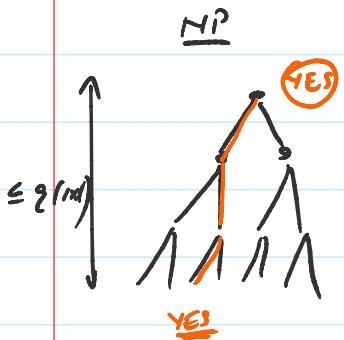
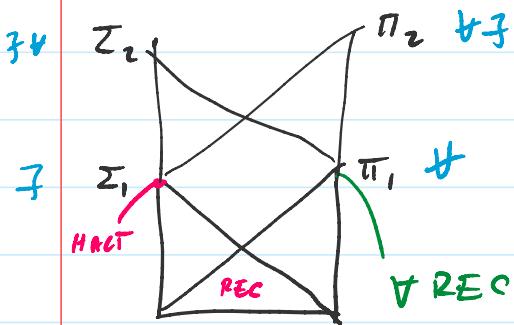


PROBLEME ÎN AFARA CLASEI NP

TAUT

Se dă  $\phi$  formulă în CNF  
Decide Este  $\phi$  Tautologie sau nu?

CALCULANILITATE

$$A \in \Sigma_1 \Leftrightarrow \exists g \in \text{REC}$$

a.i.

$$\forall x \in \Sigma^*$$

$\boxed{\exists y \quad g(x, y) = \text{TRUE}}$

$$A = L(M_i)$$

$$x \in A \Rightarrow M_i(x) \downarrow$$

$$x \notin A \Rightarrow M_i(x) \uparrow$$

$$g(x, t) = \begin{cases} 1 & \text{dacă } M_i(x) \text{ acceptă în t posă,} \\ 0 & \text{altele}\end{cases}$$

f recursive       $S_{i+1} \leftarrow U(i, x)$   
 $p^+ + p_{i+1}$  ad  $I_{i+1} M_i(x)$   
 dacă accept  $\Rightarrow$  return 1  
 altfel  $\Rightarrow$  return 0

① (FAGIN)  $A \in NP \Leftrightarrow$  există un predicat  $f$   
 calculabil în timp polinomial  
 există un polynom  $g$

$$\forall x \in \Sigma^* \quad x \in A \Leftrightarrow \exists y \quad |y| \leq g(|x|) \quad f(x, y) = \text{TRUE}$$

### ANALOGIE

$$\begin{array}{ccc} P & \hookrightarrow & REC \\ NP & \hookrightarrow & R.E. (\Sigma_1) \quad NP = \exists^{\text{poly}} P \end{array}$$

$$\begin{array}{ccc} NP \cap co-NP & & REC \cap co-REC \\ \stackrel{!}{P} & & \stackrel{!}{REC} \\ A \in NP, \bar{A} \in NP & & A \in RE, \bar{A} \in RE \end{array}$$

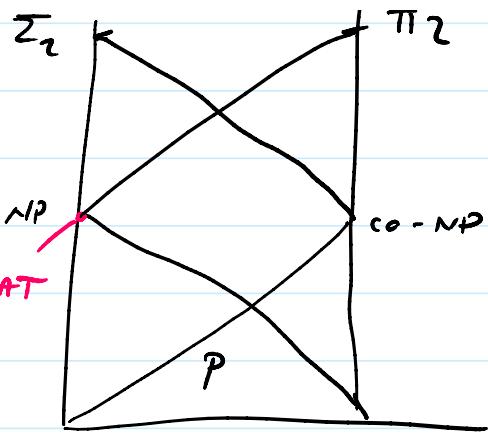
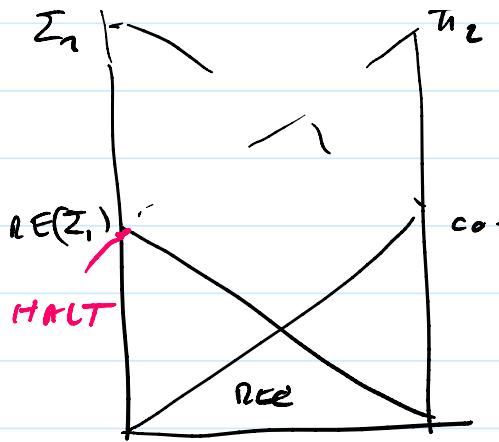
$$\begin{array}{c} co-NP = \{ A, \bar{A} \in NP \} \\ \underline{\text{Exp}} \quad T A T \in co-NP \\ co-NP \text{ complete} \end{array}$$

Nu stiu / nu se cizeză dacă

$$P \in NP \cap co-NP$$

### ANALOGIA





$\Sigma_k^P = \{ A \mid \text{exists un predicat } g \text{ calcolabil in temp polinomial}$   
 $\text{existe un polinom } q$

$$(1) \quad \forall x \in \Sigma^* \quad x \in A \Leftrightarrow (\exists y_1 \mid y_1 \in g(|x|)) \quad (\forall y_2 \mid y_2 \in g(|x|))$$

--- --- / Q  $y_k \mid y_k \in g(|x|)$

$$g(x, y_1, y_2, \dots, y_k) = \text{TRUE}$$

$\overline{\Pi}_k^P$

$$(\forall y_1 \quad ) \quad (\exists y_2 \quad \dots \quad )$$

$$(1) \rightarrow (\exists y_k \quad \dots \quad )$$

$$\Sigma_k^P \subset \Sigma_{k+1}^P \cap \overline{\Pi}_{k+1}^P$$

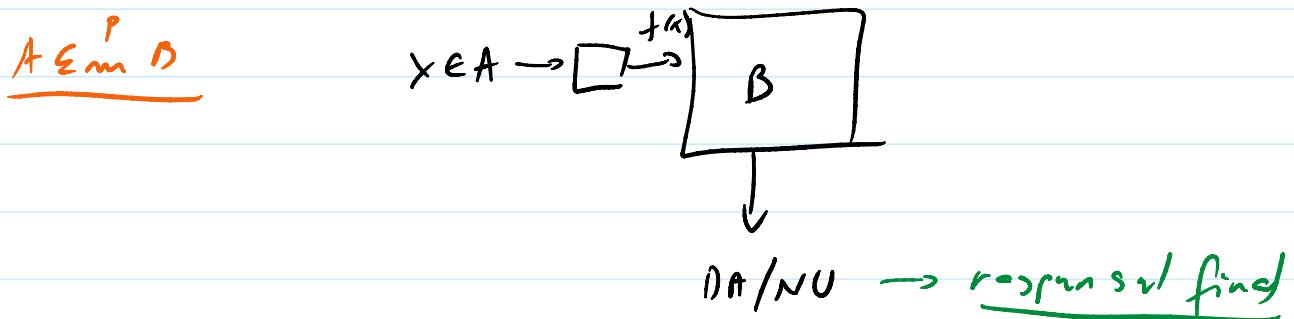
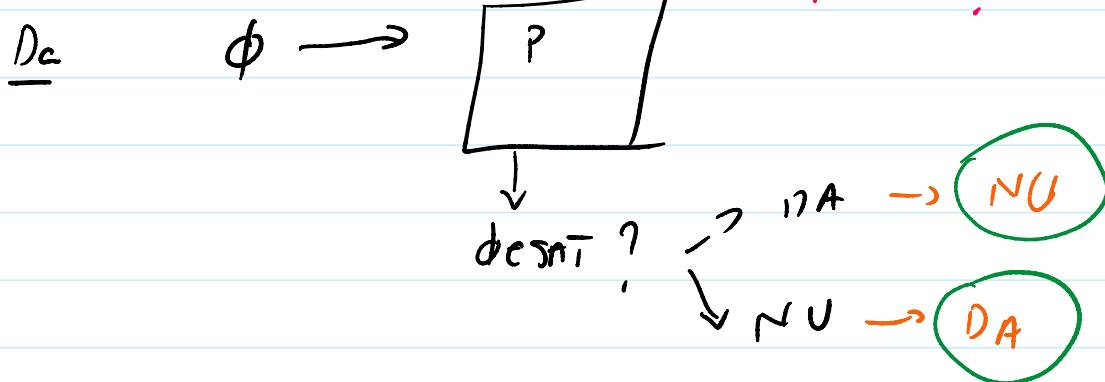
$$\overline{\Pi}_k^P \subset \Sigma_{k+1}^P \cap \overline{\Pi}_{k+1}^P$$

PH (hierarchie polynomiale)

$$PH = \bigcup_{k \geq 1} \Sigma_k^P = \bigcup_{k \geq 1} \overline{\Pi}_k^P$$

## O altă perspectivă asupra PT

Pot rezolva problema  $\overline{\text{SAT}}$  dacă am o procedură P pt SAT!



Exemplu CLIQUE ( $NP$ -completă)

$$\underline{\text{INPUT}} \quad G = (V, E) \quad O(n^k)$$

DE DECIS  $\exists S \subseteq V \quad |S| = k, S$  clique

MAX-CLIQUE

$$\underline{\text{Se dă}} \quad G = (V, E)$$

$$S \subset V,$$

DE DECIS Esta  $S$  una cláusula maximizada?

Aj

1.  $S$  no es cláusula  $\Rightarrow$  NO

2.  $(G_1, \dots, G_k) \in \text{CLIQUE}$  ?  $\rightarrow$  DA  $\rightarrow$  NO

DA  $\rightarrow$  NO

MAX-CLIQUE

$\forall S' |S'| = k+1, (G_i, S') \in \text{clique}$

$\exists v_1, v_2 \in S' v_1 \neq v_2$

$\nexists$

$\overline{Tl}_2^P$

Máquina Turing con ORACOL

①

[bande "oracol"]

②

STARÍ SPECIALÉ

QUERY

YES

NO

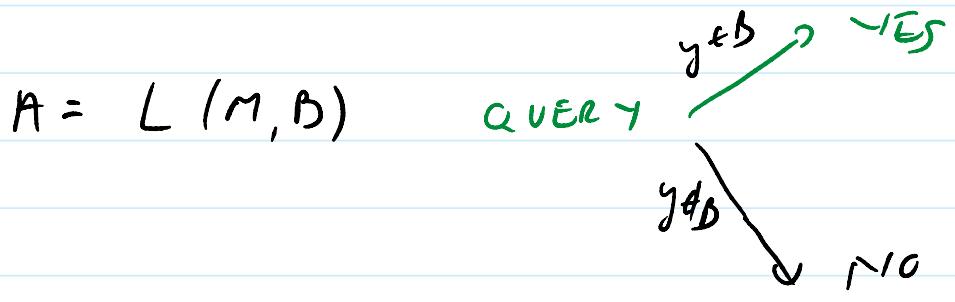
$m(x) \rightarrow$

QUERY  
[y]

YES  
NO

(si tengo bande  
oracol)

y  $\neq b \rightarrow$  YES



Exp Dacă am o procedură care decide SAT  
pot găsi o sol (două există)

$$\phi \begin{cases} \phi|_{x=0} \in \text{SAT} ? \\ \phi|_{x=1} \in \text{SAT} ? \end{cases}$$

(T)  $A \in \Sigma_2^P \Leftrightarrow$  exists a non-deterministic  
polynomial-time oracle m

$$A = L(M, \text{SAT})$$

$$A \in \Sigma_k^P \Leftrightarrow - \quad - \quad -$$

gi un oracle  $B \in \Sigma_{k-1}^P$

a.t.  $A = L(M, B)$

$$01 \quad 11 \quad 11 \quad 11 = P$$

Pb complete pt clasa  $\Sigma_2^P$

$x_1$      $x_2$      $x_3$      $x_4$

Se sforsit sa rezolvam predictat  $g(x_1, \dots, x_4)$  calculabil  
‘ In timp polynomial’ care imi spune  
cine a castigat

SE DA     $(g, q)$

DE DECIS Are primul jucator o strategie de castig  
in  $K$  pasi?

Exemplu de pb care nu poate fi rezolvata in PTIME

1. QBF    INPUT  $\phi = \exists x_1 \forall x_2 \dots \vdash Q x_n P(x_1 \dots x_n)$

↓  
quantified  
boolean  
formula

DE DECIS  $\phi$  adverzat sau nu?

2. Se da    Formula in teorie existentiale non-redu

$$x^2 > 0 \wedge xy - 1 < z^2 \dots \wedge \dots$$

De decis pot da valori sare, f x,y,z...  
care fac formula adevarata.

### 3. AET Grarer Problem

Solutie Poligon arbitrar  
k21



De decis Pot plese ic pegnici  
care sa pozezze (solatativ)  
tot interiorul ?

Cum rezolvam PB din PH?

(1) QBF solvers

(2) Answer set programming CLINEO