Simple Linear Regression

In this notebook, we'll build a linear regression model to predict `Sales` using an appropriate predictor variable.

Step 1: Reading and Understanding the Data

Let's start with the following steps:

1. Importing data using the pandas library
2. Understanding the structure of the data

In [21]:

```
# Supress Warnings

import warnings
warnings.filterwarnings('ignore')
```

In [22]:

```
# Import the numpy and pandas package

import numpy as np
import pandas as pd
```

In [23]:

```
# Read the given CSV file, and view some sample records

advertising = pd.read_csv("C:/Users/91920/Downloads/LinearRegression-master (1)/LinearRegre
advertising.drop(['Unnamed: 0'] , axis = 1 , inplace =True)
advertising.head()
```

Out[23]:

	TV	radio	newspaper	sales
0	230.1	37.8	69.2	22.1
1	44.5	39.3	45.1	10.4
2	17.2	45.9	69.3	9.3
3	151.5	41.3	58.5	18.5
4	180.8	10.8	58.4	12.9

Let's inspect the various aspects of our dataframe

In [24]:

```
advertising.shape
```

Out[24]:

```
(200, 4)
```

In [25]:

```
advertising.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 200 entries, 0 to 199
Data columns (total 4 columns):
 #   Column     Non-Null Count  Dtype  
---  ------     --------------  -----  
 0   TV         200 non-null    float64
 1   radio      200 non-null    float64
 2   newspaper  200 non-null    float64
 3   sales      200 non-null    float64
dtypes: float64(4)
memory usage: 6.4 KB
```

In [26]:

```
advertising.describe()
```

Out[26]:

	TV	radio	newspaper	sales
count	200.000000	200.000000	200.000000	200.000000
mean	147.042500	23.264000	30.554000	14.022500
std	85.854236	14.846809	21.778621	5.217457
min	0.700000	0.000000	0.300000	1.600000
25%	74.375000	9.975000	12.750000	10.375000
50%	149.750000	22.900000	25.750000	12.900000
75%	218.825000	36.525000	45.100000	17.400000
max	296.400000	49.600000	114.000000	27.000000

Step 2: Visualising the Data

Let's now visualise our data using seaborn. We'll first make a pairplot of all the variables present to visualise which variables are most correlated to `Sales` .

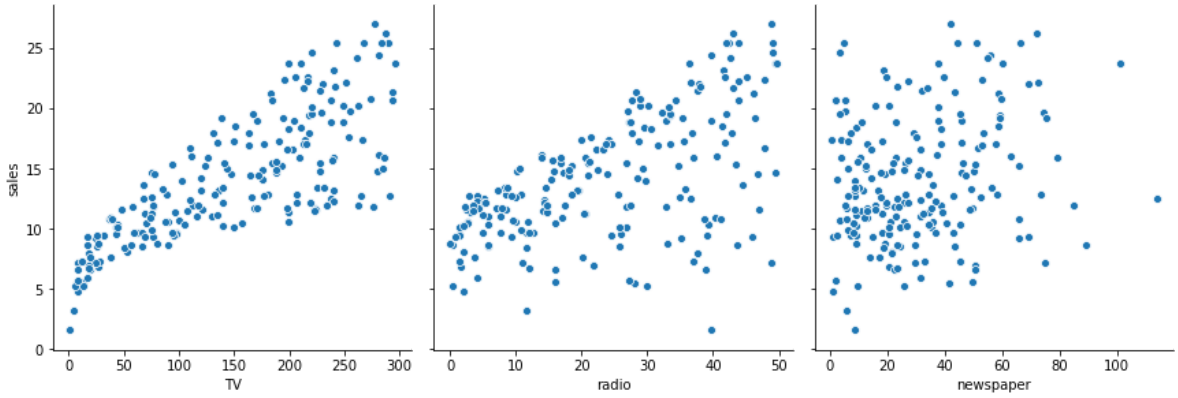
In [17]:

```
import matplotlib.pyplot as plt
import seaborn as sns
```

```
Bad key "text.kerning_factor" on line 4 in
C:\Users\91920\anaconda3\lib\site-packages\matplotlib\mpl-data\stylelib\_cla
ssic_test_patch.mplstyle.
You probably need to get an updated matplotlibrc file from
https://github.com/matplotlib/matplotlib/blob/v3.1.3/matplotlibrc.template
 (https://github.com/matplotlib/matplotlib/blob/v3.1.3/matplotlibrc.templat
e)
or from the matplotlib source distribution
```

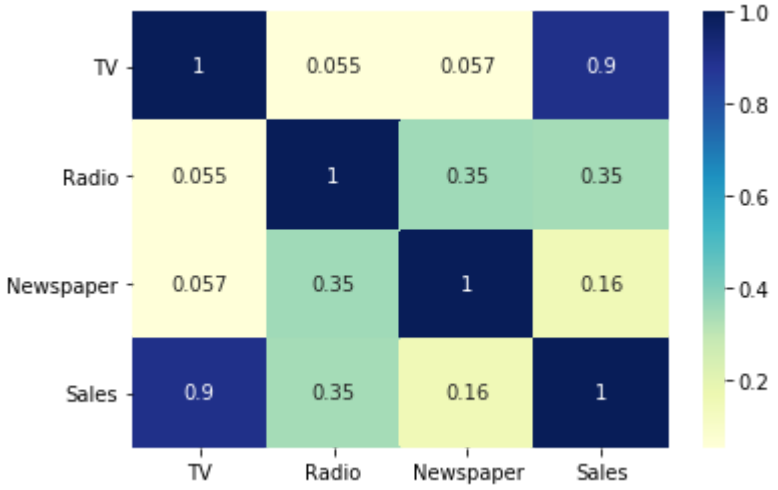
In [20]:

```
sns.pairplot(advertising, x_vars=['TV','radio','newspaper'], y_vars='sales',size=4, aspect=
plt.show()
```



In [9]:

```
sns.heatmap(advertising.corr(), cmap="YlGnBu", annot = True)
plt.show()
```



As is visible from the pairplot and the heatmap, the variable `TV` seems to be most correlated with `Sales`. So let's go ahead and perform simple linear regression using `TV` as our feature variable.

Step 3: Performing Simple Linear Regression

Equation of linear regression

$y = c + m_1 x_1 + m_2 x_2 + ... + m_n x_n$

- y is the response
- c is the intercept
- m_1 is the coefficient for the first feature
- m_n is the coefficient for the nth feature

In our case:

$y = c + m_1 \times TV$

The m values are called the model **coefficients** or **model parameters**.

Generic Steps in model building using `statsmodels`

We first assign the feature variable, `TV`, in this case, to the variable `X` and the response variable, `Sales`, to the variable `y`.

In [28]:

```
X = advertising['TV']
y = advertising['sales']
```

Train-Test Split

You now need to split our variable into training and testing sets. You'll perform this by importing `train_test_split` from the `sklearn.model_selection` library. It is usually a good practice to keep 70% of the data in your train dataset and the rest 30% in your test dataset

In [29]:

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y, train_size = 0.7, test_size = 0.3
```

In [32]:

```
# Let's now take a look at the train dataset

X_train.head()
```

Out[32]:

```
74     213.4
3      151.5
185    205.0
26     142.9
90     134.3
Name: TV, dtype: float64
```

In [33]:

```
y_train.head()
```

Out[33]:

```
74     17.0
3      18.5
185    22.6
26     15.0
90     11.2
Name: sales, dtype: float64
```

Building a Linear Model

You first need to import the `statsmodel.api` library using which you'll perform the linear regression.

In [34]:

```
import statsmodels.api as sm
```

By default, the `statsmodels` library fits a line on the dataset which passes through the origin. But in order to have an intercept, you need to manually use the `add_constant` attribute of `statsmodels` . And once you've added the constant to your `X_train` dataset, you can go ahead and fit a regression line using the `OLS` (Ordinary Least Squares) attribute of `statsmodels` as shown below

In [35]:

```
# Add a constant to get an intercept
X_train_sm = sm.add_constant(X_train)
print(X_train_sm)
# Fit the resgression line using 'OLS'
lr = sm.OLS(y_train, X_train_sm).fit()
```

```
     const     TV
74     1.0  213.4
3      1.0  151.5
185    1.0  205.0
26     1.0  142.9
90     1.0  134.3
..     ...    ...
87     1.0  110.7
103    1.0  187.9
67     1.0  139.3
24     1.0   62.3
8      1.0    8.6

[140 rows x 2 columns]
```

In [16]:

```
# Print the parameters, i.e. the intercept and the slope of the regression line fitted
lr.params
```

Out[16]:

```
const    6.948683
TV       0.054546
dtype: float64
```

In [17]:

```
# Performing a summary operation lists out all the different parameters of the regression l
print(lr.summary())
```

```
                            OLS Regression Results
==============================================================================
==
Dep. Variable:                  Sales   R-squared:                       0.8
16
Model:                            OLS   Adj. R-squared:                  0.8
14
Method:                 Least Squares   F-statistic:                     61
1.2
Date:                Thu, 13 Sep 2018   Prob (F-statistic):           1.52e-
52
Time:                        22:39:43   Log-Likelihood:                -321.
12
No. Observations:                 140   AIC:                             64
6.2
Df Residuals:                     138   BIC:                             65
2.1
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
==
                 coef    std err          t      P>|t|      [0.025      0.97
5]
------------------------------------------------------------------------------
--
const          6.9487      0.385     18.068      0.000       6.188       7.7
09
TV             0.0545      0.002     24.722      0.000       0.050       0.0
59
==============================================================================
==
Omnibus:                        0.027   Durbin-Watson:                   2.1
96
Prob(Omnibus):                  0.987   Jarque-Bera (JB):                0.1
50
Skew:                          -0.006   Prob(JB):                        0.9
28
Kurtosis:                       2.840   Cond. No.                         32
8.
==============================================================================
==

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is corre
ctly specified.
```

Looking at some key statistics from the summary

The values we are concerned with are -

1. The coefficients and significance (p-values)
2. R-squared

3. F statistic and its significance

1. The coefficient for TV is 0.054, with a very low p value

The coefficient is statistically significant. So the association is not purely by chance.

2. R - squared is 0.816

Meaning that 81.6% of the variance in `Sales` is explained by `TV`

This is a decent R-squared value.

3. F statistic has a very low p value (practically low)

Meaning that the model fit is statistically significant, and the explained variance isn't purely by chance.

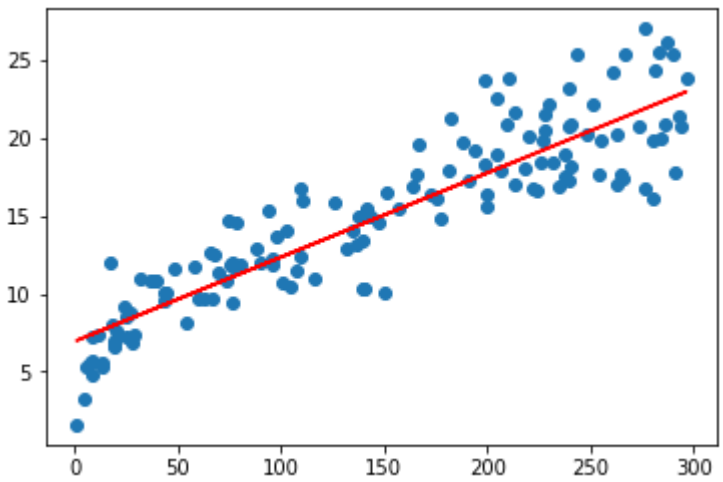
The fit is significant. Let's visualize how well the model fit the data.

From the parameters that we get, our linear regression equation becomes:

$Sales = 6.948 + 0.054 \times TV$

In [18]:

```
plt.scatter(X_train, y_train)
plt.plot(X_train, 6.948 + 0.054*X_train, 'r')
plt.show()
```



Step 4: Residual analysis

To validate assumptions of the model, and hence the reliability for inference

Distribution of the error terms

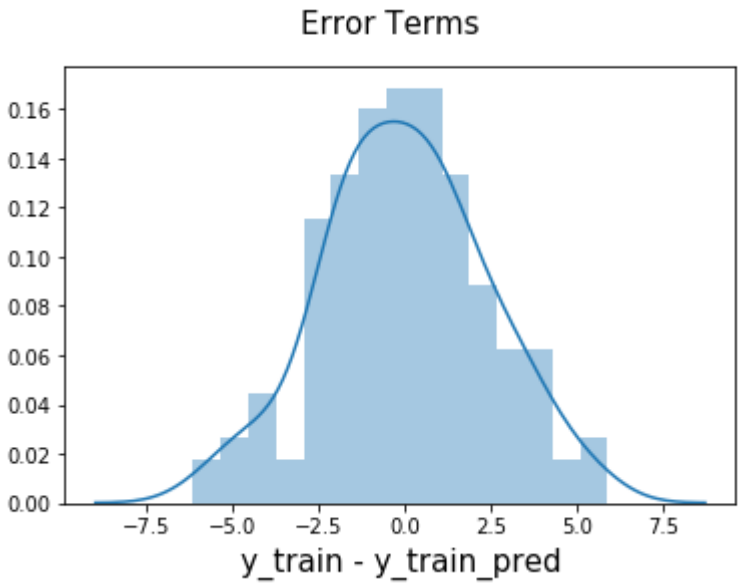
We need to check if the error terms are also normally distributed (which is infact, one of the major assumptions of linear regression), let us plot the histogram of the error terms and see what it looks like.

In [19]:

```
y_train_pred = lr.predict(X_train_sm)
res = (y_train - y_train_pred)
```

In [20]:

```
fig = plt.figure()
sns.distplot(res, bins = 15)
fig.suptitle('Error Terms', fontsize = 15)                  # Plot heading
plt.xlabel('y_train - y_train_pred', fontsize = 15)         # X-label
plt.show()
```

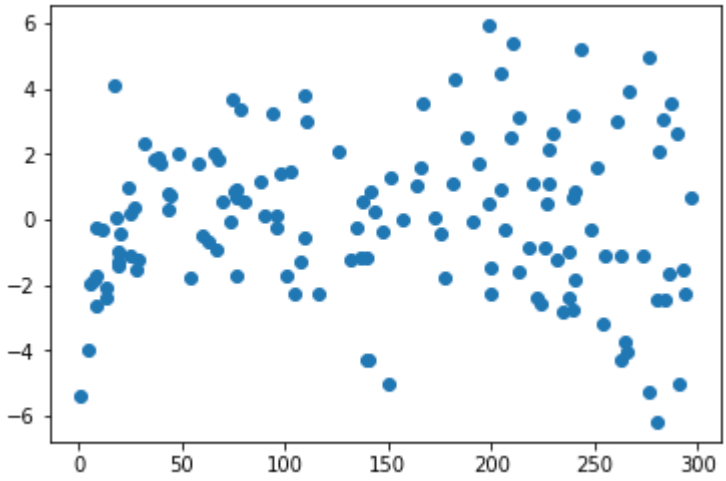


The residuals are following the normally distributed with a mean 0. All good!

Looking for patterns in the residuals

In [21]:

```
plt.scatter(X_train,res)
plt.show()
```



We are confident that the model fit isn't by chance, and has decent predictive power. The normality of residual terms allows some inference on the coefficients.

Although, the variance of residuals increasing with X indicates that there is significant variation that this model is unable to explain.

As you can see, the regression line is a pretty good fit to the data

Step 5: Predictions on the Test Set

Now that you have fitted a regression line on your train dataset, it's time to make some predictions on the test data. For this, you first need to add a constant to the `X_test` data like you did for `X_train` and then you can simply go on and predict the y values corresponding to `X_test` using the `predict` attribute of the fitted regression line.

In [22]:

```
# Add a constant to X_test
X_test_sm = sm.add_constant(X_test)

# Predict the y values corresponding to X_test_sm
y_pred = lr.predict(X_test_sm)
```

In [23]:

```
y_pred.head()
```

Out[23]:

```
126     7.374140
104    19.941482
99     14.323269
92     18.823294
111    20.132392
dtype: float64
```

In [24]:

```
from sklearn.metrics import mean_squared_error
from sklearn.metrics import r2_score
```

Looking at the RMSE

In [25]:

```
#Returns the mean squared error; we'll take a square root
np.sqrt(mean_squared_error(y_test, y_pred))
```

Out[25]:

```
2.019296008966233
```

Checking the R-squared on the test set

In [26]:

```
r_squared = r2_score(y_test, y_pred)
r_squared
```

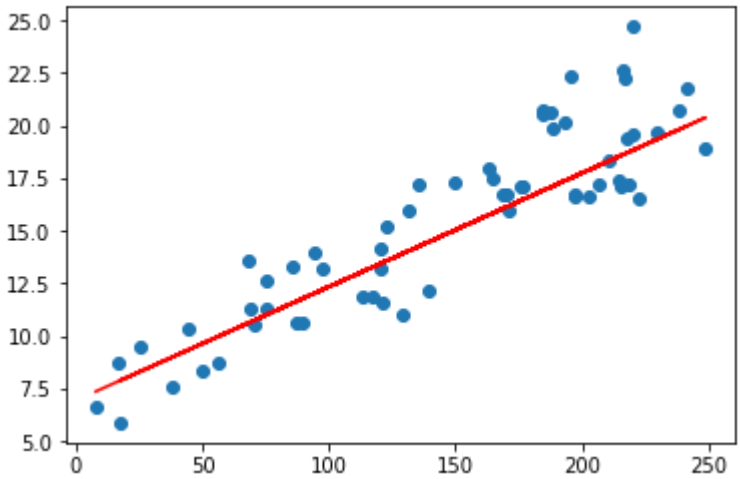
Out[26]:

```
0.7921031601245658
```

Visualizing the fit on the test set

In [27]:

```
plt.scatter(X_test, y_test)
plt.plot(X_test, 6.948 + 0.054 * X_test, 'r')
plt.show()
```



Linear Regression using `linear_model` in `sklearn`

Apart from `statsmodels` , there is another package namely `sklearn` that can be used to perform linear regression. We will use the `linear_model` library from `sklearn` to build the model. Since, we hae already performed a train-test split, we don't need to do it again.

There's one small step that we need to add, though. When there's only a single feature, we need to add an additional column in order for the linear regression fit to be performed successfully.

In [28]:

```
from sklearn.model_selection import train_test_split
X_train_lm, X_test_lm, y_train_lm, y_test_lm = train_test_split(X, y, train_size = 0.7, tes
```

In [29]:

```
X_train_lm.shape
```

Out[29]:

```
(140,)
```

In [30]:

```
X_train_lm = X_train_lm.reshape(-1,1)
X_test_lm = X_test_lm.reshape(-1,1)
```

In [31]:

```
print(X_train_lm.shape)
print(y_train_lm.shape)
print(X_test_lm.shape)
print(y_test_lm.shape)
```

```
(140, 1)
(140,)
(60, 1)
(60,)
```

In [32]:

```
from sklearn.linear_model import LinearRegression

# Representing LinearRegression as lr(Creating LinearRegression Object)
lm = LinearRegression()

# Fit the model using lr.fit()
lm.fit(X_train_lm, y_train_lm)
```

Out[32]:

```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=1, normalize=False)
```

In [33]:

```
print(lm.intercept_)
print(lm.coef_)
```

```
6.948683200001357
[0.05454575]
```

The equationwe get is the same as what we got before!

$Sales = 6.948 + 0.054 * TV$

Sklearn linear model is useful as it is compatible with a lot of sklearn utilites (cross validation, grid search etc.)

Addressing some common questions/doubts on Simple Linear Regression

Q: Why is it called 'R-squared'?

Based on what we learnt so far, do you see it? Can you answer this?

.

.

.

.

Drumroll...

.

.

.

.

In [34]:

```
corrs = np.corrcoef(X_train, y_train)
print(corrs)
```

```
[[1.         0.90321277]
 [0.90321277 1.        ]]
```

In [35]:

```
corrs[0,1] ** 2
```

Out[35]:

```
0.8157933136480384
```

Correlation (Pearson) is also called **"r"** or **"Pearson's R"**

Q: What is a good RMSE? Is there some RMSE that I should aim for?

You should be able to answer this by now!

Look at "Sharma ji ka beta"; he could answer this in a moment. How lucky is Sharma ji to have such a smart kid!

.

.

.

.

Drumroll...

.

.

.

.

The RMSE:

- depends on the units of the Y variables
- is NOT a normalized measure

While it can't really tell you of the gooodness of the particular model, it can help you compare models.

A better measure is R squared, which is normalized.

Q: Does scaling have an impact on the model? When should I scale?

While the true benefits of scaling will be apparent during future modules, at this juncture we can discuss if it has an impact on the model.

We'll rebuild the model after scaling the predictor and see what changes.

The most popular methods for scaling:

1. Min-Max Scaling
2. Standard Scaling

In [36]:

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y, train_size = 0.7, test_size = 0.3
```

SciKit Learn has these scaling utilities handy

In [37]:

```
from sklearn.preprocessing import StandardScaler, MinMaxScaler
```

In [38]:

```
# One aspect that you need to take care of is that the 'fit_transform' can be performed on
# reshape your 'X_train_scaled' and 'y_trained_scaled' data in order to perform the standar
X_train_scaled = X_train.reshape(-1,1)
y_train_scaled = y_train.reshape(-1,1)
```

In [39]:

```
X_train_scaled.shape
```

Out[39]:

```
(140, 1)
```

In [40]:

```
# Create a scaler object using StandardScaler()
scaler = StandardScaler()
#'Fit' and transform the train set; and transform using the fit on the test set later
X_train_scaled = scaler.fit_transform(X_train_scaled)
y_train_scaled = scaler.fit_transform(y_train_scaled)
```

In [41]:

```
print("mean and sd for X_train_scaled:", np.mean(X_train_scaled), np.std(X_train_scaled))
print("mean and sd for y_train_scaled:", np.mean(y_train_scaled), np.std(y_train_scaled))
```

```
mean and sd for X_train_scaled: 2.5376526277146434e-17 0.9999999999999999
mean and sd for y_train_scaled: -2.5376526277146434e-16 1.0
```

In [42]:

```
# Let's fit the regression line following exactly the same steps as done before
X_train_scaled = sm.add_constant(X_train_scaled)

lr_scaled = sm.OLS(y_train_scaled, X_train_scaled).fit()
```

In [43]:

```
# Check the parameters
lr_scaled.params
```

Out[43]:

```
array([-2.91433544e-16,  9.03212773e-01])
```

As you might notice, the value of the parameters have changed since we have changed the scale.

Let's look at the statistics of the model, to see if any other aspect of the model has changed.

In [44]:

```
print(lr_scaled.summary())
```

```
                            OLS Regression Results
==============================================================================
==
Dep. Variable:                      y   R-squared:                       0.8
16
Model:                            OLS   Adj. R-squared:                  0.8
14
Method:                 Least Squares   F-statistic:                     61
1.2
Date:                Thu, 13 Sep 2018   Prob (F-statistic):           1.52e-
52
Time:                        22:39:46   Log-Likelihood:                -80.2
33
No. Observations:                 140   AIC:                             16
4.5
Df Residuals:                     138   BIC:                             17
0.3
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
==
                 coef    std err          t      P>|t|      [0.025      0.97
5]
------------------------------------------------------------------------------
--
const      -2.914e-16      0.037  -7.98e-15      1.000      -0.072       0.0
72
x1             0.9032      0.037     24.722      0.000       0.831       0.9
75
==============================================================================
==
Omnibus:                        0.027   Durbin-Watson:                   2.1
96
Prob(Omnibus):                  0.987   Jarque-Bera (JB):                0.1
50
Skew:                          -0.006   Prob(JB):                        0.9
28
Kurtosis:                       2.840   Cond. No.                         1.
00
==============================================================================
==

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is corre
ctly specified.
```

Model statistics and goodness of fit remain unchanged.

So why scale at all?

- Helps with interpretation (we'll be able to appreciate this better in later modules)
- Faster convergence of gradient descent