Relaciones

① "R-reflexiva" $\equiv (\forall a\, aRa)$

② "R-irreflexiva" $\equiv (\forall a|: \neg(aRa))$

③ "R-Simetrica" $\equiv (\forall a,b|: aRb \equiv bRa)$

④ "R-asimetrica" $\equiv (\forall a,b|: aRb \Rightarrow \neg(bRa))$

⑤ "R-antisimetrica" $\equiv (\forall a,b|: aRb \wedge bRa \Rightarrow a=b)$

⑥ "R-transitiva" $\equiv (\forall a,b,c|: aRb \wedge bRc \Rightarrow aRc)$

⑦ "R-lineal" $\equiv (\forall a,b|: aRb \vee bRa)$

⑧ "R-conectada" $\equiv (\forall a,b|: \neg(aRb) \wedge \neg(bRa) \Rightarrow a=b)$

⑨ "R-Introvertida" $\equiv (\forall a,b|: aRb \Rightarrow a=b)$

logica

- $(\varphi \wedge \psi) \rightarrow \varphi \equiv true$
- $(\varphi \wedge \psi \equiv \varphi) \equiv (\varphi \Rightarrow \psi)$

Def.

① $aR^{\smile}b \equiv bRa$

② $a\bar{R}b \equiv \neg(aRb)$

③ $a(R\cup S)b \equiv aRb \vee aSb$

④ $a(R\cap S)b \equiv aRb \wedge aSb$

⑤ $a(R\circ S)b \equiv (\exists x|: aRc \wedge cSb)$

⑥ $R = S \equiv (\forall a,b|: aRb \equiv aSb)$

⑦ $R \subseteq S \equiv (\forall a,b|: aRb \Rightarrow aSb)$

⑧ $a\,O\,b \equiv false$

⑨ $a\,I\,b \equiv a=b$

⑩ $a\,L\,b \equiv true$

Ax (⊓) $z \preceq a \sqcap b \equiv z \preceq a \wedge z \preceq b$

Ax (⊔) $a \sqcup b \preceq z \equiv a \preceq z \wedge b \preceq z$

desigualdades indirectas

$x \preceq y \equiv (\forall z|: z \preceq x \Rightarrow z \preceq y)$

$x \preceq y \equiv (\forall z|: y \preceq z \Rightarrow x \preceq z)$

igualdad indirecta

$a = b \equiv (\forall z|: z \preceq a \equiv z \preceq b)$

cota inferior $a \sqcap b \preceq a$ y $a \sqcap b \preceq b$

cota superior $a \preceq a \sqcup b$ y $b \preceq a \sqcup b$

Conexión. $a \preceq b \equiv a \sqcap b = a$

$a \preceq b \equiv a \sqcup b = b$

Monotonia simple $(aRb \Rightarrow (a \oplus c)\, R\, (b \oplus c)) \rightarrow$ $a \preceq b \Rightarrow a \sqcap c \preceq b \sqcap c$

$a \preceq b \Rightarrow a \sqcup c \preceq b \sqcup c$

Monotonia doble $(a \preceq b \wedge c \preceq d \Rightarrow a \sqcap c \preceq b \sqcap d)$

$(a \preceq b \wedge c \preceq d \Rightarrow a \sqcup c \preceq b \sqcup d)$

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"R-irreflexiva" $\equiv (\forall a,b \mid : \neg(aRa))$

(1)

o Teo. "R-Irreflexiva" $\equiv I \cap R = \emptyset$

Dem. Transito

$I \cap R = \emptyset$

$\equiv \langle$ def (=) $\rangle$

$(\forall a,b \mid : a(I \cap R)b \equiv false)$

$\equiv \langle$ def ($\cap$) $\rangle$

$(\forall a,b \mid : aIb \wedge aRb \equiv false)$

$\equiv \langle$ def (I) $\rangle$

$(\forall a,b \mid : a=b \wedge aRb \equiv false)$

$\equiv \langle$ def de ($\neg$) $\rangle$

$(\forall a,b \mid : \neg(a=b \wedge aRb))$

$\equiv \langle \neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi \rangle$

$(\forall a,b \mid : \neg(a=b) \vee \neg(aRb))$

$\equiv \langle$ def alternativa ($\Rightarrow$) $\rangle$

$(\forall a,b \mid : a=b \Rightarrow \neg(aRb))$

$\equiv \langle$ Regla del punto $\rangle$

$(\forall a \mid : \neg(aRa))$

$\equiv \langle$ def Irreflexiva $\rangle$

"R-Irreflexiva"

o Teo. "R-lineal" $\Rightarrow$ "R-reflexiva"

dem. Transito

"R-lineal"

$\equiv \langle$ def $\rangle$

$(\forall a,b \mid : aRb \vee bRa)$

$\Rightarrow \langle$ particularización $a=b$ $\rangle$

$(\forall a,b \mid : aRa \vee aRa)$

$\equiv \langle$ logica prop. $\rangle$

$(\forall a \mid : aRa)$

$\equiv \langle$ def $\rangle$

"R-reflexiva"

○ Teo. "R-Simetrica" $\wedge$ "R-antisimetrica" $\Rightarrow$ "R-introvertida"

"R-simetrica" $\equiv$ $(\forall a,b \mid : aRb \equiv bRa)$

"R-antisimetrica" $\equiv$ $(\forall a,b \mid : aRb \wedge bRa \Rightarrow a=b)$

"R-introvertida" $\equiv$ $(\forall a,b \mid : aRb \Rightarrow a=b)$

Dem. Supongo "R-Simetrica"

basta demostrar. "R-antisimetrica" $\Rightarrow$ "R-introvertida"

Dem Transito

"R-antisimetrica"

$\equiv$ <def>

$(\forall a,b \mid : aRb \wedge bRa \Rightarrow a=b)$

$\equiv$ <sup. $aRb \equiv bRa$>

$(\forall a,b \mid : aRb \wedge aRb \Rightarrow a=b)$

$\equiv$ <logica Prop.>

$(\forall a,b \mid : aRb \Rightarrow a=b)$

$\equiv$ <def>

"R-Introvertida"

○ Teo. "R-reflexiva" $\equiv$ "R^T-reflexiva"

"R^T-reflexiva"

$\equiv$ <def>

$(\forall a \mid : aR^Ta)$

$\equiv$ <def R^T>

$(\forall a \mid : aRa)$

$\equiv$ <def>

"R-reflexiva"

○ Teo "R-reflexiva" $\equiv$ "$\bar{R}$-reflexiva"

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \rightarrow \text{No es reflexiva.}$$

"R-reflexiva" "$\bar{R}$-reflexiva"

$\rightarrow a \cdot b = b)$

○ Teo. $ant(R) \wedge ant(S) \Rightarrow ant(R \cap S)$

Dem. Sup: $ant(R) \wedge ant(S)$

basta demostrar. $ant(R \cap S)$

$a(R \cap S)b \wedge b(R \cap S)a$

$\equiv \langle def\ (\cap) \rangle$

$aRb \wedge aSb \wedge bRa \wedge bSa$

$\Rightarrow \langle sup \rangle$

$a = b \wedge a = b$

$\equiv \langle \varphi \wedge \varphi \equiv \varphi \rangle$

$a = b$

○ Teo. $ant(R) \equiv R \cap R^T \subseteq I$

$R \cap R^T \subseteq I$

$\equiv \langle def \subseteq \rangle$

$(\forall a,b \mid : a(R \cap R^T)b \Rightarrow aIb)$

$\equiv \langle def\ (\cap) \rangle$

$(\forall a,b \mid : aRb \wedge aR^Tb \Rightarrow a = b)$

$\equiv \langle def\ R^T \rangle$

$(\forall a,b \mid : aRb \wedge bRa \Rightarrow a = b)$

$\equiv$ < def "R-antisimetrica" >

"R- antisimetrica"

○ Teo. $tran(R) \equiv R \circ R \subseteq R$

$R \circ R \subseteq R$

$\equiv \langle def \subseteq \rangle$

$(\forall a,b \mid : a(R \circ R)b \Rightarrow aRb)$

$\equiv \langle def\ (\circ) \rangle$

$(\forall a,b \mid : (\exists x \mid : aRx \wedge xRb) \Rightarrow aRb)$

$\equiv \langle (\exists x \mid : P) \Rightarrow Q \equiv (\forall x \mid : P \Rightarrow Q)$; anidamiento>

$(\forall a,b,x \mid : aRx \wedge xRb \Rightarrow aRb)$

$\equiv \langle def\ tran(R) \rangle$

$Tran(R)$

○ Teo. $ant(R) \Rightarrow (R \cap \bar{I})$ asimetrica.

$asi(R \cap \bar{I})$

$\equiv$ < def asimetrica>

$(\forall a,b \mid : a(R \cap \bar{I})b \Rightarrow \neg(b(R \cap \bar{I})a))$

$\equiv \langle def\ (\cap) \rangle$

$(\forall a,b \mid : aRb \wedge a\bar{I}b \Rightarrow \neg(bRa \wedge b\bar{I}a))$

$\equiv \langle def\ alt. (\Rightarrow) \rangle$

$(\forall a,b \mid : \neg(aRb \wedge a\bar{I}b) \vee \neg(bRa \wedge b\bar{I}a))$

$\equiv$ < de morgan>

$(\forall a,b \mid : \neg(aRb) \vee \neg(a\bar{I}b) \vee \neg(bRa) \vee \neg(b\bar{I}a))$

$\equiv\langle$ doble ($\neg$), def (I), conmutat. (=) $\rangle$

$(\forall a,b \mid: \neg(aRb) \vee \neg(bRa) \vee a=b)$

$\equiv\langle$ def alt. ($\Rightarrow$) $\rangle$

$(\forall a,b \mid: aRb \wedge bRa \Rightarrow a=b)$

$\equiv\langle$ def ant (R) $\rangle$

ant (R)

○ Teo. conectada(R) $\Rightarrow$ ($\bar{R}$) antisimetrica

"$\bar{R}$ - antisimetrica"

$\equiv\langle$ def anti $\rangle$

$(\forall a,b \mid: a\bar{R}b \wedge b\bar{R}a \Rightarrow a=b)$

$\equiv\langle$ def ($\bar{R}$) $\rangle$

$(\forall a,b \mid: \neg(aRb) \wedge \neg(bRa) \Rightarrow a=b)$

$\equiv\langle$ def conec(R) $\rangle$

con (R)

○ Teo. "R - asimétrica" $\equiv$ $\boxed{\text{"R - irreflexivo"}}$

"R - asimetrica"

$\equiv\langle$ def $\rangle$

$(\forall a,b \mid: aRb \Rightarrow \neg(bRa))$

$\Rightarrow\langle$ particularización $a=b$ $\rangle$

$(\forall a,b \mid: aRa \Rightarrow \neg(aRa))$

$\equiv\langle$ logicas $\rangle$

$(\forall a,b \mid: \neg(aRa))$

$\equiv\langle$ def "$\bar{R}$ - lineal" $\rangle$

"$\bar{R}$ - lineal"

③

- Teo. "R- introvertida" $\equiv R \subseteq I$

$R \subseteq I$
$\equiv$ <def ($\subseteq$)>
$(\forall a,b \mid : aRb \Rightarrow a=b)$
$\equiv$ <def introvertida>
"R - Introvertido"

- Teo. "R - reflexiva" $\equiv I \subseteq R$

$I \subseteq R$
$\equiv$ <def $\subseteq$>
$(\forall a,b \mid : aIb \Rightarrow aRb)$
$\equiv$ <def (I)>
$(\forall a,b \mid : a=b \Rightarrow aRb)$
$\equiv$ < regla del punto>
$(\forall a \mid : aRa)$

- Teo. "R - irreflexiva" $\equiv I \cap R = \emptyset$

$I \cap R = \emptyset$
$\equiv$ < def (=), ($\cap$), (I)>
$(\forall a,b \mid : a=b \wedge aRb \equiv false)$
$\equiv$ < def alt. (¬) ; logica >
$(\forall a,b \mid : \neg(a=b) \vee \neg(aRb))$
$\equiv$ < logica >
$(\forall a,b \mid : a=b \Rightarrow \neg(aRb))$
$\equiv$ < punto u def irreflexiva >
"R - irreflexiva"

- Teo. "R-simetrica" $\equiv R = R^T$

$R = R^T$
$\equiv$ <def (=)>
$(\forall a,b \mid : aRb \equiv aR^Tb)$
$\equiv$ <def R^T >
$(\forall a,b \mid : aRb \equiv bRa)$
$\equiv$ <def>
"R-simetrica"

precedencia mutua

$(a \uparrow b) \cdot (c \uparrow \dots$

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∘ Teo. "R-asimétrica" $\equiv R \cap R^T = \emptyset$

$R \cap R^T = \emptyset$

$\equiv$ < def ($\subseteq$), ($\cap$), ($\emptyset$) >

$(\forall a,b \mid : aRb \wedge aR^Tb \equiv false)$

$\equiv$ < def R^T y ($\neg$) >

$(\forall a,b \mid : \neg(aRb) \vee \neg(bRa))$

$\equiv$ < def ($\Rightarrow$) >

$(\forall a,b \mid : aRb \Rightarrow \neg(bRa))$

$\equiv$ < def asym >

"R-asimetrica"

∘ Teo. "R-asimetrica" $\equiv R \subseteq \overline{R^T}$

$R \subseteq \overline{R^T}$

$\equiv$ < def ($\subseteq$), ($\neg$) >

$(\forall a,b \mid : aRb \Rightarrow \neg(aR^Tb))$

$\equiv$ < def (R^T) >

$(\forall a,b \mid : aRb \Rightarrow \neg(bRa))$

$\equiv$ < def >

"R-asimetrica"

∘ Teo. "R-antisimetrica" $\equiv R \cap R^T \subseteq I$

$R \cap R^T \subseteq I$

$\equiv$ < def ($\subseteq$), ($\cap$), (R^T) >

$(\forall a,b \mid : aRb \wedge bRa \Rightarrow a=b)$

$\equiv$ < def >

"R-antisimetrica"

∘ Teo. "R-transitivo" $\equiv R \circ R \subseteq R$

$R \circ R \subseteq R$

$\equiv$ < def ($\subseteq$) >

$(\forall a,b \mid : a(R \circ R)b \Rightarrow aRb)$

$\equiv$ < def ($R \circ R$) >

$(\forall a,b \mid : (\exists x \mid : aRx \wedge xRb) \Rightarrow aRb)$

$\equiv$ < $(\exists x \mid : \varphi) \Rightarrow \psi \equiv (\forall x \mid : \varphi \Rightarrow \psi)$, anidamiento >

$(\forall a,b,x \mid : aRx \wedge xRb \Rightarrow aRb)$

$\equiv$ < def >

"R-transitiva"

∘ Teo. "R-lineal" $\equiv R \cup R^T = L$

$R \cup R^T = L$

$\equiv$ < def (=), (∪), (R^T); (L) >

$(\forall a,b \mid : aRb \vee bRa \equiv true)$

$\equiv$ < logica y def lineal >

"R-lineal"

∘ Teo. "R-conectada" $\equiv \overline{R} \cap \overline{R^T} \subseteq I$

$\overline{R} \cap \overline{R^T} \subseteq I$

$\equiv$ < def (∩), (R^T), ($\overline{R}$), (I) >

$(\forall a,b \mid : \neg(aRb) \wedge \neg(bRa) \Rightarrow a=b)$

$\equiv$ < def >

"R-conectada"

∘ Teo. $a \sqcap b = b \sqcap a$

$a \sqcap b = b \sqcap a$

$\equiv$ < igualdad indirecta >

$(\forall z \mid : z \preceq a \sqcap b \equiv z \preceq b \sqcap a)$

$\equiv$ < ax ⊓ y generalización >

$z \preceq a \wedge z \preceq b \equiv z \preceq b \wedge z \preceq a$

$\equiv$ < $(a \equiv a) \equiv true$ >

true

• Teo. $a \sqcup b = b \sqcup a$

$a \sqcup b = b \sqcup a$

$\equiv$ < igualdad indirecta >

$(\forall z \mid : a \sqcup b \preceq z \equiv b \sqcup a \preceq z)$

$\equiv$ < ax. ⊔ , generalización >

$a \preceq z \wedge b \preceq z \equiv b \preceq z \wedge a \preceq z$

$\equiv$ < $(a \equiv a) \equiv true$ >

true

- Teo. $(a \sqcap b) \sqcap c = a \sqcap (b \sqcap c)$

  $(a \sqcap b) \sqcap c = a \sqcap (b \sqcap c)$

  $\equiv$ < igualdad indirecta; Ax. $\sqcap$ >

  $(\forall z|: z \preceq (a \sqcap b) \wedge z \preceq c \equiv z \preceq a \wedge z \preceq b \sqcap c)$

  $\equiv$ < Ax. $\sqcap$ y generalización >

  $z \preceq a \wedge z \preceq b \wedge z \preceq c \equiv z \preceq a \wedge z \preceq b \wedge z \preceq c)$

  $\equiv$ < $(\varphi \equiv \varphi) \equiv true$ >

  true

- Teo. $a \sqcap a = a$

  $a \sqcap a = a$

  $\equiv$ < igualdad indirecta >

  $(\forall z|: z \preceq a \sqcap a \equiv z \preceq a)$

  $\equiv$ < Ax. $\sqcap$, logica $(\varphi \wedge \varphi) \equiv \varphi$; generalización >

  $z \preceq a \equiv z \preceq a$

  $\equiv$ < $(\varphi \equiv \varphi) \equiv true$ >

  true

- Teo. $a \sqcap b \preceq a$

  $a \sqcap b \preceq a$

  $\equiv$ < desigualdad indirecta. Prede. >

  $(\forall z|: z \preceq a \sqcap b \Rightarrow z \preceq a)$

  $\equiv$ < generalización >

  $z \preceq a \sqcap b \Rightarrow z \preceq a$

  $\equiv$ < Ax. $\sqcap$ >

  $z \preceq a \wedge z \preceq b \Rightarrow z \preceq a$

  $\equiv$ < logica, debilitamiento >

  true

- Teo. $a \preceq a \sqcup b$

  $a \preceq a \sqcup b$

  $\equiv$ < desigualdad indirecta. Suces >

  $(\forall z|: a \sqcup b \preceq z \Rightarrow a \preceq z)$

  $\equiv$ < Ax. $\sqcup$; generalización >

  $a \preceq z \wedge b \preceq z \Rightarrow a \preceq z$

  $\equiv$ < logica, debilitamiento >

  true

- Teo. $a \sqcap b = a \equiv a \preceq b$

$a \sqcap b = a$

$\equiv$ < igualdad indirecta >

$(\forall z \mid: z \preceq a \sqcap b \equiv z \preceq a)$

$\equiv$ < ax. $\sqcap$, generalización >

$z \preceq a \wedge z \preceq b \equiv z \preceq a$

$\equiv$ < lógica, unión >

$z \preceq a \Rightarrow z \preceq b$

$\equiv$ < desigualdad indirecta >

$a \preceq b$

- Teo. $a \preceq b \Rightarrow a \sqcap c \preceq b \sqcap c$

$a \sqcap c \preceq b \sqcap c$

$\equiv$ < ax $\sqcap$ >

$a \sqcap c \preceq b \wedge a \sqcap c \preceq c$

$\equiv$ < cota inf >

$a \sqcap c \preceq b \wedge true$

$\Leftarrow$ < transitiva, $a \sqcap c \preceq a$ >

$a \preceq b$

- Teo. $a \sqcap b \preceq a \sqcup b$

$a \sqcap b$

$\preceq$

a

$\preceq$

$a \sqcup b$

Parcial 1° corte

1. $R \subseteq S \equiv R^T \subseteq S^T$

Dem.

$R^T \subseteq S^T$

$\equiv \langle \text{def} \subseteq \rangle$

$(\forall a,b \mid : a R^T b \Rightarrow a S^T b)$

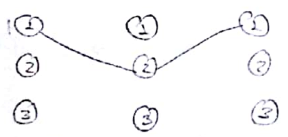
$\equiv \langle \text{def } R^T \rangle$

$(\forall a,b \mid : b R a \Rightarrow b S a)$

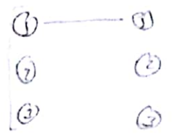
$\equiv \langle \text{def} \subseteq \rangle$

$R \subseteq S$

2. R y S irreflexiva, es $R \circ S$ irreflexiva

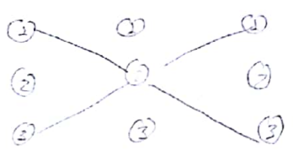


R-irr S-irr



$R \circ S$ false

3. $asim(R) \wedge asim(S) \Rightarrow asim(R \circ S)$



$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

R S

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

false

4. R y S son asimetricas, ¿$R \circ S$ es asimetrica?

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

R S $R \circ S$ false

5. $iny(R) \equiv R \circ R^T \subseteq I$

$iny(R) \equiv (\forall a,b,c \mid : aRc \wedge bRc \Rightarrow a = b)$

Dem. $R \circ R^T \subseteq I$

$\equiv \langle \text{def} \subseteq I \rangle$

$(\forall a,b \mid a(R \circ R^T)b \Rightarrow a = b)$

$\equiv \langle \text{def } R \circ S \rangle$

$(\forall a,b \mid (\exists c \mid aRc \wedge cR^T b) \Rightarrow a = b)$

$\equiv \langle \text{logica, def } R^T \rangle$

$(\forall a,b,c \mid aRc \wedge bRc \Rightarrow a = b)$

$\equiv \langle \text{def iny} \rangle$

$iny(R)$

6. $tot(R) \equiv (\forall a | : (\exists b | : aRb))$

$tot(R) \equiv I \subseteq R \circ L$

Dem.

$I \subseteq R \circ L$

$\equiv \langle def \subseteq \rangle$

$(\forall a,b | aIb \Rightarrow a(R \circ L)b)$

$\equiv \langle def\ L,\ R \circ S \rangle$

$(\forall a,b | true \Rightarrow (\exists c | aRc \wedge cLb))$

$\equiv \langle def\ L, lógica \rangle$

$(\forall a | (\exists c | aRc))$

$\equiv \langle def\ tot \rangle$

$tot(R)$

7. $c \preceq a \sqcap b \Rightarrow a \sqcup c = a$

$c \preceq a \sqcap b$

$\Rightarrow \langle monotonía \rangle$

$a \sqcup c \preceq a \sqcup (a \sqcap b)$

$\equiv \langle absorción \rangle$

$a \sqcup c \preceq a$

$\equiv \langle a \preceq a \sqcup c,\ cota\ sup \rangle$

$a \sqcup c \preceq a \wedge a \preceq a \sqcup c$

$\Rightarrow \langle antisimetría \rangle$

$a \sqcup c = a$

8. $a \sqcup b \preceq b \Rightarrow a \preceq a \sqcap b$

Sup. $a \sqcup b \preceq b \equiv a \preceq b \wedge b \preceq b$

Dem.

$a \preceq a \sqcap b$

$\equiv \langle ax\ \sqcap \rangle$

$a \preceq a \wedge a \preceq b$

$\equiv \langle orden\ parcial \rangle$

$a \preceq b$

$\equiv \langle sup \rangle$

true

Dem. $a \sqcup b \preceq b$

$\Rightarrow \langle monotonía \rangle$

$a \sqcap (a \sqcup b) \preceq a \sqcap b$

$\equiv \langle absorción \rangle$

$a \preceq a \sqcap b$