The shopkeeper finally sold the mattress to a customer at 50% profit.
- Determine how much a customer paid for the mattress that had cost the manufacturer *Ksh.* 1800 to make. (3mks)
  - A customer paid *Ksh.* 3,465 for another mattress. Determine how much it had cost the manufacturer to make the mattress. (3mks)
  - The shopkeeper bought the mattress which had cost the wholesaler *Ksh.* 3,500. He marked the mattress at a price such that after allowing his customer a discount of 10%, he would still make a profit of 50%. Find the price at which the mattress was marked. (4mks)

# Chapter Fifteen: Co-ordinates and Graphs

## Co-ordinates and Graphs

A coordinate is an ordered pair of numbers used to locate the position of a point in a plane and written in the form  $(x, y)$ . The first number,  $x$  represents the distance along the  $x$ -axis and it is called  $x$  the coordinate. The second number,  $y$  represents the distance along the  $y$ -axis and it is called the  $y$  coordinate.

The  $x$  and  $y$  axes divide the plane into four regions and each region is called a **quadrant**. The quadrants are named 1st quadrant, 2nd quadrant, 3rd quadrant, and 4th quadrant starting with the top right-hand quadrant and moving in anti-clockwise direction.

**Rectangular Cartesian co-ordinate system** is a system of locating points using two axes at right angles.

We can also use coordinate systems such as latitude and longitudes and grid references to locate places on the earth's surface.

## Solved Examples

### i Example 1

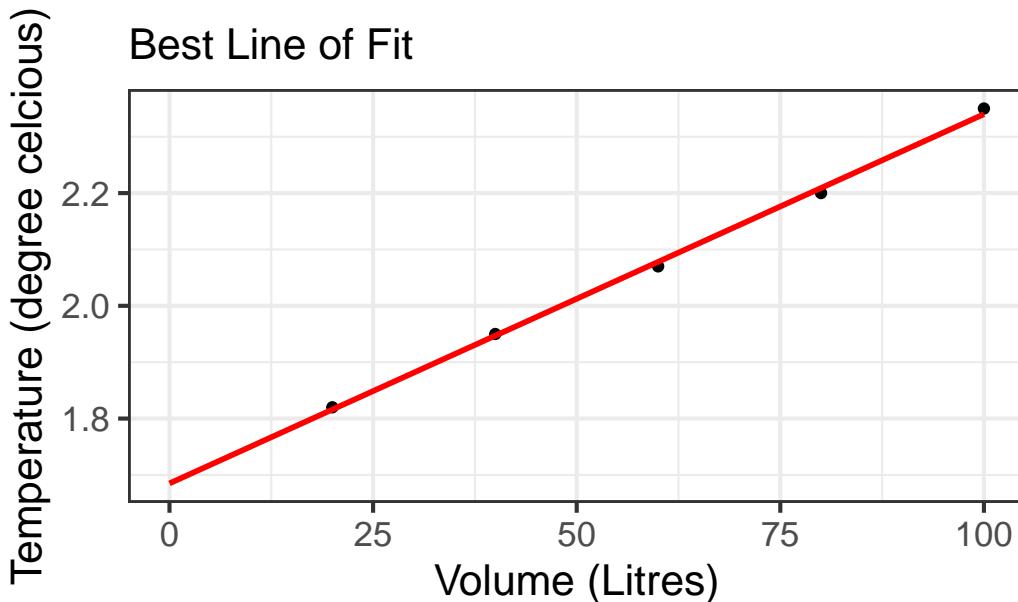
A certain quantity of gas is heated from  $0^{\circ}\text{C}$ , and the volume is measured at different temperatures. The table below gives the corresponding values:

Temperature ( $^{\circ}\text{C}$ )	20	40	60	80	100
Volume (Litres)	1.82	1.95	2.07	2.20	2.32

- Draw a graph of volume against temperature using a suitable scale.
- Use your graph to find:
  - The initial volume of the gas
  - The volume of the gas when the temperature is  $50^{\circ}\text{C}$  and  $64^{\circ}\text{C}$
  - The temperature of the gas when the volume is 1.9 litres and 2.3 litres.

### 🔥 Solution

- The graph is depicted in the figure 15.1:



The Graph of Volume against Temperature

Figure 12: The Graph of Volume against Temperature

)

- i) The initial volume is obtained by extrapolating the line to cut the y-axis. Therefore the initial volume is 1.7 litres.
- ii) The volume of the gas at  $50^{\circ}\text{C}$  is 2 litres  
The volume of the gas at  $64^{\circ}\text{C}$  is 2.1 litres.
- iii) The temperature of the gas at 1.9 litres is  $35^{\circ}\text{C}$   
The temperature of the gas at 2.3 litres is  $98^{\circ}\text{C}$

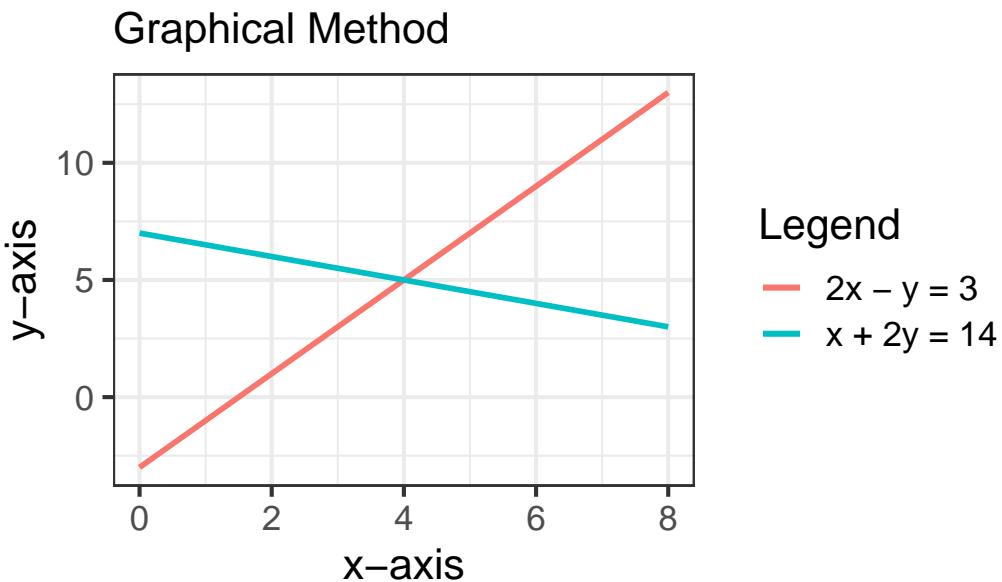
### Example 2

Solving simultaneous linear equations using graphical methods

$$\begin{aligned} 2x - y &= 3 \\ x + 2y &= 14 \end{aligned} \tag{0.44}$$

### Solution

x	0	1	2	3	4	5	6	7	8
$2x - y = 3$	-3	-1	1	3	5	7	9	11	13
$x + 2y = 14$	7	6.5	6	5.5	5	4.5	4	3.5	3



#### Solving simultaneous linear equations

Figure 13: Solving simultaneous linear equations using graphical methods

The solution of the two simultaneous equations is at the point of intersection as displayed in the figure above: From the graph, the values of x and y are:  $x = 4$  and  $y = 5$

#### i Problems to solve

1. a) Use a graphical method to solve the following simultaneous equations: (7mks)

$$\begin{aligned} 3x - y &= 4 \\ x + 4y &= 10 \end{aligned} \tag{0.45}$$

- b) If the lines cut the y-axis at points P and Q respectively, Write down the co-ordinates of the points P and Q. (3mks)

2. Copy and complete the tables below for:

- a) The linear equations  $3y = 8 + 2x$  and  $y = 5x - 6$  respectively. (4mks)

$$y = \frac{8+2x}{3}$$

x	-2	-1	0	1	2	3	4	5
y	$\frac{4}{3}$				4			

$$y = 5x - 6$$

x	-2	-1	0	1	2	3	4	5
y		-16			14			

b\)\ On a graph paper and on the same grid draw the two linear equations in (a) above. (4mks)

c\)\ What is the nature of the two graphs you have drawn? (1mk)

c\)\ Use your graphs to solve the simultaneous equations.(1mk)

$$\begin{aligned} -2x + 3y &= 8 \\ 5x - y &= 6 \end{aligned} \tag{0.46}$$

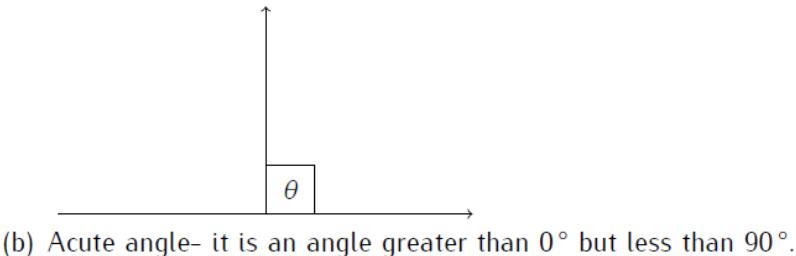
# Chapter Sixteen: Angles and Plane Figures

## Angles and Plane Figures

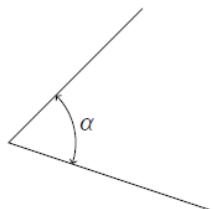
A plane is a flat surface, such as the walls of a classroom. Two planes always intersect in a straight line. A vertex is a point where two lines meet to form an angle.

### Types of angles

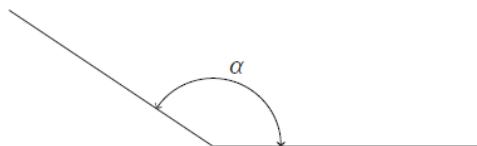
- (a) Right angle- it is an angle of  $90^\circ$



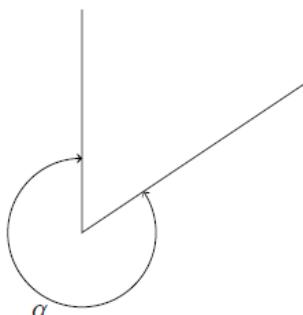
- (b) Acute angle- it is an angle greater than  $0^\circ$  but less than  $90^\circ$ .



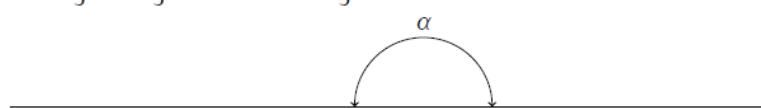
- (c) Obtuse angle-it is an angle greater than  $90^\circ$  but less than  $180^\circ$ .



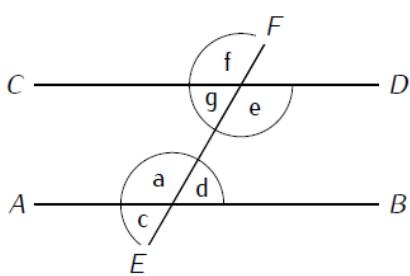
- (d) Reflex angle- it is an angle greater than  $180^\circ$ , but less than  $360^\circ$ ..



- (e) Straight angle- it is an angle of  $180^\circ$



(f) Angles formed by a transverse line across two parallel lines.



When we assume that  $AB$  and  $CD$  are parallel, i.e.,  $AB \parallel CD$ , then  $a = e$  and  $d = g$ , alternate interior angles. Similarly angle  $a$  and  $f$  are alternate exterior angles. Besides, angle  $c = d$  and  $e = f$  are vertically opposite angles and angle  $a \& d$  or  $g \& e$  add up to  $180^\circ$  (supplementary angles).

(g) Two angles that add up to  $90^\circ$  are referred to as complementary angles

### Properties of angles

- Angles on a straight line add up to  $180^\circ$
- Angles at a point add up to  $360^\circ$
- Sum of interior angles of a regular polygon is given by:

$$\text{Total interior angles} = 180^\circ(n - 2)$$

where  $n$  is the number of sides of the polygon

$$\therefore \text{Each interior angle} = \frac{180^\circ(n - 2)}{n}$$

- Sum of exterior angles of a regular polygon is equal to  $360^\circ$

$$\therefore \text{Each exterior angle} = \frac{360}{n}$$

- Sum of an interior angle and an exterior angle is equal to  $180^\circ$

## Solved Examples

### Example 1

The sum of interior angles of two regular polygons of sides;  $n$  and  $n + 2$  are in the ratio  $5 : 7$ . Calculate the sum of the interior angles of the polygon with  $n$  sides. (4mks)

### Solution

$$1^{\text{st}} \text{ Polygon} = 180^\circ(n - 2) \quad (0.47)$$

$$2^{\text{nd}} \text{ Polygon} = 180^\circ(n + 2 - 2) \quad (0.48)$$

$$\frac{180^\circ(n - 2)}{180^\circ n} = \frac{5}{7} \quad (0.49)$$

$$7n - 14 = 5n \quad (0.50)$$

$$\frac{2n}{2} = \frac{14}{2} \quad (0.51)$$

$$= 7 \text{ sides} \quad (0.52)$$

$$\therefore n \text{ sided polygon} = 180^0(5) \quad (0.53)$$

$$= 900^0 \quad (0.54)$$

### i Example 2

The exterior angle of a regular polygon is equal to two-thirds of the interior angle. Calculate the number of sides of the polygon and give its name. (4mks)

### 🔥 Solution

$$\text{Sum Exterior angles} = 360^0 \quad (0.55)$$

$$\text{Sum interior angles} = 180^0(n - 2) \quad (0.56)$$

$$\text{Exterior angle} + \text{Interior angle} = 180^0 \quad (0.57)$$

$$\text{If interior angle} = x^0, \quad (0.58)$$

$$\text{Exterior angle} = \frac{2}{3}x \quad (0.59)$$

$$x + \frac{2}{3}x = 180^0 \quad (0.60)$$

$$\frac{3}{3} \times \frac{2}{3}x = 180^0 \times \frac{3}{3} \quad (0.61)$$

$$= 36 \times 3 \quad (0.62)$$

$$\therefore \text{interior angle} = 108^0 \quad (0.63)$$

$$\text{Exterior angle} = 180^0 - 108^0 \quad (0.64)$$

$$\frac{360}{n} = 72 \quad (0.65)$$

$$n = 5; \text{ Pentagon} \quad (0.66)$$

### i Example 3

The sum of the interior angles of an n-sided polygon is  $1260^0$ . Find the value of n and hence deduce the name of the polygon. (3mks)

 Solution

$$\text{Sum interior angles} = 180(n - 2) \quad (0.67)$$

$$180(n - 2) = 1260^0 \quad (0.68)$$

$$n - 2 = 7 \quad (0.69)$$

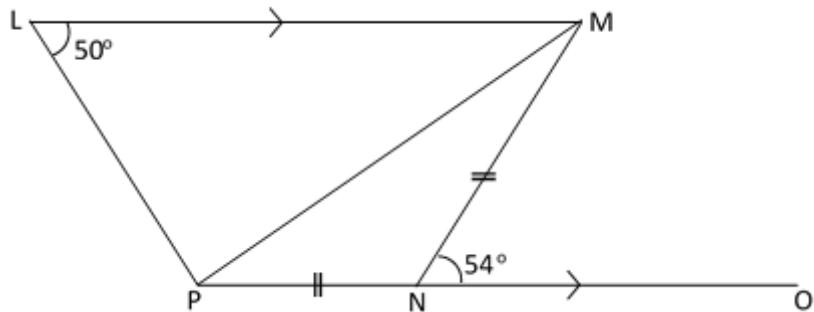
$$n = 9; \text{ Nonagon} \quad (0.70)$$

 Problems to solve

1. The size of an interior angle of a regular polygon is  $x^2$  while its exterior angle is  $3x$ . Find the number of sides of the polygon. (4mks)
2. One interior angle of a polygon is equal to  $60^\circ$  and each of the other interior angles is  $132^\circ$ . Find the number of sides and name of the polygon. (3mks)
3. The difference between the exterior and interior angle of a regular polygon is  $120^\circ$ . Determine the number of sides of the polygon. (3mks)
4. A regular polygon has internal angle of  $120^\circ$  and side of length 18cm.
  - a) Find the number of sides of the polygon. (2mks)
  - b) Find the perimeter of the polygon. (2mks)
5. The sum of the interior angles of a regular polygon is  $1800^\circ$ . Calculate
  - a) The number of sides of the polygon (2mks)
  - b) The sizes of the exterior and interior angles of the polygon. (2mks)
6. The interior angle of a regular polygon is  $30^\circ$  more than four times the exterior angle of the same polygon. Determine the number of sides of the polygon. (3mks)
7. The size of an exterior angle of a regular polygon is  $\frac{2}{3}$  times the size of its interior angle. Find the number of sides of this polygon. (3mks)
8. The sum of the interior angles of an n-sided polygon is  $1440^0$ . Find the value of n and hence deduce the name of the polygon. (3mks)
9. The sum of the interior angles of an n-sided polygon is  $1080^0$ . Find the value of n and hence give the name of the polygon. (3mks)
10. A nine-sided polygon has three of its angles equal to and the other angles are  $(2\theta - 30)$ ,  $(\theta - 28)$ ,  $3(\theta - 4)$ ,  $\theta - 45$ ,  $3\theta - 20$ , and  $(126 - \theta)$ . Calculate the value of  $\theta$ . (3mks)
11. On a graph paper,
  - a) Plot the points  $A(4, -1)$ ,  $B(5, -3)$ ,  $C(4, -4)$  and  $D(3, -3)$  and join the Points to form a polygon PQRS. State the name of the polygon formed. (2mks)

b) Write down the equation of the line of symmetry of the polygon. (1mk)

12. In the figure below  $MNO = 54^\circ$  and  $PLM = 50^\circ$ ,  $PN = NM$  and  $PO$  is parallel to  $LM$ . Find the value of  $\angle LPM$



# Chapter Seventeen: Geometrical Constructions

## Geometrical Constructions

**Note:** To circumscribe a circle around a triangle, we bisect two sides of the triangle and the point of intersection is the center of the circle.

To inscribe a circle inside a triangle, we bisect two angles of the triangle and the point of intersection is the center of the circle.

**Geometrical construction** is the drawing of accurate figures. The figures are constructed using a pair of compasses and a ruler only

### Perpendicular Lines

Figure 17.1 shows XY as a perpendicular bisector of a given line AB

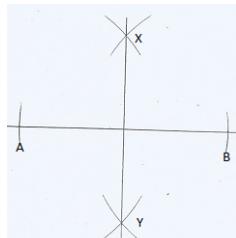


Figure 14: Figure 17.1: Perpendicular Bisector

Figure 17.2 shows PR, a perpendicular from point P to a given line AB.

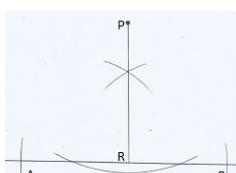


Figure 15: Figure 17.2: Perpendicular Bisector from a Point

### Constructing Angles of $60^\circ$ , $120^\circ$ , and $30^\circ$

In this section, we will consider the construction of some angles with special sizes using a pair of compasses and a ruler only.

#### Constructing a $60^\circ$ Angle

**Step 1:** Draw the line AB.

**Step 2:** Place the tip of the compass at A and draw an arc of any measure to cut the line AB at some point (say R).

**Step 3:** Keeping the width unchanged, place the tip of the compass on point R and draw another arc cutting the arc drawn in the previous step at some point (say S)

**Step 4:** Connect the points A and S with a straight line and extend it to form a line AC.

The measure of the  $\angle BAC$  is  $60^\circ$

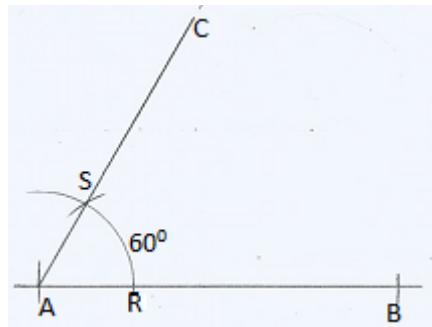


Figure 16: Figure 17.3: An Angle of  $60^\circ$

### Constructing a $120^\circ$ Angle

**Step 1:** Draw the line AB. **{Step 2:** Place the tip of the compass at A and draw an arc of any measure to cut the line AB at some point (say P). **Step 3:** Keeping the width unchanged, place the tip of the compass on point P and draw another arc cutting the arc drawn in the previous step at some point (say S) **Step 4:** With the same width place the tip of the compass at point S and draw another arc cutting the arc drawn in step 2 at some point (say T) **Step 5:** Connect the points A and T with a straight line and extend it to form a line AC.

The measure of the  $\angle BAC$  is  $120^\circ$

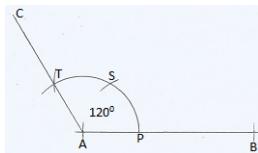


Figure 17: Figure 17.4: An Angle of  $120^\circ$

### Constructing a $30^\circ$ Angle

To construct the angle of  $30^\circ$ , Construct the angle of  $60^\circ$  as described above and then bisect it as shown below.

The measure of the  $\angle BAC$  is  $30^\circ$

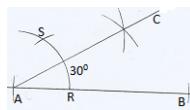


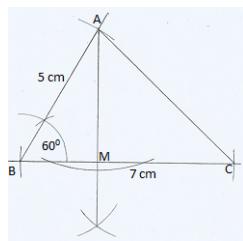
Figure 18: Figure 17.5: An Angle of  $30^\circ$

## Solved Examples

### i Example 1

Using a ruler and pair of compasses only, construct triangle ABC in which  $AB = 5\text{cm}$ ,  $BC = 7\text{cm}$  and angle  $ABC = 60^\circ$ . Drop a perpendicular from A to meet BC at M. Measure AM and AC. (3mks)

 Solution



$$AM = 4.4 \pm 0.1 \text{ cm}$$

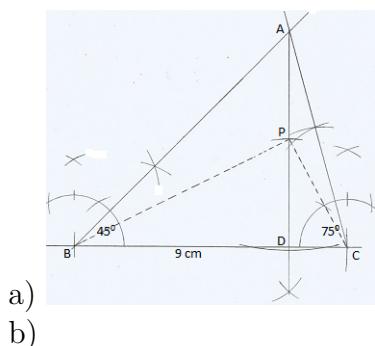
$$AC = 6.3 \pm 0.1 \text{ cm}$$

 Example 2

Use a ruler and a pair of compasses only for all constructions in this question.

- Construct a triangle ABC in which  $BC = 9 \text{ cm}$ , angle  $ABC = 45^\circ$  and angle  $ACB = 75^\circ$  (2mks)
- Measure AB and AC. (2mks)
- At A drop a perpendicular to meet BC at D. (1mk)
- Measure AD and hence calculate the area of triangle ABC. (3mks)
- Mark a point P on DA produced such that the area of triangle BPC is half the area of triangle ABC. (1mk)
- Complete triangle BPC and measure PC (1mk)

 Solution



- 
- 

$$AB = 10 \pm 0.1 \text{ cm}$$

$$AC = 7.4 \pm 0.1 \text{ cm}$$

- 

$$AD = 7.1 \pm 0.1 \text{ cm}$$

$$\text{Area of Triangle } ABC = 31.95 \pm 0.45 \text{ cm}^2$$

- 

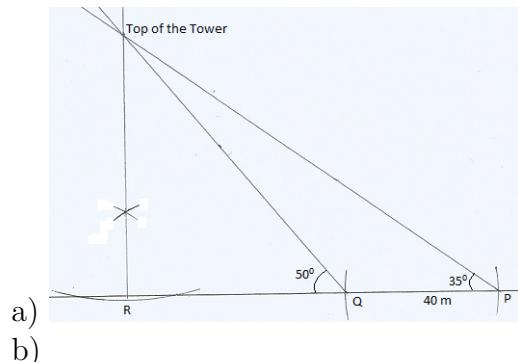
$$PC = 4.1 \pm 0.1 \text{ cm}$$

**i** Example 3

Use a ruler and a pair of compasses only for all constructions in this question.

- Construct triangle PQR in which QR=8.5cm, PQ=7cm and angle PQR=  $60^\circ$  (3mks)
- Measure PR and angle QPR. (2mks)
- Construct a circle which is inscribed inside triangle PQR. (4mks)
- Measure the radius of this circle. (1mk)

**fire** Solution



a)  
b)

$$PR = 7.8 \pm 0.1 \text{ cm}$$

$$\angle QPR = 70 \pm 1^\circ$$

c) Radius=  $2.2 \pm 0.1 \text{ cm}$

**i** Problems to solve

- a) Using a pair of compasses and a ruler only construct a triangle ABC such that  $AB = 6.5\text{cm}$ ,  $BC = 5\text{cm}$  and angle  $ABC = 135^\circ$  (2mks)  
b) Construct the height of triangle ABC in (a) above taking AB as the base, hence calculating the area of triangle ABC. (2mks)
- Using compass and ruler only construct a triangle ABC such that  $AB = 7\text{cm}$ ,  $BC = 6\text{cm}$  and angle  $ABC = 67.5^\circ$  measure the length of AC. (4mks)
- Use a ruler and a pair of compasses only for all the constructions in this question.
  - Construct a triangle PQR in which  $PQ=9 \text{ cm}$ ,  $QR=6 \text{ cm}$  and angle  $PQR=30^\circ$  (3mks)
  - From P drop a perpendicular to meet QR produced at S. (1mk)
  - Measure PS and hence calculate the area of triangle PQR. (2mks)
  - Locate a point T on SP produced such that the area of triangle QTR is  $\frac{4}{3}$  times the area of triangle PQR. (2mks)
  - Complete triangle QTR and measure RT and angle TQR. (2mks)

4. Use a ruler and a pair of compasses only for all constructions in this question.
  - a) Construct triangle ABC in which BC=AC=7 cm and angle ACB=  $37.5^0$  (3mks)
  - b) Measure AB. (1mk)
  - c) From A drop a perpendicular to meet BC produced at  
D. (1mk)
  - d) Measure AD and hence calculate the area of triangle ABC. (2mks)
  - e) Mark a point P on AD such that the area of triangle PBC is half the area of triangle ABC. (1mk)
  - f) Complete triangle PBC and measure angle PBC (2mks)
5. Use a ruler and a pair of compasses only for all constructions in this question.
  - a) Construct triangle PQR in which QR=5cm, PR=7 cm and PRQ=  $150^0$  (3mks)
  - b) Measure PQ and angle PQR. (2mks)
  - c) From P drop a perpendicular to meet QR produced at  
D. (1mk)
  - d) Mark a point T on DP produced such that the area of triangle TQR is twice the area of triangle PQR (2mks)
  - e) Complete triangle TQR and measure angle QRT. (2mks)
6. Use a ruler and a pair of compasses only for all constructions in this question
  - a) Construct triangle ABC in which BC=7 cm, and angle ABC=  $45^0$  and angle ACB=  $60^0$ . (3mks)
  - b) Measure AB and AC (2mks)
  - c) Construct a circle that touches BC at B and passes through  
A. (4mks)
  - d) Measure the radius of the circle (1mk)
7. Use a ruler and a pair of compasses only for all constructions in this question.
  - a) Construct triangle ABC in which BC=7cm, angle ABC=  $22.50$  and angle ACB=  $120^0$ . (4mks)
  - b) Measure AC (1mk)
  - c) Produce AB to D and AC to E and bisect angle CBD and angle BCE. (2mks)
  - d) Construct a circle which touches all three sides AD, BC, and AE. (2mks)
  - e) What is the radius of this circle? (1mk)

# Chapter Eighteen: Scale Drawing

## Scale Drawing

### Bearing and Distance, Angles of Elevation and Depression

#### Types of bearing

- **True Bearing-** It can only measured from North pole in a clockwise direction and given in three digits format. For example given that  $065^0$  is the bearing between two points, say  $A$  from  $B$ ; the angle is obtained at point  $B$  from north pole in a clockwise direction to the line joining  $A$ .
- **Compass bearing-** It can be measured from either North or South pole in either clockwise or anticlockwise direction depending on the side the target object is located. For example if point  $A$  is  $60^0$  North-West of point  $B$ , then point  $A$  is on a compass bearing of  $N60^0W$  from point  $B$ .

## Solved Examples

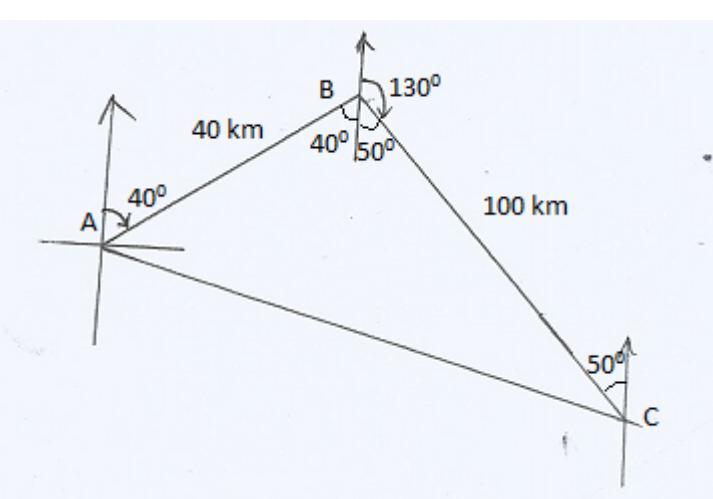
### i Example 1

Three towns A, B, and C are situated such that town B is 40km on a bearing of 0400 from town A. Town C is 100 km on a bearing of 1300 from town B.

- (a) Draw a sketch showing the positions of towns A, B, and C. (1mk)
- (b) Calculate:
  - i. The size of angle ABC (1mk)
  - ii. The distance of C from A to 1 decimal place (2mk)

### 🔥 Solution

- (a)



(b) i.  $\angle ABC = 40^\circ + 50^\circ = 90^\circ$

ii.

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 40^2 + 100^2$$

$$AC^2 = 11,600$$

$$AC = 107.7 \text{ Km}$$

### **i Example 2**

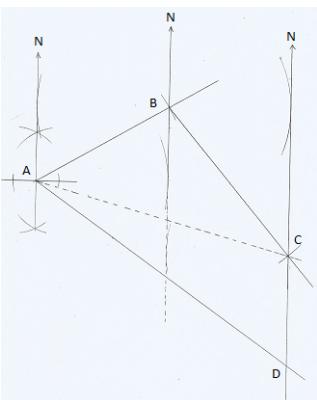
A dove flies from a tree A to another tree B which is 80 m on a bearing of  $060^\circ$  from A. From B the dove flies 100 m to tree C which is on a compass bearing of  $S40^\circ E$  from tree B and finally flies due south to another tree D which is on a bearing of  $1^\circ$  from A. a) Using a ruler and a pair of compasses only construct an accurate scale drawing showing the positions of A, B, C, and D. (*scale : 1 cm = 10 m*)

b) By measurement from your scale drawing determine:

- (i) The distance and compass bearing of A from C. (2mks)
- (ii) The distance of D from C. (1mks)
- (iii) The distance and compass bearing of A from D (2mks)

### **🔥 Solution**

(a)



(b)

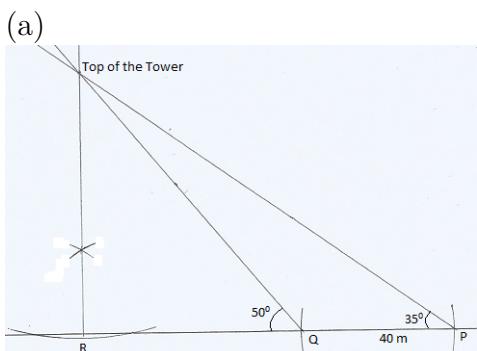
- (i) Distance of A from C =  $139\pm 1m$  ;  
Compass bearing of A from C =  $N75^0\pm 10W$
- (ii) Distance of D from C =  $58\pm 1m$
- (iii) Distance of A from D =  $164\pm 1m$  ;  
Compass bearing of A from D =  $N55^0\pm 10W$

### **i Example 3**

The angle of elevation of the top of a vertical tower from a point P is  $35^0$ . The angle of elevation of the top of the tower from another point Q which is nearer the foot R of the tower is  $50^0$ . The distance between Q and P is 40 m and the points P, Q, and R are on the same straight line on level ground.

- (a) Use a scale of 1cm to represent 10 m, draw an accurate scale drawing to represent the above information. (4mks)
- (b) Use your scale drawing to determine:
  - (i) The height of the tower (2mks)
  - (ii) The distance QP (2mks)
  - (iii) The distance of P from the top of the tower (2mks)

### **🔥 Solution**



(b)

- (i) Height of the tower =  $68 \pm 1m$
- (ii)  $RQ = 58 \pm 1m$
- (iii) Distance of P from top of the tower =  $120 \pm 1m$

**i** Example 4

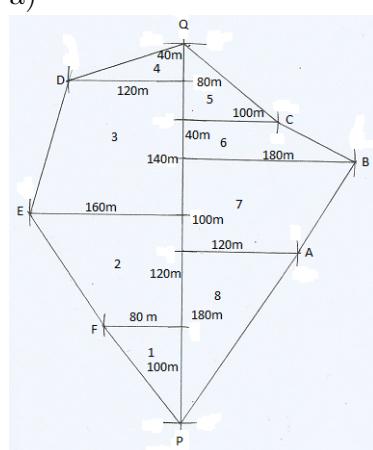
The following measurement were recorded in a book of a virgin land using PQ as the base line. And  $PQ = 400\text{ m}$

		Y
C	80	400
	360	160 D
	300	100 E
	240	200 F
B	100	180
A	140	120
	X	

- Using a scale of 1: 4000, draw an accurate map of the farm. (4mks)
- Determine the actual area of the farm in hectares.(4mks)
- If the farm is on sale at 320,000 per hectare, Find how much the farm costs.(2mks)

**🔥 Solution**

a)



- The Area of the farm is given as follows:

$$\begin{aligned}
 \text{Label 1} &\Rightarrow \frac{1}{2} \times 100 \times 50 = 4,000 \text{ m}^2 \\
 \text{Label 2} &\Rightarrow \frac{1}{2} \times 120 \times 60 = 14,400 \text{ m}^2 \\
 \text{Label 3} &\Rightarrow \frac{1}{2} \times 140 \times 70 = 19,600 \text{ m}^2 \\
 \text{Label 4} &\Rightarrow \frac{1}{2} \times 40 \times 20 = 2,400 \text{ m}^2 \\
 \text{Label 5} &\Rightarrow \frac{1}{2} \times 80 \times 40 = 4,000 \text{ m}^2 \\
 \text{Label 6} &\Rightarrow \frac{1}{2} \times 40 \times 20 = 5,600 \text{ m}^2 \\
 \text{Label 7} &\Rightarrow \frac{1}{2} \times 100 \times 50 = 15,000 \text{ m}^2 \\
 \text{Label 8} &\Rightarrow \frac{1}{2} \times 180 \times 120 = 10,800 \text{ m}^2
 \end{aligned}$$

$$\therefore \text{The total Area of the Farm} \Rightarrow \frac{75,800}{10,000} = 7.58 \text{ ha}$$

c) The farm cost is given by:

$$\Rightarrow 7.58 \times 320,000 = \text{Ksh.} 2,425,600$$

### Problems to Solve

- The scale of a map is given as 1 : 50,000. Find the actual area in hectares of a region represented by a right-angled triangle of base 5 cm and height 6 cm. (3 mks)
- The angle of elevation of the top of a flag post from point A on level ground is  $24^\circ$ . The angle of elevation of the top of the flag post from another point B, nearer the flag post and 20 m from A, is  $36^\circ$ .
  - Use a scale of 1 cm to represent 5 m, and draw an accurate scale drawing to represent the above information. (4 mks)
  - Find the height of the flag post. (2 mks)
  - Calculate the distance from point B to the top of the flag post. (2 mks)
  - Determine the distance of A from the top of the flag post. (2 mks)
- A plane leaves town P for town Q on a bearing of  $120^\circ$  and a distance of 300 km. It then flies 400 km on a bearing of  $050^\circ$  to town R. By scale drawing, find the distance and compass bearing of P from R. (3 mks)

4. Three ports A, B, and C are situated such that port A is 140 km on a compass bearing of  $N60^{\circ}E$  from port B. Port C is 180 km on a compass bearing of  $S30^{\circ}E$  from A. A ship S is docked 90 km on a bearing of  $190^{\circ}$  from port B.
  - a) Using a scale of 1 cm to represent 20 km, draw a diagram to show the position of ports A, B, C, and ship S. (4 mks)
  - b) Using your diagram, find:
    - c) The distance between the ship and port A. (1 mk)
    - ii) The distance and bearing of the ship from port C. (2 mks)
    - iii) The distance from B to C. (1 mk)
    - iv) The compass bearing of S from A. (2 mks)
5. Four schools - Mucagara, Kerugoya, Kiamutugu, and Kiburia - are situated such that Kerugoya is 22 km from Mucagara on a bearing of  $220^{\circ}$ . Kiamutugu is east of Mucagara and 6 km away, while Kiburia is 8 km on a compass bearing of  $S42^{\circ}E$  from Kiamutugu.
  - a) Using a scale of 1:200,000, draw a scale diagram showing the relative positions of the four schools. (5 mks)
  - b) Using your diagram, determine:
    - c) The distance and bearing of Kiburia from Kerugoya. (2 mks)
    - ii) The distance and compass bearing of Kerugoya from Kiamutugu. (3 mks)
6. Four towns - P, R, T, and S - are situated such that R is 90 km directly north of P, and T is on a bearing of  $295^{\circ}$  from P at a distance of 75 km. S is on a compass bearing of  $N30^{\circ}W$  from T and a distance of 40 km.
  - a) Using a scale of 1 cm to represent 10 km, make an accurate scale drawing to show the relative positions of the towns. (4 mks)
  - b) Find:
    - c) The distance and bearing of R from T. (2 mks)
    - ii) The distance and bearing of S from R. (2 mks)
    - iii) The compass bearing of P from S. (2 mks)
7. Two airplanes, T and S, leave airport A at the same time. S flies on a bearing of  $062^{\circ}$  at 600 km/h, while T flies on a bearing of  $290^{\circ}$  at 750 km/h.
  - a) Use a suitable scale to draw a diagram showing the relative positions of the airplanes after two hours. (3 mks)
  - b) Use your diagram to determine:
    - c) The distance between the two airplanes. (2 mks)
    - ii) The bearing of T from S. (1 mk)
    - iii) Airplane T later flew east at the same speed for one hour. Show its final position on the diagram and determine:
      - iv) Its final distance from A. (2 mks)
      - v) Its final bearing from S. (1 mk)

# SECTION TWO

## Model Samples Papers

### **i Model Sample Paper 1**

#### **SECTION A: (50 MARKS)**

**Answer all the question in this section**

- Evaluate without using tables or calculators. (3mks)

$$\frac{\frac{3}{7} \text{ of } 28 \div 80 \times -\frac{40}{3}}{-2 \times 5 + (14 \div 7) \times 3}$$

- Felix has four times as many ducks as hens and three-quarters as many turkeys as ducks.
  - If he has  $x$  hens, write down a simplified expression in  $x$  for the total number of birds (2mks)
  - Find the total number of birds given that the Felix has 45 turkeys.(2mks)
- Given that  $x = 4$ ,  $y = -2$ , and  $z = -3$  evaluate. (3mks)

$$\frac{2(x+z)^2 - (x-y)(y-z)}{4(x+y) - 2(y-z)}$$

- Using a ruler and a pair of compasses only, construct triangle PQR in which  $PQ = 5.2\text{cm}$   $QR = 7.5\text{cm}$  and Angle  $PQR = 45^0$ . By construction bisect angle PQR to meet line PR at a point M. find the ratio  $PM : MR$ .(3mks)
- Use square and square root table to evaluate to 4 significant figures, the expression. (3mks)

$$\sqrt{24.640 - (4.362)^2}$$

- The cost of a TV outside is US\$ 1200. Kelvin decides to buy one TV through an agent who deals with Japanese Yen. The agent charges him a commission of 5% on the price of the TV and further 2,000 Yen as import tax. To the nearest Ksh. how much will he need to send to the agent to obtain the TV, given that:- (3mks)

$$1 \text{ US\$} = 110.95 \text{ Yen}$$

$$1 \text{ US\$} = \text{Kshs. } 102.80$$

7. A two -digit number is such that the sum of the ones digit and the tens digit is 4. If the digits are reversed, the number formed exceeds the original number by 18. Find the number. (3mks)
8. Metal block of side 5.6 cm was melted and the molten material used to make a sphere. In three significant figures, find the radius of the sphere in metres (take  $\pi = \frac{22}{7}$ ) (3mks)
9. Solve the simultaneous equations (3mks)

$$\begin{aligned}x + 3y &= 9 \\4x - 8y &= -4\end{aligned}$$

10. Kinyua bought soya and millet at Ksh. 70 per kg and Ksh. 40 per kg respectively. He then mixed them and sold the mixture at Ksh. 60 per kg making a profit of 20%. Determine the ratio of soya to millet in mixture. (3mks)
11. An aircraft left Nairobi at 2245h on Monday and arrived in Cape Town on Tuesday at 0300h. It departed from Cape Town at 0330h and arrived in Washington DC at 0630h on Wednesday. Find the travel ling time for the whole journey from Nairobi to Washington DC took? (3mks)
12. A town B is 250 km due east of town A. Another town C is 200 km on a compass bearing of  $S40^0E$  from town B. use scale drawing to find the distance and bearing of town C from A. (4mks)
13. 1.784 kg of sugar whose density is  $1.08g/cm^3$  and 0.744kg of salt whose density is  $1.04g/cm^3$  are mixed together for a certain experiment. What is the density of the mixture in  $kg/m^3$ ? ( Give the answer to 4. s.f) (3mks)
14. The interior angle of a regular polygon is 5 times the exterior angle. How many sides does the polygon have? (2mks)
15. Find the least number of steps in staircase if, when I go up 3 steps at a time, 4 steps at a time or 6 steps at a time, there is always 1 step remaining at the top. (3mks)
16. An arc of a circle of length 37.4cm subtends an angle of  $153^0$  at the centre of the circle. Calculate the area of the sector bounded by this arc. Take  $\pi = \frac{22}{7}$ . (4mks)

**SECTION B (50 MARKS)**  
*Attempt all the questions*

17. Kirote and Kanze bought a bus which could carry 50 passengers when full. The bus uses Nairobi-Machakos route and charges Ksh. 160 per passengers for one way. The bus makes three trips between the two towns daily. The cost of fuel was Ksh. 2500 per day. The driver and the conductor are paid

## SECTION TWO

allowances of Ksh. 1500 and Ksh. 800 respectively. A further of Ksh. 5,000 per day are set aside for maintenance.

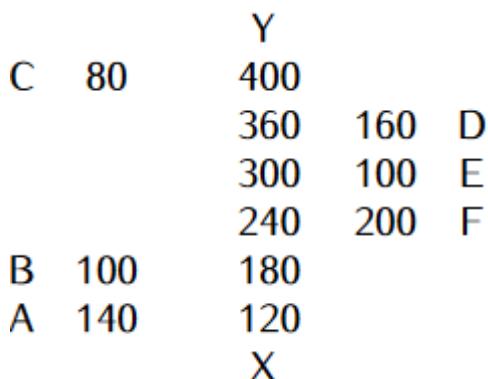
a) One day the bus was full on every trip.

i) How much money was collected from the passengers that day? (2mks)

ii) How much was the net profit. (3mks)

b) On another day, the minibus was 80% full on average for the trips how much did Kanze get if the days profit was shared to the ratio 2:3? (5mks)

18. The following measurement were recorded in a field book of a farm using  $XY = 450\text{ m}$  as the base line



a) Using a scale, draw an accurate map of the farm. (3mks)

b) Determine the actual area of the farm in hectares. (4mks)

c) If the farm is on sale at Ksh. 280,000 per hectare, find how much the farm costs. (3mks)

19. Four ships are at sea such that a steam-liner W is 250 km on a bearing of  $030^0$  from a cargo ship Q. A trawler M is 350 km on a bearing of  $145^0$  from the cargo ship Q and a yacht R is due west of Q and on a compass bearing of  $N60^0W$  from M

a) Using a scale of 1 cm=50 km, draw an accurate scale drawing showing the positions of W, Q, M, and R. (5mks)

b) By measurement from your scale drawing determine:

i) The distance and bearing of R from W. (3mks)

ii) The distance WM. (1mk)

iii) The distance RM. (1mk)

20. An electronics manufacturer makes speakers and sells them to a distributor at a profit of 20%. The distributor sells the speakers to a retailer at a profit of 25%. The retailer finally sells the speakers to customers at a profit of 40%.

a) A customer paid Ksh. 1,680 for a portable speaker. Find how much it had cost the manufacturer to make the speaker. (3mks)

b) A retailer bought a speaker which had cost the manufacturer Ksh. 5,600 to make. Calculate the amount he paid for it. (3mks)

- c) A customer bought a pioneer speaker at Ksh. 6,300. Calculate the much the distributor had paid for the same radio. (2mks)
- d) Express as a percentage the amount the customer paid for the speaker in (c) above to the amount the distributor paid for it. (2mks)
21. Four trucks A, B, C, and D are used to transport 42,000 bags of maize to a depot. However, trucks A and B together take 40 days to transport the same number of bags while trucks C and D together take 25 days. Truck A carries  $1\frac{1}{2}$  times the number of bags B carries and C carries  $1\frac{4}{5}$  times as much as D.
- a) Determine the number of bags of maize transported by each truck per day.(5mks)
- b) All the trucks A, B, C, and D work together for 5 days, after which truck C and D are withdrawn. A and B work together for another 5 days after which truck A breaks down. How long does truck B take to complete the rest of the remaining bags? (5mks)

**i Model Sample Paper 2**

**SECTION A: (50 MARKS)**

**Answer all the question in this section**

1. Use the tables of squares and square roots only to find the value of; (3mks)

$$(0.0847)^{\frac{1}{2}} + (2.35)^2$$

2. Without using calculator, evaluate.(3mks)

$$\frac{2\frac{4}{5} + 3\frac{1}{5} \div \frac{7}{8} \text{ of } 4\frac{4}{7} - \frac{3}{5}}{1\frac{3}{4} \div 3\frac{1}{2} - \frac{5}{12} + \frac{2}{3}}$$

3. Two years ago, Musa was three times as old as Ahmed. In four years' time, Musa will be twice as old as Ahmed. Find their present ages (4mks)
4. On a map with a scale of 1:16,000, a banana plantation covers an area of  $70cm^2$ . Find the area of the plantation in hectares. (3mks)
5. A Canadian tourist came to Kenya with sterling pounds 4500 which he exchanged to Kenyan shillings. He spent a quarter of the money and exchanged the rest to sterling pounds on leaving. How much in sterling pounds did he receive? (4mks)

Exchange rate in Ksh. per pound

Buying	Selling
119.74	119.88

## SECTION TWO

6. The sums of interior angles of two regular polygons of sides' n-1 and n are in the ratio 3:4. Calculate;
  - a) The value of n. (2mks)
  - b) The size of interior angle of each polygon. (2mks)
7. John bought six exercise books and three text books for Ksh. 660. If he had bought three similar exercise books and six text books, he would have paid Ksh. 210 more. How much would he pay for five exercise books and five text books? (3mks)
8. Find the least number of biscuits that can be packed into carton boxes which contain either 4 or 9 or 24 or 40 with none left over.(3mks)
9. In order to complete a certain job in 10 days, a company employs 30 workers to work at the rate of 8 hours a day. Determine how long it would take 20 workers working at the rate of 12 hours a day to complete the same job. (2mks)
10. Simplify the expression. (3mks)

$$\frac{y^2 - 4x - 4xy + y}{(y + 1)(4x^2 - xy)}$$

11. A two digit number is such that the sum of the ones and the tens digit is 11. If the Digits are reversed; the original number exceeds the new number formed by 9. Find the number. (3mks)
12. Joyce on her cycling practice cycled on a bearing of  $125^0$  for 5.5km, then on a bearing of  $180^0$  for 6.7km finally he turned northwards for 12.5km, by scale drawing determine the distance and compass bearing of her final position from the starting point. (4mks)
13. Njoki bought Mike a suit for Ksh. 3600. This price was such that the shopkeeper had allowed a discount of 10% on the marked price in order to make a profit of 20%. Calculate both the marked price and the buying price of the suit. (3mks)
14. Bronze is made by mixing tin, brass, and zinc in the ratio 3:5:4. A piece of bronze contains 7.2 kg of tin. Determine the total mass of brass and zinc in that piece of steel. (2mks)
15. A cylindrical solid of length 40cm and radius 7cm is melted to form 10 similar spherical solids. Determine the radius of each spherical solid. (3mks)
16. Starting from midnight the minute hand of a clock moved so that the clock is showing 24 minutes past midnight.
  - a) Find the angle through which the minute hand has moved. (1mk)
  - b) Given that the minute hand is 14 cm long, calculate the length of the arc it describes in that time. (2mks)

**SECTION B (50 MARKS)***Attempt all the questions*

17. Simon sold an article at Ksh. 5,100 after allowing his customer a 15% discount on the marked price of the article. In so doing he made a profit of 25%.
- Calculate:
    - The marked price of the article. (2mks)
    - The price at which Simon had bought the article (2mks)
  - If Simon had sold the same article without giving a discount. Calculate the percentage profit he would have made to three significant figures. (3mks)
  - To clear his stock, Simon decided to sell the remaining articles at a loss of 20%. Calculate the price at which he sold each article. (3mks)
18. (a) Using a ruler and a pair of compasses only construct triangle ABC in which  $BC = 8\text{cm}$ ,  $AB = 6\text{cm}$  angle  $ABC = 67.5^0$  (4mks)
- Measure AC and angle ACB (2mks)
  - Construct a circle that passes through AB, AC and BC (3mks)
  - What is the radius of this circle? (1mk)
19. A bus had 48 passengers at the begining of the journey, 20 passengers alighted at the first stop while 12 boarded. 8 of those who boarded at the first stop alighted at the second stop and 16 got in. The bus did not stop again upto the final destination. The charges from the starting point were Ksh. 100 upto the first stop, Ksh. 150 upto the second stop, and Ksh. 220 upto the final destination.
- How many passengers alighted at the final destination? (3mks)
  - How many passengers were ferried by the bus through the journey? (2mks)
  - How much money was collected during the trip? (5mks)
20. (a) The angle of elevation of the top of a tree from a point P on the horizontal ground  $28.5^0$ . From another point Q, five meters from P towards the base of the tree, the angle of elevation of the top of the tree is  $37.2^0$ . By scale drawing calculate to one decimal place the height of the tree.(4mks)
- Four points A, B, C and D lie on the same plane. Point A is due southwest of point B. Point C is 70 Km on a bearing of  $S60^0E$  from B. Point D is equidistant from B, Q and C.
    - Using the Scale: 1 cm represents 10km, construct a diagram showing the position of B, C, Q and D. (4mks)
    - Determine the distance between A and B (1mk)
    - Determine the bearing of D from B. (1mk)

## SECTION TWO

21. Water flows through a cylindrical pipe of diameter 2.8 cm at a speed of 70m/min.
- Calculate the volume of the water delivered by the pipe per minute in litres. (3mks)
  - A cylindrical storage tank of depth 5m is filled by water from this pipe and at the same rate of flow. Water begins flowing into the empty storage tank at 9.30 p.m. and is full by 2.10 a.m. Calculate the area of the cross-section of this tank in  $m^2$ . (4mks)
  - A family consumes the capacity of this tank in one month. The cost of water is Ksh. 25 per thousand litres plus a fixed basic charge of Ksh. 1800.60. Calculate the cost of this family's water bill for the month. (3mks)

### i Model Sample Paper 3

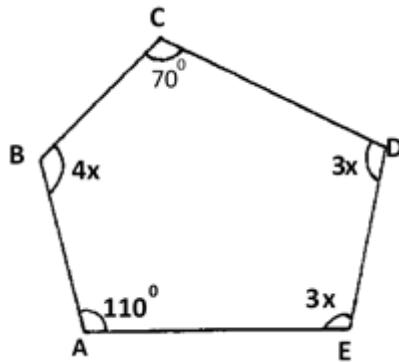
#### **SECTION A: (50 MARKS)**

**Answer all the question in this section**

- Without using a calculator or mathematical tables, evaluate: (3mks)

$$\frac{15 - 6 \times -14 - 21 \div -3}{9 \times 3 + -8(5 - (-2))}$$

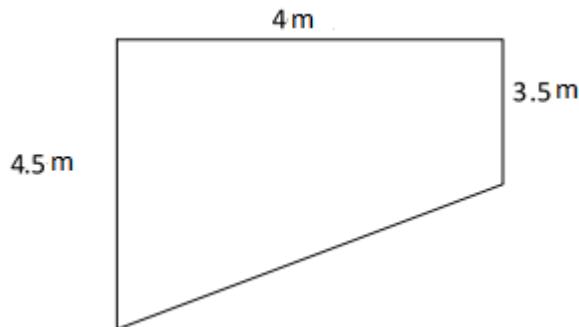
- Hannah finds that she needs 29 beacons placed 42 m apart when she surveys a length of road. If she was to place the beacons 49 m apart,
  - How many beacons would she need? (2mks)
  - What is the shortest distance that can be divided into exact portions of 25, 30 or 40 metres giving a remainder of 3 metres. (2mks)
- Two friends Wakuraya and Muchoki have goats. Wakuraya has more goats than Muchoki and if Muchoki gives Wakuraya one of his goats, Wakuraya will have twice as many goats as Muchoki. If Wakuraya gives Muchoki one of his goat, they will have an equal number of goats. How many goats does each have. (3mks)
- Simiyu has six times as many one-shilling coins as twenty-shillings coins, a third as many five-shillings coins as one-shilling coins and four times as many ten-shillings coins as twenty-shilling coins. If in total he has Ksh. 228, find the number of coins he has. (3mks)
- The figure below shows the angles of a polygon ABCDE.



Obtain the size of each of the following angles,

- $\text{CBA}$  (2mks)
- $\text{CDE}$  (1mk)

6. The figure below is a cross-section of a swimming pool.



- Find the capacity of the pool in litres given that its length is 27m. (3mks)
  - Given that the water has a density of  $1g/cm^3$ , calculate the mass of water in the pool (2mks)
7. Food aid 492,850 French Franc was donated to the Turkana drought stricken area. The food was purchased from United States of America (USA) and paid for in US dollars. Calculate the exact value of the food aid in dollars if: (3mks)  
 $1 \text{ French Franc} = \text{ksh } 12.80$  and  $1 \text{ Us dollars} = \text{ksh } 102.50$
8. Munene is paid commission at the rate of 5 cents in every shilling on all goods he sells. During one month he sold 15 computers at Ksh. 14,200 each, 6 DVD players at Ksh. 6,800 each and 8 laptops at Ksh. 31,500 each. Calculate the total commission he earned in that month. (3mks)
9. Sarah bought 3 plates and 6 jugs at a total cost of Ksh. 324. If she had bought 1 plate more and 2 jug less, she would have spent Ksh. 48 less. On another occasion Sarah bought 5 plates and 5 jugs at the same prices. Find how much she spent on the second occasion. (3mks)
10. A boat Q is 300m due west of boat P. Another boat R is 240m on a bearing of  $155^{\circ}$  from boat Q. Using scale drawing, find the distance ad bearing of boat R from P. (4mks)

## SECTION TWO

11. Two coils which are made by winding copper wire of different gauges and length have the same mass. The first coil is made by winding 250 metres of wire with cross sectional diameter 2.1mm while the second coil is made by winding a certain length of wire with cross-sectional diameter 1.4mm. Find the length of wire in the second coil. (3mks)
12. To prepare hay for the daily animals, green grass is dried and then processed into bails. In the process the mass of green grass decreases in the ratio 5:17. Determine the mass of green grass which must be processed to produce 1.2 tonnes of dry hay.(3mks)
13. Evaluate using squares and square root tables. (4mks)

$$[\sqrt{27.47} + (0.701)^2]^2$$

14. Simplify: (3mks)

$$\frac{(4a+b)^2 - (b-4a)^2}{(a+b)^2 - (b-a)^2}$$

15. The radius of a cylindrical tin is increased by 24% while its height is reduced by 18%. In 4 significant figures find the percentage increase in the volume of the milk in the tin.(3mks)
16. Taps A and B can fill a water tank in 40 minutes and 30 minutes respectively while C can empty in 20 minutes. If the three taps are turned on for 15 minutes then B and C closed. How long would it take before the tank is filled? (3mks)

## **SECTION B (50 MARKS)**

***Attempt all the questions***

17. A rectangular aluminum sheet whose density is  $2.2g/cm^3$  is 1.2 m long, 80 cm wide, and 1.5 mm thick. A square of side 10 cm is cut off from each of the four corners of the rectangle and the remaining part folded into an open cuboid.
  - a) Calculate:
    - i) To the mass of the empty cuboid to the nearest Kg. (4mks)
    - ii) The capacity of the cuboid in litres (2mks)
  - b) If the cuboid is filled with alcohol whose density is  $0.75g/cm^3$ , calculate the mass of the cuboid when full of alcohol. (4mks)
18. A bus left Dodoma on Thursday evening and traveled to Mombasa according to the travel table below and arrived there on Saturday morning.

<b>Dodoma</b>	<b>Departure</b>	<b>2015h</b>
<b>Arusha</b>	<b>Arrival</b>	<b>0045h</b>
	<b>Departure</b>	<b>0125h</b>
<b>Namanga</b>	<b>Arrival</b>	<b>1140h</b>
	<b>Departure</b>	<b>1220h</b>
<b>Nairobi</b>	<b>Arrival</b>	<b>1950h</b>
	<b>Departure</b>	<b>2130h</b>
<b>Mombasa</b>	<b>Arrival</b>	<b>0650h</b>

- a) Calculate the total:
- b) Travelling time for the whole journey. (3mks)
- i. Stoppage time in all stations (3mks)
  - ii. Time taken for the whole journey (2mks)
- c) Given that the average speed of the bus for the whole journey is 60km/h, calculate the distance between Dodoma and Mombasa. (2mks)
19. Three cargo ships P, Q, and R are at sea such that ship Q is 300 km on a compass bearing of  $N35^0W$  from ship P. ship R is 420 km on a bearing of  $220^0$  from ship Q. Another ship S is reported to be 475 km from ship R and due south of ship P.
- a) Draw an accurate scale drawing showing the positions of ships P, Q, R, and S. (use scale 1cm=50 km) (5mks)
- b) Use your scale drawing to determine the:
- i) Distance of ship S from P. (1mk)
  - ii) Distance and bearing of ship S from ship Q (2mks)
  - iii) Compass bearing of ship S from ship R (2mks)
20. (a) A solution whose volume is 50 litres is made up of 40% water and 60% alcohol. When n litres of water are added the percentage of alcohol drops to 50%. Find the value of n. (4mks)
- (b) 15 litres of water is added to the new solution. Calculate the percentage of alcohol in the resulting solution. (2mks)
- (c) If 6 litres of the solution in (b) above is added to 3 litres of the original solution, calculate in the simplest form, the ratio of water to alcohol in the resulting solution. (4mks)
21. Gabriel, an artisan made an article and sold it to a wholesaler to a profit of 25%. The wholesaler sold it to a retailer at a profit of 32%. The retailer finally sold the article to a customer at a profit of 45%.
- a) If Gabriel used sh 1200 to make the article, find how much the customer paid for it. (3mks)

## SECTION TWO

- b) A customer paid sh 4785 for another article. Calculate how much the wholesaler had paid for it. (3mks)
- c) During a clearance sale the retailer reduced his prices by 20%. Find the percentage profit the retailer made on an article which had cost Gabriel sh 4000 to make it.(4mks)

### **i Model Sample Paper 4**

#### **SECTION A: (50 MARKS)**

**Answer all the question in this section**

- Evaluate without using a calculator or Mathematical tables leaving your answer in the simplest form. (3mks)

$$\frac{\frac{3}{9} \text{ of } \left(\frac{2}{5} - \frac{1}{10}\right)}{\left(4 + \frac{2}{3}\right) \div \left(1 + \frac{4}{3}\right)}$$

- Three similar 21 inch television sets and five similar 17 inch television cost Ksh. 145,000. The difference between the cost of two 21 inch television sets and three 17 inch television Sets is Ksh. 8,000. Calculate the price of a 21-inch television set and that of 17-inch Television set. (3mks)
- A Kenya bank buys and sells foreign currencies as shown.

	Buying (Ksh)	Selling (Ksh)
1 Euro	115.15	115.26
100 Japanese Yen	90.37	90.45

A Japanese traveling from France to Kenya had 4500 Euros. He converted all the 4500 Euros to Kenya shilling at the bank. While in Kenya, he spent a total of Ksh.225,600 and then converted the remaining Kenya shilling to Japanese Yens at the bank. Calculate the amount in Japanese Yen that he received. (3mks)

- Use tables of square and square roots to evaluate.(4mks)

$$(0.3264)^2 + \sqrt{364.5}$$

- A business woman usually makes a profit of 32% by selling a pair of shoes at sh 1650. If she reduces the price of the shoes by sh 250, calculate the percentage profit she will now make. (3mks)
- Kinyua bought soya and millet at Ksh.75 per kg and Ksh.45 per kg respectively. He then mixed them and sold the mixture at Ksh.65 per kg making a profit of 30%. Determine the ratio of soya to millet in mixture. (3mks)

7. The curved surface area of a cylindrical container is  $2540\text{cm}^2$ . If the radius of the container is 28cm, calculate to one decimal place the capacity of the container in litres ( $\text{Take}\pi = \frac{22}{7}$ ). (4mks)
8. The radius of a cylindrical tin is decreased by 20% while its height is increased by 12%. In 4 significant figures find the percentage change in the volume of the milk in the tin and indicate whether it is an increase or a decrease. (3mks)
9. Wang'ombe, a farmer, uses  $\frac{1}{2}$  of his land to plant cassava,  $\frac{1}{5}$  for planting sweet potatoes,  $\frac{1}{3}$  of the remainder for grazing and the rest for maize plantation. If he uses 10 hectares for grazing, determine how much land he uses for maize plantation. (4mks)
10. Pauline bought 7 kg of rice and 3 KDF cakes at a total cost of Ksh. 450. Wathitha bought 5 kg of rice and 8 KDF cakes at the same prices and spent Ksh. 70 less than Pauline. Calculate the cost of each item. (3mks)
11. (a) Using a pair of compasses and a ruler only construct a triangle ABC such that  $AB = 6\text{cm}$ ,  $BC = 8\text{cm}$  and  $\angle ABC = 135^\circ$ . (2mks)  
 (b) Construct the height of triangle ABC in (a) above taking BC as the base and measure the height. (2mks)
12. One interior angle of a polygon is equal to  $40^\circ$  and each of the other interior angles are  $125^\circ$ . Find the number of sides of the polygon. (3mks)
13. Given that  $x = 2$ ,  $y = -1$ , and  $z = 3$ , find the value of; (3mks)

$$\frac{5x^2 - 4y^2z + 6y}{4x^2z + 5y^3 - z^3}$$

14. Three boats X, Y, and Z are situated in such a way that boat Y is 450 m on a compass bearing of  $S55^\circ E$  from boat X. Boat Z is 600 m on a bearing of  $035^\circ$  from boat Y.  
 a) Draw a sketch showing the positions of X, Y, and Z. (1mk)  
 b) Calculate the distance of boat Z from boat X. (2mks)
15. Pipe P can fill a tank in 6 minutes while pipe Q can fill the same tank in 10 minutes. On the other hand, a drainage pipe R, can empty the same full tank in 5 minutes. If Pipes P and Q are opened and left running for 3 minutes and then the drainage pipe R is then opened and all three left running, determine how many more minutes it takes to fill the drum. (4mks)

**SECTION B (50 MARKS)**  
*Attempt all the questions*

16. Mucagara secondary school hired a number of buses and matatus to ferry the students for an academic trip to Mombasa. The number of buses hired were two-third as many as matatus and no vehicle made a double trip. Each

## SECTION TWO

Matatu can carry a maximum of 18 students and a bus can carry three times as many students. The hire charges were Ksh. 8,400 per bus and Ksh. 4,800 per matatu. A total of Ksh. 62,400 was spent on transport for the trip.

- a) Calculate the number:
- Of buses hired (4mks)
  - Of matatus hired (1mk)
- b) Calculate the number of students who took the trip if each vehicle was filled to capacity.(3mks)
- c) Each student contributed Ksh 150 towards the cost of the trip and the school paid the remaining amount. Calculate the amount the school paid. (2mks)
17. Three business dealers Kavula, Nzuki, and Ngina decided to buy a bus. They agreed to pay for the bus in the ratio 6:7:5. Using the marked price of the bus, Kavula was supposed to contribute Ksh. 120,600 more than Ngina. However, the sales agent allowed them a 10% discount after paying it on cash.
- a) Determine:
- The marked price of the bus. (3mks)
  - The price at which they bought the bus. (2mks)
  - How much more Kavula and Ngina paid than Nzuki? (2mks)
- c) The dealers agreed that they would share monthly profits from the bus in the ratio of their contribution after setting aside 10% of the profits. If in one month the bus realized Ksh. 120,900 as the profit, how much did Ngina got? (3mks)
18. A train left Nairobi on Monday evening and traveled to Mombasa according to the travel time table below. The train arrived in Lamu on Wednesday morning of the same week.
- | Station     | Departure | Arrival   |
|-------------|-----------|-----------|
| Nairobi     | 1830h     |           |
| Mtito Andei | 0120h     | Arrival   |
|             | 0225h     | Departure |
| Voi         | 0845h     | Arrival   |
|             | 1130h     | Departure |
| Mombasa     | 1710h     | Arrival   |
|             | 1805h     | Departure |
| Lamu        | 0620h     | Arrival   |
- a) Calculate the time the train took to travel between: (4mks)
- Nairobi and Mtito Andei
  - Mtito Andei and Voi
  - Voi and Mombasa
  - Mombasa and Lamu

- b) Determine the total time for the whole journey. (4mks)
- c) Given that the railway road distance between Nairobi and Lamu is 1505 Km, calculate the average speed for the whole journey. (2mks)
19. The following measurements were recorded in a field book using XY as the base line.  $XY = 400m$ .
- |       | Y   |       |
|-------|-----|-------|
| C 40  | 360 |       |
|       | 320 | 120 D |
|       | 240 | 160 E |
|       | 200 | 160 F |
| B 80  | 160 |       |
| A 100 | 60  |       |
|       | X   |       |
- a) Using a scale of 1: 4000, draw an accurate map of the farm. (4mks)
- b) Determine the actual area of the farm in hectares. (4mks)
- c) If the farm is on sale at Ksh. 100,000 per hectare, find how much the farm costs. (2mks)
20. Karani bought some eggs at Ksh 96 per dozen and sold three-quarter of them at Ksh 280 per tray and the remainder at Ksh 270 per tray. In so doing she made a profit of Ksh 675. Given that one tray holds 30 eggs, calculate:
- a) The number of eggs she bought. (4mks)
- b) The percentage profit she made giving your answer to three significant figures. (3mks)
- c) The percentage profit she would have made if she sold all the eggs at sh 280 per tray. (3mks)

**i Model Sample Paper 5**

**SECTION A: (50 MARKS)**

**Answer all the question in this section**

1. Evaluate. (3mks)

$$\frac{(32 - (-16))}{-8} - \frac{(18 - (-)(-6))}{3}$$

2. The cost of 5 skirts and 3 blouses is Ksh. 1750. Afro's wife bought three skirts and one of the blouses for Ksh. 850. Find the cost of each item. (3mks)
3. Last year, Gaceru was four times as old as his son, Kairu; in four years' time the sum of their ages will be 55. Determine their present ages. (3mks)

## SECTION TWO

4. Given that  $x = 4$ ,  $y = -3$ , and  $z = -1$  evaluate. (3mks)

$$\frac{3(x+z)^2 - 2(x-y)(y-z)}{3(x+z) - (y-z)}$$

5. A plastic test tube is made up of hemispherical bottom and a cylindrical stem, both of internal radius 0.7cm.
- Calculate the capacity of the test tube, given that its height is 12cm. (3mks)
  - Given that the test tube is filled with a liquid of density  $0.75g/cm^3$ , calculate the mass in g of the liquid in the test tube (1mk)
6. A saleslady earns commission at the rate of 3.5 cents per shilling for all sales up to Ksh 350,000 and at the rate of 4.5 cents per shilling on any sales above that. During a certain month she sold 26 pieces of second hand computers at Ksh 14,500 per computer, 6 laptop batteries at Ksh. 4,500 each and 60 pieces of laptop chargers at Ksh 1,200 each. Calculate the total commission she earned in that month. (3mks)
7. Water and milk are mixed such that the ratio of the volume of water to that of milk are 4:1. Taking the density of water as  $1g/cm^3$  and that of milk as  $1.25g/cm^3$ , find the mass in grams of 2.4 litres of the mixture. (3mks)
8. An arc of a circle of length 9.24cm subtends an angle of  $108^0$  at the centre of the circle. Calculate the area of the sector bounded by this arc. Take  $\pi = \frac{22}{7}$ . (3mks)
9. When maize seeds are dried up after harvest the mass decreases in the ratio 7:15. Find the mass of dried maize which is obtained after drying 1725 kg of green maize. (3mks)
10. Bag X contains 672 sweets, bag Y 504 sweets and bag Z 360 sweets all of different types. The sweets are to be shared among a group of students in such a way each gets the same number of each type. Find the largest possible number of students.(3mks)
11. On a map with a scale of  $1 : 50000$ , a banana plantation covers an area of  $100 cm^2$ . Find the area of the plantation in hectares. (3mks)
12. The radius of a cylindrical tin is increased by 30% while its height is decreased by 25%. If the capacity of the old tin is  $400cm^3$ , find the capacity of the new tin. (3mks)
13. Three towns P, Q, and R are such that tow Q is 90 km on a bearing of  $147^0$  from town P and town R is 120 km on a bearing of  $057^0$  from town Q.
- Draw a sketch showing the positions of the three towns. (1mk)
  - Calculate the distance between town P and R. (2mks)
14. Find the exact value of: (4mks)  
 $1.\dot{3}\dot{2} - 0.5\dot{3}$

15. Simplify the following expression by reducing it to a single fraction (3mks)

$$\frac{4a - 5}{5} - \frac{2a - 3}{3} = \frac{1 - a}{6}$$

16. Starting from midnight the minute hand of a clock moved so that the clock is showing quarter to one.

a) Find the angle through which the minute hand has moved. (1mk)

b) Determine the area of the sector described by the minute hand given that its length is 10.5 cm to 4 s.f. (2mks)

### **SECTION B (50 MARKS)**

*Attempt all the questions*

17. Using the linear equations below:

a) Complete the tables below and then write down the point in coordinate form (x,y) as shown. (4mks)

$$2x + y = 6$$

x	0	1	2	3	4	5
$y=6-2x$	6			0		
Point	(0,6)			(3,0)		

$$3x - 4y = 9$$

x	-1	1	3	5	7	9
$y = \frac{3x-9}{4}$		-1.5			3	
Point		(1,-1.5)			(7,3)	

b) Use graphical method by plotting the two linear equations on the same grid to solve the simultaneous equations: (4mks)

$$\begin{aligned} 2x + y &= 6 \\ 3x - 4y &= 9 \end{aligned} \tag{0.71}$$

c) The line  $2x + y = 6$  meets x-axis and the line y-axis at the points A and B respectively. From the graph state the co-ordinates of A and B. (2mks)

18. A rectangular sheet measuring 80 cm by 50 cm and 2mm thick is made of copper whose density is  $2.5g/cm^3$ . A square of side 5 cm is removed from each corner of the rectangle and the remaining part folded to form an open cuboid.

a) Calculate:

i) The area of the copper which forms the cuboid.(2mks)

## SECTION TWO

- ii) The mass of the empty cuboid in Kilograms. (4mks)
- b) If the cuboid is filled with water whose density is  $1g/cm^3$ , calculate the mass of the cuboid when full of water. (4mks)
19. At 1400hr, two ships A and B leave port P and sail out to sea. Ship A sails at a steady speed of 55km/h on a bearing of  $060^0$  while ship B sails at a steady speed of 40km/h on a bearing of  $140^0$ . At 1800hr both ships radio back to port giving their positions. At the same time a third ship C gives its position as 300km due east of P.
- a) Using a ruler and a pair of compass only, construct a scale drawing showing the positions of P, A, B, and C at 1800hr. (4mks)
- b) Use your scale drawing to determine:
- Distance and compass bearing of B from A (4mks)
  - Distance of C from A (2mks)
  - Distance Of B from C (2mks)
20. In the year 2016 Muriithi had 40 more hens than cocks and half as many ducks as cocks. In the year 2017 his hens increased by 60%, his ducks increased by 40% and his cocks decreased by 20%. At the end of 2017 all his birds were 1335. Determine the percentage increase in the number of his birds in the year 2017. (10mks)
21. (a) Using a ruler and a pair of compasses only, construct triangle  $ABC$  in which  $BC = 6$  cm,  $AB = 8.8$  cm and angle  $ABC = 22.5^0$ . (4mks)
- (b) Measure AC and angle ACB. (2mks)
- (c) Construct a circle that passes through A, B and C. (3mks)
- (d) What is the radius of this circle? (1mk)

### **i Model Sample Paper 6**

#### **SECTION A: (50 MARKS)**

**Answer all the question in this section**

1. Evaluate: (3mks)

$$\frac{8 \times \frac{1}{3} of 9 \div 2 - \frac{2}{3} of 144 \div 12 + 2 \times 3}{\frac{3}{4} of 36 \div 3 - 4 \div \frac{2}{5} of 10 + 3 \times -2}$$

2. Nyamu bought 5 pairs of trousers and 3 pairs of socks for sh 2,540. If the cost of three pairs of trousers and 5 pairs of socks is sh 1780, calculate the cost of two pairs of trousers and 6 pairs of socks. (4mks)

3. Simplify: (3mks)

$$\frac{(4a+b)^2 - (b-4a)^2}{(a+b)^2 - (b-a)^2}$$

4. Taps P and Q can fill a water tank in 25 minutes and 20 minutes respectively while R can empty in 15 minutes. If the three taps are turned on for 12 minutes then Q and R closed. How long would it take before the tank is filled? (3mks)
5. The G.C.D of three numbers is 42 and their L.C.M is 1764. If two of the numbers are 84 and 294, what is the other smallest number? (3mks)
6. Gatungo sold a pullover to a customer for Ksh. 1240 after allowing her a 20% discount on the marked price. Find the price at which the pullover was marked. (2mks)
7. After harvesting the green maize, the seed are dried, treated and then packed in 70 kg bags. The mass of undried maize decreases in the ratio 4:7. Calculate the mass of the undried maize that must be dried to produce 8 such bags when packed. (3mks)
8. Hannah paid rent which was  $\frac{3}{10}$  of her net salary. She used  $\frac{1}{2}$  the remaining amount to make a down payment for a plot. She gave her mother Ksh. 3,200 and did shopping worth Ksh. 5,000 for herself. She saved the remainder which was Ksh. 12,800. How much was the down payment that she made. (4mks)
9. The interior angles of an hexagon are  $(2x + 30)^0$ ,  $(3x - 15)^0$ ,  $(2x + 45)^0$ ,  $3x^0$ ,  $(3x - 40)^0$  and  $x^0$ . Find the value of the smallest exterior angle. (3mks)
10. A Kenya company received y US Dollars. The money was converted into Kenya Shillings in a bank which buys and sells foreign currencies.

	Buying (in Ksh)	Selling (in Ksh)
1 Sterling Pound	125.78	126.64
1 Us Dollar	102.66	102.86

- a) If the company received Ksh.12,452,658, calculate the amount, y received to the nearest US Dollar (2mks)
- b) The company exchanged the above Kenya shillings into Sterling pounds to buy a car in Britain. Calculate the cost of the car to the nearest Sterling pound. (2mks)
11. item Juma, Ali and Hassan share the profit of their business in the ratio 2: 3: 5 respectively. If Juma receives ksh. 56, 000. How much profit did the hassan get. (3mks)
12. Three police posts are such that Q is on a bearing of  $220^0$  and 14 km from P while R is on a bearing of  $145^0$  and 10 km from P.
  - a) Using a suitable scale, draw a diagram to represent the above situation. (2mks)
  - b) From the scale drawing determine:
    - i) The distance and bearing of Q from R (2mks)

## SECTION TWO

13. Eighteen laborers takes 15 days to plough 6 acres of land. Find the number of laborers required to plough 8 acres in 12 days. (2mks)
14. Starting from noon the minute hand of a clock moved so that the clock is showing 18 minutes to one.
  - a) Find the angle through which the minute hand has moved. (1mk)
  - b) Given that the minute hand is 15 cm long, find the length of the arc it describes in that time. (2mks)
15. Two coils of the same mass are made by winding brass wire of different gauges and length. If the first coil is made by winding 324 m of the wire with 3.36 mm cross-sectional diameter and the second coil is made by winding a certain length of the wire with a cross-sectional diameter of 2.52 mm, find the length of the second coil wire. (3mks)
16. A Juakali artisan reduced the base radius of a cone shaped container by 15% but, increased its height by 45%. Find the in three s.f percentage change in its volume and state whether if the volume increased or decreased.(volume of a cone=  $\frac{1}{3}\pi r^2 h$ ) (3mks)

## **SECTION B (50 MARKS)**

***Attempt all the questions***

17. From a reservoir, water flows through a cylindrical pipe of diameter 0.3m at a rate of 0.28m per second.
  - a) Determine the number of litres of water discharged from reservoir in one hour. (4mks)
  - b) The water flows from the reservoir for 15 hours per day for 22 days per month and serves a population of 3,000 families. Determine the average consumption of water per family per month giving your answer to the nearest 1 litres. (4mks)
  - c) The water is charged at the rate of Ksh. 10.50 per 100 litres. Calculate to the nearest Kenya shilling the average water in a family per month. (2mks)
18. At noon, three ships P, Q, and R start together from port A and sail out to sea. Ship P sails at a steady speed of 40 km/h on a bearing of  $055^0$ . Ship Q sails steadily at a speed of 60km/h due east of A and ship R sails steadily at 50km/h on a bearing of  $152^0$ . At 1530hr all three ships radio back to port giving their positions.
  - a) Draw a sketch diagram showing the position of ships P, Q, and R at 1500hr. (1mk)
  - b) Use a ruler and a pair of compasses only to construct a scale drawing showing the positions of the ships P, Q, and R with respect to port A at 1500 hrs (1cm=25km). (5mks)
  - c) By measurement use your scale drawing to determine:
    - i) The distance and bearing of ship P from ship Q. (2mks)

- ii) The distance of ship R from ship Q. (1mk)  
 iii) The distance of ship P from ship R. (1mk)
19. (a) A small field was surveyed and the measurements recorded in a surveyor's field book as in the table below.
- |             | <b>180</b> | <b>0 F</b>  |
|-------------|------------|-------------|
|             | <b>150</b> | <b>42 E</b> |
| <b>C 75</b> | <b>120</b> |             |
|             | <b>90</b>  | <b>60 D</b> |
| <b>B 45</b> | <b>60</b>  |             |
| <b>A 0</b>  | <b>0</b>   |             |
- Using a scale of 1cm to 15 m make an accurate drawing of the map of the field. (4mks)
- (b)
- i) Find the area of the field. (3mks)  
 ii) Assuming that the baseline in (a) runs in a northern direction give the position of D relative to A using compass bearing and distance. (3mks)
20. Four trucks A, B, C and D are used to transport 27,000 bags of maize to a depot. However, trucks A and B together take 24 days to transport the same number of bags while trucks C and D together take 15 days. Truck A carries  $1\frac{1}{4}$  times the number of bags B carries and C carries  $1\frac{2}{5}$  times as much as D.
- a) Determine the number of bags of maize transported by each truck per day. (5mks)  
 b) All the trucks A, B C and D work together for 5 days, after which truck C and D are withdrawn. A and B work together for another 5 days after which truck A breaks down. How long does truck B take to complete the rest of the remaining bags? (5mks)
21. A solution whose volume is 160 litres is made up of 40% water and the rest alcohol. When  $x$  litres of water is added the percentage of alcohol drops to 30%
- a) Find the value of  $x$  (4mks)  
 b) If 1 litres of alcohol are added to the new solution, calculate the percentage of water in the resulting solution. (2mks)  
 c) A blend is made by mixing 25 litres of the solution in (b) above with 20 litres of the original solution. Calculate in the simplest form, the ratio of water to that of alcohol in the blend. (4mks)

## SECTION TWO

### Model Sample Paper 7

#### **SECTION A: (50 MARKS)**

**Answer all the question in this section**

- Evaluate (3mks)

$$\frac{1\frac{1}{4} \text{ of } 20 + 3\frac{3}{4} \div \frac{3}{8} - 4\frac{1}{2} \times 3\frac{1}{3}}{5\frac{4}{9} \times 1\frac{2}{7} - 4 \div \frac{2}{3} + \frac{3}{4} \text{ of } 12}$$

- Use squares and square root tables to evaluate. (4mks)

$$(0.9233)^2 + \sqrt{15.2453} - 0.223$$

- A UK tourist comes to Kenya with £, 65, 000. He pays 25% commission at the airport and his expenses in Kenya amounted to Ksh. 890,000. How much money did he remain with in Ksh? (Take 1 UK£ = Ksh.130.50) (3mks)
- All prime numbers less than 10 are arranged in a descending order to form a number which forms a quotient of 1883 with a certain number. Calculate the number (3mks)
- Solve the equation (2mks)

$$\frac{1}{2y} = \frac{3}{4y+1}$$

- Daniel deposited 48 different notes in the bank. He had eight times as many two-hundred shilling notes as one-thousand shilling notes and half as many one-hundred shilling notes as two-hundred shilling notes. The rest were fifty shilling notes. If he deposited a total of Ksh. 9,450, find the number of fifty shilling notes he deposited. (3mks)
- Esther bought maize and sorghum flour from a vendor. She then mixed them in the ratio 4:3. She bought the maize flour at ksh.50 per kg and the millet flour at ksh. 71 per kg. If she was to sell and make a profit of 25%. What should be the selling price of 1kg of the mixture? Give you answer correct to the nearest 10 cent. (3mks)
- Teresia sold a dress at Ksh. 1,290 after allowing a discount of Ksh. 160. If she did not allow any discount to the customer, she would have made a profit of 25%. Calculate the percentage profit she made. (3mks)
- Nyambura bought 4 sufurias and 6 cups for Ksh. 850 from a hawker. If she had bought 3 sufurias and 9 cups she would have saved Ksh. 55. Calculate the cost of one sufuria. (3mks)

10. Omondi is now two-third as old as his sister and twice as old as his younger brother. In six years' time Omondi's age will be 26 years less than the sum of ages of his sister and brother. Determine Omondi's present age. (3mks)
11. A Juakali artisan reduced the base radius of a cone shaped container by 15% but, increased its height by 45%. Find the in three s.f percentage change in its volume and state whether if the volume increased or decreased (volume of a cone =  $\frac{1}{3}\pi r^2 h$ ) (3mks)
12. The sum of the interior angles of an n-sided polygon is  $720^0$ . Find the value of n and hence give the name of the polygon. (3mks)
13. A rectangular water tank has a base measuring 4.5m by 2m. This tank has water to a height of 90cm. Water is then pumped into this tank continuously from 2030 hours to 2140 hours at the rate of 1.2 litres per second. Find the new depth of water in the tank after this period of time giving your result in metres. (3mks)
14. Four strings measuring 15cm, 20cm, 25cm and 30cm are cut into pieces of equal length so that exact number of pieces is obtained from each string without wastage. Find the longest length of each string. (2mks)
15. The number of students at Lorna Waddington High School is 600. On a particular day  $\frac{1}{5}$  of the boys and  $\frac{1}{4}$  of the girls attended a sports meeting. If 468 students were left behind, find how many more boys than girls attended the meeting. (4mks)
16. Using ruler and a pair of compasses only:
  - i) Construct triangle ABC in which BC = 7.5 cm and angle  $ABC = 105^0$  and angle  $BAC = 30^0$ . (3mks)
  - ii) Drop a perpendicular from A to meet line BC at P. Determine the area of triangle ABC. (2mks)

### **SECTION B (50 MARKS)**

***Attempt all the questions***

17. A protestant church hired a number of buses and matatus to transport a group of youths to Embu for a youths rally. The number of matatus was four times the number of buses. The hire charges were Ksh. 5,200 per matatu and Ksh. 9,000 per bus. The total cost of hiring the vehicles was Ksh. 59,600. Each matatu can carry 14 youths while a bus can carry three times as many.
  - a) Calculate:
    - i) The number of buses hired (4mks)
    - ii) The number of matatus hired (1mk)
  - b) Calculate the number of youths ferried to Embu if each vehicle was filled to capacity and no vehicle made a double trip. (3mks)
  - c) Each youth contributed Ksh. 200 towards the cost of the trip and the church paid the remaining amount. How much did the church pay? (2mks)

## SECTION TWO

18. Three business partners Muhammed, Maimuna , and Hasan decided to start a business. Muhammed contributed Ksh. 25,000, Maimuna contributed Ksh. 35,000, and Hasan contributed Ksh. 45,000 as business capital exclusive of rental fee. They rented a business premises that needed to be paid Ksh. 15,000 which was to be shared equally among them. They agreed to put 25% of the profit obtained annually back in to the business and share the rest in the ratio of their contributions. The business realized Ksh. 128,000 as gross profits.
- Find the ratio in which they contributed business capital and rental fees. (3mks)
  - Calculate:
    - The profits shared. (2mks)
    - Each partner's share of the profits. (3mks)
    - The percentage of Hasan share to the sum of Maimuna and Muhammed share. (2mks)
19. In 2008, the number of students in Mucagara Secondary School was 240, a 20% increase over the number of students in 2007. The number of students dropped by 5% in 2009 but, increased by 25% in the year 2010.
- Determine the number of students:
    - In the year 2007 (2mks)
    - In the year 2009 (2mks)
    - In the year 2010 (2mks)
  - Express as a percentage the increase in the number students in the year 2010 over the that in the year 2008 (2mks)
  - What was the percentage increase in student population between 2007 and 2009. (2mks)
20. Use the given linear equations to:
- Complete the table below (4mks)
- $$3x + y = 9$$
- | x             | -1      | 0 | 1 | 2     | 3 | 4 |
|---------------|---------|---|---|-------|---|---|
| $y = -3x + 9$ | 12      |   |   | 3     |   |   |
| Point         | (-1,12) |   |   | (2,3) |   |   |
- $$x - 2y = -4$$
- | x                   | -4     | -2 | 0 | 2 | 4     | 6 |
|---------------------|--------|----|---|---|-------|---|
| $y = \frac{x+4}{2}$ | 0      |    |   |   | 4     |   |
| Point               | (-4,0) |    |   |   | (4,4) |   |
- Using graph solve the simultaneous equations: (4mks)

$$\begin{aligned}x - 2y &= -4 \\3x + y &= 9\end{aligned}\tag{0.72}$$

- c) If the line  $3x + y = 9$  cuts the x-axis and y-axis at A and B respectively, state the coordinates of A and B. (2mks)
21. A Parrot flies from a tree X to another tree Y which is 70m on a bearing of  $035^0$  from X. From Y the dove flies 100 m due west to another tree Z and finally flies due south to another tree W which is on a bearing of  $230^0$  from X.
- a) Using a ruler and a pair of compasses only construct an accurate scale drawing showing the positions of X, Y, Z, and W. (scale : 1 cm=10 cm) (4mks)
- b) By measurement from your scale drawing determine:
- i) The distance and bearing of Z from X. (2mks)
  - ii) The distance of W from Z (1mk)
  - iii) The distance of W from X (1mk)
- c) The compass bearing of W from Y. (2mks)

### **i Model Sample Paper 8**

#### **SECTION A: (50 MARKS)**

**Answer all the question in this section**

1. Evaluate: (3mks)

$$\frac{6 \times \frac{1}{3} \text{ of } 12 \div 2 - \frac{1}{3} \text{ of } 36 \div 6 + 3 \times 4}{\frac{1}{4} \text{ of } 24 \div 2 - 24 \div \frac{3}{5} \text{ of } 10 + 5 \times (-2)}$$

2. Factorize completely (3mks)

$$(x + y)(4x - 5y) - (x + y)^2$$

3. Murimi, a cattle keeping farmer, has twenty five times as many cows as goats, and three-fifth as many sheep as goats.

- a) If there are g goats, write down a simplified expression in g for the total number of animals Murimi has. (1mk)
- b) Given that there are 75 sheep, calculate as a percentage the sum of goats and sheep to the number of cows Murimi had. (3mks)

4. Four interior angles of a hexagon are  $120^0$ ,  $115^0$ ,  $135^0$  and  $80^0$ . The fifth interior angle is twice times the sixth. Find, in degrees the sixth interior angle. (3mks)

## SECTION TWO

5. Munyiti uses  $\frac{1}{3}$  of his farm for planting mangoes,  $\frac{1}{4}$  for planting macadamia and  $\frac{2}{5}$  of the remainder for grazing and home stead. He still has 6 hectares of unused land. Find the size of land Munyiti used for planting mangoes. (3mks)
6. Use tables and squares roots to evaluate to 4 significant figures. (4mks)

$$(\sqrt{245.6} + (4.436)^2)^{\frac{1}{2}}$$

7. A speaker coil which is made by winding 750 m of copper wire of cross-sectional diameter of 1.89 mm has the same mass as another coil of copper wire with cross-sectional diameter of 2.1 mm. Find the length of the wire making the second coil. (3mks)
8. Three children shared some money. Rose got 0.5 of the money and Mutua 0.3 of the remainder. Loice received the rest which was Ksh. 400. How much did Mutua get? (4mks)
9. The sum of the interior angles of an n-sided polygon is  $1080^0$ . Find the value of n and hence calculate the size of each exterior angle. (3mks)
10. Three metal rods of lengths 243cm, 270cm and 198cm were cut into shorter pieces all of the same length to make window grills. Calculate the length of the longest piece that can be cut from each of the rods and hence the total number of pieces that can be obtained from the rods. (3mks)
11. A blend of juice is made from pineapple and passion. The cost of two limes of pineapple is Ksh. 162 and three limes of passion is Ksh. 414. In what ratio should the juice be mixed such that by selling the mixture at Ksh. 140 per lime a profit of 40% is realized? (3mks)
12. Awino bought a shirt for Ksh. 1400 and sold it to a customer at a profit of 35%. What is the marked price of the shirt if selling the shirt she had allowed her customer a 10% discount on the marked price? (4mks)
13. Sarah bought 3 plates and 6 jugs at a total cost of Ksh. 324. If she had bought 1 plate more and 2 jug less, she would have spent Ksh. 48 less. On another occasion Sarah bought 5 plates and 5 jugs at the same prices. Find how much she spent on the second occasion. (3mks)
14. A Kenyan tourist left Berlin, Germany for Nairobi, Kenya through Geneva, Switzerland. While in Geneva, he bought a watch worth 110.6 Deutsche marks. Using the exchange rates below:
  - 1 Swiss Franc = 1.58 Deutsche marks.
  - 1 Swiss Franc = 100.98 Kenya shillings.
15. Find the value of the watch in:
  - a) Swiss Francs (2mks)
  - b) Kenya shillings (1mk)

16. Using a ruler and a pair of compasses only construct triangle ABC such that  $AB = 5.8\text{ cm}$ ,  $BC = 7\text{ cm}$  and angle  $CBA = 75^\circ$ . Measure angle  $CAB$ . (3mks)
17. Two pipes A and B each running alone can fill a Jerican in 6 hours and 10 hours respectively. a drainage pipe C can empty the full Jerican in 15 hours. If pipe A and B are turned on and left running for  $1\frac{1}{2}$  hours and then the drainage pipe C is opened and all three left running, find how much longer it takes to fill the Jerican. (4mks)

**SECTION B (50 MARKS)***Attempt all the questions*

18. Peter bought a second hand probox and later sold it through a sales agent who charges 6.5% commission on the price of the probox. He received Ksh. 392,700 from the agent after the latter had deducted his commission. Peter incurred a loss of 30% on the price at which he had bought the probox.
  - a) Determine the price at which the agent sold the probox. (2mks)
  - b) Find the price at which Peter had bought the probox (3mks)
  - c) If the amount Peter paid for the probox was 25% less than the price of the new probox, calculate the price of the new probox. (3mks)
  - d) Express as a percentage the amount Peter received for his probox to its price when new. (2mks)
19. Nyakio and Anyango went to buy clothes for their business. Nyakio spent Ksh. 20,150 to buy a number of dresses and skirts from a wholesaler A at Ksh. 450 per dress and Ksh. 200 per skirt. Anyango bought the same number of dresses and skirts from wholesaler B where she paid 20% less per dress and 10% more per skirt. It was found that Anyango spent Ksh. 2,710 less than Nyakio.
  - a) Determine the number of dresses and skirts each cloth dealer bought.(4mks)
  - b) Nyakio sold all her clothes at a profit of 20% per dress and 40% per skirt. How much profit did she make? (3mks)
  - c) Anyango also sold all her clothes at a profit of 20% per dress and 40% per skirt. Calculate to three significant figures the percentage profit she made. (3mks)
20. A metal sheet measuring  $1\text{ m}$  long and  $80\text{ cm}$  wide is  $2\text{ mm}$  thick and its density is  $2.5\text{ g/cm}^3$ . A square of side  $x\text{ cm}$  is removed from each of the four corners and the remaining part folded to form an open cuboid.
  - a) Calculate the area of the remaining part in terms of  $x$ . (2mks)
  - b) Given that the area of the remaining part is  $0.76\text{ m}^2$ , calculate the value of  $x$  and hence state the internal dimensions of the cuboid.(3mks)
  - c) Calculate the mass in kg of the empty cuboid to two significant figures.(2mks)
  - d) If the cuboid is filled with a liquid of density  $d\text{ g/cm}^3$  and its mass when full of the liquid is 39.8 kg, calculate the value of  $d$ .(3mks)

## SECTION TWO

21. Use a ruler and a pair of compasses only for all the constructions in this question.
  - a) Construct a triangle  $ABC$  in which  $BC = 7\text{cm}$ ,  $AC = 9\text{cm}$ , and angle  $ACB = 135^0$  (3mks)
  - b) Measure AB and angle ABC (2mks)
  - c) From A drop a perpendicular to meet BC produced at D. (1mk)
  - d) Measure AD and hence calculate the area of triangle ABC. (2mks)
  - e) Mark a point E on AD such that area of triangle BEC is 1.5 the area of triangle ABC. (1mk)
  - f) Complete triangle BEC and measure EC. (1mk)
22. Town B is 250 km from a bearing of  $050^0$  from town A. Town C is 350 km from town B and on a compass bearing of  $S57^0E$  from town B. A fourth town D is on a bearing of  $240^0$  from town C and due south of town A.
  - a) Using a scale of  $1\text{cm} = 50\text{km}$ , draw an accurate scale drawing showing the positions of towns A, B, C, and D. (5mks)
  - b) By measurement from your scale drawing, determine:
    - i) The distance AC (1mk)
    - ii) The distance AD (1mk)
    - iii) The distance CD (1mk)
    - iv) The bearing of B from C (2mks)

### Model Sample Paper 9

#### **SECTION A: (50 MARKS)**

**Answer all the question in this section**

1. Evaluate: (3mks)

$$\frac{\frac{3}{5} \text{ of } 30 + 5\frac{5}{6} \div \frac{7}{12} - 2\frac{2}{3} \times 1\frac{1}{2}}{5\frac{5}{8} \times 1\frac{7}{9} - 1\frac{1}{4} \text{ of } 4\frac{4}{5} + 2\frac{4}{5} \div \frac{7}{10}}$$

2. A law firm bought 60 files at a total cost of Ksh. 10,000. Some files cost Ksh. 150 each while others cost Ksh. 250 each. Find the number of files which were bought at Ksh. 150 each. (3mks)
3. Mutheu is now 10 years older than her younger brother. Six years ago the She was three times as old as his brother. Find their present ages. (3mks)
4. Wambui marked a skirt at Ksh. 400 and sold it to a customer at a discount of 15%. Find the percentage profit She made if she had bought the skirt at Ksh. 280. (3mks)
5. Use tables to evaluate:- (4mks)

$$(6.342 + 3.289)^2 - \sqrt{432.85 - 124.45}$$

6. Osinya constructed a closed wooden box with external measurements 1.5 metres long, 1.4 metres wide and 0.6 metres high. The wood used in constructing the box was 10.0cm thick and has a density of  $0.75g/cm^3$ . Determine the:
- Volume in  $cm^3$  of the wood used in constructing the box. (3mks)
  - Mass of the box in kilograms correct to 1 decimal place. (1mk)
7. Factorize completely (3mks)

$$(x - 3y)(4x + 3y) - (x - 3y)^2$$

8. 16 workers working at the rate of 9 hours a day can complete a piece of work in 14 days. How many more workers working at the rate of 7 hours a day would complete the same job in 12 days. (3mks)
9. In a form one class there are 5 more boys than girls. On a certain day one-quarter of the boys and one-fifth of the girls went for a football games. If 8 students from this class went for the football game, find the number of students in the class. (3mks)
10. Kamau toured Switzerland from Germany. In Switzerland he bought his wife a present worth 150 Deutsche marks. If
- 1 Swiss Franc = 1.65 Deutsche marks
- 1 Swiss Franc = 100.90 Kenya shillings
- Find the value of the present in:
- Swiss Francs correct to 2 decimal places. (1mk)
  - Kenya shillings correct to the nearest Ksh (2mks)
11. Find the least number of biscuits that can be packed into carton boxes which contain either 10 or 15 or 21 or 24 with leaving 5 biscuits unpacked. (3mks)
12. A polygon of n sides has half of the interior angles  $140^0$  each and the rest  $160^0$  each. Find the value of n. (2mks)
13. A coffee dealer mixes two brands of coffee, P and Q to obtain 50kg of the mixture worth Ksh. 60. If brand P is valued at Ksh. 80 per kg and brand Q is valued at Ksh. 50 per kg. Calculate the ratio in its simplest form in which brands P and Q are mixed. (4mks)
14. In a 3 digit number, the hundreds digits is 3 more than the units digit and the tens digit is thrice the hundreds digit. If the sum of the digits is 12, find the three digits. Write the number. (3mks)
15. Three similar pieces of timber of length 150cm, 140cm and 180cm are cut into equal pieces. Find the largest possible area of a square which can be made from any of the three pieces. (3mks)

## SECTION TWO

16. Arrange the following fractions in descending order. (3mks)

$$\frac{3}{5}, \frac{8}{9}, \frac{1}{3}, \frac{4}{7}, \frac{3}{4}$$

### **SECTION B (50 MARKS)**

*Attempt all the questions*

17. Melvin bought a second hand minibus at later sold it through a sales agent who charged 8% commission on the price at which she sold the minibus. She received Ksh. 699,200 from the agent after she had deducted her commission. Melvin made a profit of 15% on the price at which she had bought the vehicle.

- a) Calculate the price at which the sales agent sold the minibus. (2mks)
- b) Find the amount Melvin paid for the minibus. (3mks)
- c) If the amount Melvin paid was 50% less than the price of the new minibus, calculate its price when new. (3mks)
- d) Express as a percentage the amount Melvin received for the minibus to its price when new. (2mks)

18. Given linear equations  $2x + y = 7$  and  $5x - 3y = 12$ :

- a) Complete the following tables: (4mks)

$$2x + y = 7$$

x	-1	1	3	5	7
$y = 7 - 2x$	9				
Points	(-1,9)				

$$5x - 3y = 12$$

x	-3	0	3	6	9
$y = \frac{5x-12}{3}$			1		
Points			(3,1)		

- b) Use graphical method to solve the simultaneous equations: (4mks)

$$\begin{aligned} 2x + y &= 7 \\ 5x - 3y - 12 &= 0 \end{aligned} \tag{0.73}$$

- c) If the line  $2x + y - 7 = 0$  cuts x-axis and y-axis at point A and B respectively, state the coordinates of A and B. (2mks)

19. (a) Solution whose volume is 100 litres is made up of 30% water and 70% milk. When y litres of water are added the percentage of milk drops to 40%. Find the value of y. (4mks)

- (b) Twenty five litres of water is added to the new solution. Calculate the percentage of milk in the resulting solution. (2mks)
- (c) If 8 litres of the solution in (b) above is added to 16 litres of the original solution, calculate in the simplest form, the ratio of water to milk in the resulting solution. (4mks)
20. Kiptanui's farm produced 13,600 bags of wheat in 2015 which was a decrease of 15% over the production in 2014. In 2016, the production was a 25% increase of the previous year but, in 2017 Kiptanui farm produced 16,405 bags of wheat.
- Calculate the number of bags of wheat Kiptanui's farm produced:
    - In 2014 (2mks)
    - In 2016 (2mks)
  - What was the percentage decrease in production in 2017 over that of the previous year? (2mks)
  - Determine the percentage increase in production in 2017 over that in 2014. Give your answer in 3 significant figure (2mks)
  - Calculate the percentage increase in production in 2016 over that in 2014. (2mks)
21. The external measurements of a closed wooden box are 1.0 m long, 70 cm wide, and 40 cm high. The wood used in making the box is 1.5 cm thick and has a density of  $0.8g/cm^3$ . Given that the box contains 25 packets of tools and each packet holds a dozen tools each weighing 108.5 g, calculate:
- The volume of wood used in making the box. (4mks)
  - The mass of the empty box in kilograms to four significant figures. (3mks)
  - The total mass of the box in kilograms to 3 significant figures. (3mks)

**i Model Sample Paper 10**

**SECTION A: (50 MARKS)**

**Answer all the question in this section**

1. Without using a calculator, evaluate: (3mks)

$$\frac{3\frac{1}{5} + \frac{2}{7} \text{ of } 1\frac{3}{4} - \frac{7}{10}}{1\frac{3}{4} - 1\frac{4}{5} \div 3\frac{3}{5} + 3\frac{3}{4}}$$

2. The sum of the ages of three friends Kiama, Murugi, and Naomi is 78 years. Murugi is two and a third as old as Naomi and seven years older than Kiama. Calculate their ages. (3mks)
3. A shirt which is marked for ksh. 750 is sold to a customer for Ksh. 673.50. What percentage discount is the customer allowed? (2mks)

## SECTION TWO

4. Starting from noon the minute hand of a clock moved so that the clock is showing 25 minutes to one.
  - a) Find the angle through which the minute hand has moved. (1mk)
  - b) Given that the minute hand is 14 cm long, find the length of the arc it describes in that time. (2mks)
5. A cylindrical tank of diameter 2.8 m and height 2.0 m is three-quarter full of water. This water is transferred to an empty rectangular container measuring 1.4 m long and 80 cm wide. Calculate the height of the water in the container in centimeters. (3mks)
6. Three towns P, Q, and R are situated such that town Q is 40 km on a bearing of  $060^0$  from town P. Town R is 90 km on a bearing of  $150^0$  from town Q.
  - a) Draw a sketch showing the positions of towns P, Q, and R. (1mk)
  - b) Calculate:
    - i) The size of angle PQR (1mk)
    - ii) The distance of R from P (2mks)
7. A school bought 40 books at a total cost of Ksh 10,800. Some books cost Ksh 200 each while others cost Ksh 300 each. Find the number of books which were bought at Ksh 200 each. (3mks)
8. Muthoni is now four times as old as his daughter. In sixteen years' time she will be twice as old as her daughter. Find their present ages. (3mks)
9. Ali sold goods which were marked at Ksh. 450, 000 allowing a discount of 5% to the customer. If he received Ksh. 35,055 as a commission for this sale, calculate the percentage rate of commission he was paid. (3mks)
10. Solve the following expression (3mks)
$$\frac{3y - 4}{2} - \frac{1 - 2y}{3} = \frac{2y - 1}{4}$$

11. (a) The angles of a triangle are  $(2x - 15)^0$ ,  $(x + 35)^0$  and  $(5x - 40)^0$ . Write down an equation in x with three terms only for the sum of the angles of the triangle. (1mk)  
(b) Solve the equation in (a) above and hence find the size of the largest angle. (2mks)
12. Gladys house is 25 km from the office where she works. She uses her car to travel to and from her office every day for 5 days a week. Her car consumes petrol at the rate of 1 litre for every 12 km and petrol costs Ksh. 112.20 per litre. Allowing 4 weeks for holidays in a year, calculate the amount of money Gladys spends on petrol going to and from her office in one year. (3mks)

13. The radius of a water can in form of cylinder is increased by 20% while its height decreased by 15%. If the capacity of the old can is  $250\text{cm}^3$ , find the capacity of the new can. (3mks)
14. In a certain school there are 30 more boys than girls. One-quarter of the boys and two-thirds of the girls are boarders. If there are 255 boarders, find the number of students in that school. (3mks)
15. After work a hawker had four times as many ten-shilling coins as twenty-shilling coins, eight times as many five-shilling coins as twenty-shilling coins and thrice as many one-shilling coins as ten-shilling coins. After counting his money he found that he had a total of Ksh. 560. Calculate the number of coins he had. (3mks)
16. Given that  $x = 4$ ,  $y = -3$ , and  $z = -1$  evaluate. (3mks)

$$\frac{3x^2yz^2 - 4xy^2z^2 + 5x^2y^2z^2}{4xy^2z + 3x^2yz - x^2y^2z}$$

### **SECTION B (50 MARKS)**

*Attempt all the questions*

17. Tum bought a number of T-shirts and a number of trousers at Ksh 100 and Ksh 250 respectively from a wholesaler in which he spend Ksh 9,500. Kerich bought the same number of T-shirts and trousers from another wholesaler where he paid 25% more for a T-shirt and 14% less for a trouser. Therefore, Kerich spent Ksh 550 less than Tum.
- Determine the number of T-shirts and trousers each guy bought. (4mks)
  - If Tum sold all his clothes at a profit of 40% per T-shirt and 20% per trouser, determine the profit he would make. (3mks)
  - Similarly, if kerich sold all his clothes at a profit of 20% per T-shirt and 40% per trouser, calculate the percentage profit he would make on the sale of all his clothes. (3mks)
18. A rectangular sheet is 90 cm long and 70 cm wide. The sheet is 1.8 mm thick and made of material whose density is  $2.5\text{g/cm}^3$ . A square of side  $t$  cm is removed from each corner and the remaining part folded to form an open cuboid.
- Given that the area of the remaining part is  $A \text{cm}^2$ , write down an equation for  $A$  in terms of  $t$ . (2mks)
  - Calculate to one decimal place the mass of the empty cuboid in kg given that  $t = 5\text{cm}$ . (3mks)
  - Find the dimensions of the cuboid and hence calculate its capacity in litres. (3mks)
  - If the cuboid is filled with liquid whose density is  $d \text{g/cm}^3$  and its mass found to be 19.6 kg, calculate the value of  $d$ . (2mks)

## SECTION TWO

19. Boat A is 160 m from the base of a vertical cliff. From A the angle of elevation of the top of the cliff is  $38^{\circ}$ . From another boat B nearer to the base of the cliff, the angle of elevation of the top of the cliff is  $60^{\circ}$ . The two boats are on the same straight line in front of the cliff.
- Using a scale of  $1\text{cm} = 20\text{m}$ , draw an accurate scale drawing to represent the above information. (4mks)
  - Use your scale drawing to determine:
    - The height of the cliff (2mks)
    - The distance between the two boats (2mks)
    - The distance of boat B from the base of the cliff (2mks)
20. A truck left Uganda on Wednesday evening and travelled to Mombasa according to the travel time table below arriving there on Friday morning.

<b>Kampala</b>	<b>Departure</b>	<b>1925 h</b>
<b>Busia</b>	<b>Arrival</b>	<b>0430 h</b>
	<b>Departure</b>	<b>0605 h</b>
<b>Naivasha</b>	<b>Arrival</b>	<b>1315 h</b>
	<b>Departure</b>	<b>1425 h</b>
<b>Nairobi</b>	<b>Arrival</b>	<b>1645 h</b>
	<b>Departure</b>	<b>1910 h</b>
<b>Mombasa</b>	<b>Arrival</b>	<b>0440 h</b>

- Calculate:
    - Kampala to Busia
    - Busia to Naivasha
    - Naivasha to Nairobi
    - Nairobi to Mombasa
  - The time taken by the truck to travel from: (4mks)
  - The total travelling time between Kampala and Mombasa. (2mks)
  - The total stoppage time during the whole journey (2mks)
  - The average speed for the whole journey given that the distance between Kampala and Mombasa is 1995 km (2mks)
21. Wanja, Murimi, Kareb, and Elisha decided to buy a minibus. They acceded to pay for the cost of the minibus in the ratio  $9 : 7 : 4 : 5$ . Basing their calculations on the marked price of the minibus, they found that Wanja would pay Ksh. 320,500, more than Kareb. The sales operator however allowed them a 20% discount on cash payment.
- Calculate:
    - The marked price of the minibus. (2mks)

- ii) How much more Murimi and Kaleb paid than Wanja.(5mks)
- b) The four friends agreed to divide their profit obtained from the minibus in the ratio of their contributions after setting aside 15% of the profits for maintenance. If the minibus made Ksh. 96,500 as the profit during one month, how much did Elisha received that month? (3mks)

# SECTION THREE

## Answers to Problems to Solve

### 🔥 chapter 1: Natural Numbers

1. (a) 210  
(b) 200
2. (a) 19,171,311  
(b) 100,000
3. 7
4. 82
5. 480
6. 420
7. 331
8. 69
9. 45
10. 46

### 🔥 Rounding off

1. (a) 37,600,000  
(b) 320,000  
(c) 46.19
2. 400,000
3. (b) 149,680
4. 6.65
5. Grace (with 0.2)

 Operations

1. (a) Ksh. 1,374  
(b) Ksh. 50
2. (a) 904 cabbages  
(b) Ksh. 13,560
3. (a) 46  
(b) 374 rem 1036
4. (a) 1,226 cartons  
(b) 2,452 Kg  
(c) 11,034 Kg
5. (a) 2,250 Kg  
(b) 180 Kg

**6. Solution**

$$\text{No. of Passengers} = 23$$

At 1st stop:  $\Rightarrow$  12 alighted

$\Rightarrow$  9 boarded

The amount of money collected in the first stop

$$\Rightarrow 12 \times 50 = \text{Ksh.} 600$$

At 2nd stop:

6 of those in the 1st stop alighted ( $9 - 6 = 3$  remained)

$$\text{Money collected: } \Rightarrow 6 \times (70 - 50) = \text{Ksh.} 120$$

In the final destination:

$\Rightarrow$  11 boarded from the beginning of the journey

$\Rightarrow$  3 boarded from the first stop

$\Rightarrow$  12 boarded from the second stop

(a) No. Passengers alighted at final destination:

$$\Rightarrow (11 + 3 + 12) = 26$$

(b) Total passengers:  $\Rightarrow (23 + 9 + 12) = 44$

(c) Money collected in the trip: 1st stop = Ksh.600

2nd stop = Ksh.120

Final destination:  $\Rightarrow 11 \times 85 = \text{Ksh.} 935$

$$\Rightarrow 3 \times (85 - 50) = \text{Ksh.} 105$$

$$\Rightarrow 12 \times (85 - 70) = \text{Ksh.} 180$$

$\therefore$  Money collected in the whole trip:

$$\Rightarrow (600 + 120 + 935 + 105 + 180) = \text{Ksh.} 1,940$$

## SECTION THREE

7. (a) (i) Tens  
(ii) Tenth  
(iii) Ones  
(iv) Hundredth  
(b) Thousands

### 🔥 chapter 2: Factors

1. (a)  $11 \times 13^2$   
(b)  $3 \times 5 \times 7$   
(c)  $2^2 \times 3^2 \times 5^2$   
(d)  $2 \times 5^2 \times 7$   
(e)  $7^2 \times 11^2$   
(f)  $2 \times 7^2 \times 11$   
(g)  $11^2 \times 17$   
(h)  $2 \times 3^2 \times 7 \times 11$   
(i)  $11^2 \times 13$   
(j)  $3 \times 331$

### 🔥 chapter 3 Divisibility Test

1. a) Yes

- b) No  
c) No  
d) Yes  
e) Yes

2. 5172354 (iii)

3. a)

- i) 2  
ii) 1  
iii) 7  
iv) 3  
b) i) 4

ii) 5

4. a) Yes

- b) Yes  
c) No

 Chapter 4: G.C.D and L.C.M

1. 90, 180, and 450

2. 5 and 12

3. 24

4. 5:45 pm

5. a) 90

b) 24

6. 21

7. 12:30 pm

8. 8:25 am

9. 275

10. 11

11.  $17.64m^2$

12. 46 pieces

13. 7

14. 1764

15. 1700 ml

16.  $0.16m^2$

17. n=1540

18. a)  $48 = 2^4 \times 3$

b)  $60 = 2^2 \times 3 \times 5$

19.  $144 m^2$

 Chapter 5: Integers

1. -2

2. Ksh. 8,280

3. -2

4.  $\frac{19}{39}$

5.  $4\frac{4}{19}$

## SECTION THREE

6. a) -3
- b) -5
- c) 0

### 🔥 Chapter 6: Fractions

1. Ksh. 200,000
2. Ksh. 240,000
3. Ksh. 22,500
4. ksh. 1,200
5.  $\frac{4}{51}$
6.  $3\frac{4}{5}$
7. 4
8. 400%
9. 2
10.  $\frac{3}{5}$
11.  $\frac{2}{135}$
12. a) 4  
    b) 2
13. Ksh. 18,750
14. 420 students
15. a) Ksh. 48,000  
    b) Ksh. 12,000  
    c) Jane=Ksh. 4,000  
        Jepchonge=Ksh. 2,666.70  
        Chepkoech=Ksh. 1,333.30

### 🔥 Chapter 7: Decimals, Squares, and Square Roots

1. 0.2334
2.  $\frac{5}{8}$
3. 68.24

4.  $\frac{5}{9}$
5.  $2\frac{1}{11}$
6.  $5\frac{9}{11}$
7.  $\frac{3}{50}$
8.  $\frac{41}{110}$
9. 1827.3
10. 0.6717
11.  $\frac{1567}{2475}$
12. 0.04666
13. 0.001903
14.  $r = 0.977 \text{ cm}$
15.  $T = 0.3079\pi \text{ sec}$

### Chapter 8: Algebraic Expression

1. 4
2. 3
3.  $3.4m$
4. Ksh. 400
5.  $\frac{1}{80}$
6. Albina=48 years  
Son=18 years
7. 4
8. a)  $\frac{29}{4}x + 48$   
b) 512
9. Mother=64 years;  
Jane=16 years
10.  $2(p - 2q)(p + 2q)$
11.  $\frac{2}{3}$
12. 125 coins

### SECTION THREE

13. Osinya= 96 years  
Son=32 years
14. 24 years
15. 50 years
16. 51 men
17. 36 women
18. Awiti=32 years \ Wafula=24 years\ Najala=12 years
19. 32 years
20. Beans=150 bags; Peas=200 bags
21. a) 4  
b) 3
22. 4.36%
23. (a) i)  $(x + 9)$  years  
ii)  $(4x + 18)$  years  
iii) 7 years or 2 years  
iv) 35 years or 30 years  
v) 46 years or 26 years
24. (a) i)  $\frac{480,000}{x}$   
ii)  $\frac{480,000}{x-4}$   
(b) 8 Women  
(c) 2:3  
(d) 0.75 ha



### Chapter 9: Rate, Ratio, Proportion, and Percentage

1. 7 : 5
2. 4 days
3. 3 : 1
4. Ksh. 108
5. 15 hours 9 minutes
6. 5 days

7. 40 m
8. 108 mangoes
9. 24 days
10. 4 more days
11. 60
12. 18 women
13. 2 : 1
14. 2 : 3
15. a) 8.16%  
b) 12.49%
16. 11.11%
17. 3 : 7
18. Ksh. 690
19. 5 days
20. 21 days
21. 3 : 2
22. 27 men
23. 13 hours 20 minutes
24. 36 minutes
25.  $324\text{ cm}^3$
26. 39.26%
27. a) 16 minutes 40 seconds or  $16\frac{2}{3}$  minutes  
b) 15 minutes
28.  $480\text{ cm}^3$
29. a)  $58,520\text{ cm}^3/\text{s}$   
b) 6.2 m  
c) 16 minutes 44 seconds

### SECTION THREE

30. a) Fraction of work of potter C alone:  $\frac{9}{40}$

Fraction of all potters working together:  $\frac{3}{5}$

Fraction of work they did for 40 minutes:  $\frac{3}{5} \times \frac{4\theta}{6\theta} = \frac{2}{5}$

Fraction of work undone:  $\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$

Fraction of work done by A and C in 20 min:

$$\Rightarrow \left(\frac{1}{a} + \frac{9}{40}\right) \frac{2\theta}{6\theta} = \left(\frac{1}{3a} + \frac{3}{40}\right)$$

Time taken by potter A to complete the remaining work:  $\frac{53}{30}$

$$\therefore \left[\frac{3}{5} - \left(\frac{1}{3a} + \frac{3}{40}\right)\right] \div \frac{1}{a} = \frac{53}{30}$$

$$\Rightarrow \left(\frac{21}{40} - \frac{1}{3a}\right) \times a = \frac{53}{30}$$

$$\Rightarrow \frac{40}{21} \times \frac{21}{40} a = \frac{21}{10} \times \frac{4\theta}{21} = 4$$

$\therefore A = 4$  hours working alone.

b)  $\left(\frac{9}{40} + \frac{1}{4} + \frac{1}{b}\right) = \frac{3}{5}$

$$\Rightarrow \frac{1}{b} = \left(\frac{3}{5} - \frac{19}{40}\right) = \frac{1}{8}$$

$\therefore B = 8$  hours working alone.

c)  $\frac{9}{40} + \frac{1}{4} = \frac{19}{40}$

$\therefore A$  and  $C$  alone takes :  $2\frac{2}{19}$  hours or 2hrs 6 min

31. a) Ksh. 19.50

- b) i) Ksh. 84.20

- ii) 22.02%

32. a)  $x = 40$  litres

- b) 50%

- c) 1 : 2

33. a)  $P = 84$  litres

$$Q = 112$$
 litres

$$R = 224$$
 litres

- b) ksh. 142.30

- c) 44.1%

34. a) i) 2,500 cows

- ii) 3,100 cows

- iii) 3,720 cows

- b) i) 24%

- ii) 48.8%

 Chapter 10: Length, Area, Volume, and Capacity

1.  $\left(\frac{180x}{p}\right) \text{ cm}$
2.  $\left(\frac{9p}{\pi}\right) \text{ cm}$
3. 56.0 m
4. 22 cm
5. 22.2 litres
6. 10 cm
7. a)  $61,288 \text{ cm}^3$   
b) 45.97 Kg
8. 8.96 Kg
9. 5040 litres
10.  $1,264 \text{ cm}^2$
11.  $QR = 12 \text{ cm}$

$$PQ = 8 \text{ cm}$$

$$PR = 18 \text{ cm}$$

12. a) i) 16.23 cm  
ii)  $539.46 \text{ cm}^2$
- b) 5.99 cm
- c)  $450.86 \text{ cm}^2$

 Chapter 11: Mass, Weight, and Density

1.  $36.24 \text{ cm}$
2.  $5 \text{ g/cm}^3$
3.  $1.5 \text{ m}$
4.  $945 \text{ Kg}$
5.  $7,529 \text{ Kg/m}^3$
6. 240 m
7. a)  $29,508 \text{ cm}^3$   
b) 28.03 Kg  
c) 73.0 Kg

## SECTION THREE

### 🔥 Chapter 12: Time

1. 2:51 pm
2. 2:05 pm
3. 5:51 pm
4. 5 times
5.  $60 \text{ km/h}$

### 🔥 Chapter 13: Linear Equations

1.  $x = 3$   
 $y = -\frac{1}{3}$
2.  $x = -1$   
 $y = 2$
3.  $x = 0$   
 $y = 2$
4. Pen=Ksh. 15  
Exercise book=Ksh. 20
5. Kiama=Ksh. 45,000
6. Plate=Ksh. 15  
Spoon=Ksh. 9
7. Socks=Ksh. 220  
Trouser=Ksh. 355
8. 100 mangoes
9. a)  $(9x + 6)$  cows  
b) 32 cows
10. Cup=Ksh. 60  
Plate=Ksh. 36
11. Blade=Ksh. 6  
Pen=Ksh. 30
12. a) Pencil=Ksh. 3  
Blade=Ksh. 9  
b) 16 Pencils
13. Ksh. 1,860

 Chapter 14: Commercial Arithmetic

1. 2,093 Japanese Yens
2. 2,930 US dollars
3. 1,046 Us dollars
4. 17,030 South African Rands
5. Ksh. 345
6. Ksh. 370,354
7. a) 24%  
b) Ksh. 2,880
8. 56.25%
9. 38.89%
10. Ksh. 680,000
11. Ksh. 41,310
12. a) Ksh. 300  
b) Ksh. 200
13. Paying through his account in UK by Ksh. 5,131,971
14. a) Ksh. 3,148,535  
b) 112,710 South African Rands
15. 6.18%
16. Ksh. 2500
17. Ksh. 840
18. Ksh. 16,200
19. Ksh. 17,000
20. 8.5%
21. a) Ksh. 4,500  
b) Ksh. 100,500  
c) Ksh. 18,736  
d) Ksh. 1,642,105.30

### SECTION THREE

22. a) 215 Kg  
b) i) 45.3%  
ii) 50%
23. a) Ksh. 4,455  
b) Ksh. 1,400  
c) Ksh. 7,700

#### 🔥 Chapter 15: Co-ordinates and Graphs

1. (a)  $x=2$ ;  $y=2$   
(b) Q(0,2.5); P(0,-4)
2. a)  $y = \frac{8+2x}{3}$

<b>x</b>	-2	-1	0	1	2	3	4	5
<b>y</b>	$\frac{4}{3}$	2	$\frac{8}{3}$	$\frac{10}{3}$	4	$\frac{14}{3}$	$\frac{16}{3}$	6

$$y = 5x - 6$$

<b>x</b>	-2	-1	0	1	2	3	4	5
<b>y</b>	-16	-11	-6	-1	4	9	14	19

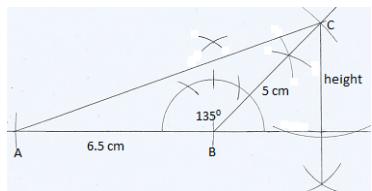
- c) The two graphs are straight lines  
d)  $x=2$ ;  $y=4$

#### 🔥 Chapter 16: Angles and Plane Figures

1. 5 sides  
2. 6 sides  
3. 12 sides  
4. a) 6 sides  
b) 108 cm  
5. a) 12 sides  
b)  $150^0$   
6. 12 sides  
7. 5 sides  
8.  $n=10$ ; Decagon  
9.  $n=9$ ; Octagon  
10.  $\theta = 141^0$

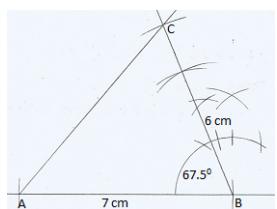
 Chapter 17: Geometrical Constructions

1. a)



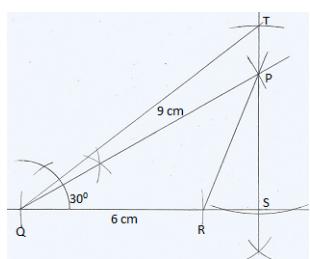
b)  $\text{Area} = 11.375 \pm 0.325 \text{ cm}^2$

2. a)



$AC = 7.3 \pm 0.1 \text{ cm}$

3. a)



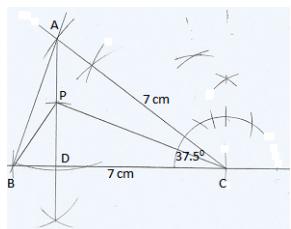
c)  $PS = 4.5 \pm 0.1 \text{ cm}$

$\text{Area} = 13.5 \pm 0.3 \text{ cm}^2$

e)  $RT = 6.3 \pm 0.1 \text{ cm}$

$\angle TQR = 38 \pm 1^\circ$

4. a)



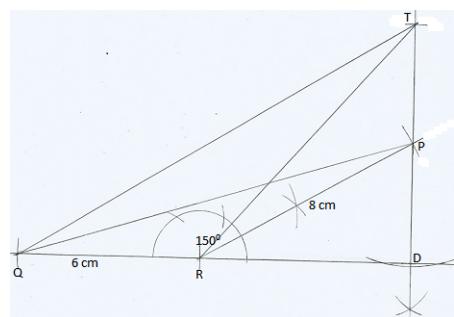
b)  $AB = 4.4 \pm 0.1 \text{ cm}$

d)  $AD = 4.2 \pm 0.1 \text{ cm}$

f)  $\angle ABC = 56 \pm 1^\circ$

### SECTION THREE

5. a)

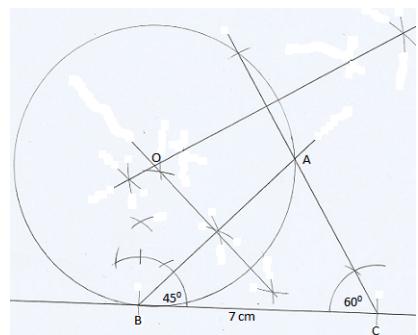


b)  $PQ = 13.6 \pm 0.1 \text{ cm}$

$\angle PQR = 17 \pm 1^\circ$

e)  $\angle QRT = 132 \pm 1^\circ$

6. a)

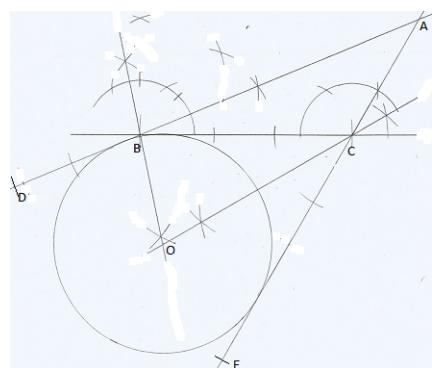


b)  $AB = 6.3 \pm 0.1 \text{ cm}$

$AC = 5.2 \pm 0.1 \text{ cm}$

d)  $Radius = 4.1 \pm 0.1 \text{ cm}$

7. a)



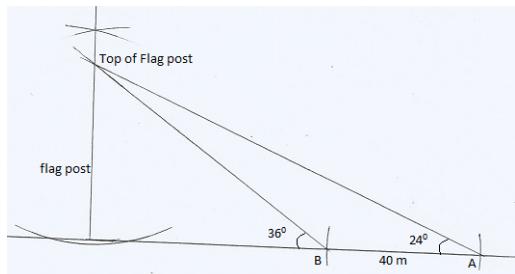
b)  $AC = 4.4 \pm 0.1 \text{ cm}$

e)  $Radius = 3.6 \pm 0.1 \text{ cm}$

 Chapter 18: Scale Drawing

1. 375 ha

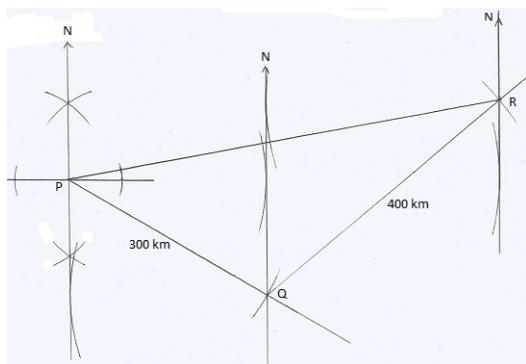
2. a)



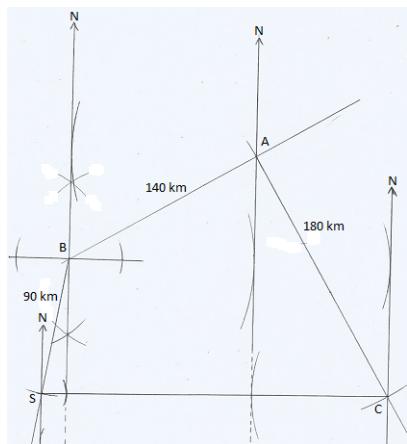
b) Height =  $23 \pm 1$  m

c)  $31.5 \pm 1$  m

d)  $39 \pm 1$  m

 3.  $PR = 58 \pm 1$  m; Compass bearing =  $S80 \pm 1^{\circ}W$ 


4. a)



b) i)  $212 \pm 2$  Km

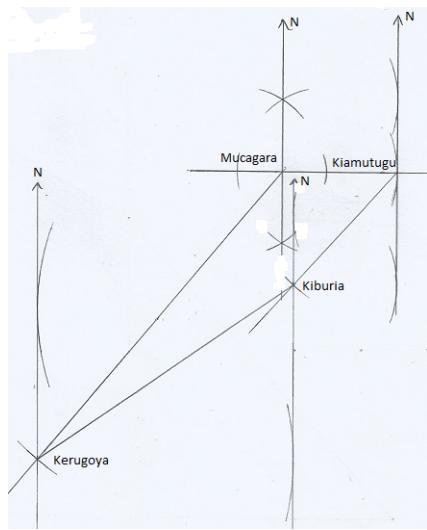
ii)  $228 \pm 2$  Km

iii)  $230 \pm 2$  Km

iv)  $S41 \pm 1^{\circ}W$

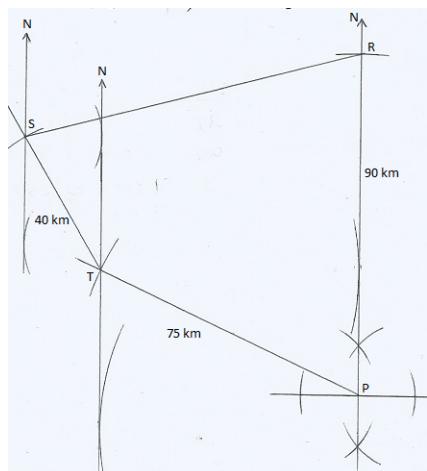
### SECTION THREE

5. a)



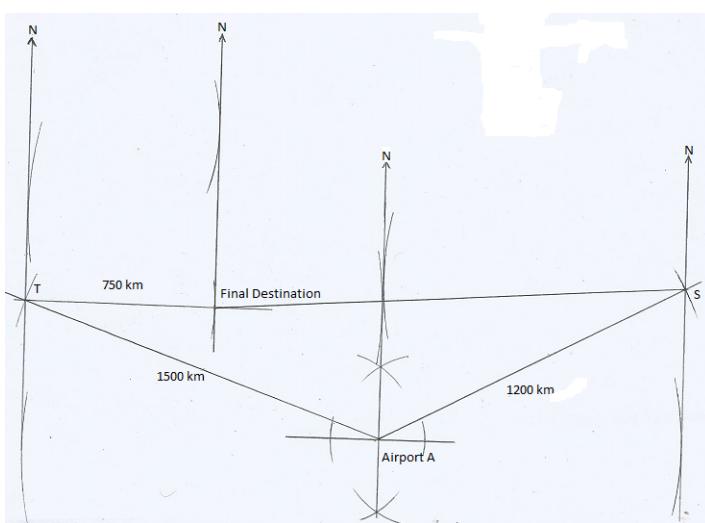
- b)  $16.4 \pm 0.2 \text{ km}; 056 \pm 1^\circ$   
 c)  $24.4 \pm 0.2 \text{ km}; S51 \pm 1^\circ W$

6. a)



- b) i)  $100 \pm 1 \text{ Km}; 50 \pm 1^\circ$   
 ii)  $92 \pm 1 \text{ Km}; 256 \pm 1^\circ$   
 iii) Compass bearing =  $N53 \pm 1^\circ W$

7. a)



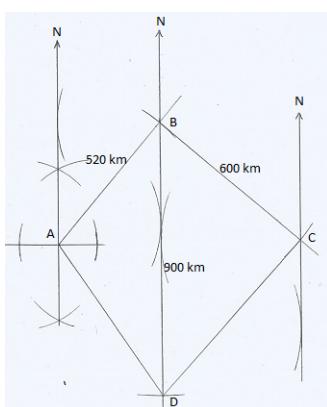
b) i)  $2625 \pm 15 \text{ Km}$

ii)  $268 \pm 1^\circ$

c) i)  $840 \pm 15 \text{ km}$

ii)  $266 \pm 1^\circ$

8. a)



b) i)  $N34 \pm 1^\circ W$

ii)  $S42 \pm 1^\circ W$

c) i)  $600 \pm 10 \text{ Km}$

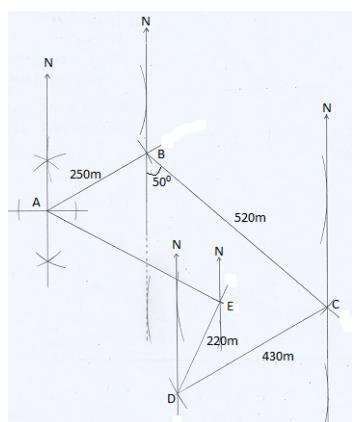
ii)  $690 \pm 10 \text{ Km}$

d) i)  $310 \pm 1^\circ$

ii)  $269 \pm 1^\circ$

9. a)

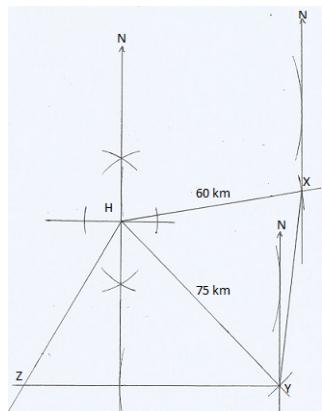
### SECTION THREE



c) i)  $430 \pm 5 m$ ;  $298 \pm 1^{\circ}$

ii)  $S35 \pm 1^{\circ}E$

10. a)

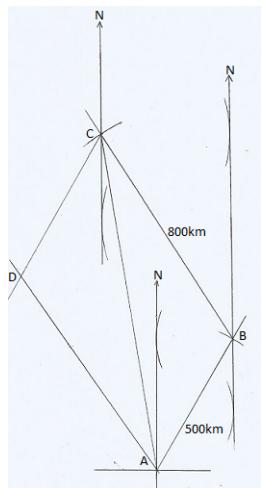


b) i)  $64 \pm 1 Km$ ;  $186 \pm 1^{\circ}$

ii)  $63 \pm 1 Km$

iii)  $112 \pm 1 Km$ ; Compass bearing =  $S55 \pm 1^{\circ}W$

11. a)

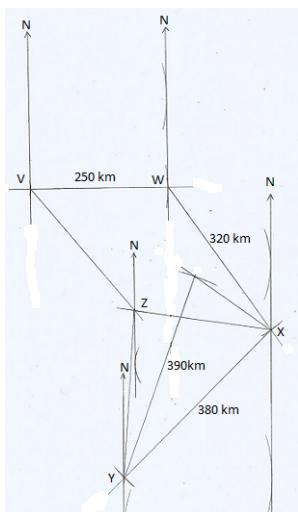


b)  $5 hrs 36 \pm 3 min$

c)  $171 \pm 1^{\circ}$

e)  $287 \pm 1^{\circ}$

12. a)

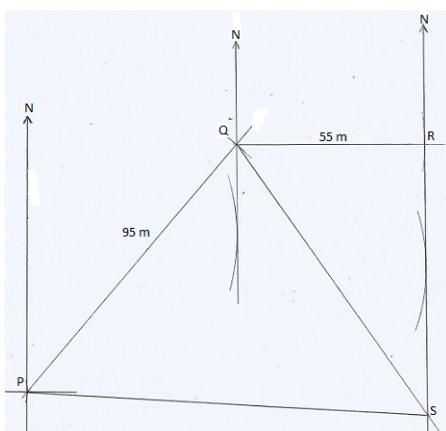


b) i)  $295 \pm 5 Km$

ii) Compass bearing of Y from W is  $S9 \pm 1^{\circ}W$

c)  $170 \pm 5 Km$

13. a)



b) i)  $RP = 138 \pm 1 m$ ; Compass bearing of P from R is  $S58 \pm 1^{\circ}W$

ii)  $RS = 79 \pm 1 m$

iii)  $PS = 118 \pm 1 m$ ;  $093 \pm 1^{\circ}$

# SECTION FOUR

## Answers to Model Sample Papers

🔥 Model Sample Paper 1

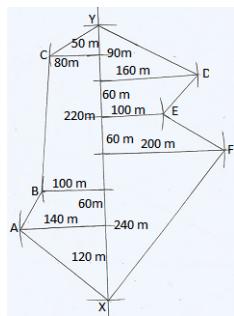
### SECTION A (50 MARKS)

1.  $\frac{1}{2}$
2. a)  $8x$   
b) 120 birds
3.  $-\frac{2}{3}$
4. PM:MR=3:7
5. 2.3691
6. Ksh. 131,381
7. 13
8. 0.0347 m
9.  $x = 3; y = 2$
10. 1 : 2
11. 31 hrs 15 minutes
12.  $AC = 410 \pm 5 \text{ km}$
13.  $1068 \text{ kg/m}^3$
14. n=12 sides
15. 13 steps
16.  $616 \text{ cm}^2$

### SECTION B (50 MARKS)

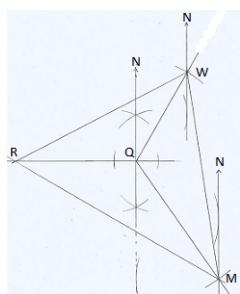
17. a) i) Ksh. 24,000  
ii) Ksh. 14,200  
b) Ksh. 5,640

18. a)



- b) 8.54 ha  
c) Ksh. 2,391,200

19. a)



- b) i)  $RW = 475 \pm 5 \text{ km}$ ; Bearing:  $242 \pm 1^\circ$   
ii)  $WM = 510 \pm 5 \text{ km}$   
iii)  $RM = 575 \pm 5 \text{ km}$

20. a) Ksh. 800

- b) Ksh. 8,400  
c) Ksh. 3,600  
d) 175%

21. a) A=630 bags

$$B=420 \text{ bags}$$

$$C=1080 \text{ bags}$$

$$D=600 \text{ bags}$$

- b) 55 days

### Model Sample Paper 2

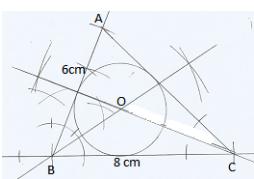
#### SECTION A (50 MARKS)

1. 5.814
2. 4

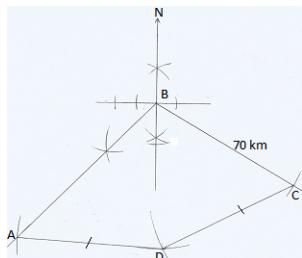
## SECTION FOUR

3. Musa=20 years \ Ahmed=8 years
4. 179.2 ha
5. 3,371 Sterling Pounds
6. a) n=6 sides  
b) for n sides,  $interior\angle = 120^0$   
for n-1 sides,  $interior\angle = 108^0$
7. Ksh. 850
8. 360 biscuits
9. 10 days
10.  $-\frac{1}{x}$
11. 65
12.  $5.2 \pm 0.1 km$ ;  $S60 \pm 1^0 W$
13. Marked Price= Ksh. 4,000  
Buying Price= Ksh. 3,000
14. 21.6 kg
15. radius=5.278 cm
16. a)  $144^0$   
b) 35.2 cm

## SECTION B (50 MARKS)

17. a) i) Ksh. 6,000  
ii) Ksh. 4,080  
iii) 47.1%  
iv) Ksh. 3,264
18. a)  
  
b)  $AC = 8 \pm 1 cm$ ;  $\angle ACB = 44 \pm 1^0$   
d)  $r = 2.0 \pm 0.1 cm$

19. a) 48 Passengers  
 b) 28 Passengers  
 c) Ksh. 10,160
20. a)  $height = 5.6 \pm 0.1 m$   
 b) i)



- ii)  $AB = 85 \pm 1 km$   
 iii)  $177 \pm 1^\circ$
21. a) 43.12 litres  
 b)  $2.415 m^2$   
 c) Ksh. 2,102.40

### Model Sample Paper 3

#### SECTION A (50 MARKS)

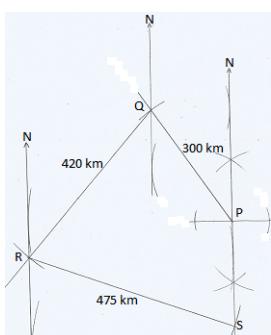
1. -2
2. a) 25 beacons  
 b) 603 metres
3.  $Muchoki = 5 \text{ goats}$   
 $Wakuraya = 7 \text{ goats}$
4. 39 coins
5. a)  $\angle CBA = 144^\circ$   
 b)  $\angle CDE = 108^\circ$
6. a) 432,000 litres  
 b) 432,000 Kg
7. \$61,546
8. Ksh. 25,290
9. Ksh. 345

## SECTION FOUR

10.  $RP = 256 \pm 4 \text{ km}$ ; Bearing:  $220 \pm 1^\circ$
11. 562.5 m
12. 4.08 tonnes
13. 32.86
14. 4
15. 26.08%
16. 35 minutes

## SECTION B (50 MARKS)

17. a) i) 3 kg  
ii) 60 litres  
b) 48 kg
18. a) i) 31 hrs 35 minutes  
ii) 3 hrs  
iii) 34 hrs 35 minutes  
b) 2075 km
19. a)



- b) i)  $SP = 230 \pm 5 \text{ km}$   
ii)  $SQ = 510 \pm 5 \text{ km}$   
Bearing:  $160 \pm 1^\circ$   
iii) Compass bearing of S from R:  $S70 \pm 1^\circ E$
20. a) n=10 litres  
b) 40%  
c) 8 : 7
21. a) Ksh. 2,871  
b) Ksh. 2,500  
c) 16%

 Model Sample Paper 4

**SECTION A (50 MARKS)**

1.  $\frac{1}{20}$
2. 21-inch TV= Ksh. 25,000  
17-inch TV= Ksh. 14,000
3. 323,466 Japanese yens
4. 19.20
5. 12% profit
6. 1:5
7. 35.56 litres
8. 28.32%;decrease
9. 20 ha
10. Rice=Ksh. 60  
KDF cake=Ksh. 10
11. b)  $h = 4.2 \pm 0.1 \text{ cm}$
12. n=5 sides
13.  $\frac{1}{8}$
14. b) 750 m
15. 3 minutes

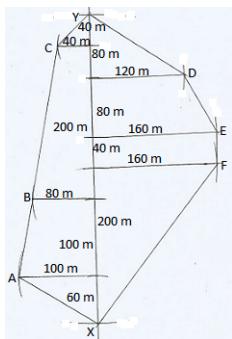
**SECTION B (50 MARKS)**

16. a) i) 4 buses  
ii) 6 matatus  
b) 324 students  
c) Ksh. 13,800
17. a) i) Ksh. 2,170,800  
ii) Ksh. 1,953,720  
iii) Ksh. 434,160  
b) Ksh. 30,225

## SECTION FOUR

18. a) i) 6 hours 50 minutes  
ii) 6 hours 20 minutes  
iii) 5 hours 40 minutes  
iv) 12 hours 15 minutes  
b) 35 hours 50 minutes  
c) 42 km/h

19. a)



- b) 6.32 ha  
c) Ksh. 632,000
20. a) 540 eggs  
b) 15.6%  
c) 16.67%

### 🔥 Model Sample Paper 5

## SECTION A (50 MARKS)

1. 2
2. Skirt=Ksh. 200  
Blouse=Ksh. 250
3. Gaceru=37 years  
Kairu=10 years
4. 5
5. a)  $19.199 \text{ cm}^3$   
b) 14.4 g
6. Ksh. 17,920
7. 2520 g
8.  $22.638 \text{ cm}^3$

9. 805 Kg  
 10. 24 students  
 11. 2,500 ha  
 12.  $507 \text{ cm}^3$   
 13. b) 150 Km  
 14.  $\frac{391}{495}$   
 15.  $a = \frac{5}{9}$   
 16. a)  $270^\circ$   
 b)  $259.9 \text{ cm}^2$

**SECTION B (50 MARKS)**

17. a)

$$2x + y = 6$$

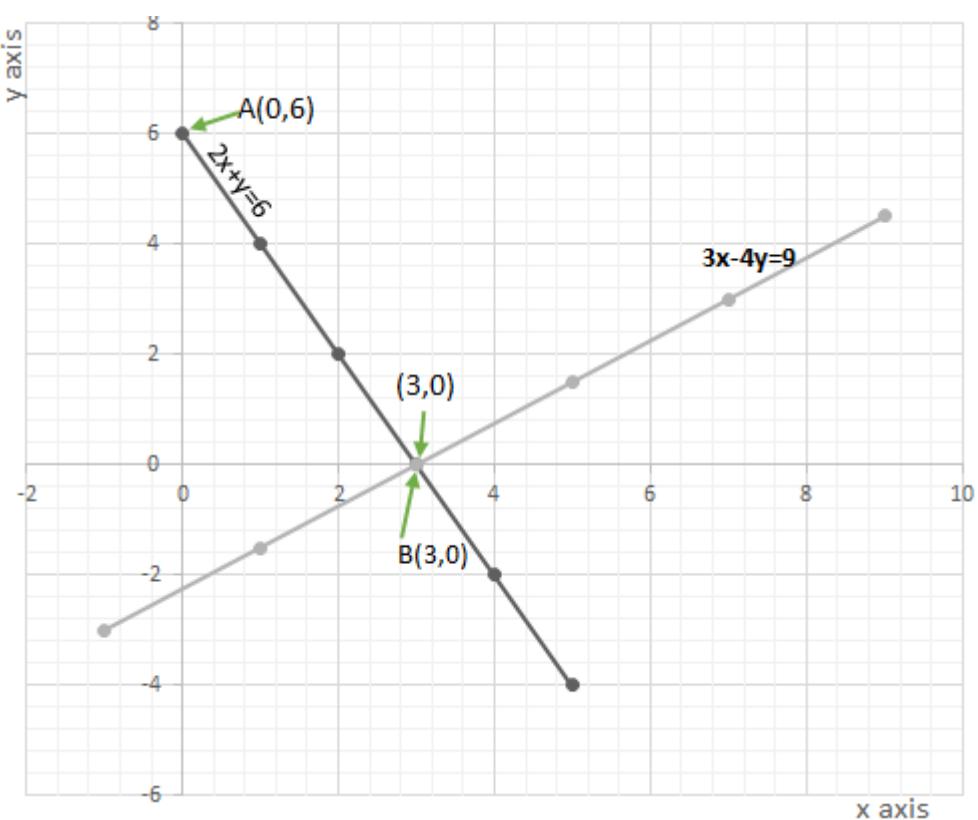
x	0	1	2	3	4	5
$y=6-2x$	6	4	2	0	-2	-4
Point	(0,6)	(1,4)	(2,2)	(3,0)	(4,-2)	(5,-4)

$$3x - 4y = 9$$

x	-1	1	3	5	7	9
$y = \frac{3x-9}{4}$	-3	-1.5	0	1.5	3	4.5
Point	(-1,-3)	(1,-1.5)	(3,0)	(5,1.5)	(7,3)	(9,4.5)

- b)

## SECTION FOUR



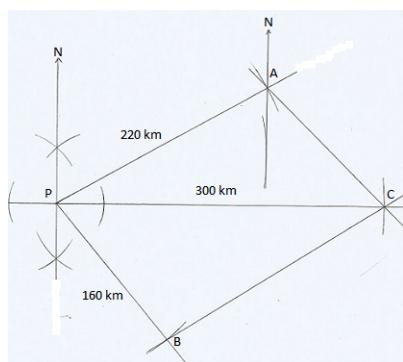
$$\therefore x = 3; y = 0$$

18. a) i)  $3,900\text{cm}^2$

ii)  $1.97 \text{ kg}$

b)  $15.97 \text{ kg}$

19. a)



b) i)  $BA = 247.5 \pm 2.5 \text{ km}$

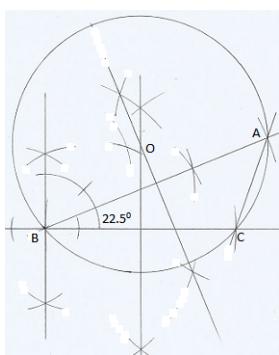
Compass bearing =  $S21 \pm 1^{\circ}W$

ii)  $AC = 152 \pm 2.5 \text{ km}$

iii)  $BC = 232.5 \pm 2.5 \text{ km}$

20.  $25.35\%$

21. a)



b)  $AC = 5.5 \pm 0.1 \text{ cm}$ ;  $\angle ACB = 110 \pm 1^\circ$

d)  $r = 4.6 \pm 0.1 \text{ cm}$

### Model Sample Paper 6

#### SECTION A (50 MARKS)

1. 5
2. Ksh.1,400
3. 4
4. 18 minutes
5. 240 and 300
6. Ksh.1,550
7. 980 kg
8. Ksh.21,000
9.  $x = 40^\circ$
10. a) Ksh.121,300  
b) Ksh.98,331
11. Ksh. 140,000
- 12 b)  $14.6 \pm 0.2 \text{ km}$ ;  $260 \pm 1^\circ$
13. 30 laborers
14. a)  $252^\circ$   
b) 70.4 cm.
15. 576 m
16. 4.76%, increase

## SECTION FOUR

### SECTION B (50 MARKS)

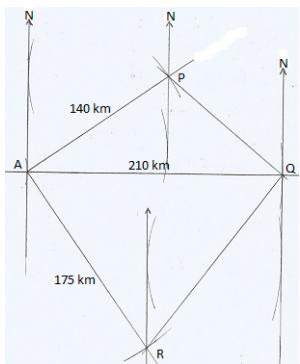
17. a) 71,280 litres

b) 7,800 litres

c) Ksh.819

18. a) Sketch

b)

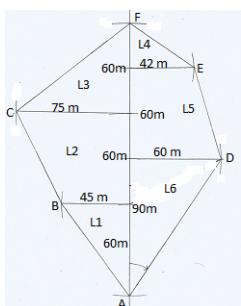


c) i)  $125 \pm 2.5$  km;  $310 \pm 1^0$

ii)  $181 \pm 2.5$  km

iii)  $225 \pm 2.5$  km

19. a)



b) i) Area is given as follows:

$$\begin{aligned}
 L1 &\Rightarrow \frac{1}{2} \times 60^{\cancel{30}} \times 45 & = 1350 \text{ } m^2 \\
 L2 &\Rightarrow \frac{1}{2} \times 60^{\cancel{30}} \times (45 + 75) & = 3600 \text{ } m^2 \\
 L3 &\Rightarrow \frac{1}{2} \times 60^{\cancel{30}} \times 75 & = 2250 \text{ } m^2 \\
 L4 &\Rightarrow \frac{1}{2} \times 30^{\cancel{15}} \times 42 & = 630 \text{ } m^2 \\
 L5 &\Rightarrow \frac{1}{2} \times 60^{\cancel{30}} \times (42 + 60) & = 3060 \text{ } m^2 \\
 L6 &\Rightarrow \frac{1}{2} \times 90^{\cancel{45}} \times 60 & = 2700 \text{ } m^2 \\
 \text{Total} &\Rightarrow 1350 + \dots + 2700 & = 13,590 \text{ } m^2
 \end{aligned}$$

∴ The area of the field is equal to  $13,590 \text{ } m^2$

ii)  $108 \pm 1.5 \text{ m}$ ;  $N34 \pm 1^0 E$

20. a) A=625 bags

B=500 bags

C=1050 bags

D=750 bags

b)  $13\frac{1}{2}$  days

21. a)  $x=160$  litres of water

b) 44.8%

c) 32 : 43



## Model Sample Paper 7

### SECTION A (50 MARKS)

1. 2

2. 4.5346

3. Ksh. 5,471,875

4. 4

5.  $y = \frac{1}{2}$

6. 9 notes

7. Ksh. 73.80

## SECTION FOUR

8. 11.21%
9. Ksh. 160
10. 20 years
11. 4.76% *increase*
12. n=6; Hexagon
13. 1.46 m
14. 5 cm
15. 12 more boys
16. b)  $38.25 \pm 0.375 \text{ cm}^2$

## SECTION B (50 MARKS)

17. a) i) 2 buses  
ii) 8 matatus  
b) 196 youths  
c) Ksh.20,400
18. a) 3:4:5  
b) i) Ksh.96,000  
ii) Muhammed = Ksh 24,000  
Maimuna=Ksh.32,000  
Hasan=Ksh.40,000  
iii) 71.43%
19. a) i) item 200 Students  
ii) 228 students  
iii) 285 Students  
b) 18.75%  
c) 14%
20. a)

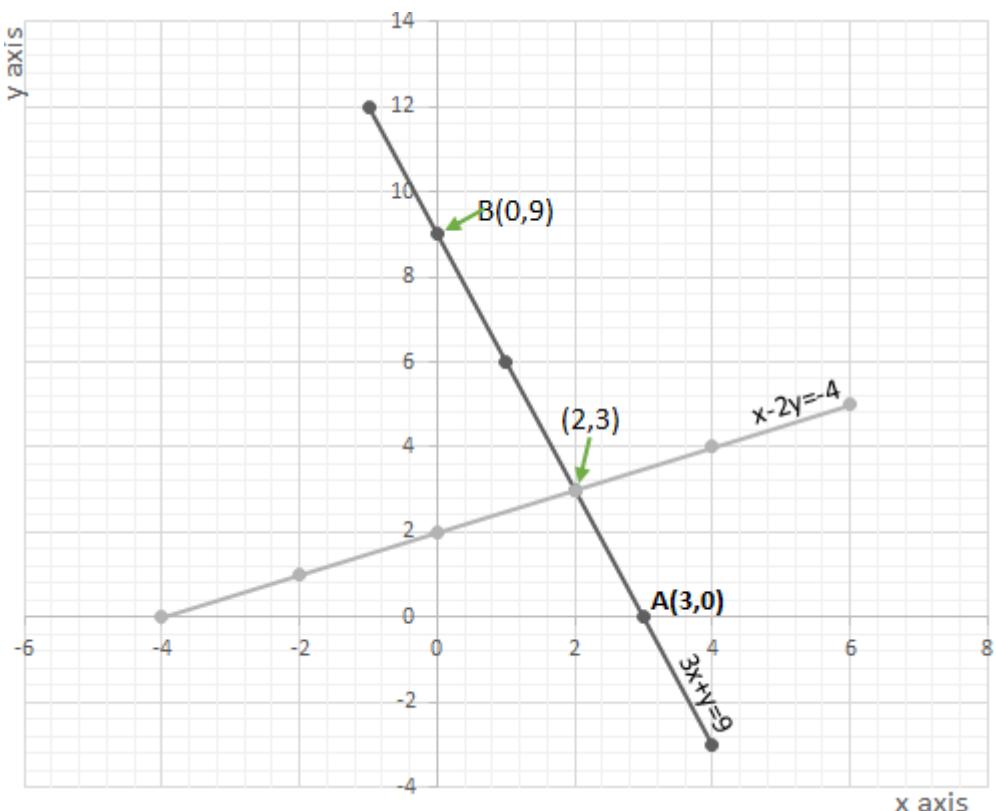
$$3x + y = 9$$

x	-1	0	-1	2	3	4
y = -3x + 9	12	9	6	3	0	-3
Point	(-1, 12)	(0, 9)	(1, 6)	(2, 3)	(3, 0)	(4, -3)

$$x - 2y = -4$$

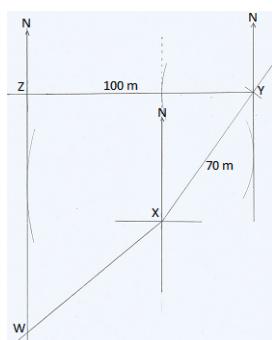
x	-4	-2	0	2	4	6
y = $\frac{x+4}{2}$	0	1	2	3	4	5
Point	(-4, 0)	(-2, 1)	(0, 2)	(2, 3)	(4, 4)	(6, 5)

b)



c)  $A(3,0); B(0,9)$

21. a)



b) i)  $ZX = 81 \pm 1$  m;  $313 \pm 1^\circ$

ii)  $WZ = 105 \pm 1$  m

iii)  $WX = 77 \pm 1$  m

iv)  $S44 \pm 1^{\circ}W$



Model Sample Paper 8

## SECTION A (50 MARKS)

1. -2

2.  $3(x + y)(x - 2y)$

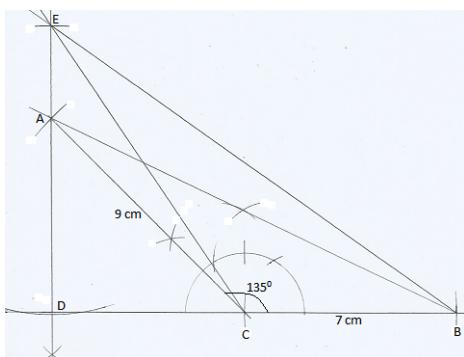
## SECTION FOUR

3. a)  $\frac{133}{5} g$   
b) 6.4%
4.  $90^0$
5. 8 ha
6. 5.946
7. 607.5 m
8. Ksh.1200
9. n=8; octagon  
 $Exterior \angle = 45^0$
10. 79 pieces
11. 2:1
12. Ksh.2,100
13. Ksh. 345
14. a) 70 Swiss Francs  
b) Ksh.7,068.60
15. b)  $\angle CAB = 59 \pm 1^0$
16. 3 hrs

## SECTION B (50 MARKS)

17. a) Ksh.420,000  
b) Ksh 561,000  
c) 748,000  
d) 52.5%
18. a) 35 dresses; 22 skirts  
b) Ksh.4,910  
c) 25.6%
19. a)  $A = (8000 - 4x^2) cm^2$   
b)  $x = 10cm$ ; \, 80 cm  $\times$  60 cm  $\times$  10 cm  
c) 3.8 kg  
d)  $0.75g/cm^3$

20. a)

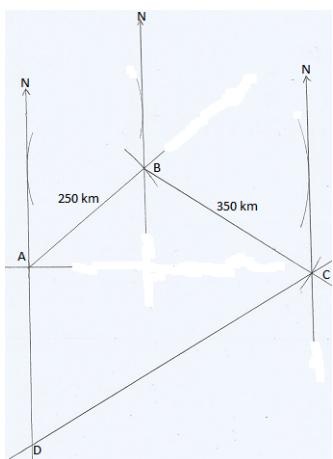


b)  $AB = 14.8 \pm 0.1 \text{ cm}$ ;  $ABC = 25 \pm 1^\circ$

d)  $AD = 6.4 \pm 0.1 \text{ cm}$ ;  $22.4 \pm 0.35 \text{ cm}^2$

f)  $EC = 11.4 \pm 0.1 \text{ cm}$

21. a)



b)

i)  $AC = 465 \pm 5 \text{ km}$

ii)  $AD = 293 \pm 3 \text{ km}$

iii)  $CD = 540 \pm 5 \text{ km}$

iv)  $204 \pm 1^\circ$



### Model Sample Paper 9

#### SECTION A (50 MARKS)

1. 3

2. 50 Files

3. Mutheu= 21 years

Brother=11 years

4. 21.4%

## SECTION FOUR

5. 75.199
6. a)  $636,000\text{cm}^3$   
b) 477.0 kg
7.  $3(x-3y)(x+2y)$
8. 8 more workers
9. 35 students
10. a) 90.91 Swiss Franks  
b) Ksh.9,173
11. 845 biscuits
12. 12 sides
13. 1:2
14. 390
15.  $100\text{cm}^2$
16.  $\frac{8}{9}, \frac{3}{4}, \frac{3}{5}, \frac{4}{7}, \frac{1}{3}$

## SECTION B (50 MARKS)

17. a) Ksh. 760,000  
b) Ksh. 608,000  
c) Ksh. 1,216,000  
d) 57.5%
18. a)

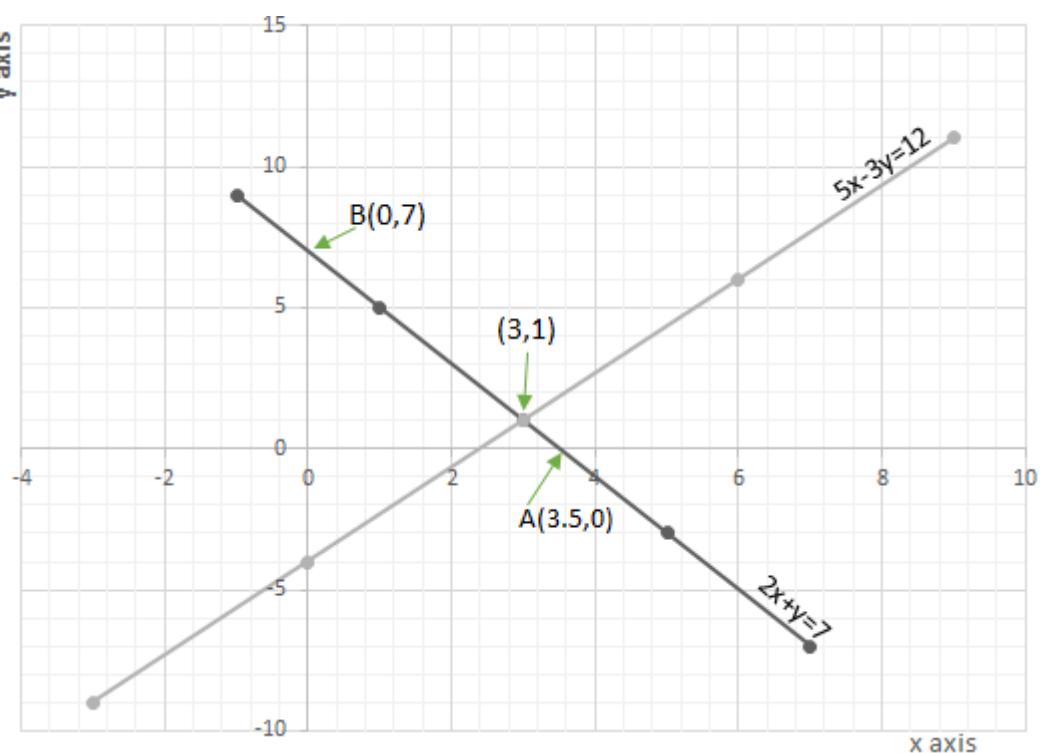
$$2x + y = 7$$

x	-1	1	3	5	7
$y = 7 - 2x$	9	5	1	-3	-7
Points	(-1,9)	(1,5)	(3,1)	(5,-3)	(7,-7)

$$5x - 3y = 12$$

x	-3	0	3	6	9
$y = \frac{5x-12}{3}$	-9	-4	1	6	11
Points	(-3,-9)	(0,-4)	(3,1)	(6,6)	(9,11)

b)



- c) A(3.5,0); B(0,7)
19. a)  $y=75$  litres  
 b) 35%  
 c) 5:7
20. a)  
 i\)\) 16,000 bags  
 ii\)\) 17,000 bags  
 b\)\) \$3.5\%\\$\n  
 c\)\) \$2.53\%\\$\n  
 d\)\) \$6.25\%\\$\n
21. a)  $39,537 \text{ cm}^3$   
 b) 31.63 kg  
 c) 64.2 kg

Model Sample Paper 10

**SECTION A (50 MARKS)**

1.  $\frac{3}{5}$

## SECTION FOUR

2. Kiama=28 years  
Murugi=35 years  
Naomi=15 years

3. 10.2%

4. a)  $210^0$   
b) 51.33 cm

5. 825 cm

6. a)  $90^0$   
b) 98.5 km

7. 12 books

8. Mother=32 years  
Daughter=8 years

9. 8.2%

10.  $y = \frac{5}{4}$

11. a)  $(8x - 20)^0$   
b)  $85^0$

12. Ksh. 112,200

13.  $306 \text{ cm}^3$

14. 570 students

15. 125 coins

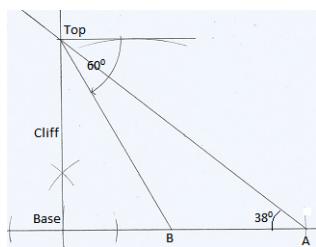
16. 3

## SECTION B (50 MARKS)

17. a) 30 Trousers; 20 T-shirts  
b) Ksh. 2,300  
c) 34.41%

18. a)  $A = (6300 - 4t^2) \text{ cm}^2$   
b) 2.8 kg  
c)  $80 \times 60 \times 5; 24,000 \text{ cm}^3$   
d)  $0.7 \text{ g/cm}^3$

19. a)



b)

- i) Height =  $126 \pm 2$  m
- ii)  $AB = 88 \pm 2$  m
- iii)  $72 \pm 2$  m

20. a)

- i) 9 hrs 5 min
- ii) 7 hrs 10 min
- iii) 2 hrs 20 min
- iv) 9 hrs 30 min
- b) 28 hrs 05 min
- c) 5 hrs 10 min
- d) 60 km/hr

21. a)

- i) Ksh. 1,602,500
- ii) Ksh. 102,560
- b) Ksh. 16,405

