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ejercicios 3.4
7. Sean $ : da, --, on una secuencia de proposiciones y u una
   valuación. Demuestre o refute:
 a) V (oper (0, ->, n)) = T Sii V (Qi) = T para algun 0 = i + n
  Q(n) = V(oper (0, ->, n))= ( sii V(0j)= T para alyon 0 = j = n
 Caso base n=0
 Q(0) = V(oper(0, ->,0))= 7 Si V(0;)= 7 para algón 0 = 1 = 0
  * V (oper (0, ->, 0)) = T enfonces V(Qj) = T para agon 0 = j = 0
    v (oper (0, ->, 0)) = v (00) = T < Por definición >
     0 = j = 0 enfonces j=0
     V(Q_i) = V(Q_0) = T \vee
    V((Qj) = T para algún 0 = j = 0 entonces V(oper ((), ->, 0)) = T
     0 < j < 0 entonces j = 0
     V(Q_i) = V(Q_0) = T
   (Por definición) V(\text{oper}(\Phi, \rightarrow, 0)) = V(\text{O}_0) = T
    cpor reemplazo>
 Se supone Q(n)
 Caso inductivo n= n+1
Q(n+1) = V(oper (1, ->, n+1)) = T sir V(Q) = T para algun 0 = j = n+1
· V (oper (0, ->, n+1)) = + entonces V(Qj) = + para algón 0 = j = n+1
   V (Oper (Q, ->, N+1)) = V (Oper (Q, ->, n) -> On+1) = T < POr definition>
                   por metateorema 2-23 (->)
                                                  < POR reemplazo hipotesis induc ->
                         V (Qn+1) = T
         0 = j = n+1 1 = n+1
                V(Q_i) = V(Q_{n+1}) = T
      Por reemplazo
                             T = T
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V(oper (I, ->, n+1) = [
   V(Q1) = T Para algón 0 = 3 = N+1
                                    entonces
     0 \le \hat{J} \le n+L, J = n+L
     V(Q_i) = V(Q_{n+1}) = T
    cpor definición> v(oper (0, 7), n+1) = v(oper (0, +2, n) -> (n+2) = T
    por reemplazo de hipotesis inductiva
                                      V ( T -> (Pn+L) = T
     por reemplazo devlant
                                            V (T->T)=T
b) v (oper (0, ->, n)) = T sii v (0, i) = T para cada o ≤ i ≤ n
Q(n) = Vloper(0, ->, n)) = t sii V(4j) = T para cadd o = j = n
caso base n=0
 Q(0) = V(oper(0, ->, 0))=T Sii V(0;) = T para cada 0 = 5 = 0
 · V (oper (0, ->, o)) = r enforces V(Qj) = r para cada 0 4 5 40
(por definicion) vioper (0, ->, a) = via) = T
  O < j < 0 entonces j=0
    V(Q_1) = V(Q_0) = T
                           spor reemplace>
 · V (Qj) = T para cada 0 ≤ j ≠ 0 entonces vloper (0, +>, 0))=T
    OSISO entonces j=0
   V(Q_i) = V(Q_0) = T
    por definición
    V (Oper ( 0, ->, 0)) = V ((00) = T
    Por reemplazo
          V(Qu) = T
            T = T
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Suponemos Q(n)
 Casa inductivo n= h+1
 Q(n+1) = V(oper(0, ->, n+1)) = T sii V(0)) = T para cada 0 = i = n+1
 · V (oper (0, ->, n+1)) = P entonces V(0) = T para cada 0 = j = n+1
   Por definición
    V(oper (Q, ->, n+1) = V(oper (Q, ->, n) +> Qn+1) = T
     Por reemplazo de VIT-> (n+1) = T
       hipotesis inductiva
     Por metalogrema 2:13 V (Qn+1) = T
   0 \le j \le n+1, j = n+1
      V(Qi) = V(Qn+L) = T
 Por reemplaza
 · V(Qi) = T para cada 0 = j = n+1
                                   entonces V(oper (0, ->, n+1) = T
    0 \le j \le n+1 j = n+1
    V(Qs) = V(Qn+2) = T
   Por definición
     V (oper (0, ->, n +1)) = V (oper(0, ->, n) -> (n+1) = T
                              V (T -> Qn+2) = 7
    Por reemplazo de hipotesis
                               V (T->T)=T
     box reemblaso en Mouth
3. Sean Co, -- , Qn proposiciones . Demoestie :
 a) = ( Ni=0 (i-> (i), para cada 0 = i=n
   Por definición
Qn: V (oper (0, n, n) -> Qi) = T para cada O = i = n
  Caso Lase n=0
 Q(0) = V (OPER (0, 1,0)-> 0) = T para rada 0 = 1 = 0
   Por def V (Qc -> Qo) = T
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se supone Q(n)
Caso inductivo n= N+1
  Q(n+1): V(Oper(Q, N, n+1) +> Q() = T para dada 0 = J = n+1
  Por defi -
    1 ((Oper (Q, N, N-) -> Qn+1) -> Qn+1)= T
     · I Por hipotesis inductiva. I
        V ( Oper (Q, N, n +) (n+1)) = T
        V((Oper (Q, 1, 1) -> (n+1) +> (n+1) = T
b) = (Qj -> Vieo Qi) para rada
                              OFJEN
  Por def.
 Q(n) = V(Q1 -) Oper(Q,V,n))=1 para cada O & j & n
 caso base n=0
 Q(0)= V(Q; -> 0,001 (Q, V,0))= T parci cada 0 = 5 = 0
  POS deF.
        V(Q_0) = V(Q_0) = V
 Suponemos Q(n)
 COO INDUCTION N= N+1
  Q(n+1) = V(Q; -> OPEr(Q, V, n+1)) = T Para cada 0 = 5 = n+1
   Por de E.
       V ( C(n+1) -> (Oper (Q, V, n) -> On+1)) = 7
        Por hipotesus inductiva
        V ( Qn+1 -> Oper (Q, V, n) = P
        V ((Qn+1 > OPer(Q,V,N) -> Qn+1) =F
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a) = (V10 (Q1 V V1) = (V10 Q1 V V10 V1))
 Q(n): V(oper (QV4, V, n) = (oper (Q, V, n) V oper (d, V, n)
 caso base n= 0
 Q(0) = V (oper (OVY, V, O) = (Oper (O, V, O) V OPEN (Y, V, O))=1
   Por def
        7 = ((0+40)) = (0+40)) + T
           V ( ( 0 V Y ) = V ( ( 0 V Y ) = F . V
 Se Sipone QIn
 Caso inductivo M= N+1
  Q(n+1) = V(oper (Qviv, N, n+1) = (oper (Q, V, n+1) V oper (iv, N, n+1)) = (
 Por des
V((oper. (QVV, V, n) V (Opt V + n+1)) = ((oper (Q, V, n) V (Oper (+, V, n) V (Pn+1)))= 1
 Por hipotes indudical
N ((Oper (QVY, V, N) = (oper (Q, V, N) V oper (Y, V, N)) = P
V ((Qn+1 V Yn+1) = (Qn+1 V Yn+1)=7
      V((n+1) Yn+1) = V((n+1) Yn+1) = 7 /
b) = ( \(\hat{i}_0 \) (\(\text{0}_i \) \(\text{1}_i \) = ( \(\hat{i}_0 \) (\(\text{0}_i \) \(\hat{i}_{i0} \) \(\text{1}_i \))
Q(n) = V(oper(Qn+, n, n) = (oper(Q, n, n) n oper(+, n, n))) = 1
 coso buse n=0
Q(0) = V(Oper(QnY, n, 0) = (oper(Q, n, 0) n oper (Y, 1,0))) = T
 Por def-
      V ((QO N YO) = (QO N YO)) = F
 caso incluctivo n= n+1
Q(n+1) = V(qper(Ont, n, n+1) = (oper(Q, n, n+1) 1 oper(Y, n, n+1)) = T
 V ((Oper (( 1, 1, 1) ) N (( n+1 ) Yn+1)) = (( oper (( 1, 1, 1) ) ) ( ( oper (( 1, 1, 1) ) ) ) = 1
 By hipoteau inductive
V (oper ( ( nx, n, n) = 6 per ( ( n, n) , oper ( x, n, n)) = 1
V((Qn+1 N Yn+1) = (Qn+1 N Yn+1)) = T
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ejercicios 4.1 S. Demoestre $\models (\psi \equiv \varphi)$ $V = (\phi = \phi) = V$ Por metaleorema 2-23 V(Q) = V(Q)51 U(Q) = T SI VIQ = F . 7 = (T = T) U 7 = (7 = F) = T 7 = 7 T = T 6. encuentre proposiciones Q, Y, Y para las cuales = (Q[P== 4] = Q[P== 1]) Pero no = (4=7) Q = (Pvtrue) Y = true 7 = false = ((Putrue)[P:= true] = (Putrue)[P:= fonce]) = 1 (true v true) = (false v true)) Por definición N ((true v true) = (fanse v true)) = T H= W(true vilue), v (false vilue)) = 7 7 = (T : T) = H 7 = 7. 7 (Y = Y) = F V (true = false) = F F = F ejercicios 4.2 4. Hos true 1. ((true = true) = true) Ax3 2. (true = true) identidad tive identidad

$$\begin{aligned} & +_{\overline{b}_{S}} \quad ((\mathring{\phi} = \mathring{\psi}) \equiv \text{true}) \\ & \perp ((\mathring{\phi} = \text{true}) \equiv \mathring{\psi}) \\ & \geq ((\mathring{\phi} = (\mathring{\phi} = \text{true}))) \quad \text{conmoded windowd} (1) \\ & \geq ((\mathring{\phi} = \mathring{\psi}) \equiv \text{true}) \quad \text{Associative dad} (2) \end{aligned}$$

$$\begin{aligned} & +_{\overline{b}_{S}} \quad (\mathring{\psi} \equiv \mathring{\psi}) \\ & \geq ((\mathring{\phi} \equiv \mathring{\psi})) \quad \text{identified of (1)} \end{aligned}$$

$$\begin{aligned} & +_{\overline{b}_{S}} \quad (\mathring{\psi} \equiv \mathring{\psi}) \quad \text{identified of (1)} \end{aligned}$$

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$$\begin{aligned} & +_{\overline{b}_{S}} \quad (\mathring{\psi} \equiv \text{true}) \quad \text{identified (1)} \end{aligned}$$

$$\begin{aligned} & +_{\overline{b}_{S}} \quad (\mathring{\psi} \equiv \text{true}) \quad \text{identified (2)} \end{aligned}$$

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- 6. Sean ϕ y ψ proposiciones. Demuestre:
 - a) Si ϕ es un axioma, entonces $\vdash_{DS} \phi[p := \psi]$.
 - b) Si $\vdash_{DS} \phi$, entonces $\vdash_{DS} \phi[p := \psi]$.

b)

como φ es teorema, entonces φ es tautología y por definición de sustitución textual φ [$p:=\psi$] es tautología y al ser tautología también es teorema del sistema formal DS.