# Análise de Séries Temporais

0.8 - Aula Prática

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## Modelo autorregressivo

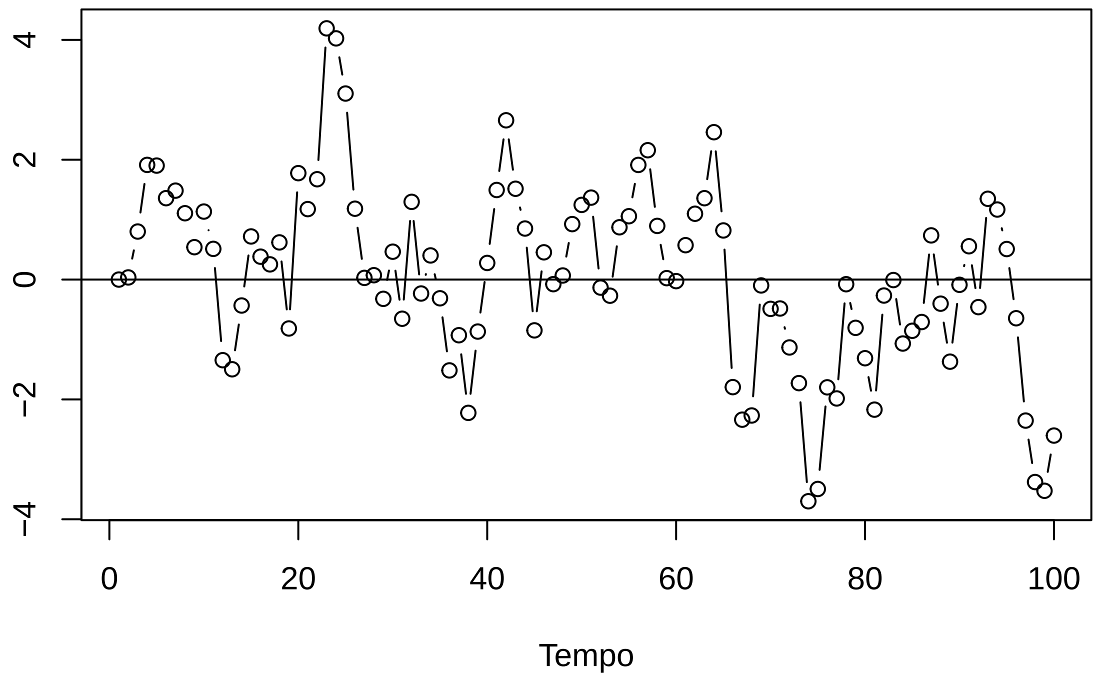
### Exemplo 1

Vamos simular uma série temporal com 100 observações de acordo com o modelo AR(1) dado por

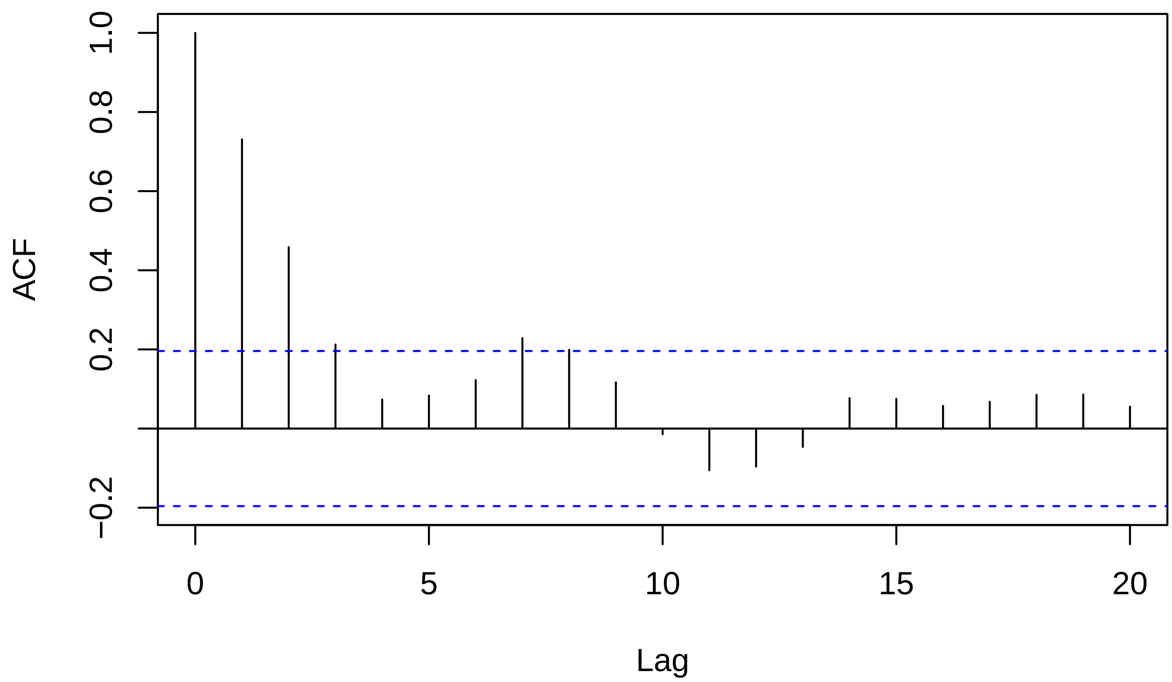
$$Z_t = 0,8Z_{t-1} + a_t,$$

sendo que $a_t \sim N(0, 1)$.

```r
set.seed(2025)

n <- 100
a <- rnorm(n)
z <- numeric(n)
z[1] <- 0

for(t in 2:n){
  z[t] <- 0.8*z[t-1] + a[t]
}

# plot da serie temporal simulada

plot(z, type="b", ylab="", xlab="Tempo")
abline(h=0)
```




```r
# acf da serie simulada
acf(z, main="")
```




```r
# ajuste
# usando a funcao do pacote basico do R
arima(z, order=c(1, 0, 0)) # AR(1) com intercepto
```

```
##
## Call:
## arima(x = z, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.7465    -0.0283
## s.e.  0.0668     0.3884
##
## sigma^2 estimated as 1.022:  log likelihood = -143.39,  aic = 292.77
```

```r
arima(z, order=c(1, 0, 0), include.mean = F) # AR(1) sem intercepto
```

```
##
## Call:
## arima(x = z, order = c(1, 0, 0), include.mean = F)
##
## Coefficients:
##          ar1
##       0.7463
## s.e.  0.0667
##
## sigma^2 estimated as 1.022:  log likelihood = -143.39,  aic = 290.78
```

```r
# usando o pacote forecast
library(forecast)
```

```
## Warning: pacote 'forecast' foi compilado no R versão 4.4.3

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```r
Arima(z, order=c(1, 0, 0)) # AR(1) com intercepto
```

```
## Series: z
## ARIMA(1,0,0) with non-zero mean
##
## Coefficients:
##          ar1     mean
##       0.7465  -0.0283
## s.e.  0.0668   0.3884
##
## sigma^2 = 1.043:  log likelihood = -143.39
## AIC=292.77   AICc=293.02   BIC=300.59
```

```r
Arima(z, order=c(1, 0, 0), include.mean = F) # AR(1) sem intercepto
```

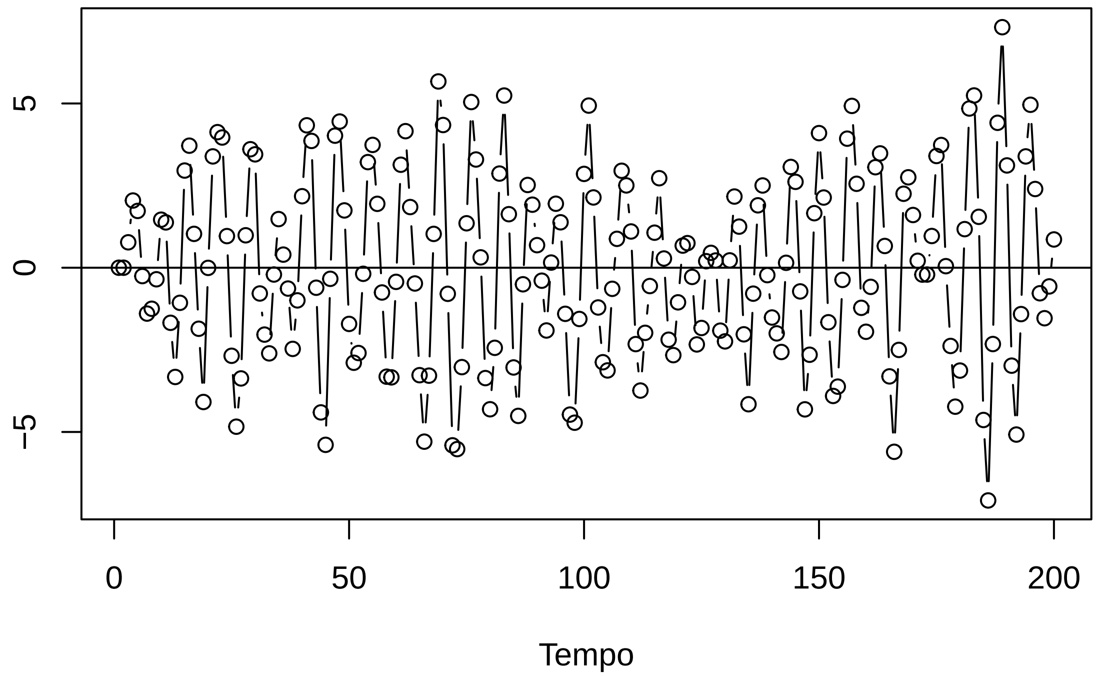
```
## Series: z
## ARIMA(1,0,0) with zero mean
##
## Coefficients:
##          ar1
##       0.7463
## s.e.  0.0667
##
## sigma^2 = 1.032:  log likelihood = -143.39
## AIC=290.78   AICc=290.9   BIC=295.99
```

### Exemplo 2

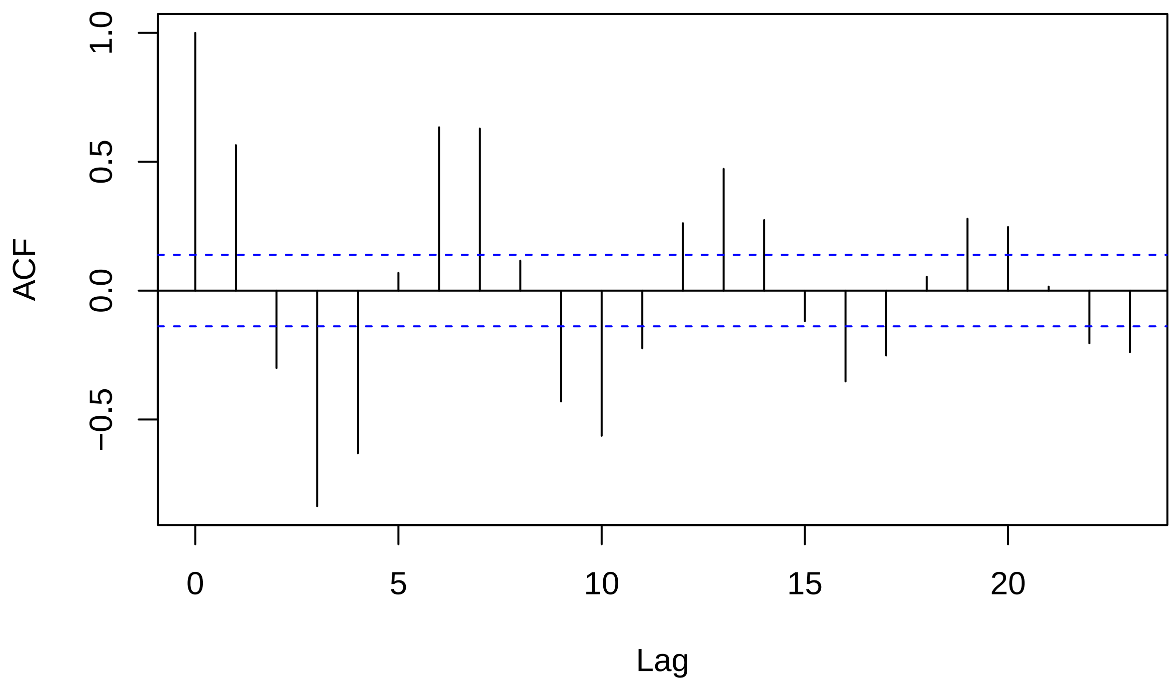
Vamos agora simular uma série temporal com 200 observações de acordo com o modelo AR(2) dado por $z_t = 1,0z_{t-1} - 0,89z_{t-2} + a_t$.

```r
set.seed(2025)

n <- 200
a <- rnorm(n)
z <- numeric(n)

z[1] <- z[2] <- 0

for(t in 3:n){
  z[t] <- 1*z[t-1] -0.89*z[t-2]+ a[t]
}


# plot da serie temporal simulada

plot(z, type="b", ylab="", xlab="Tempo")
```

```r
abline(h=0)
```




```r
# acf da serie simulada
acf(z, main="")
```




```r
# usando o pacote forecast
library(forecast)

Arima(z, order=c(2, 0, 0)) # AR(2) com intercepto
```

```
## Series: z
## ARIMA(2,0,0) with non-zero mean
##
## Coefficients:
##          ar1      ar2    mean
##       1.0743  -0.9023  -0.001
## s.e.  0.0290   0.0281   0.083
##
## sigma^2 = 0.95:  log likelihood = -279.02
## AIC=566.04   AICc=566.25   BIC=579.23
```

```r
Arima(z, order=c(2, 0, 0), include.mean = F) # AR(2) sem intercepto
```

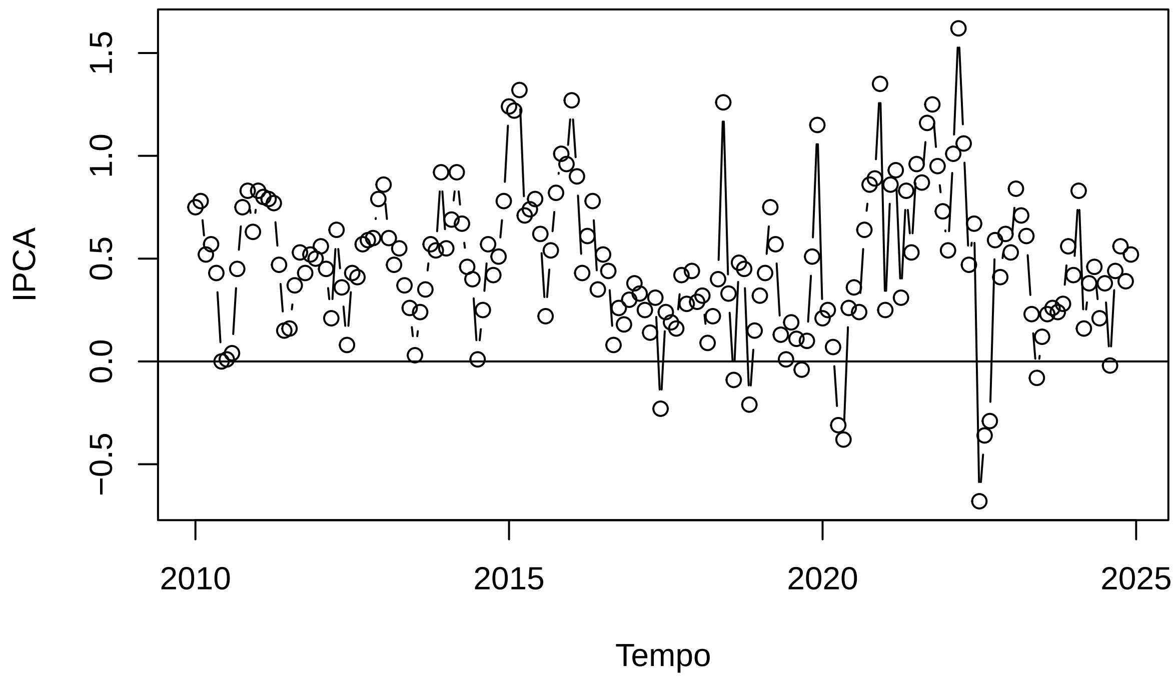
```
## Series: z
## ARIMA(2,0,0) with zero mean
##
## Coefficients:
##          ar1      ar2
##       1.0743  -0.9023
## s.e.  0.0290   0.0281
##
## sigma^2 = 0.9452:  log likelihood = -279.02
```

```
## AIC=564.04   AICc=564.16   BIC=573.94
```

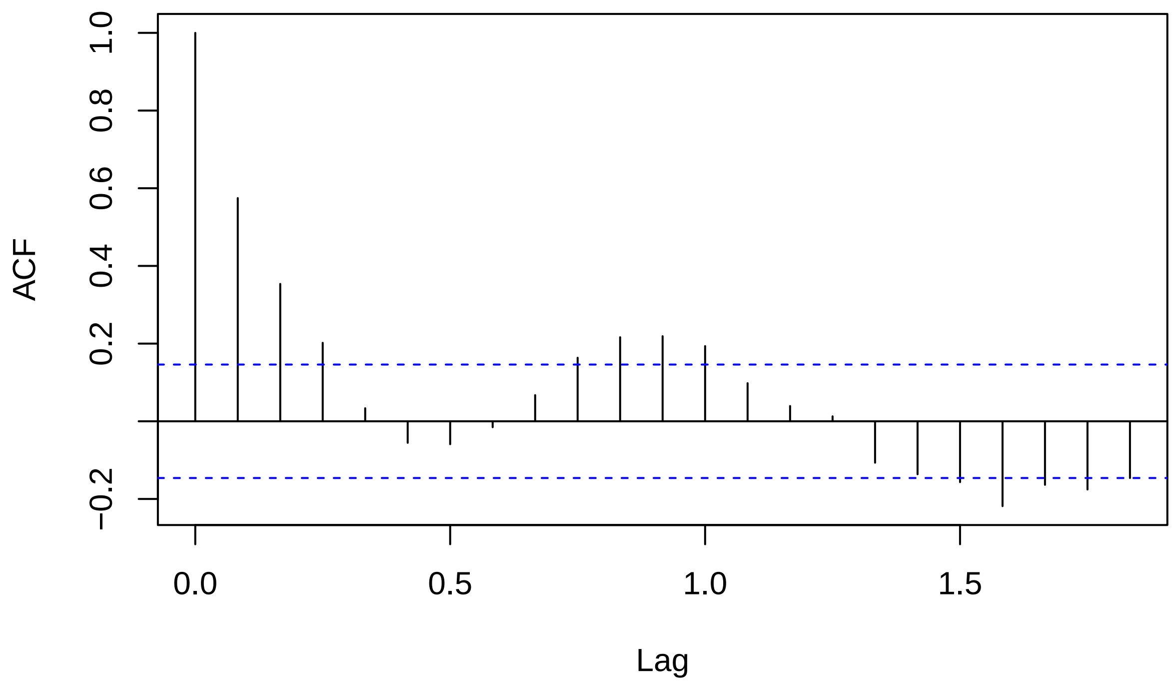
### Exemplo 3

Vamos agora utilizar a série temporal para ajustar algumas ordens dos modelos AR. Só para lembrar, a série temporal a seguir é do IPCA (índice nacional de preços ao consumidor amplo) do período de janeiro de 2010 até dezembro de 2024.

```r
# serie temporal do IPCA

serie_ipca <- c(0.75, 0.78, 0.52, 0.57, 0.43, 0.00, 0.01, 0.04, 0.45, 0.75, 0.83, 0.63,
                0.83, 0.80, 0.79, 0.77, 0.47, 0.15, 0.16, 0.37, 0.53, 0.43, 0.52, 0.50,
                0.56, 0.45, 0.21, 0.64, 0.36, 0.08, 0.43, 0.41, 0.57, 0.59, 0.60, 0.79,
                0.86, 0.60, 0.47, 0.55, 0.37, 0.26, 0.03, 0.24, 0.35, 0.57, 0.54, 0.92,
                0.55, 0.69, 0.92, 0.67, 0.46, 0.40, 0.01, 0.25, 0.57, 0.42, 0.51, 0.78,
                1.24, 1.22, 1.32, 0.71, 0.74, 0.79, 0.62, 0.22, 0.54, 0.82, 1.01, 0.96,
                1.27, 0.90, 0.43, 0.61, 0.78, 0.35, 0.52, 0.44, 0.08, 0.26, 0.18, 0.30,
                0.38, 0.33, 0.25, 0.14, 0.31,-0.23, 0.24, 0.19, 0.16, 0.42, 0.28, 0.44,
                0.29, 0.32, 0.09, 0.22, 0.40, 1.26, 0.33,-0.09, 0.48, 0.45,-0.21, 0.15,
                0.32, 0.43, 0.75, 0.57, 0.13, 0.01, 0.19, 0.11,-0.04, 0.10, 0.51, 1.15,
                0.21, 0.25, 0.07,-0.31,-0.38, 0.26, 0.36, 0.24, 0.64, 0.86, 0.89, 1.35,
                0.25, 0.86, 0.93, 0.31, 0.83, 0.53, 0.96, 0.87, 1.16, 1.25, 0.95, 0.73,
                0.54, 1.01, 1.62, 1.06, 0.47, 0.67,-0.68,-0.36,-0.29, 0.59, 0.41, 0.62,
                0.53, 0.84, 0.71, 0.61, 0.23,-0.08, 0.12, 0.23, 0.26, 0.24, 0.28, 0.56,
                0.42, 0.83, 0.16, 0.38, 0.46, 0.21, 0.38,-0.02, 0.44, 0.56, 0.39, 0.52)

ipca_ts <- ts(serie_ipca, start= c(2010, 1), frequency = 12)

# grafico da serie
plot(ipca_ts, type="b", ylab="IPCA", xlab="Tempo")
abline(h=0)
```




```r
# acf da serie
acf(ipca_ts, main="")
```




```
# usando o pacote forecast
library(forecast)

ajuste1 <- Arima(ipca_ts, order=c(1, 0, 0)) # AR(1) com intercepto
summary(ajuste1)
```

```
## Series: ipca_ts
## ARIMA(1,0,0) with non-zero mean
##
## Coefficients:
##          ar1    mean
##       0.5734  0.4795
## s.e.  0.0607  0.0509
##
## sigma^2 = 0.08706:  log likelihood = -34.9
## AIC=75.8   AICc=75.93   BIC=85.38
##
## Training set error measures:
##                          ME      RMSE       MAE  MPE MAPE      MASE        ACF1
## Training set -0.001133749 0.2934154 0.2209792 -Inf  Inf 0.6500526 -0.0192077
```

```
confint(ajuste1)
```

```
##                2.5 %    97.5 %
## ar1       0.4544142 0.6922987
## intercept 0.3797570 0.5792230
```

```r
ajuste2 <- Arima(ipca_ts, order=c(2, 0, 0)) # AR(2) com intercepto
summary(ajuste2)
```

```
## Series: ipca_ts
## ARIMA(2,0,0) with non-zero mean
##
## Coefficients:
##          ar1     ar2    mean
##       0.5525  0.0363  0.4797
## s.e.  0.0742  0.0743  0.0527
##
## sigma^2 = 0.08743:  log likelihood = -34.78
## AIC=77.56   AICc=77.79   BIC=90.33
##
## Training set error measures:
##                        ME      RMSE       MAE  MPE MAPE      MASE        ACF1
## Training set -0.001248374 0.2932193 0.2206198 -Inf  Inf 0.6489955 0.002370084
```

```r
confint(ajuste2)
```

```
##                2.5 %    97.5 %
## ar1        0.4069917 0.6979181
## ar2       -0.1093055 0.1818487
## intercept  0.3764420 0.5830374
```

# Modelo de Médias Móveis

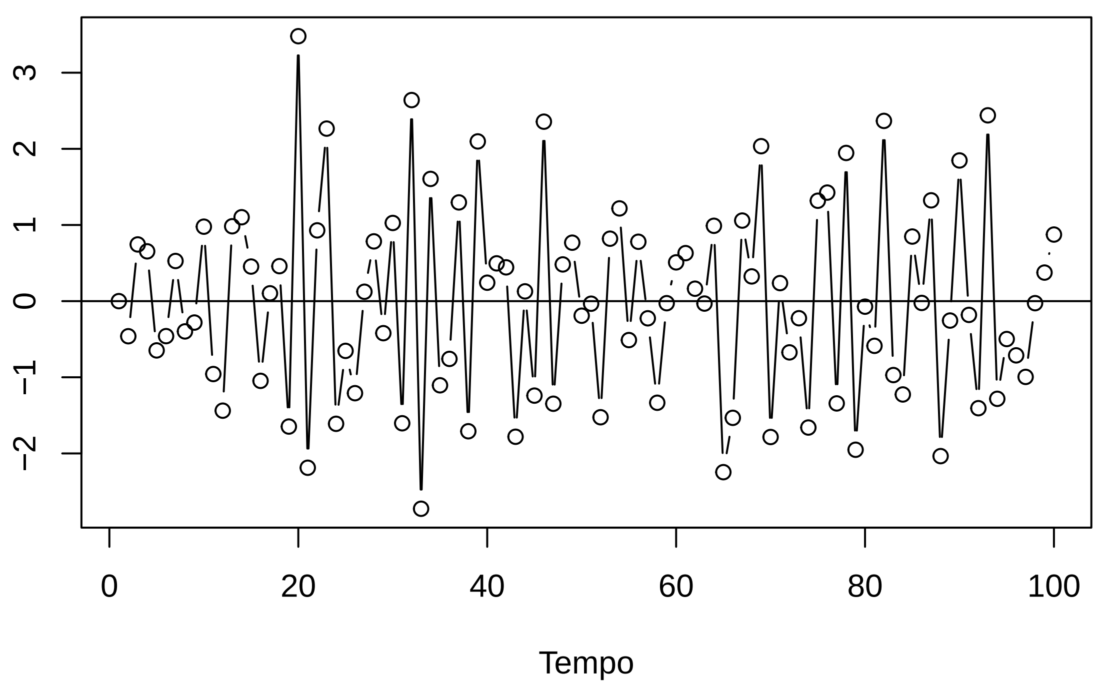
## Exemplo 1

Vamos agora simular uma série temporal com 100 observações de acordo com o modelo MA(1) dado por

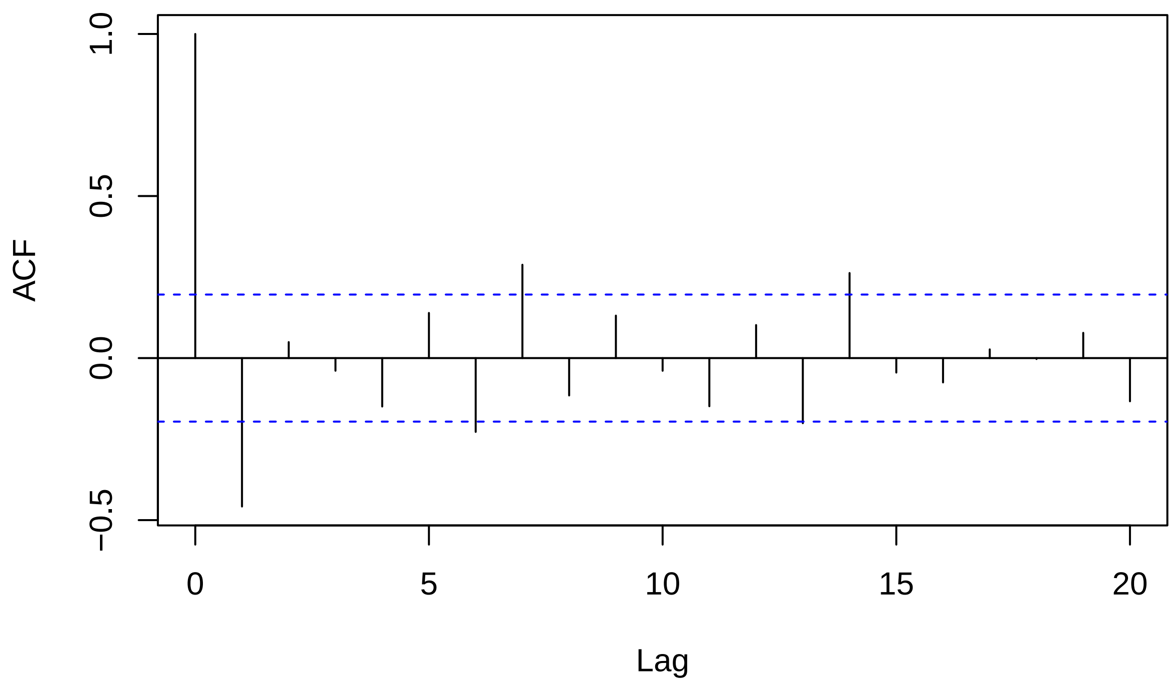
$$Z_t = a_t - 0,8a_{t-1},$$

sendo que $a_t \sim N(0,1)$.

```r
set.seed(2025)

n <- 100
a <- rnorm(n)
z <- numeric(n)
z[1] <- 0

for(t in 2:n){
  z[t] <- a[t] - 0.8*a[t-1]
}

# plot da serie temporal simulada

plot(z, type="b", ylab="", xlab="Tempo")
abline(h=0)
```



```r
# acf da serie simulada
acf(z, main="")
```




```r
# usando o pacote forecast
library(forecast)

Arima(z, order=c(0, 0, 1)) # MA(1) com intercepto
```

```
## Series: z
## ARIMA(0,0,1) with non-zero mean
##
## Coefficients:
##           ma1     mean
##       -0.8558  -0.0071
## s.e.   0.0595   0.0155
##
## sigma^2 = 1.026:  log likelihood = -142.85
## AIC=291.7   AICc=291.95   BIC=299.51
```

```r
Arima(z, order=c(0, 0, 1), include.mean = F) # MA(1) sem intercepto
```

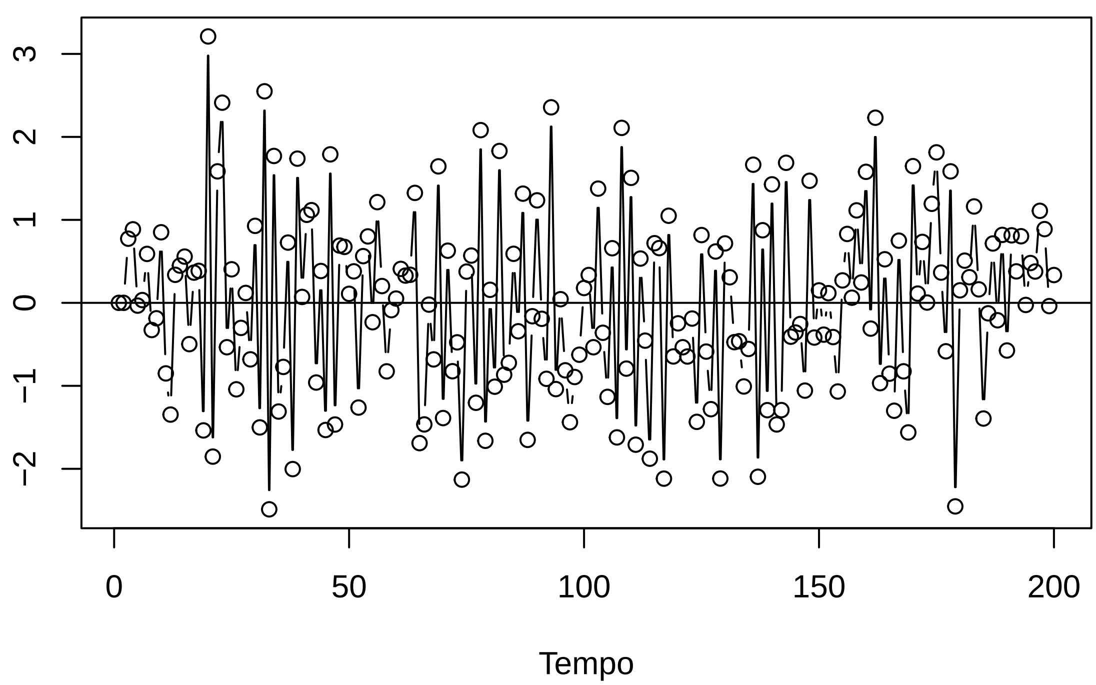
```
## Series: z
## ARIMA(0,0,1) with zero mean
##
## Coefficients:
##           ma1
##       -0.8539
```

```
## s.e.   0.0600
##
## sigma^2 = 1.018:  log likelihood = -142.95
## AIC=289.91   AICc=290.03   BIC=295.12
```

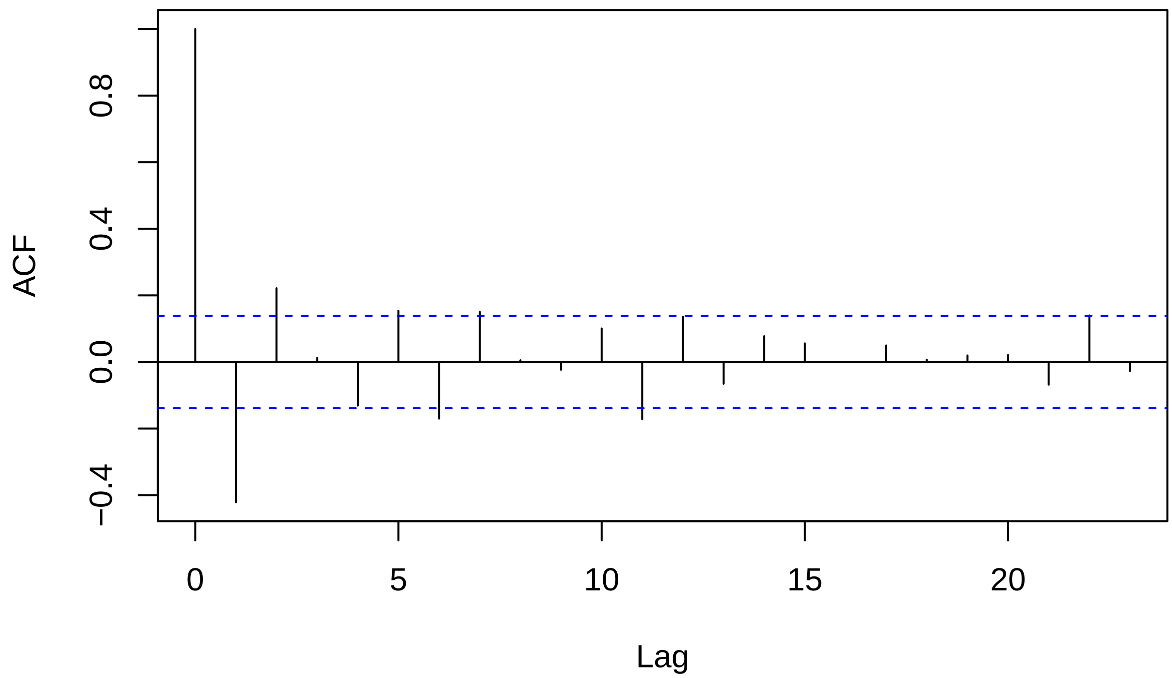
#Exemplo 2

Vamos agora simular uma série temporal com 200 observações de acordo com o modelo MA(2) dado por $z_t = a_t - 0,5a_{t-1} + 0,3a_{t-2}$.

```
set.seed(2025)

n <- 200
a <- rnorm(n)
z <- numeric(n)

a[1] <- a[2] <- 0
z[1] <- z[2] <- 0

for(t in 3:n){
z[t] <- a[t] - 0.5*a[t-1] + 0.3*a[t-2]
}

# plot da serie temporal simulada
plot(z, type="b", ylab="", xlab="Tempo")
abline(h=0)
```



```r
# acf da serie simulada
acf(z, main="")
```




```r
# usando o pacote forecast
library(forecast)
Arima(z, order=c(0, 0, 2)) # MA(2) com intercepto
```

```
## Series: z
## ARIMA(0,0,2) with non-zero mean
##
## Coefficients:
##           ma1     ma2     mean
##       -0.3773  0.2723  -0.0010
## s.e.   0.0659  0.0730   0.0619
##
## sigma^2 = 0.9718:  log likelihood = -279.54
## AIC=567.07   AICc=567.28   BIC=580.27
```

```r
Arima(z, order=c(0, 0, 2), include.mean = F) # MA(2) sem intercepto
```

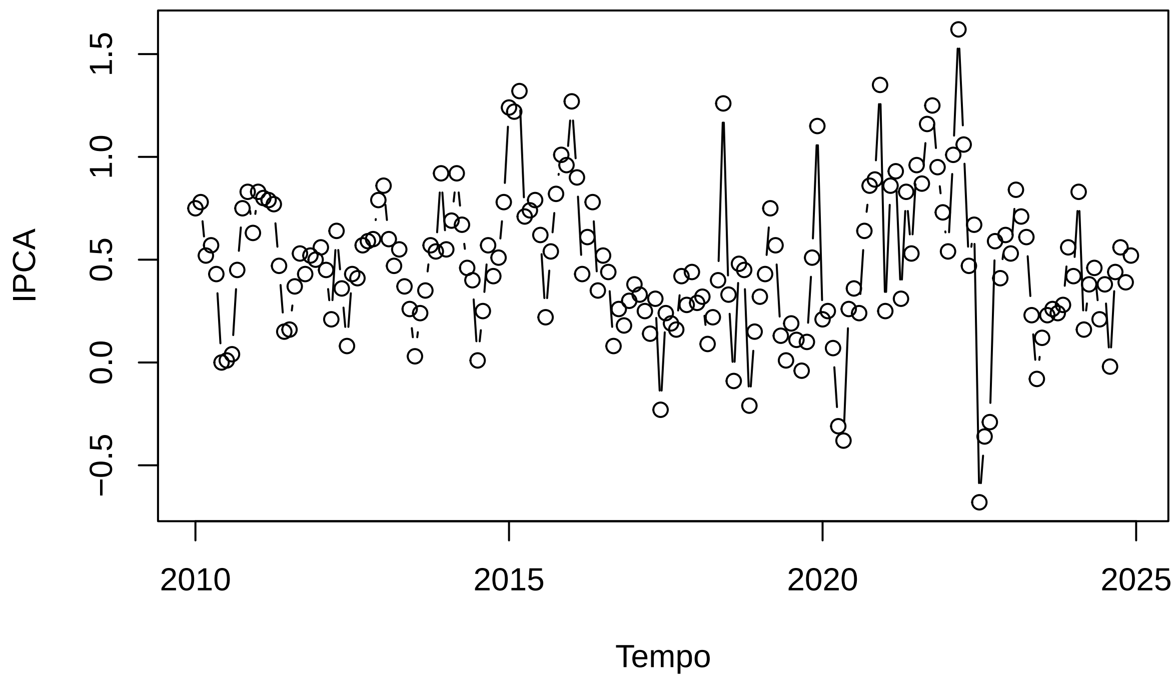
```
## Series: z
## ARIMA(0,0,2) with zero mean
##
## Coefficients:
##           ma1     ma2
##       -0.3773  0.2723
## s.e.   0.0659  0.0730
```

```
##
## sigma^2 = 0.9669:  log likelihood = -279.54
## AIC=565.07   AICc=565.2   BIC=574.97
```

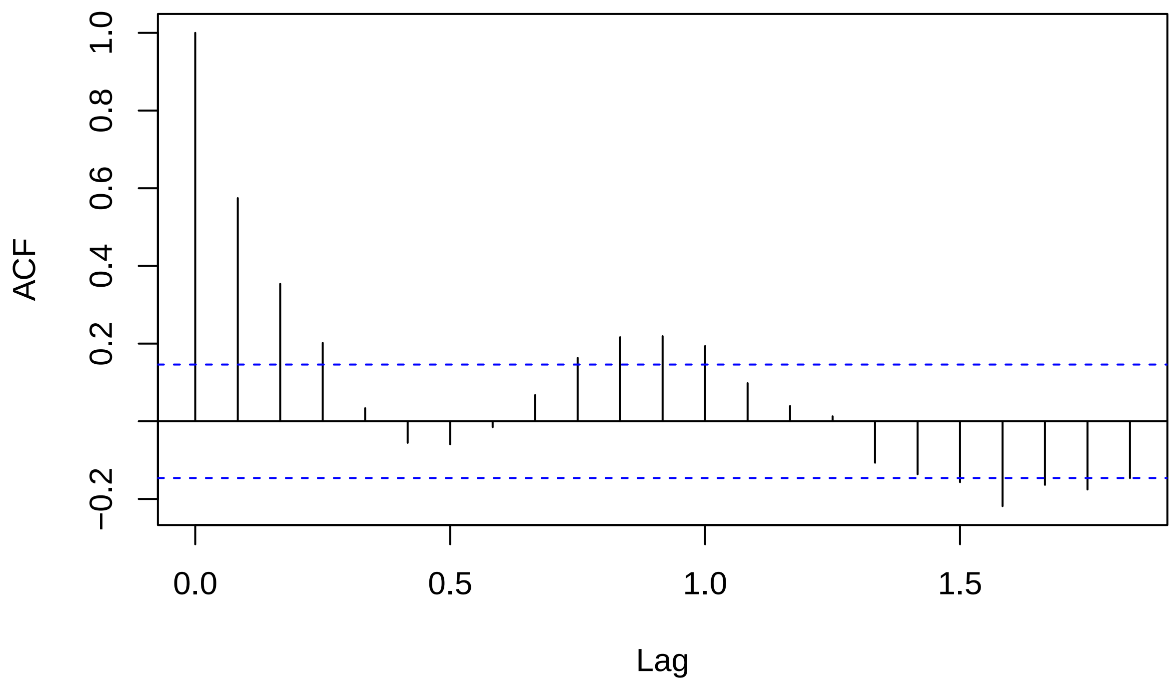
### Exemplo 3

Vamos agora utilizar a série temporal de IPCA e ajustar algumas ordens dos modelos MA.

```r
# serie temporal do IPCA
ipca_ts <- ts(serie_ipca, start= c(2010, 1), frequency = 12)

# grafico da serie
plot(ipca_ts, type="b", ylab="IPCA", xlab="Tempo")
```




```r
# acf da serie
acf(ipca_ts, main="")
```




```r
# usando o pacote forecast
library(forecast)

ajuste1 <- Arima(ipca_ts, order=c(0, 0, 1)) # MA(1) com intercepto
summary(ajuste1)
```

```
## Series: ipca_ts
## ARIMA(0,0,1) with non-zero mean
##
## Coefficients:
##          ma1    mean
##       0.4737  0.4777
## s.e.  0.0559  0.0339
##
## sigma^2 = 0.09671:  log likelihood = -44.28
## AIC=94.57   AICc=94.7   BIC=104.15
##
## Training set error measures:
##                         ME      RMSE       MAE  MPE MAPE      MASE      ACF1
## Training set -0.0003045201 0.3092434 0.2355548 -Inf  Inf 0.6929296 0.1282103
```

```r
confint(ajuste1)
```

```
##                2.5 %    97.5 %
## ma1       0.3641277 0.5831885
## intercept 0.4112778 0.5441903
```

```r
ajuste2 <- Arima(ipca_ts, order=c(0, 0, 2)) # MA(2) com intercepto
summary(ajuste2)
```

```
## Series: ipca_ts
## ARIMA(0,0,2) with non-zero mean
##
## Coefficients:
##          ma1     ma2    mean
##       0.5204  0.2256  0.4783
## s.e.  0.0728  0.0655  0.0389
##
## sigma^2 = 0.09142:  log likelihood = -38.74
## AIC=85.47   AICc=85.7   BIC=98.25
##
## Training set error measures:
##                         ME      RMSE       MAE  MPE MAPE      MASE       ACF1
## Training set -0.0006271665 0.2998192 0.2285121 -Inf  Inf 0.6722121 0.04850939
```

```r
confint(ajuste2)
```

```
##                2.5 %    97.5 %
## ma1       0.37784485 0.6630390
## ma2       0.09724735 0.3539142
## intercept 0.40205818 0.5545396
```

```r
ajuste3 <- Arima(ipca_ts, order=c(0, 0, 3)) # MA(3) com intercepto
summary(ajuste3)
```

```
## Series: ipca_ts
## ARIMA(0,0,3) with non-zero mean
##
## Coefficients:
##          ma1     ma2     ma3    mean
##       0.5255  0.3183  0.2146  0.4785
## s.e.  0.0730  0.0713  0.0715  0.0447
##
## sigma^2 = 0.08773:  log likelihood = -34.58
## AIC=79.16   AICc=79.5   BIC=95.12
##
## Training set error measures:
##                         ME      RMSE      MAE  MPE MAPE      MASE      ACF1
## Training set -0.0009065868 0.2928912 0.220635 -Inf  Inf 0.6490402 0.0246864
```

```r
confint(ajuste3)
```

```
##                2.5 %    97.5 %
## ma1       0.38237713 0.6685366
## ma2       0.17859629 0.4580371
## ma3       0.07447166 0.3548243
## intercept 0.39088356 0.5661817
```

```r
ajuste4 <- Arima(ipca_ts, order=c(0, 0, 4)) # MA(4) com intercepto
summary(ajuste4)
```

```
## Series: ipca_ts
## ARIMA(0,0,4) with non-zero mean
##
```

```
## Coefficients:
##          ma1     ma2     ma3     ma4    mean
##       0.5430  0.3491  0.2753  0.1170  0.4791
## s.e.  0.0735  0.0796  0.0789  0.0764  0.0492
##
## sigma^2 = 0.0871:  log likelihood = -33.43
## AIC=78.87   AICc=79.35   BIC=98.02
##
## Training set error measures:
##                         ME      RMSE       MAE  MPE MAPE     MASE       ACF1
## Training set -0.0009509513 0.2909927 0.2198949 -Inf  Inf 0.646863 0.00488913
```

```r
confint(ajuste4)
```

```
##                 2.5 %    97.5 %
## ma1        0.39890104 0.6871654
## ma2        0.19307372 0.5050929
## ma3        0.12065344 0.4298863
## ma4       -0.03262943 0.2667096
## intercept  0.38255282 0.5755948
```

# Modelo Autorregressivo e de Médias Móveis

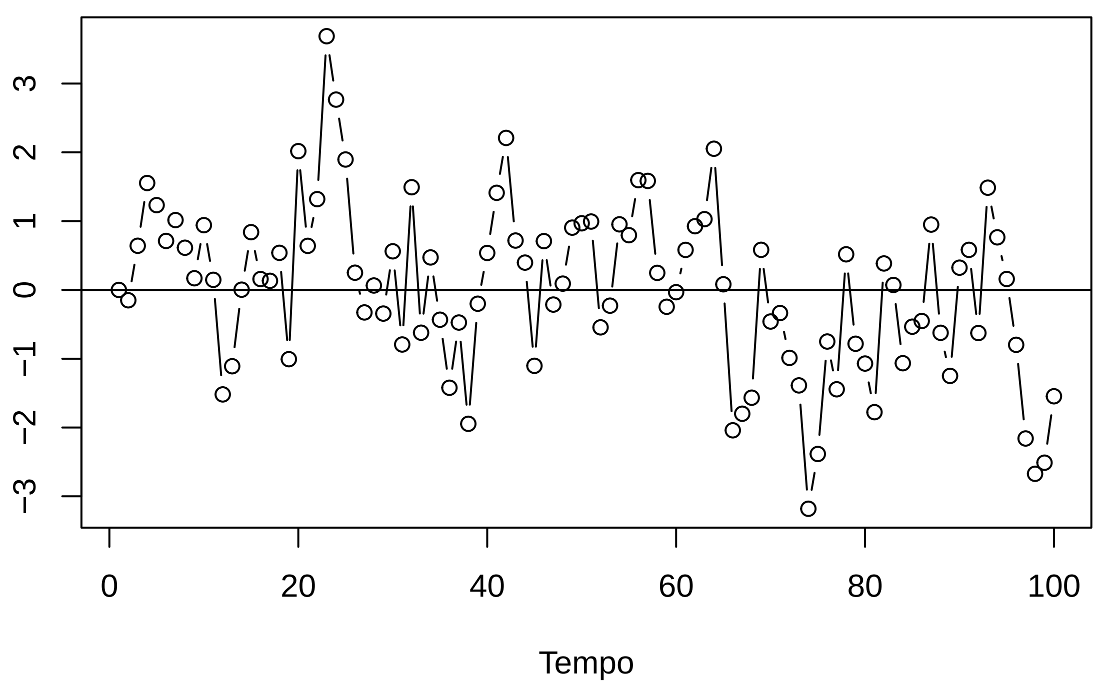
## Exemplo 1

Vamos agora simular uma série temporal com 100 observações de acordo com o modelo ARMA(1,1) dado por

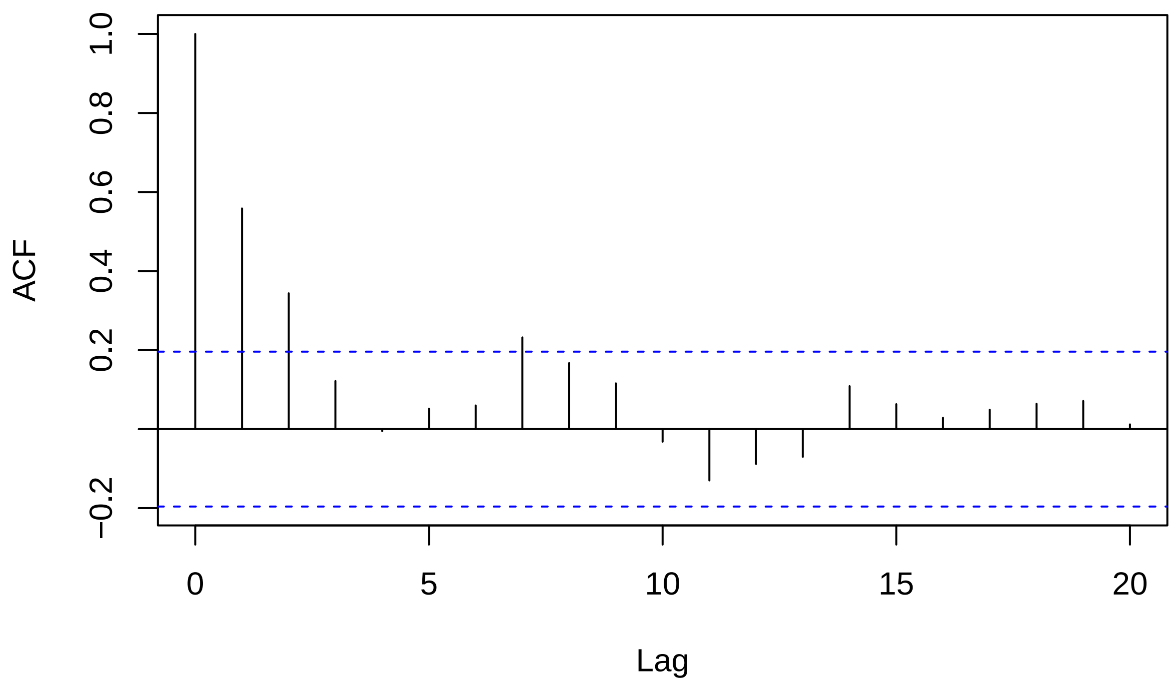
$$Z_t = 0,8Z_{t-1} + a_t - 0,3a_{t-1},$$

sendo que $a_t \sim N(0,1)$.

```r
set.seed(2025)

n <- 100
a <- rnorm(n)
z <- numeric(n)
z[1] <- 0

for(t in 2:n){
  z[t] <- 0.8*z[t-1] + a[t] - 0.3*a[t-1]
}

# plot da serie temporal simulada

plot(z, type="b", ylab="", xlab="Tempo")
abline(h=0)
```



```r
# acf da serie simulada
acf(z, main="")
```




```r
# usando o pacote forecast
library(forecast)

Arima(z, order=c(1, 0, 1)) # ARMA(1,1) com intercepto
```

```
## Series: z
## ARIMA(1,0,1) with non-zero mean
##
## Coefficients:
##          ar1      ma1     mean
##       0.6167  -0.0779  -0.0090
## s.e.  0.1262   0.1490   0.2371
##
## sigma^2 = 1.029:  log likelihood = -142
## AIC=292   AICc=292.42   BIC=302.42
```

```r
Arima(z, order=c(1, 0, 1), include.mean = F) # ARMA(1,1) sem intercepto
```

```
## Series: z
## ARIMA(1,0,1) with zero mean
##
## Coefficients:
##          ar1      ma1
##       0.6164  -0.0777
```

```
## s.e.  0.1261   0.1488
##
## sigma^2 = 1.019:  log likelihood = -142
## AIC=290   AICc=290.25   BIC=297.82
```

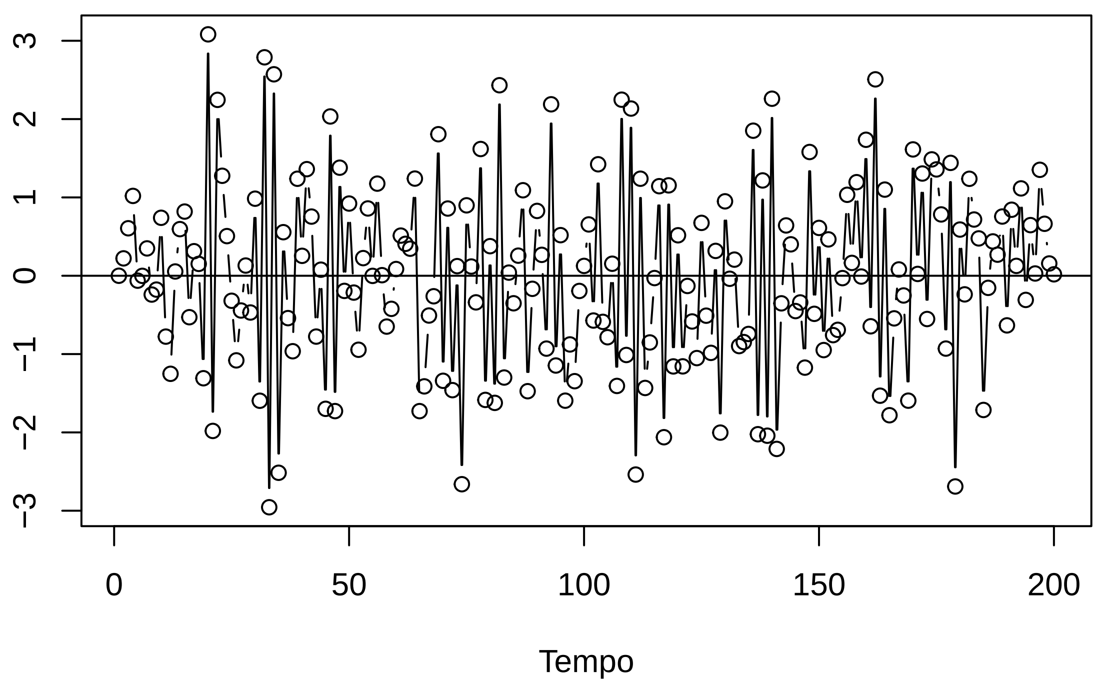
### Exemplo 2

Vamos agora simular uma série temporal com 200 observações de acordo com o modelo ARMA(1,1) dado por

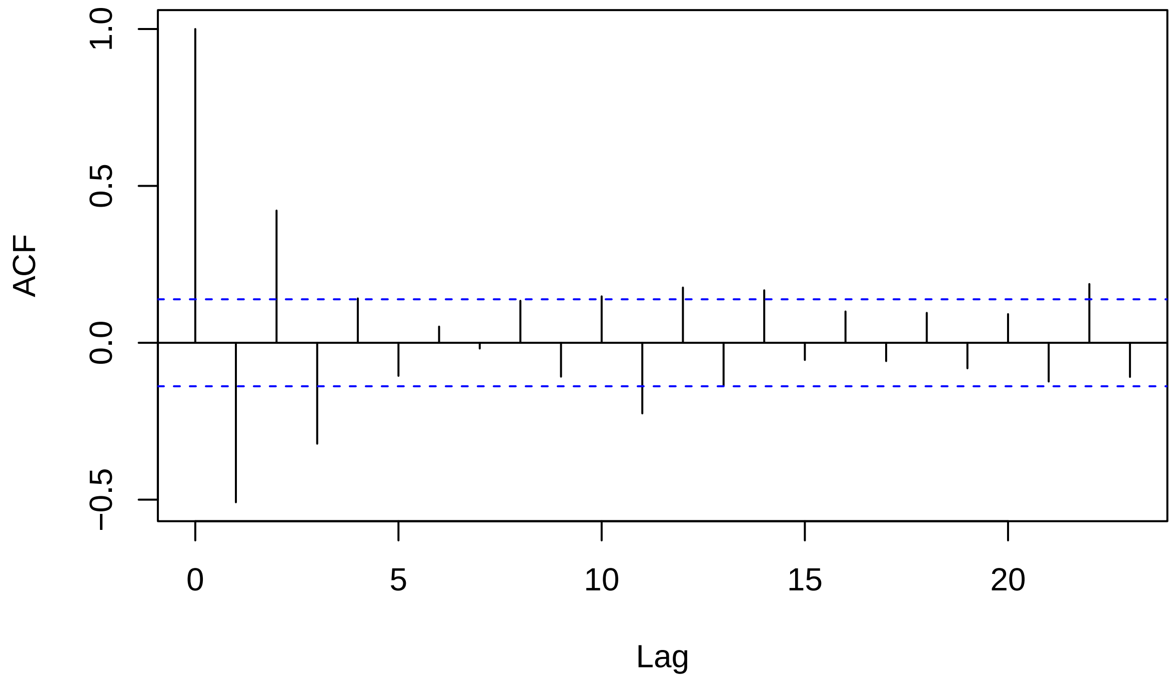
$$Z_t = -0,8Z_{t-1} + a_t + 0,3a_{t-1},$$

sendo que $a_t \sim N(0,1)$.

```
set.seed(2025)

n <- 200
a <- rnorm(n)
z <- numeric(n)
z[1] <- 0

for(t in 2:n){
  z[t] <- -0.8*z[t-1] + a[t] + 0.3*a[t-1]
}

# plot da serie temporal simulada

plot(z, type="b", ylab="", xlab="Tempo")
abline(h=0)
```



```r
# acf da serie simulada
acf(z, main="")
```




```r
# usando o pacote forecast
library(forecast)

Arima(z, order=c(1, 0, 1)) # ARMA(1,1) com intercepto
```

```
## Series: z
## ARIMA(1,0,1) with non-zero mean
##
## Coefficients:
##           ar1     ma1     mean
##       -0.7552  0.3402  -0.0003
## s.e.   0.0735  0.1002   0.0531
##
## sigma^2 = 0.979:  log likelihood = -280.35
## AIC=568.69   AICc=568.9   BIC=581.88
```

```r
Arima(z, order=c(1, 0, 1), include.mean = F) # ARMA(1,1) sem intercepto
```

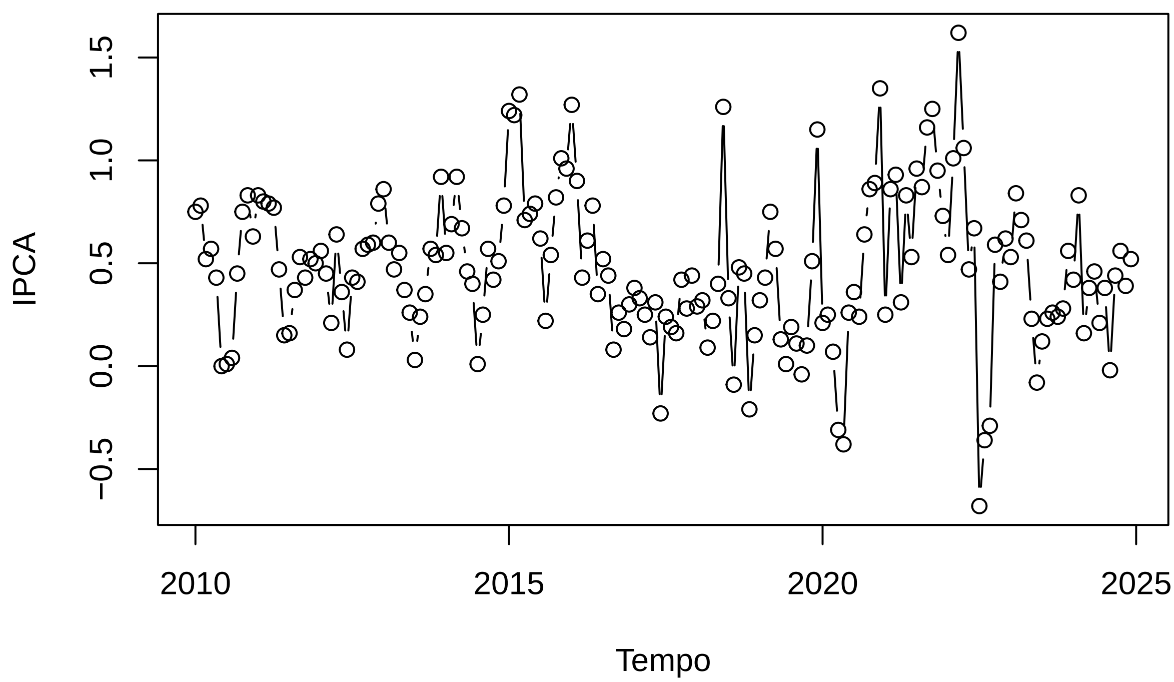
```
## Series: z
## ARIMA(1,0,1) with zero mean
##
## Coefficients:
##           ar1     ma1
##       -0.7552  0.3403
```

```
## s.e.   0.0734  0.1002
##
## sigma^2 = 0.9741:  log likelihood = -280.35
## AIC=566.69   AICc=566.81   BIC=576.59
```

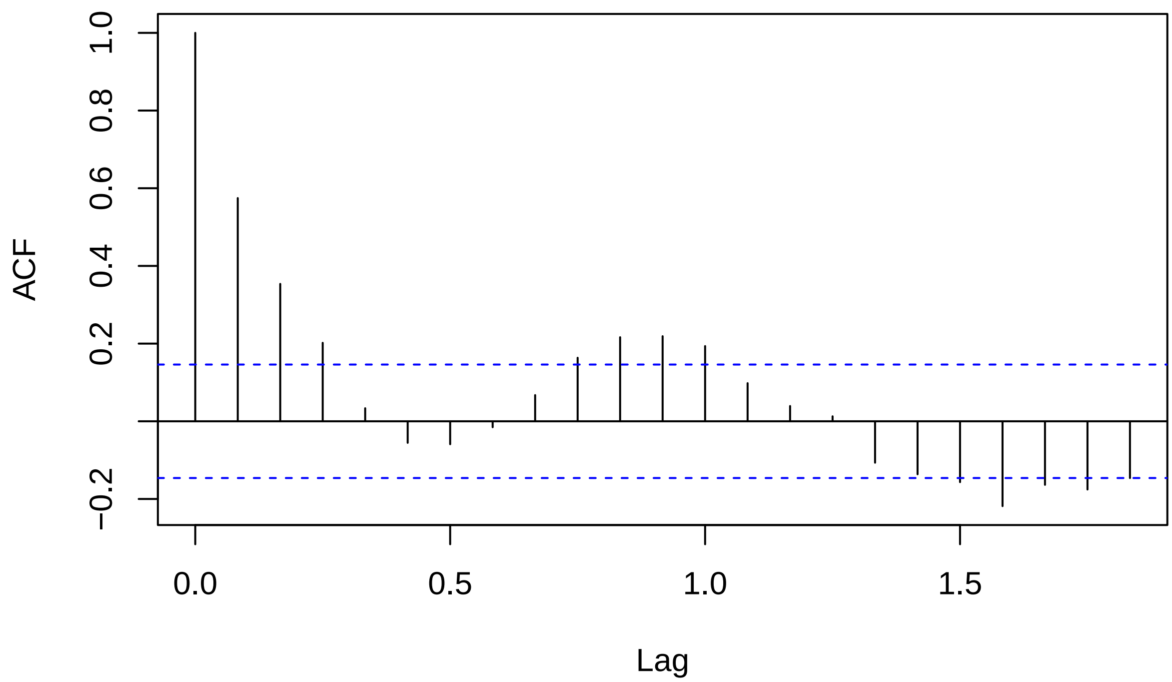
### Exemplo 3

Vamos agora utilizar novamente a série temporal de IPCA e ajustar algumas ordens dos modelos ARMA.

```r
# serie temporal do IPCA
ipca_ts <- ts(serie_ipca, start= c(2010, 1), frequency = 12)

# grafico da serie
plot(ipca_ts, type="b", ylab="IPCA", xlab="Tempo")
```




```r
# acf da serie
acf(ipca_ts, main="")
```




```r
# usando o pacote forecast
library(forecast)

ajuste1 <- Arima(ipca_ts, order=c(1, 0, 1)) # ARMA(1,1) com intercepto
summary(ajuste1)
```

```
## Series: ipca_ts
## ARIMA(1,0,1) with non-zero mean
##
## Coefficients:
##          ar1      ma1    mean
##       0.6110  -0.0563  0.4797
## s.e.  0.0972   0.1200  0.0526
##
## sigma^2 = 0.08745:  log likelihood = -34.79
## AIC=77.58   AICc=77.81   BIC=90.35
##
## Training set error measures:
##                         ME      RMSE       MAE  MPE MAPE      MASE         ACF1
## Training set -0.001227434 0.2932383 0.2206587 -Inf  Inf 0.6491098 0.000189028
```

```r
confint(ajuste1)
```

```
##                 2.5 %    97.5 %
## ar1        0.4204985 0.8015882
## ma1       -0.2913739 0.1788574
## intercept  0.3766025 0.5827848
```

```r
ajuste2 <- Arima(ipca_ts, order=c(1, 0, 2)) # ARMA(1,2) com intercepto
summary(ajuste2)
```

```
## Series: ipca_ts
## ARIMA(1,0,2) with non-zero mean
##
## Coefficients:
##          ar1      ma1     ma2    mean
##       0.5756  -0.0288  0.0458  0.4796
## s.e.  0.1313   0.1411  0.1010  0.0520
##
## sigma^2 = 0.08784:  log likelihood = -34.69
## AIC=79.38   AICc=79.73   BIC=95.35
##
## Training set error measures:
##                        ME      RMSE       MAE  MPE MAPE      MASE        ACF1
## Training set -0.001214972 0.2930729 0.2205499 -Inf  Inf 0.6487898 0.004813304
```

```r
confint(ajuste2)
```

```
##                2.5 %    97.5 %
## ar1        0.3182785 0.8330143
## ma1       -0.3054346 0.2478068
## ma2       -0.1521092 0.2437920
## intercept  0.3777977 0.5814459
```

```r
ajuste3 <- Arima(ipca_ts, order=c(2, 0, 1)) # ARMA(2,1) com intercepto
summary(ajuste3)
```

```
## Series: ipca_ts
## ARIMA(2,0,1) with non-zero mean
##
## Coefficients:
##          ar1     ar2     ma1    mean
##       0.4719  0.0830  0.0802  0.4798
## s.e.  0.7577  0.4376  0.7565  0.0526
##
## sigma^2 = 0.08793:  log likelihood = -34.77
## AIC=79.55   AICc=79.89   BIC=95.51
##
## Training set error measures:
##                        ME      RMSE      MAE  MPE MAPE      MASE        ACF1
## Training set -0.001255014 0.2932093 0.220634 -Inf  Inf 0.6490371 0.002351602
```

```r
confint(ajuste3)
```

```
##                2.5 %    97.5 %
## ar1       -1.0131626 1.9569284
## ar2       -0.7747512 0.9407501
## ma1       -1.4024161 1.5629118
## intercept  0.3766463 0.5828585
```

```r
ajuste4 <- Arima(ipca_ts, order=c(2, 0, 2)) # ARMA(2,2) com intercepto
summary(ajuste4)
```

```
## Series: ipca_ts
## ARIMA(2,0,2) with non-zero mean
```

```
##
## Coefficients:
##           ar1     ar2     ma1      ma2    mean
##       -0.1012  0.4453  0.6584  -0.0587  0.4797
## s.e.   0.8684  0.5258  0.8670   0.1140  0.0529
##
## sigma^2 = 0.08841:  log likelihood = -34.76
## AIC=81.51   AICc=82   BIC=100.67
##
## Training set error measures:
##                        ME      RMSE       MAE  MPE MAPE     MASE         ACF1
## Training set -0.001240821 0.2931807 0.2202213 -Inf  Inf 0.647823 -0.001110801
```

```r
confint(ajuste4)
```

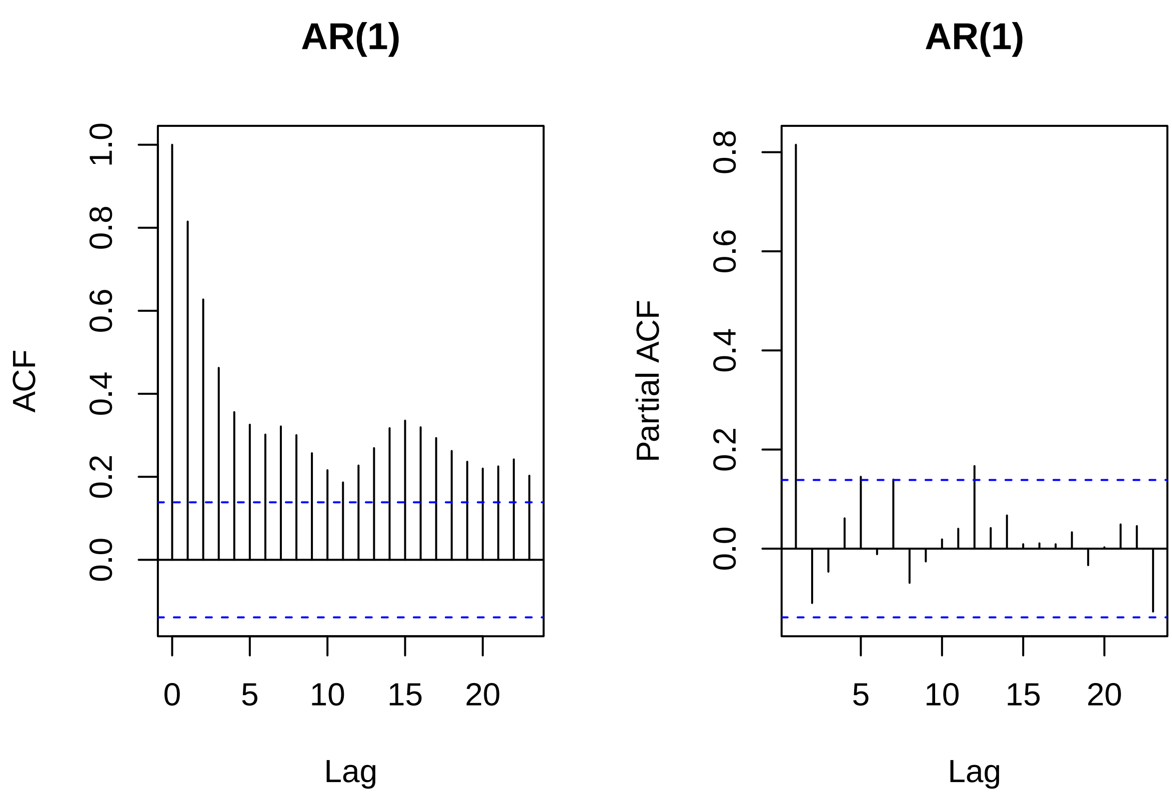
```
##                2.5 %    97.5 %
## ar1       -1.8033188 1.6009067
## ar2       -0.5852159 1.4758822
## ma1       -1.0409755 2.3577811
## ma2       -0.2821431 0.1647403
## intercept  0.3761211 0.5833080
```

## Identificação

### FACP de séries simuladas

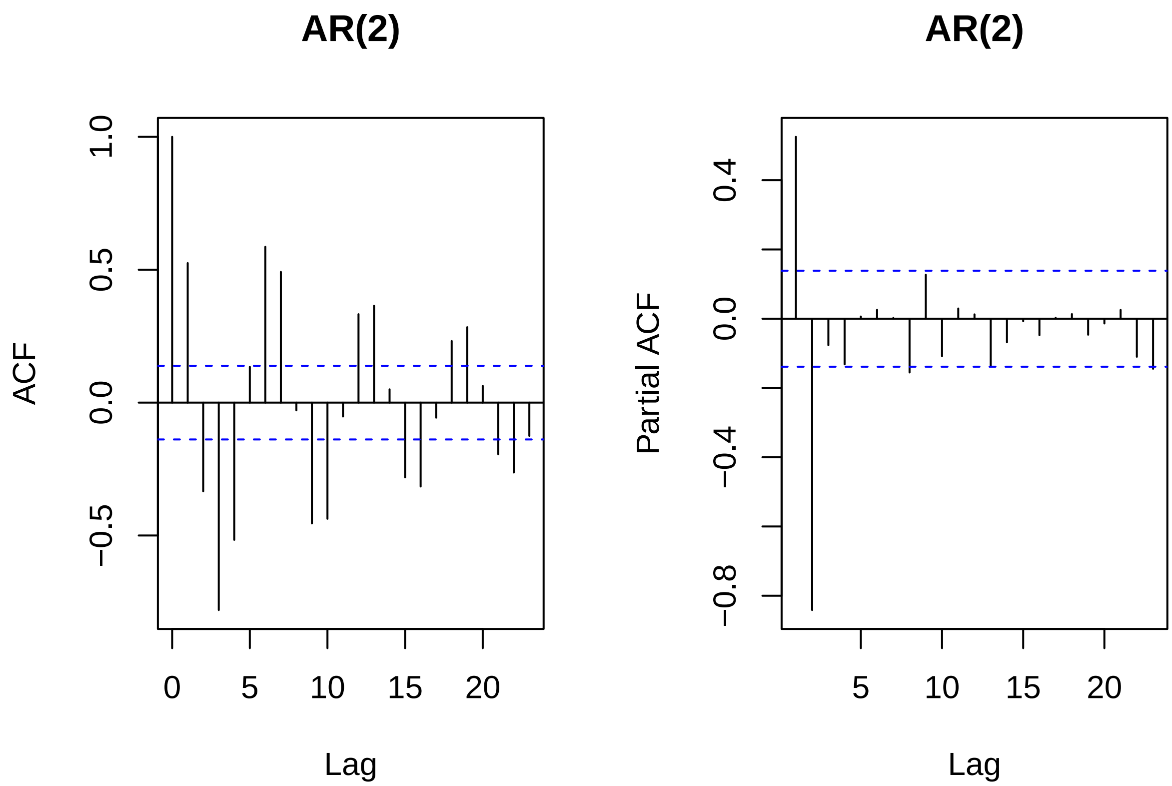
Vamos utilizar as séries simuladas anteriormente e obter o gráfico das funções FAC e FACP.

```r
set.seed(2025)
n <- 200

# modelo ar(1)

a <- rnorm(n)
z <- numeric(n)
z[1] <- 0

for(t in 2:n){
  z[t] <- 0.8*z[t-1] + a[t]
}

par(mfrow=c(1,2))
acf(z, main="AR(1)")
pacf(z, main="AR(1)")
```

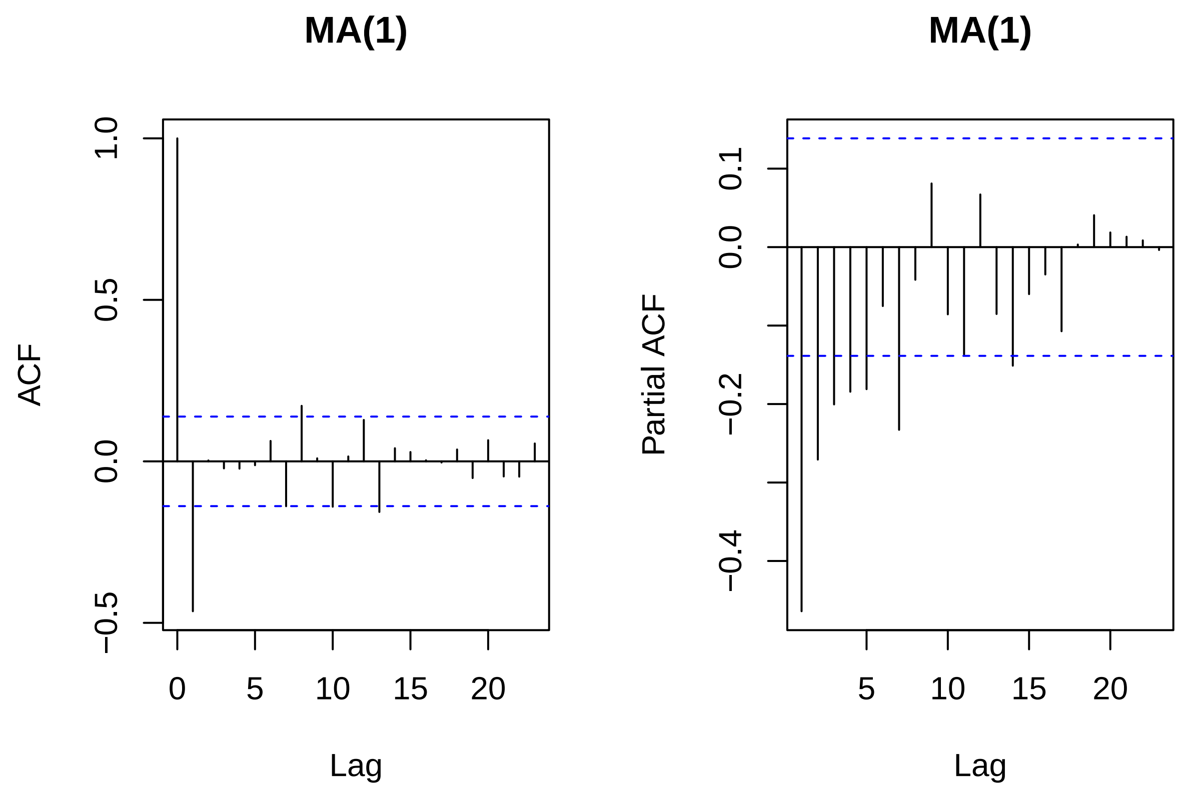



```r
# modelo ar(2)

a <- rnorm(n)
z <- numeric(n)
```

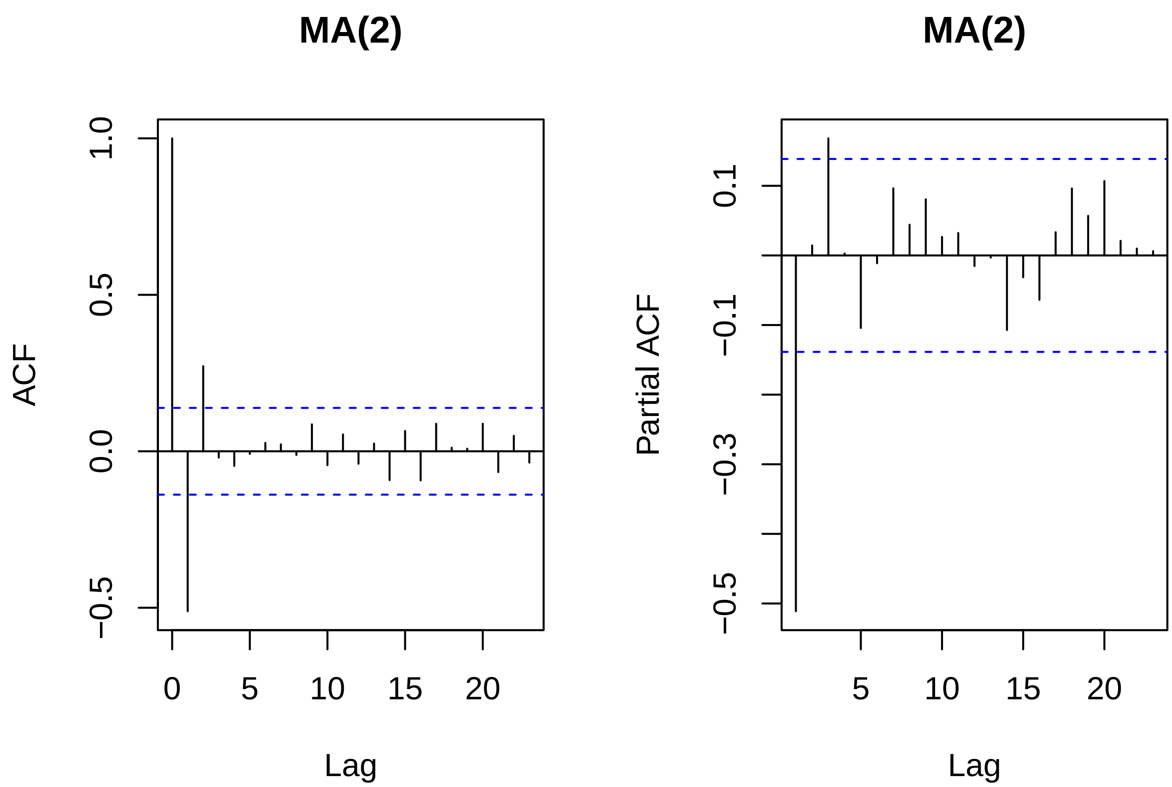
```
z[1] <- z[2] <- 0

for(t in 3:n){
  z[t] <- 1*z[t-1] -0.89*z[t-2]+ a[t]
}

par(mfrow=c(1,2))
acf(z, main="AR(2)")
pacf(z, main="AR(2)")
```



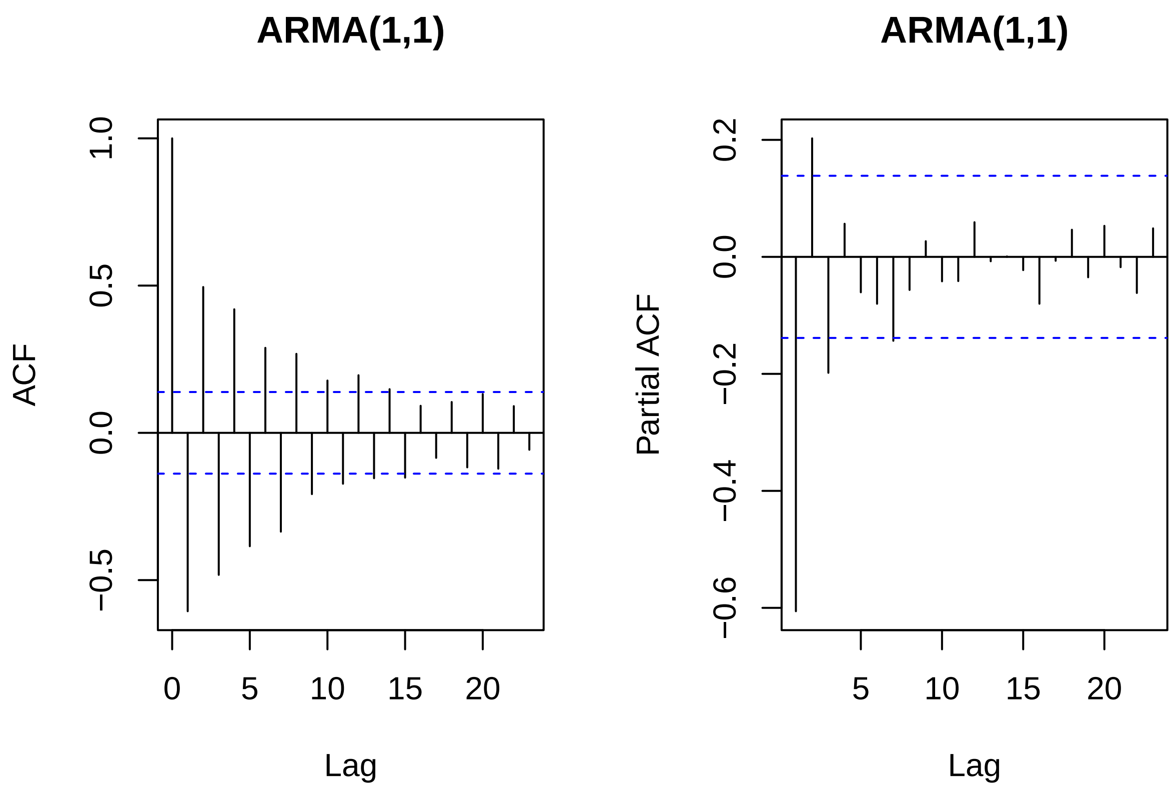

```
# Modelo MA(1)

a <- rnorm(n)
z <- numeric(n)
z[1] <- 0

for(t in 2:n){
  z[t] <- a[t] - 0.8*a[t-1]
}

par(mfrow=c(1,2))
acf(z, main="MA(1)")
pacf(z, main="MA(1)")
```




```
# Modelo MA(2)

a <- rnorm(n)
z <- numeric(n)

a[1] <- a[2] <- 0
z[1] <- z[2] <- 0

for(t in 3:n){
z[t] <- a[t] - 0.5*a[t-1] + 0.3*a[t-2]
}

par(mfrow=c(1,2))
acf(z, main="MA(2)")
pacf(z, main="MA(2)")
```



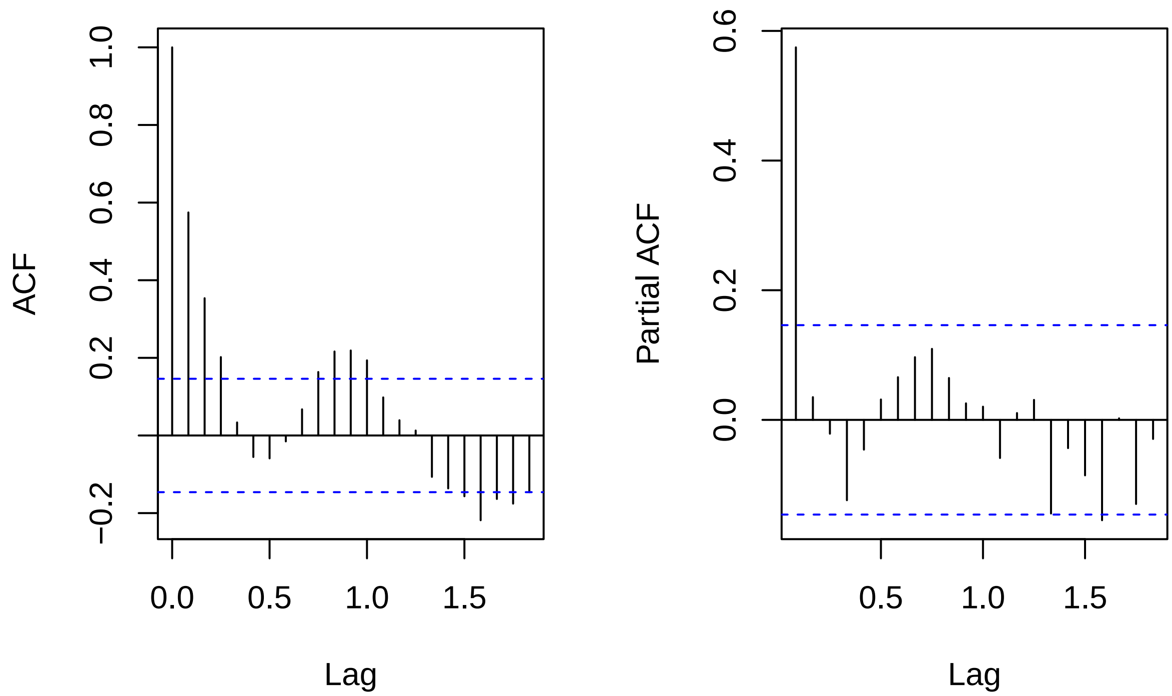

```
# Modelo ARMA(1,1)

a <- rnorm(n)
z <- numeric(n)
z[1] <- 0

for(t in 2:n){
  z[t] <- -0.8*z[t-1] + a[t] + 0.3*a[t-1]
}

par(mfrow=c(1,2))
acf(z, main="ARMA(1,1)")
pacf(z, main="ARMA(1,1)")
```




## Exemplo 2

Vamos agora obter a FACP da série temporal de IPCA e tentar obter o modelo a ser ajustado utiliando as funções FAC e FACP.

```
# serie temporal do IPCA
ipca_ts <- ts(serie_ipca, start= c(2010, 1), frequency = 12)

par(mfrow=c(1,2))
# acf da serie
acf(ipca_ts, main="")
pacf(ipca_ts, main="")
```




```r
# indicacao - ARMA(1,0)

# Analisando o AIC
# ARMA(1,0) - AIC=75.80
# ARMA(2,0) - AIC=77.56
# ARMA(0,1) - AIC=94.57
# ARMA(0,2) - AIC=85.47
# ARMA(0,3) - AIC=79.16
# ARMA(0,4) - AIC=78.87
# ARMA(1,1) - AIC=77.58
# ARMA(1,2) - AIC=79.38
# ARMA(2,1) - AIC=79.55
# ARMA(2,2) - AIC=81.51
# Observe que o menor AIC indica do modelo ARMA(1,0)

# nao e preciso ajustar todos os modelos de forma manual

# ajustando todos os modelos
# max.p = 5 e max.q=5
auto.arima(ipca_ts, d = 0, seasonal = FALSE)
```

```
## Series: ipca_ts
## ARIMA(1,0,0) with non-zero mean
##
## Coefficients:
##          ar1    mean
```

```
##        0.5734  0.4795
## s.e.  0.0607  0.0509
##
## sigma^2 = 0.08706:  log likelihood = -34.9
## AIC=75.8   AICc=75.93   BIC=85.38
```

```r
# usando a estrategia stepwise
auto.arima(ipca_ts, d = 0, seasonal = FALSE, stepwise = FALSE)
```

```
## Series: ipca_ts
## ARIMA(1,0,0) with non-zero mean
##
## Coefficients:
##          ar1    mean
##       0.5734  0.4795
## s.e.  0.0607  0.0509
##
## sigma^2 = 0.08706:  log likelihood = -34.9
## AIC=75.8   AICc=75.93   BIC=85.38
```

# Diagnóstico

# Exemplo 1

Vamos continuar analisando a série temporal de IPCA, agora vamos verificar a adequação do modelo ARMA(1,0) aos dados, por meio dos gráficos de dispersão, FAC e FACP, dos resíduos.

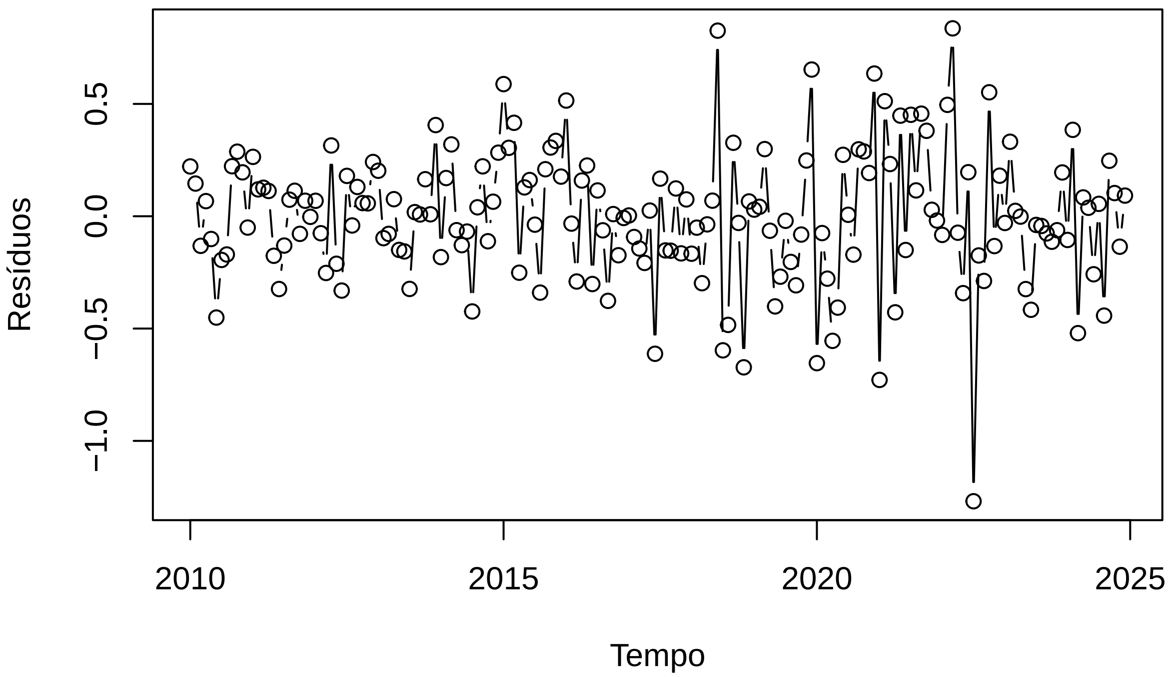
```
# usando o pacote forecast
library(forecast)

# o ajuste
ajuste1 <- Arima(ipca_ts, order=c(1, 0, 0)) # AR(1) com intercepto
summary(ajuste1)
```

```
## Series: ipca_ts
## ARIMA(1,0,0) with non-zero mean
##
## Coefficients:
##          ar1    mean
##       0.5734  0.4795
## s.e.  0.0607  0.0509
##
## sigma^2 = 0.08706:  log likelihood = -34.9
## AIC=75.8   AICc=75.93   BIC=85.38
##
## Training set error measures:
##                        ME      RMSE       MAE  MPE MAPE      MASE       ACF1
## Training set -0.001133749 0.2934154 0.2209792 -Inf  Inf 0.6500526 -0.0192077
```

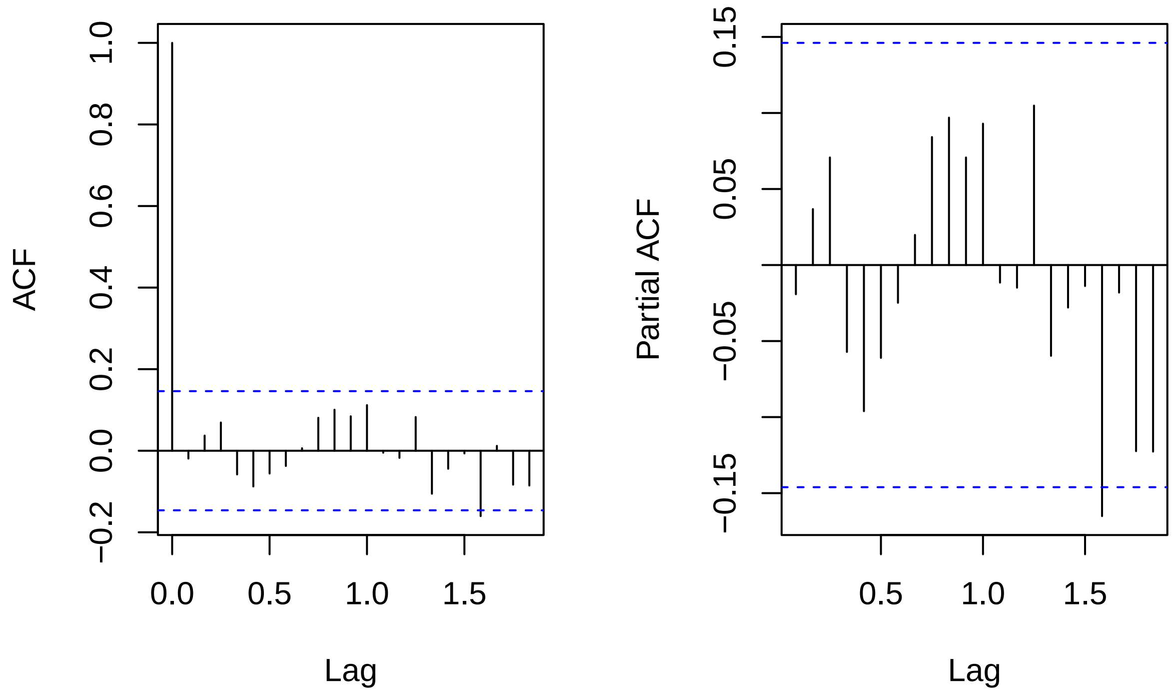
```
confint(ajuste1)
```

```
##               2.5 %    97.5 %
## ar1       0.4544142 0.6922987
## intercept 0.3797570 0.5792230
```

```
# grafico dos residuos
plot(ajuste1$residuals, xlab="Tempo", ylab="Resíduos", type="b")
```




```
# acf e pacf dos residuos
par(mfrow=c(1,2))
# acf da serie
acf(ajuste1$residuals, main="")
pacf(ajuste1$residuals, main="")
```




```r
# observe que nao existe lags "baixos" significativos, nem na acf, nem na pacf
# indicando assim que houve um bom ajuste do modelo aos dados

# teste de Box-Pierce
Box.test(ajuste1$residuals, type="Box-Pierce") # lag = 1
```

```
##
##  Box-Pierce test
##
## data:  ajuste1$residuals
## X-squared = 0.066408, df = 1, p-value = 0.7966
```

```r
# O parametro lag determina quantos defasagens (lags)
# da autocorrelacao vamos considerar ao testar se os
# residuos sao ruido branco

# exemplo: lag = 12 implica que o teste avalia se as autocorrelacoes
# dos residuos nos lags 1 a 12 sao, conjuntamente, estatisticamente
# diferentes de zero

# teste de Box-Pierce
Box.test(ajuste1$residuals, lag=12, type="Box-Pierce") # lag = 12
```

```
##
##  Box-Pierce test
##
## data:  ajuste1$residuals
```

```
## X-squared = 10.519, df = 12, p-value = 0.5705
```

```r
# observacao:
# se o p-value > 0.05, existe evidencias para aceitarmos
# H0: os residuos sao ruido branco (modelo pode estar adequado)
# se o p-value < 0.05, existe evidencias para rejeitarmos
# H0: os residuos tem autocorrelacao (modelo pode estar mal especificado)

# vamos agora utilizar o teste Ljung-Bo

Box.test(ajuste1$residuals, type="Box-Pierce") # lag = 1
```

```
##
##  Box-Pierce test
##
## data:  ajuste1$residuals
## X-squared = 0.066408, df = 1, p-value = 0.7966
```

```r
Box.test(ajuste1$residuals, lag=12, type="Box-Pierce") # lag = 12
```

```
##
##  Box-Pierce test
##
## data:  ajuste1$residuals
## X-squared = 10.519, df = 12, p-value = 0.5705
```

```r
# aceitamos a hipotese nula em ambos os teste
# indicando assim que o modelo pode estar adequado

# finalmente chegou a hora de realizar previsoes!!!

forecast(ajuste1, h = 3)
```

```
##          Point Forecast      Lo 80     Hi 80       Lo 95    Hi 95
## Jan 2025      0.5027167 0.12458308 0.8808503 -0.07558874 1.081022
## Feb 2025      0.4928072 0.05692924 0.9286851 -0.17381059 1.159425
## Mar 2025      0.4871255 0.03386868 0.9403823 -0.20607098 1.180322
```

```r
# Observe que no lag 19
# tanto a fac quanto a facp é
# estatisticamente diferente de zero
# vamos proceder com um ajuste de um modelo AR(19) incompleto
# com os termos \phi_1 e \phi_{19}

vetor <- c(NA, rep(0, 17), NA, NA)
ajuste_inc <- Arima(ipca_ts, order=c(19, 0, 0), fixed = vetor)
ajuste_inc
```

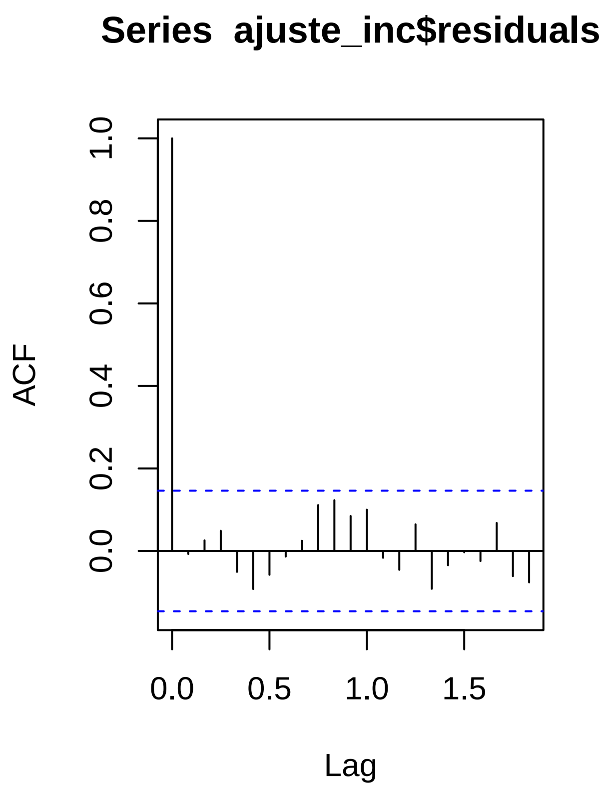
```
## Series: ipca_ts
## ARIMA(19,0,0) with non-zero mean
##
## Coefficients:
##          ar1  ar2  ar3  ar4  ar5  ar6  ar7  ar8  ar9  ar10  ar11  ar12  ar13
##       0.5510    0    0    0    0    0    0    0    0     0     0     0     0
## s.e.  0.0605    0    0    0    0    0    0    0    0     0     0     0     0
##       ar14  ar15  ar16  ar17  ar18     ar19    mean
##          0     0     0     0     0  -0.1375  0.4817
```

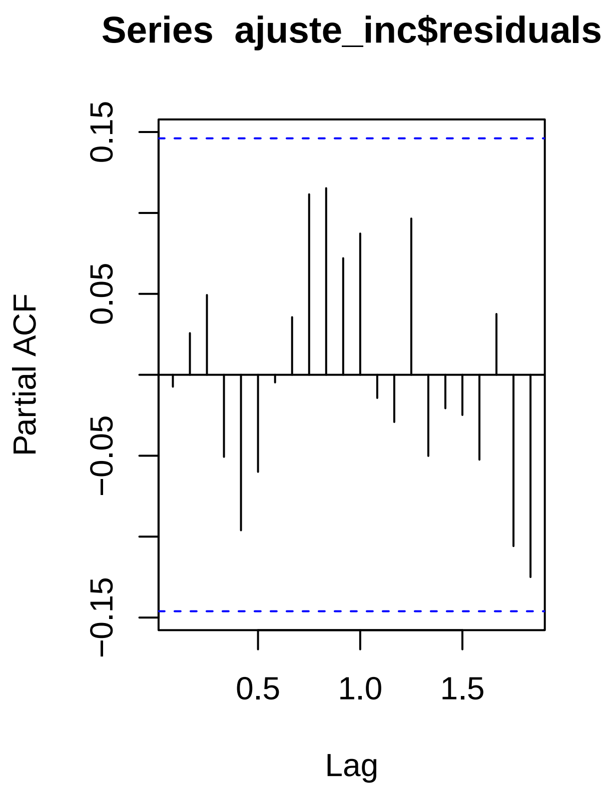
```
## s.e.      0      0      0      0      0   0.0613  0.0375
##
## sigma^2 = 0.08495:  log likelihood = -32.43
## AIC=72.86   AICc=73.09   BIC=85.63
```

```r
confint(ajuste_inc)
```

```
##                2.5 %      97.5 %
## ar1        0.4323182  0.66961915
## ar2               NA          NA
## ar3               NA          NA
## ar4               NA          NA
## ar5               NA          NA
## ar6               NA          NA
## ar7               NA          NA
## ar8               NA          NA
## ar9               NA          NA
## ar10              NA          NA
## ar11              NA          NA
## ar12              NA          NA
## ar13              NA          NA
## ar14              NA          NA
## ar15              NA          NA
## ar16              NA          NA
## ar17              NA          NA
## ar18              NA          NA
## ar19      -0.2576288 -0.01741723
## intercept  0.4082209  0.55513326
```

```r
acf(ajuste_inc$residuals)
pacf(ajuste_inc$residuals)
```




```r
# teste de Box-Pierce
Box.test(ajuste_inc$residuals, type="Box-Pierce") # lag = 1
```

```
##
##  Box-Pierce test
##
## data:  ajuste_inc$residuals
## X-squared = 0.0097596, df = 1, p-value = 0.9213
```

```r
Box.test(ajuste_inc$residuals, type="Box-Pierce", lag = 12) # lag = 12
```

```
##
##  Box-Pierce test
##
## data:  ajuste_inc$residuals
## X-squared = 11.352, df = 12, p-value = 0.499
```

```r
Box.test(ajuste_inc$residuals, type="Box-Pierce", lag = 24) # lag = 24
```

```
##
##  Box-Pierce test
##
## data:  ajuste_inc$residuals
## X-squared = 16.966, df = 24, p-value = 0.8501
```

```r
Box.test(ajuste_inc$residuals, type="Box-Pierce", lag = 36) # lag = 36
```

```
##
##  Box-Pierce test
```

```
##
## data:  ajuste_inc$residuals
## X-squared = 28.329, df = 36, p-value = 0.8153
```

```r
# teste de Box-Pierce
Box.test(ajuste_inc$residuals, type="Ljung-Box") # lag = 1
```

```
##
##  Box-Ljung test
##
## data:  ajuste_inc$residuals
## X-squared = 0.0099232, df = 1, p-value = 0.9206
```

```r
Box.test(ajuste_inc$residuals, type="Ljung-Box", lag = 12) # lag = 12
```

```
##
##  Box-Ljung test
##
## data:  ajuste_inc$residuals
## X-squared = 12.065, df = 12, p-value = 0.4405
```

```r
Box.test(ajuste_inc$residuals, type="Ljung-Box", lag = 24) # lag = 24
```

```
##
##  Box-Ljung test
##
## data:  ajuste_inc$residuals
## X-squared = 18.382, df = 24, p-value = 0.7841
```

```r
Box.test(ajuste_inc$residuals, type="Ljung-Box", lag = 36) # lag = 36
```

```
##
##  Box-Ljung test
##
## data:  ajuste_inc$residuals
## X-squared = 32.274, df = 36, p-value = 0.6465
```

```r
# Comparacao dos modelos
AIC(ajuste1)
```

```
## [1] 75.79674
```

```r
AIC(ajuste_inc)
```

```
## [1] 72.86248
```

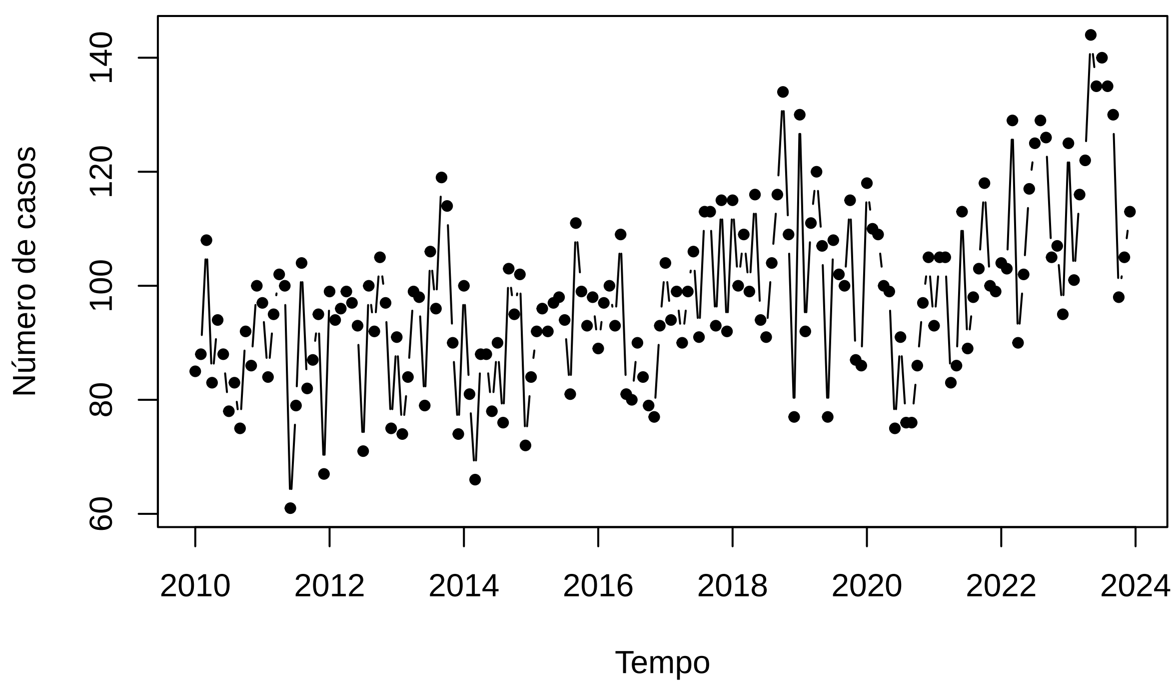
```r
# Analisando os graficos das acf e pacf, tem-se que o modelo AR(19) e o melhor
# Observando o criterio AIC, o modelo AR(19) tbm e o melhor
# Mas faz sentido ter um termo com 19 meses de defasagem?


forecast(ajuste_inc, h = 3)
```

```
##          Point Forecast      Lo 80    Hi 80       Lo 95    Hi 95
## Jan 2025      0.5800353 0.2065217 0.953549  0.00879551 1.151275
## Feb 2025      0.5856083 0.1591535 1.012063 -0.06659809 1.237815
## Mar 2025      0.5735513 0.1322802 1.014822 -0.10131456 1.248417
```

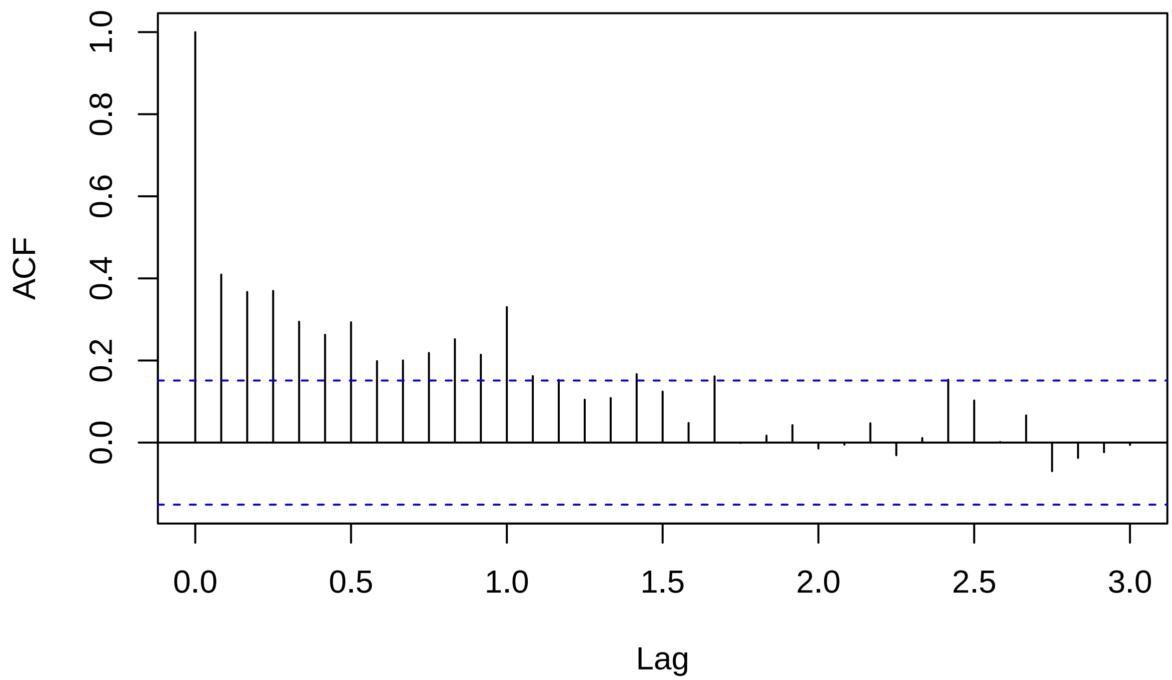
## Análise da série temporal de tuberculose

Considerando a série temporal do número de casos de tuberculose no Estado de Goiás, com o número de casos mensal de janeiro de 2010 a dezembro de 2023, vamos obter o gráfico das funções FAC e FACP, na sequência ajustar alguns modelos da família ARMA.

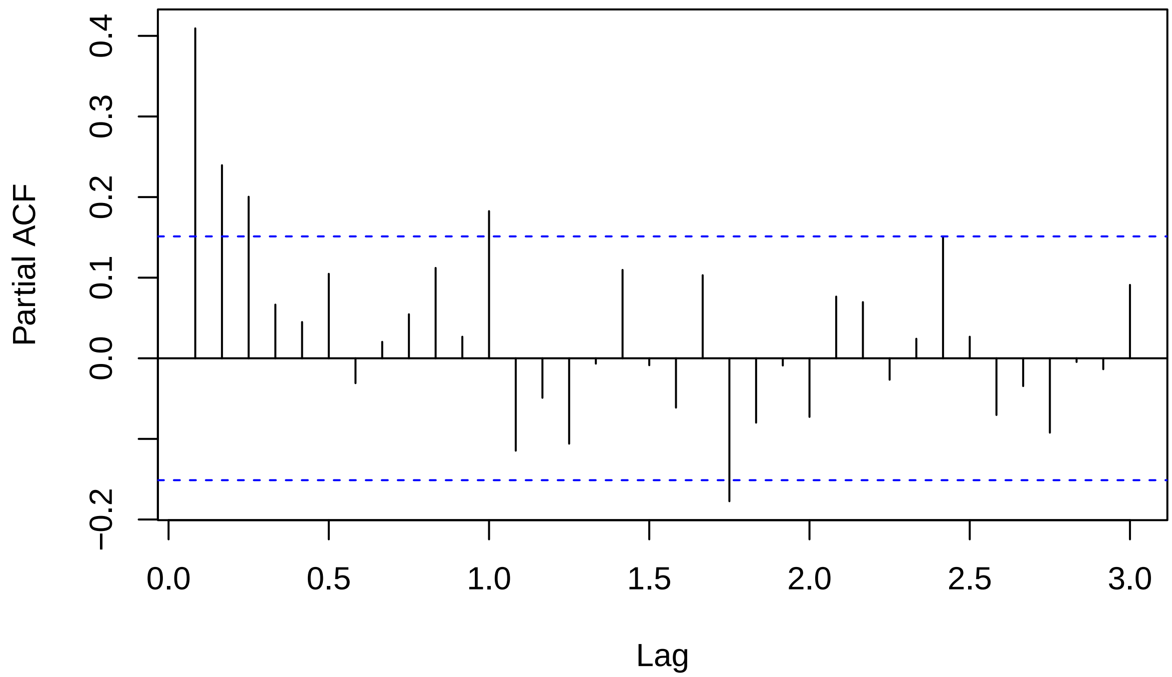
```
# serie temporal do numero de casos de tuberculose em Goias

serie_tuberculose <- c(85,88,108,83,94,88,78,83,75,92,86,100,
                       97,84,95,102,100,61,79,104,82,87,95,67,
                       99,94,96,99,97,93,71,100,92,105,97,75,
                       91,74,84,99,98,79,106,96,119,114,90,74,
                       100,81,66,88,88,78,90,76,103,95,102,72,
                       84,92,96,92,97,98,94,81,111,99,93,98,
                       89,97,100,93,109,81,80,90,84,79,77,93,
                       104,94,99,90,99,106,91,113,113,93,115,92,
                       115,100,109,99,116,94,91,104,116,134,109,77,
                       130,92,111,120,107,77,108,102,100,115,87,86,
                       118,110,109,100,99,75,91,76,76,86,97,105,
                       93,105,105,83,86,113,89,98,103,118,100,99,
                       104,103,129,90,102,117,125,129,126,105,107,95,
                       125,101,116,122,144,135,140,135,130,98,105,113)

tuberculose_ts <- ts(serie_tuberculose, start= c(2010, 1), frequency = 12)

plot(tuberculose_ts, type="b", pch=20 , ylab="Número de casos", xlab="Tempo")
```



```r
#par(mfrow=c(1,2))
# acf da serie
acf(tuberculose_ts, main="", lag.max=36)
```




```r
# pacf da serie
pacf(tuberculose_ts, main="", lag.max=36)
```




```r
# ajuste de alguns modelos da familia ARMA

ajuste100 <- Arima(tuberculose_ts, c(1, 0, 0))
ajuste100
```

```
## Series: tuberculose_ts
## ARIMA(1,0,0) with non-zero mean
##
## Coefficients:
##          ar1     mean
##       0.4110  97.8607
## s.e.  0.0703   1.8230
##
## sigma^2 = 197.6:  log likelihood = -681.53
## AIC=1369.07   AICc=1369.21   BIC=1378.44
```

```r
ajuste200 <- Arima(tuberculose_ts, c(2, 0, 0))
ajuste200
```

```
## Series: tuberculose_ts
## ARIMA(2,0,0) with non-zero mean
##
## Coefficients:
##          ar1     ar2     mean
##       0.3123  0.2407  97.8593
## s.e.  0.0747  0.0747   2.3158
##
```

```
## sigma^2 = 187.2:  log likelihood = -676.51
## AIC=1361.02   AICc=1361.27   BIC=1373.52
```

```r
ajuste300 <- Arima(tuberculose_ts, c(3, 0, 0))
ajuste300
```

```
## Series: tuberculose_ts
## ARIMA(3,0,0) with non-zero mean
##
## Coefficients:
##          ar1     ar2     ar3     mean
##       0.2628  0.1789  0.1979  97.8906
## s.e.  0.0756  0.0770  0.0756   2.7891
##
## sigma^2 = 180.8:  log likelihood = -673.16
## AIC=1356.33   AICc=1356.7   BIC=1371.95
```

```r
ajuste400 <- Arima(tuberculose_ts, c(4, 0, 0))
ajuste400
```

```
## Series: tuberculose_ts
## ARIMA(4,0,0) with non-zero mean
##
## Coefficients:
##          ar1     ar2     ar3     ar4     mean
##       0.2493  0.1656  0.1777  0.0734  97.9230
## s.e.  0.0768  0.0780  0.0785  0.0781   2.9871
##
## sigma^2 = 181:  log likelihood = -672.72
## AIC=1357.45   AICc=1357.97   BIC=1376.19
```

```r
ajuste001 <- Arima(tuberculose_ts, c(0, 0, 1))
ajuste001
```

```
## Series: tuberculose_ts
## ARIMA(0,0,1) with non-zero mean
##
## Coefficients:
##          ma1     mean
##       0.2989  97.8574
## s.e.  0.0633   1.4421
##
## sigma^2 = 210.2:  log likelihood = -686.64
## AIC=1379.28   AICc=1379.43   BIC=1388.66
```

```r
ajuste002 <- Arima(tuberculose_ts, c(0, 0, 2))
ajuste002
```

```
## Series: tuberculose_ts
## ARIMA(0,0,2) with non-zero mean
##
## Coefficients:
##          ma1     ma2     mean
##       0.2803  0.1942  97.8537
## s.e.  0.0817  0.0721   1.5983
##
## sigma^2 = 202.1:  log likelihood = -682.86
```

```
## AIC=1373.73   AICc=1373.97   BIC=1386.22
```

```r
ajuste003 <- Arima(tuberculose_ts, c(0, 0, 3))
ajuste003
```

```
## Series: tuberculose_ts
## ARIMA(0,0,3) with non-zero mean
##
## Coefficients:
##          ma1     ma2     ma3     mean
##       0.2761  0.2311  0.1995  97.8459
## s.e.  0.0767  0.0709  0.0656   1.7981
##
## sigma^2 = 192.8:  log likelihood = -678.46
## AIC=1366.92   AICc=1367.29   BIC=1382.54
```

```r
ajuste004 <- Arima(tuberculose_ts, c(0, 0, 4))
ajuste004
```

```
## Series: tuberculose_ts
## ARIMA(0,0,4) with non-zero mean
##
## Coefficients:
##          ma1     ma2     ma3     ma4     mean
##       0.2824  0.2125  0.2131  0.1202  97.8323
## s.e.  0.0786  0.0842  0.0663  0.0757   1.9087
##
## sigma^2 = 191:  log likelihood = -677.14
## AIC=1366.28   AICc=1366.8   BIC=1385.03
```

```r
ajuste101 <- Arima(tuberculose_ts, c(1, 0, 1))
ajuste101
```

```
## Series: tuberculose_ts
## ARIMA(1,0,1) with non-zero mean
##
## Coefficients:
##          ar1      ma1     mean
##       0.9652  -0.7888  99.1243
## s.e.  0.0362   0.0833   5.5643
##
## sigma^2 = 174.4:  log likelihood = -670.84
## AIC=1349.67   AICc=1349.92   BIC=1362.17
```

```r
ajuste102 <- Arima(tuberculose_ts, c(1, 0, 2))
ajuste102
```

```
## Series: tuberculose_ts
## ARIMA(1,0,2) with non-zero mean
##
## Coefficients:
##          ar1      ma1      ma2     mean
##       0.9733  -0.7579  -0.0556  99.4009
## s.e.  0.0304   0.0838   0.0785   6.1832
##
## sigma^2 = 174.9:  log likelihood = -670.59
## AIC=1351.18   AICc=1351.55   BIC=1366.8
```

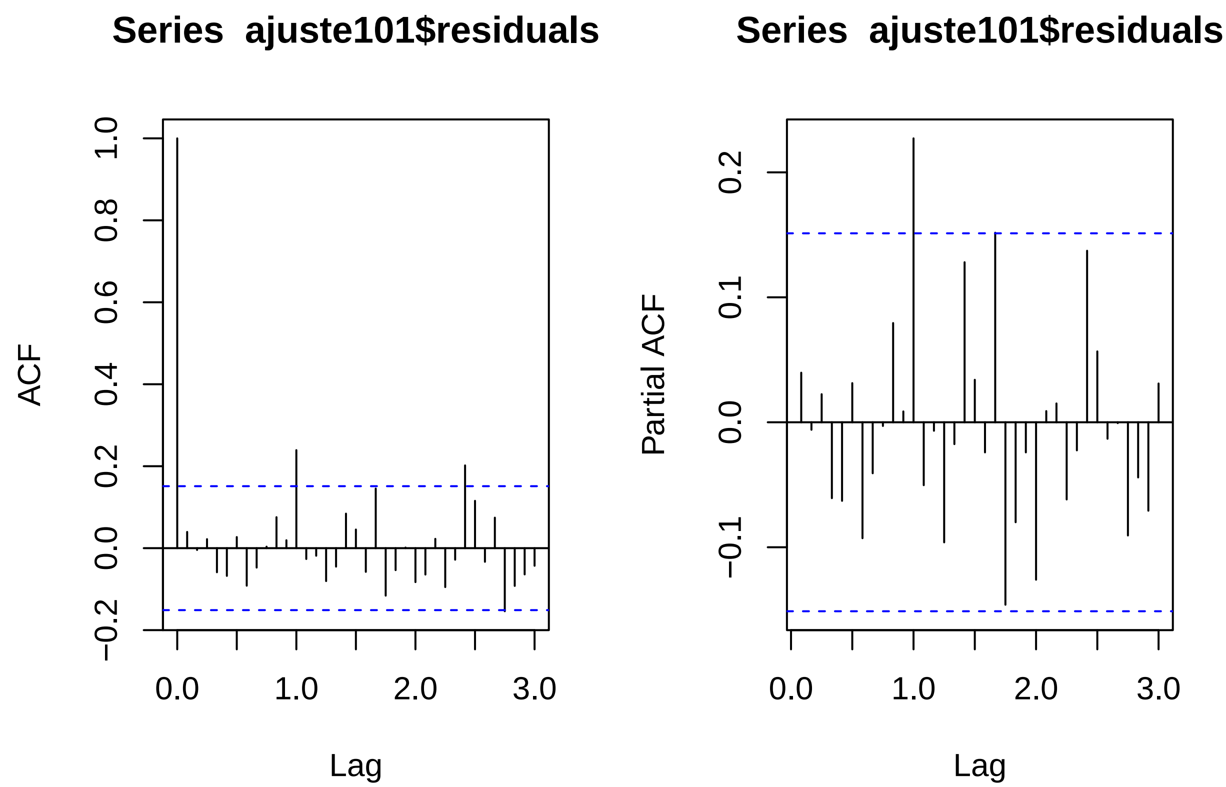
```r
ajuste201 <- Arima(tuberculose_ts, c(2, 0, 1))
ajuste201
```

```
## Series: tuberculose_ts
## ARIMA(2,0,1) with non-zero mean
##
## Coefficients:
##          ar1      ar2      ma1     mean
##       1.0461  -0.0705  -0.8286  99.4163
## s.e.  0.1109   0.0965   0.0790   6.2051
##
## sigma^2 = 174.9:  log likelihood = -670.57
## AIC=1351.15   AICc=1351.52   BIC=1366.77
```

```r
# o "melhor" modelo e o ARMA(1,1)

# vamos buscar o "melhor" modelo usando o auto.arima
auto.arima(tuberculose_ts, d=0, seasonal = FALSE)
```

```
## Series: tuberculose_ts
## ARIMA(1,0,1) with non-zero mean
##
## Coefficients:
##          ar1      ma1     mean
##       0.9652  -0.7888  99.1243
## s.e.  0.0362   0.0833   5.5643
##
## sigma^2 = 174.4:  log likelihood = -670.84
## AIC=1349.67   AICc=1349.92   BIC=1362.17
```

```r
# confirmado que o modelo ARMA(1,1)
# e o "melhor" modelo

# vamos analisar a qualidade do ajuste
par(mfrow=c(1,2))
# acf da serie
acf(ajuste101$residuals, lag.max = 36)
pacf(ajuste101$residuals, lag.max = 36)
```




```r
# observe que tanto a acf quanto a pacf apresentaram
# lag=12 significativo
# vamos proceder com um ajuste de um modelo AR(12) incompleto
# com os termos \phi_1,  \phi_{12}, \theta_1 e media

vetor <- c(NA, rep(0, 10), NA, NA, NA)
ajuste_inc <- Arima(tuberculose_ts, order=c(12, 0, 1), fixed = vetor)
ajuste_inc
```

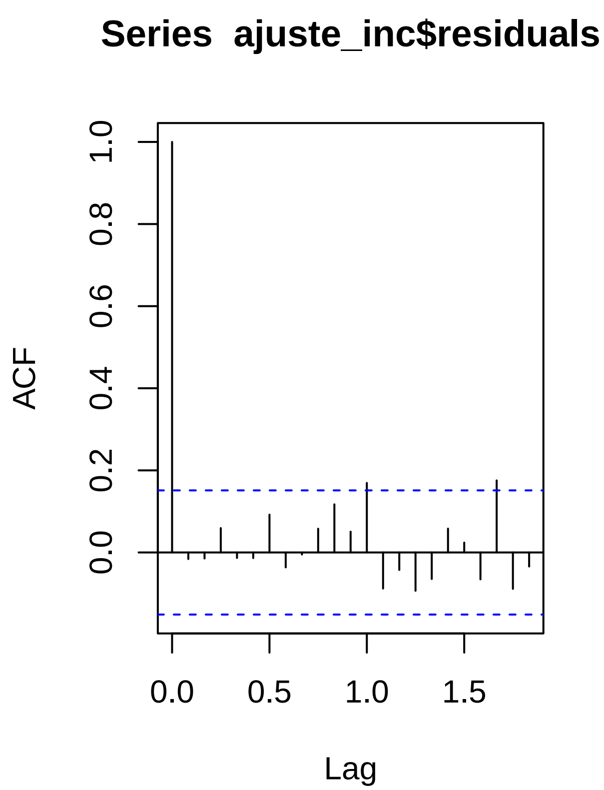
```
## Series: tuberculose_ts
## ARIMA(12,0,1) with non-zero mean
##
## Coefficients:
##          ar1  ar2  ar3  ar4  ar5  ar6  ar7  ar8  ar9  ar10  ar11    ar12
##       0.7849    0    0    0    0    0    0    0    0     0     0  0.1431
## s.e.  0.1486    0    0    0    0    0    0    0    0     0     0  0.1074
##           ma1     mean
##       -0.5361  99.4312
## s.e.   0.1958   5.5574
##
## sigma^2 = 172.9:  log likelihood = -669.77
## AIC=1349.54   AICc=1349.91   BIC=1365.16
```

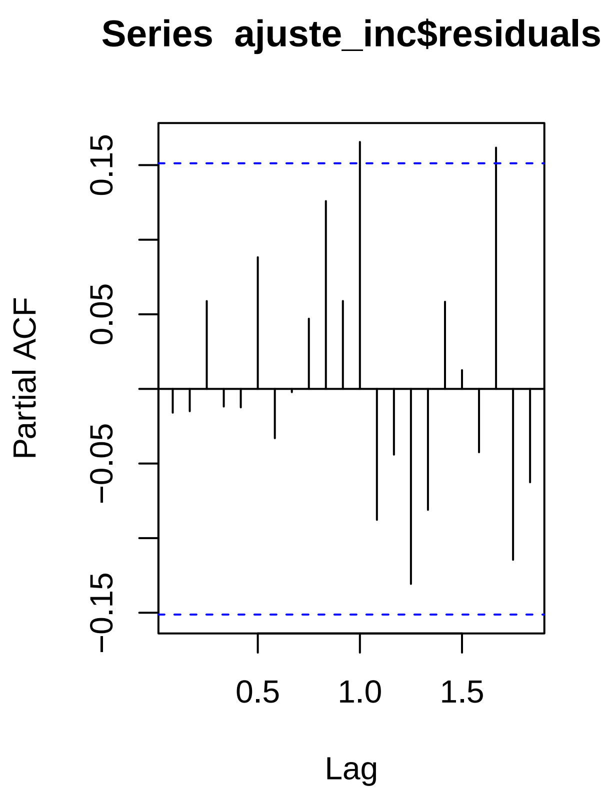
```r
confint(ajuste_inc)
```

```
##                 2.5 %     97.5 %
## ar1        0.49363765  1.0761039
```

```
## ar2                   NA          NA
## ar3                   NA          NA
## ar4                   NA          NA
## ar5                   NA          NA
## ar6                   NA          NA
## ar7                   NA          NA
## ar8                   NA          NA
## ar9                   NA          NA
## ar10                  NA          NA
## ar11                  NA          NA
## ar12         -0.06742744   0.3536904
## ma1          -0.91979527  -0.1524444
## intercept    88.53898810 110.3234897
```

```r
# analisando acf e pacf
acf(ajuste_inc$residuals)
pacf(ajuste_inc$residuals)
```




```r
# esse e um exemplo de que devemos adotar uma modelagem
# que considera o comportamento sazonal - SARIMA ;)
```