# Risk Measurements for Investment Portfolios

Risk measurement is a key component of portfolio construction and ongoing performance assessment. Risk can be assessed in many ways within a portfolio. The five most common risk measurements are Alpha, Beta, Standard Deviation, Sharpe Ratio and R-Squared. These statistical measures have historically been used to help assess investment risk/volatility.

#### **ALPHA**

Alpha is a risk ratio that measures the performance of an investment against a market index that is used as a benchmark. The excess return relative to the benchmark is called the investment's alpha. Alpha can be either positive or negative depending on the performance and is represented as a single number.

$$a = R_n - [R_{rf} + (R_m - R_{rf}) * B]$$

 $R_n = realized return of the portfolio$ 

 $R_{rf} = risk-free rate of return (e.g., 3-month T-Bill)$ 

R<sub>m</sub> = investment manager return

B = beta

# **BETA**

Beta is a risk ratio that measures the systematic risk (e.g. volatility) of an investment in comparison to the market as a whole. Beta is represented as a positive integer, with 1 as a baseline measure representing perfect matching of the security and market price movements. Any number greater than one indicates the security will have more movement than the market, and a beta less than 1 indicates the security will move less than the market.

$$\beta = \frac{(n \sum XY - (\sum X)(\sum Y))}{n \sum X^2 - (\sum X)^2} \quad (or) \quad \beta = \frac{cov(Y, X)}{var(X)}$$

X = benchmark returns

Y = investment returns

## VOLATILITY

## STANDARD DEVIATION

Standard deviation, the square root of variance, is arguably the most commonly cited statistic regarding the degree of dispersion for a population of observations.

Mathematically, the fact that you square the term (xi-X) so that all numbers are positive illustrates the fact that you are not distinguishing between "good" and "bad" deviations from the mean.

If your objective is to look at investment return volatility, standard deviation is a good measure. If your focus is on returns that are below average, you'll likely want to augment standard deviation with measures and statistics that specifically address downside risk.

Standard Deviation<sub>(SD)</sub> = 
$$\left[\frac{\sum (x_i - X)^2}{n-1}\right]^{1/2}$$

Annualized SD = SD x 
$$\sqrt{N_v}$$

x, = the *ith* observation

X = the mean (arithmetic average) return

n = the number of observations

N<sub>y</sub> = the number of periods in a year (e.g., 4 for quarterly data, 12 for monthly data)



## **SHARPE RATIO**

The Sharpe ratio is a direct measure of excess returns over a risk-free rate versus volatility. It allows you to quickly assess the risk/reward profile of one asset class (or manager) versus another as long as you accept return volatility as a proxy for risk.

Sharpe Ratio = Ann Rtn 
$$(R_m)$$
 - Ann Rtn  $(R_{rf})$ 
Annualized SD  $(R_m)$ 

Ann Rtn = annualized return

 $R_m$  = investment manager return

R\_ = risk-free rate of return (e.g., 3-month T-Bill)

# DOWNSIDE RISK & CAPITAL PRESERVATION

#### MAX DRAWDOWN

A visual depiction may illustrate maximum drawdown more effectively than a formula.



The term "high water mark" relates to the manager's highest cumulative return prior to the lowest value. The use of an absolute value will always result in a positive number for the maximum drawdown.

$$Max Drawdown_p = Max [ABS (Cumulative R_m - HW_m)]$$
  
 $HW_m$ 

ABS = absolute value

R<sub>m</sub> = investment manager return

HW<sub>m</sub> = High water mark of manager return for the time period

P = Time period of evaluation

## **DOWNSIDE CAPTURE RATIO**

Downside capture ratio is an effective measure of comparing a manager's negative returns versus that of a stated benchmark. Multiplying the term (1+R) for each applicable period results in a compounded (or annualized) rate of return over the time period. A ratio greater than 1 signifies the manager had a larger compounded negative return than the benchmark. For capital preservation-minded investors, downside capture ratio is a good summary statistic over a specified time period.

Downside Capture Ratio = 
$$[[(1+R_{m_1})*(1+R_{m_i})]^{(1/N)}]-1$$
  
 $[[(1+R_{h_1})*(1+R_{h_i})]^{(1/N)}]-1$ 

R<sub>m</sub> = investment manager return

R<sub>b</sub> = benchmark return

Both R<sub>m</sub> and R<sub>h</sub> refer to the series of negative returns over the time periods (1, .., N)

## **DOWNSIDE DEVIATION | MINIMUM ACCEPTABLE RATE (MAR)**

Downside deviation focuses on a very specific set of returns, those that do no meet a minimum acceptable rate of return (MAR). For capital preservation investors, simply set the MAR = 0. If your focus is an evaluation versus money market or other "risk-free" investments, a 30 or 90 day Treasury bill return serves as an appropriate MAR. For real return investors, the MAR should set to a measure of inflation, be that realized inflation or a simplified assumption, e.g. a constant rate of 2.5%.

Investors should be mindful of time period definition. Be sure to include periods that include negative returns.

Downside Deviation (DD)= 
$$\left[ \frac{\sum (R_{m_i} - MAR)^2}{n} \right]^{1/2}$$

Annualized DD = DD x 
$$\sqrt{N_y}$$

 $R_{mi}$  = manager returns for the *ith* observation where performance is below the MAR

MAR = the minimum acceptable rate of return (i.e., the hurdle rate of return)

n = the number of observations

 $N_y$  = the number of periods in a year (e.g., 4 for quarterly data, 12 for monthly data)

## **SORTINO RATIO**

The Sortino ratio might be considered the "downside risk" version of the Sharpe ratio. It evaluates excess returns over a risk-free rate versus downside deviation. In periods of muted downside deviation, Sortino ratios tend to be very high. Hence, peer percentile rankings as well as rolling period analyses through a variety of market cycles are helpful in gauging the persistency of downside risk management.

Sortino Ratio = 
$$\frac{\text{Ann Rtn (R}_{m}) - \text{Ann Rtn (R}_{rf})}{\text{Annualized DD (R}_{m})}$$

Ann Rtn = annualized return

R<sub>m</sub> = investment manager return

 $R_{rf}$  = risk-free rate of return (e.g., 3-month T-Bill)

# ACTIVE RISK VS. BENCHMARK

#### TRACKING ERROR

Tracking error, a common measure of "active (manager) risk," is defined as the standard deviation of excess returns versus a benchmark. As with the Sharpe ratio, tracking error is a measure of volatility of excess returns and does not distinguish between upside and downside deviation.

Tracking Error<sub>(TE)</sub> = 
$$\left[ \frac{\sum (R_{m_i} - R_{b_i})^2}{n-1} \right]^{1/2}$$
 (or Standard Deviation of Excess Returns)

Annualized TE = TE x 
$$\sqrt{N_y}$$

= the *ith* observation

R<sub>m</sub> = investment manager return

R<sub>b</sub> = benchmark return

n = the number of observations

 $N_{v}$  = the number of periods in a year (e.g., 4 for quarterly data, 12 for monthly data)

## **INFORMATION RATIO**

Information ratio, a common measure of "manager efficiency", evaluates excess returns over a benchmark versus tracking error. It directly addresses the question, "did the manager add value relative to his active risk?" For benchmark-centric investors, information ratio is a commonly cited measure of risk-adjusted returns.

Information Ratio = 
$$\frac{\text{Ann Rtn (R}_{m}) - \text{Ann Rtn (R}_{b})}{\text{Tracking Error}}$$

Ann Rtn = annualized return

R<sub>m</sub> = investment manager return

R<sub>b</sub> = risk-free rate of return (e.g., 3-month T-Bill)

## **ACTIVE SHARE**

Active share is used to determine the percentage of holdings within a fund that differ from the stated benchmark index. By comparing the actual holdings within a portfolio to those of the index, an investor can get a clearer picture of what is driving performance. The higher the active share, the more a portfolio differentiates from its benchmark index.

Active Share = 
$$1/2 \sum_{i=1}^{N} |W(fund), i - W(index), i|$$

N = Number of observations/holdings W (fund) = Weight of the holding within the fund W (index) = Weight of the holding within the index

## **R-SQUARED**

R-squared is used as a measure of how much a fund's movements can be attributed to movements in its benchmark index. R-squared is typically presented as a percentage and ranges from 0-100%. A fund with a 100% R-squared moves in line with its index, while a 0% R-squared indicates no correlation to its benchmark.

$$r^{2} = \frac{(n \sum RxRy - \sum Rx \sum Ry)^{2}}{[n \sum (Rx)^{2} - \sum (Rx)^{2}] [n \sum (Ry)^{2} - (\sum Ry)^{2}]}$$

N = Number of observations

Rx = Market excess return

Ry = Portfolio excess return

