Temă seminarul 10

1. $(\mathbb{R}^3/\mathbb{R}, \langle , \rangle)$ să se construiască o bază ortonormată pornind de la baza $B=\{f_1=(1,1,1), f_2=(1,1,-1), f_3=(1,-1,-1)\} \subset \mathbb{R}^3$ folosind Procedeul de ortonormalizare Gram-Schmidt.

$e_1' = f_1 = (1,1,1)$

$e_2' = f_2 - \dfrac{\langle f_2, e_1'\rangle}{\|e_1'\|^2} e_1' = (1,1,-1) - \dfrac{1}{3}(1,1,1) = \left(\dfrac{2}{3}, \dfrac{2}{3}, -\dfrac{4}{3}\right) = \dfrac{2}{3}(1,1,-2)$

$e_3' = f_3 - \dfrac{\langle f_3, e_1'\rangle}{\|e_1'\|^2} e_1' - \dfrac{\langle f_3, e_2'\rangle}{\|e_2'\|^2} e_2' =$

$= (1,-1,-1) + \dfrac{1}{3}(1,1,1) - \dfrac{2}{3}\cdot 2 \cdot \dfrac{1}{\frac{4}{9}\cdot 6} \cdot \dfrac{2}{3}(1,1,-2) =$

$= (1,-1,-1) + \dfrac{1}{3}(1,1,1) - \dfrac{1}{3}(1,1,-2) = (1,-1,0)$

$\|e_1'\| = \sqrt{3}$

$\|e_2'\| = \dfrac{2\sqrt{6}}{3}$

$\|e_3'\| = \sqrt{2}$

$e_1 = \dfrac{e_1'}{\|e_1'\|} = \dfrac{1}{\sqrt{3}}(1,1,1)$

$e_2 = \dfrac{3}{2\sqrt{6}} \cdot \dfrac{2}{3}(1,1,-2) = \dfrac{1}{\sqrt{6}}(1,1,-2)$

$e_3 = \dfrac{1}{\sqrt{2}}(1,-1,0) \Rightarrow \{e_1, e_2, e_3\}$ bază ortonormată

Temă seminarul 11

1. $E_3 = (\mathbb{R}^3/\mathbb{R}, \langle , \rangle)$ și baza ortonormată $B' = \{ e_1 = \frac{1}{\sqrt{2}}(0,1,1), e_2 = \frac{1}{\sqrt{6}}(2,-1,1), e_3 = \frac{1}{\sqrt{3}}(1,1,-1)\}$. Să se determine coordonatele vectorilor următori:

b) $w = (-1, 1, 2)$

$w = w_1 e_1 + w_2 e_2 + w_3 e_3$ scriere unică

$\langle e_i, e_j \rangle = \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$

$[w]_{B'} = (w_1, w_2, w_3)$

$\langle w, e_1 \rangle = w_1 \underbrace{\langle e_1, e_1 \rangle}_{=1} + w_2 \underbrace{\langle e_2, e_1 \rangle}_{=0} + w_3 \underbrace{\langle e_3, e_1 \rangle}_{=0} \Rightarrow w_1 = \langle w, e_1 \rangle$

$\langle w, e_2 \rangle = w_1 \underbrace{\langle e_1, e_2 \rangle}_{=0} + w_2 \underbrace{\langle e_2, e_2 \rangle}_{=1} + w_3 \underbrace{\langle e_3, e_2 \rangle}_{=0} \Rightarrow w_2 = \langle w, e_2 \rangle$

$\langle w, e_3 \rangle = w_1 \underbrace{\langle e_1, e_3 \rangle}_{=0} + w_2 \underbrace{\langle e_2, e_3 \rangle}_{=0} + w_3 \underbrace{\langle e_3, e_3 \rangle}_{=1} \Rightarrow w_3 = \langle w_1, e_3 \rangle$

$w_1 = \langle w, e_1 \rangle = \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$w_2 = \langle w, e_2 \rangle = -\cancel{\frac{2}{\sqrt{6}}} - \frac{1}{\sqrt{6}} + \cancel{\frac{2}{\sqrt{6}}} = -\frac{1}{\sqrt{6}}$

$w_3 = \langle w, e_3 \rangle = -\cancel{\frac{1}{\sqrt{3}}} + \cancel{\frac{1}{\sqrt{3}}} - \frac{2}{\sqrt{3}} = -\frac{2}{\sqrt{3}}$

$[w]_{B'} = \left(\frac{3}{\sqrt{2}}, -\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{3}} \right)$

Temă seminarul 12

1. $P=(0,1,-1)$
$Q=(1,2,-2)$
$R=(3,1,2)$

a) pct necoliniare
b) ec param și implicite pt planul (PQR)

a) $\left.\begin{array}{l} \overrightarrow{PQ} = Q-P = (1,1,-1) \\ \overrightarrow{PR} = R-P = (3,0,3) \\ \frac{1}{3} \neq \frac{1}{0} \neq -\frac{1}{3} \end{array}\right\} \Rightarrow \nexists\, \alpha \in \mathbb{R}$ a.î. $\overrightarrow{PQ} = \alpha \overrightarrow{PR} \Rightarrow P, Q, R$ necoliniare

b) $(PQR): \begin{vmatrix} x-0 & y-1 & z+1 \\ 1 & 1 & -1 \\ 3 & 0 & 3 \end{vmatrix} = 0 \Leftrightarrow 3x - 3(y-1) - 3(z+1) - 3(y-1) = 0$

$\Leftrightarrow 3x - 3y + \cancel{3} - 3z - \cancel{3} - 3y + 3 = 0 \Leftrightarrow 3x - 6y - 3z + 3 = 0 \Leftrightarrow x - 2y - z + 1 = 0$

$y = \alpha,\ z = \beta,\ \alpha, \beta \in \mathbb{R}$

$\begin{cases} x = 2\alpha + \beta - 1 \\ y = \alpha \\ z = \beta \end{cases}, \ \alpha, \beta \in \mathbb{R}$

2. $\pi = \{(2-s-t,\ 3-s,\ t,\ 3s-2t) \mid s, t \in \mathbb{R}\}$
$\pi' = \{(1-2s,\ 2+2t,\ 3+\cancel{s}-5t) \mid s, t \in \mathbb{R}\}$

$\pi = \pi'$?

$\pi: \begin{cases} x = 2-s-t \\ y = 3-s+t \\ z = 3s-2t \end{cases} \Rightarrow \begin{cases} x+y = 5-2s \\ x-y = -1-2t \\ z = 3s-2t \end{cases} \Rightarrow \begin{cases} s = \frac{x+y-5}{-2} \\ t = \frac{x-y+1}{-2} \\ z = 3s-2t \end{cases} \Rightarrow$

$\Rightarrow z = -3 \cdot \frac{x+y-5}{2} + 2 \cdot \frac{x-y+1}{2} \Rightarrow 2z = 3x - 3y + 15 + 2x - 2y + 2 \Rightarrow -x - 5y - 2z + 17 = 0 \Rightarrow x + 5y + 2z - 17 = 0 : \pi$

$\pi': \begin{cases} x = 1-2s \\ y = 2+2t \\ z = 3+5-5t \end{cases} \Rightarrow \begin{cases} s = \frac{1-x}{2} \\ t = \frac{y-2}{2} \\ z = 3 + \frac{1-x}{2} - 5\frac{y-1}{2} \end{cases} \Rightarrow 2z = 6 + 1 - x - 5y + 10 \Rightarrow$

$\Rightarrow x + 5y + 2z - 17 = 0 : \pi'$

$\Rightarrow \pi = \pi'$

2. $A(-1, 2, 3)$

$B(3, 2, -1)$

$C(-1, -1, -3)$

$P(3, -5, 2)$

$\pi : Ax + By + Cz + D = 0$

$\vec{n} = (A, B, C)$, $\vec{n}$ = normala la plan

$\vec{OP}$ = normala la plan $= (3, -5, 2)$

$\pi : 3x - 5y + 2z + D = 0$

$P \in \pi \Rightarrow 3 \cdot 3 - 5(-5) + 2 \cdot 2 + D = 0 \Rightarrow 9 + 25 + 4 + D = 0 \Rightarrow D = -38$

$\pi : 3x - 5y + 2z - 38 = 0$