Portfolio Credit Risk models

Živa Artnak, Alja Dostal, Manca Kavčič, Anamarija Potokar

1. Commodity selection and material cost modelling

The company under consideration is a US-based non-financial firm whose production process is highly exposed to fluctuations in commodity prices. At current prices, the cost of the core input material represents approximately 69% of the final product price, making commodity price risk the dominant source of operating risk. As a first step, an appropriate traded commodity must therefore be selected to proxy the dynamics of the company's input costs.

1.1. Choice of commodity and data

Copper was selected as the reference commodity. This choice is economically justified as copper is a widely traded industrial metal, commonly used as an input in manufacturing processes, and exhibits substantial price volatility. Copper spot prices quoted in USD were obtained from a publicly available data source (index mundi) at monthly frequency, covering the period from December 2010 to June 2025. Using spot prices is consistent with the objective of capturing the underlying commodity price dynamics that affect the firm's cost base.

Let P_t^{Cu} denote the spot price of copper at time t.

1.2. Mapping commodity prices to material costs

At current prices, annual sales of the company amount to USD 920 million. Given that the core material represents 69% of the sales price, the current annual material cost is

$$CM_0 = 0.69 \times 920\,\text{million USD} = 634{,}8\,\text{million USD}.$$

To link historical copper prices to the firm's material costs, we assume a linear relationship between the commodity price and the cost of materials. Specifically, the material cost at time t is modelled as

$$CM_t = k \cdot P_t^{Cu},$$

where the scaling factor k represents the effective quantity of copper embedded in annual production. The parameter k is calibrated such that the material cost at the current copper price matches the observed current material cost:

$$k = \frac{CM_0}{P_0^{Cu}} = 64544{,}53,$$

where $P_0^{Cu} = 9835{,}07$ is the most recent observed copper price.

This calibration ensures that the constructed material cost series is fully consistent with the company's current profit and loss statement while preserving the historical dynamics of copper prices.

1.3. Commodity price dynamics

To analyse risk and to support later modelling steps, logarithmic returns of copper prices were computed as

$$r_t^{Cu} = \ln(P_t^{Cu}) - \ln(P_{t-1}^{Cu}).$$

Since the material cost is a linear transformation of the copper price, monthly logarithmic returns of copper prices are computed, as risk measurement and hedging effectiveness depend on price changes rather than price levels. Log returns are used due to their additive properties over time, which facilitates aggregation and historical simulation–based risk measures.

The resulting time series of material costs captures the historical variability of input costs faced by the firm and forms the basis for analysing operating risk, pricing pass-through mechanisms, and hedging strategies in the subsequent sections.

2. Sales modelling under alternative pass-through assumptions

Having established a time series for the cost of the core input material based on copper prices, the next step is to model the company's sales dynamics. The objective is to assess how different degrees of commodity price pass-through to customers affect operating performance and risk. Two conceptually distinct sales regimes are considered.

2.1. Case (a): Fixed sales (no pass-through)

In the first scenario, sales are assumed to be independent of commodity price movements. Formally, annual sales are assumed to be constant at

$$S_t = S_0 = 920\,\text{million USD} \quad \text{for all } t.$$

In this case, sales exhibit no variability and are uncorrelated with copper price movements. All variability in operating performance therefore arises exclusively from fluctuations in material costs. This scenario serves as a benchmark representing maximum exposure to commodity price risk on the cost side.

2.2. Case (b): Correlated sales (partial pass-through)

In the second scenario, sales are allowed to co-move with commodity prices, reflecting the presence of partial price pass-through mechanisms. Rather than modelling prices directly, sales dynamics are generated by constructing a time series of sales returns that is correlated with copper price returns. Let r_t^{Cu} denote the logarithmic return of the copper price. Sales returns are modelled as

$$r_t^S = \rho\, r_t^{Cu} + \sqrt{1-\rho^2}\, \varepsilon_t,$$

where ρ is the target correlation coefficient between sales returns and commodity price returns, and ε_t is an independent standard normal shock. This construction ensures that sales returns exhibit the desired correlation with copper prices while retaining an idiosyncratic component.

Two levels of pass-through are considered:

- $\rho_1 = 0.74$, representing moderate pass-through to customers,
- $\rho_2 = 0.92$, representing strong pass-through and high pricing flexibility.

Given the simulated sales returns, the corresponding annual sales levels are obtained recursively as

$$S_t = S_{t-1} \exp(r_t^S),$$

with initial sales set equal to the current annual sales level, $S_0 = 920$ million USD. As a result, sales are no longer constant and may deviate from the initial level depending on the realised commodity price dynamics.

In contrast to the fixed-sales case, sales now fluctuate over time and respond positively to increases in copper prices. Higher values of the correlation parameter ρ lead to stronger co-movement between sales and commodity prices, resulting in greater variability of sales levels.

2.3. Empirical properties of simulated sales

The constructed sales return series, display several economically intuitive properties. First, sales returns are centred around zero, indicating the absence of deterministic growth or decline in sales. Second, volatility increases with the degree of pass-through, reflecting stronger sensitivity of revenues to commodity price movements. Finally, the correlation structure ensures that periods of rising copper prices are, on average, associated with increasing sales revenues, providing a natural hedge against rising input costs.

This modelling approach deliberately reverses the usual empirical estimation framework. Instead of estimating pass-through from observed sales data, sales are generated directly from the commodity price process. This allows for a controlled analysis of how different degrees of pass-through affect operating profitability and risk, which is the primary objective of the present study.

3. Simulation of EBIT and comparison with the point estimate

Based on the simulated time series of material costs and sales described in the previous sections, the company's operating performance is analysed by constructing a time series of Earnings Before Interest and Taxes (EBIT). At this stage, the financial part of the profit and loss statement is deliberately excluded in order to focus exclusively on operating risk arising from commodity price fluctuations and pricing pass-through mechanisms.

3.1. EBIT definition and cost structure

EBIT at time t is defined as

$$\text{EBIT}_t = S_t - CM_t - OC_t - DL_t - OH,$$

where:

- S_t denotes annual sales,
- CM_t denotes the cost of the core material,

- $OC_t = 0.06\, S_t$ represents other variable material costs,
- $DL_t = 0.14\, S_t$ represents direct labour costs,
- $OH = 77$ million USD denotes annual overhead costs.

Material costs are modelled as a linear function of the copper price, while other cost components are proportional to sales or fixed. This structure reflects the firm's strong exposure to commodity price risk, as nearly 70% of the sales price is driven by the core input material.

3.2. Point estimate of EBIT

Using current prices and the given profit and loss data, the point estimate of EBIT is

$$\text{EBIT}_0 = 24.2 \text{ million USD.}$$

This value reflects the company's operating profitability at the most recent observed copper price and serves as a benchmark against which simulated EBIT outcomes are compared.

3.3. Simulated EBIT under alternative sales regimes

EBIT is simulated for each historical observation using the corresponding material cost and sales levels under the three sales regimes introduced previously:

- fixed sales (no pass-through),
- partial pass-through with $\rho_1 = 0.74$,
- strong pass-through with $\rho_2 = 0.92$.

For each scenario, a time series of annual EBIT levels is obtained. Table 1 summarises the average simulated EBIT and the EBIT corresponding to the most recent observation (current prices).

Table 1: Simulated EBIT under alternative sales regimes

Scenario	**Average EBIT** (USD million)	**Current EBIT** (USD million)
Fixed sales (no pass-through)	183.2	24.2
Partial pass-through ($\rho_1 = 0.74$)	117.5	7.3
Strong pass-through ($\rho_2 = 0.92$)	96.8	69.4

3.4. Interpretation

Several important insights emerge from the EBIT simulations. First, the current EBIT level is substantially lower than the historical average under all scenarios, indicating that the firm is currently operating in an unfavourable commodity price environment. This is particularly evident in the fixed-sales case, where elevated copper prices significantly compress margins.

Second, sales pass-through acts as a natural hedge against commodity price risk. Strong pass-through ($\rho_2 = 0.92$) substantially improves current EBIT relative to the fixed-sales case, raising EBIT from USD 24.2 million to USD 69.4 million at current prices. However, partial pass-through ($\rho_1 = 0.74$) proves insufficient to offset high input costs and results in even weaker current EBIT.

Finally, increasing pass-through compresses the EBIT distribution, while strong pass-through improves downside protection, it also reduces average EBIT by limiting upside potential when commodity prices are low. This trade-off highlights the importance of evaluating operating performance not only in terms of expected profitability but also in terms of downside risk, which is analysed formally in the next section using Value-at-Risk and Expected Tail Loss measures.

4. Value-at-Risk, Expected Tail Loss and management implications

While average EBIT provides useful information about expected operating performance, it does not adequately capture downside risk arising from adverse commodity price movements. To assess the firm's exposure to extreme but plausible outcomes, this section evaluates the risk of EBIT using Value-at-Risk (VaR) and Expected Tail Loss (ETL).

4.1. Risk measures

Value-at-Risk at confidence level $\alpha = 5\%$ is defined as the lower α-quantile of the EBIT distribution:

$$\mathrm{VaR}_{5\%} = \inf\{x : \mathbb{P}(\mathrm{EBIT} \leq x) \geq 5\%\}.$$

It represents the EBIT level that will not be exceeded with 95% confidence.

Expected Tail Loss (ETL), also referred to as Expected shortfall, is defined as the average EBIT conditional on being in the worst 5% of outcomes:

$$\mathrm{ETL}_{5\%} = \mathbb{E}[\mathrm{EBIT} \mid \mathrm{EBIT} \leq \mathrm{VaR}_{5\%}].$$

ETL provides a more conservative measure of risk by capturing the severity of losses beyond the VaR threshold.

Both measures are computed directly from the simulated EBIT distributions obtained in the previous section.

4.2. Empirical results

Table 2 reports the VaR and ETL at the 5% level for all three sales regimes.

Table 2: VaR and ETL at the 5% level

Scenario	**VaR (5%)** (USD million)	**ETL (5%)** (USD million)
Fixed sales (no pass-through)	29.5	13.9
Partial pass-through ($\rho_1 = 0.74$)	-34.8	-52.2
Strong pass-through ($\rho_2 = 0.92$)	29.2	21.3

4.3. Interpretation

The results highlight substantial differences in downside risk across sales regimes. Under fixed sales, the firm remains profitable even in the lower tail of the EBIT distribution, although margins are significantly compressed. Introducing partial pass-through with $\rho_1 = 0.74$ markedly worsens downside risk, both VaR and ETL become negative, indicating that the firm would incur operating losses in severe but plausible scenarios. This outcome reflects the fact that moderate pass-through is insufficient to offset high commodity input costs, while simultaneously reducing upside potential when commodity prices are low.

In contrast, strong pass-through ($\rho_2 = 0.92$) significantly improves downside protection. Both VaR and ETL are positive and exceed those observed in the fixed-sales case, demonstrating that high pricing flexibility acts as an effective natural hedge against commodity price risk.

4.4. Recommendations to the management board

Several recommendations emerge from the analysis. First, reliance on partial pass-through pricing mechanisms should be avoided, as they may increase downside risk and expose the firm to operating losses in adverse commodity price environments. Second, where market conditions permit, the firm should aim to implement pricing structures with strong commodity price indexation, as high pass-through substantially stabilizes EBIT and improves tail outcomes. Third, even under strong pass-through, residual risk remains. Therefore, pricing policy should be complemented by a formal commodity hedging program using financial derivatives. Such a program should be designed with the explicit objective of reducing VaR and ETL rather than maximizing expected profits. Aligning pricing flexibility with financial hedging would allow the firm to achieve a more resilient operating performance and improve planning certainty.

Overall, the results demonstrate that effective management of commodity risk requires a coordinated approach combining commercial pricing decisions with financial risk management instruments.

5. Hedging setup and methodology

5.1. Scope and objective

This report presents the results for Parts 5 and 6 of the assignment, where the firm is exposed to copper price fluctuations through its material costs. The goal is to design a hedge and evaluate how hedging affects the distribution of EBIT under three operating cases: (a) fixed sales, and (b) sales correlated with the commodity with correlation parameters $\rho = 0.74$ and $\rho = 0.92$. Downside risk is measured by the 5% Value-at-Risk ($\mathrm{VaR}_{0.05}$) and the Expected Tail Loss ($\mathrm{ETL}_{0.05}$). All risk numbers below are reported in USD millions, so a larger VaR/ETL corresponds to less severe downside outcomes for profitability.

5.2. Hedge construction

Hedges are constructed using the minimum-variance hedge ratio applied to first differences. Let ΔCM_t denote the change in the firm's total material costs (USD), and let ΔX_t denote the change in the hedge instrument price expressed in USD per ton. The hedge quantity (in tons) is estimated as

$$h^* = \frac{\mathrm{Cov}(\Delta CM, \Delta X)}{\mathrm{Var}(\Delta X)}.$$

Hedge P&L is then computed as $\mathrm{PnL}_t = h^* \Delta X_t$ and added to each EBIT series. Economically, the firm is harmed by rising copper prices through higher input costs, so taking a *long* position in the hedge instrument provides positive hedge P&L when copper prices rise. VaR and ETL are computed by historical simulation from the empirical EBIT distribution.

5.3. Unit conversion and contract sizing

Copper futures (HG=F) are quoted in USD per pound. Prices are converted to USD per metric ton using

$$1 \text{ metric ton} = 2204.6226 \text{ lb}, \qquad F_t^{\text{ton}} = F_t^{\text{lb}} \cdot 2204.6226.$$

A standard contract covers 25,000 lb, corresponding to approximately 11.3398 metric tons.

5.4. Implementation note: synthetic derivative construction in Parts 5 and 6

Two slightly different constructions are used in the code. In Part 5, the derivative *changes* are generated as a linear combination

$$\Delta D_t = \rho \, \Delta CM_t + \sqrt{1-\rho^2}\, \varepsilon_t,$$

with ε_t drawn from a normal distribution scaled to the volatility of ΔCM_t. Because the sample is finite and the noise is not explicitly orthogonalized to ΔCM_t, the realized *sample* correlation can deviate from the target; in the reported run, the target is 0.75 but the realized sample correlation equals 0.717.

In Part 6, the derivative changes are generated using an orthogonalization step that removes the projection of the random noise on standardized cost changes. This construction ensures that the realized *sample* correlation matches the target correlation levels (0.60, 0.75, 0.90) much more tightly, which is preferable for a clean correlation sensitivity analysis.

6. Results for Parts 5 and 6

6.1. Hedge using actual copper futures

The estimated hedge quantity using copper futures is

$$h^*_{\text{fut}} = 32{,}193.71 \text{ tons}, \qquad N \approx \frac{32{,}193.71}{11.3398} \approx 2{,}839 \text{ contracts}.$$

Table 3: Part 5A: EBIT risk (USD millions), unhedged vs. hedged with copper futures.

Case	Strategy	α	VaR_α	ETL_α
a) fixed sales	unhedged	0.05	28.6	12.6
b) $\rho = 0.74$	unhedged	0.05	-38.2	-53.9
b) $\rho = 0.92$	unhedged	0.05	28.3	20.3
a) fixed sales	hedged (copper futures)	0.05	24.1	12.0
b) $\rho = 0.74$	hedged (copper futures)	0.05	-36.8	-61.7
b) $\rho = 0.92$	hedged (copper futures)	0.05	21.3	10.4

Table 3 compares $\mathrm{VaR}_{0.05}$ and $\mathrm{ETL}_{0.05}$ for unhedged and futures-hedged EBIT.

The futures hedge does not lead to uniform risk reduction in EBIT. In case (a) fixed sales, both VaR and ETL decrease after hedging, indicating worse left-tail profitability. In case (b) with $\rho = 0.74$, VaR improves slightly (from -38.2 to -36.8), but ETL becomes more negative (from -53.9 to -61.7), meaning the most adverse outcomes in the far tail deteriorate. In case (b) with $\rho = 0.92$, both VaR and ETL fall strongly, which is consistent with an operational natural hedge: when revenues co-move strongly with copper, the unhedged EBIT already benefits from offsetting movements, and a financial hedge can remove part of this beneficial co-movement.

6.2. Hedge using a synthetic derivative (target corr 0.75, realized sample corr 0.717)

A synthetic derivative instrument is constructed to target correlation 0.75 with ΔCM, but in the reported run the realized sample correlation between ΔCM and ΔD equals 0.717. The estimated hedge quantity is

$$h^*_{\text{deriv}} = 24{,}360.62 \text{ tons}, \qquad N \approx \frac{24{,}360.62}{11.3398} \approx 2{,}148 \text{ contracts (futures-equivalent).}$$

Table 4 reports the corresponding VaR/ETL results.

Table 4: Part 5B: EBIT risk (USD millions), unhedged vs. hedged with synthetic derivative (target corr 0.75, realized sample corr 0.717).

Case	Strategy	α	VaR_α	ETL_α
a) fixed sales	unhedged	0.05	28.6	12.6
b) $\rho = 0.74$	unhedged	0.05	-38.2	-53.9
b) $\rho = 0.92$	unhedged	0.05	28.3	20.3
a) fixed sales	hedged (deriv corr$\sim$0.75)	0.05	28.9	13.5
b) $\rho = 0.74$	hedged (deriv corr$\sim$0.75)	0.05	-31.1	-57.0
b) $\rho = 0.92$	hedged (deriv corr$\sim$0.75)	0.05	20.8	8.70

Relative to the futures hedge, the synthetic derivative hedge behaves differently. In the fixed-sales case, VaR and ETL increase slightly (from 28.6 to 28.9 and from 12.6 to 13.5), indicating a modest improvement in the left tail of EBIT. In the $\rho = 0.74$ operating case, the derivative hedge improves VaR substantially (from -38.2 to -31.1), but ETL becomes more negative (from -53.9 to -57.0), which means that extremely adverse outcomes in the far tail remain severe and can even worsen. In the high revenue–commodity correlation case ($\rho = 0.92$), hedging lowers both VaR and ETL, reinforcing the interpretation that the firm already has a strong natural hedge through revenues and that financial hedging can remove beneficial co-movement between revenues and costs.

6.3. Correlation sensitivity analysis (synthetic derivative)

Part 6 repeats the derivative hedge for correlations $\rho_{\text{deriv}} \in \{0.60, 0.75, 0.90\}$. Derivative changes are constructed so that their *sample* correlation with ΔCM equals the target correlation, and the volatility is scaled to be comparable across correlation levels.

To evaluate hedge quality on the cost exposure alone, hedge effectiveness is computed as

$$\text{HE} = 1 - \frac{\text{Var}(\Delta CM - h^* \Delta D)}{\text{Var}(\Delta CM)}.$$

Table 5 reports the resulting hedge ratios and hedge effectiveness.

Table 5: Part 6: Hedge effectiveness on material-cost changes.

ρ_{deriv}	Sample corr	h^* (tons)	Hedge effectiveness
0.60	0.60	30,377	0.360
0.75	0.75	37,971	0.562
0.90	0.90	45,565	0.810

As expected, hedge effectiveness increases strongly with correlation: moving from correlation 0.60 to 0.90 increases the fraction of cost-change variance removed by the hedge from 36% to 81%.

6.4. EBIT VaR/ETL across correlations

Table 6 reports EBIT VaR/ETL for each correlation level.

Table 6: Part 6: EBIT risk (USD millions) across derivative correlations.

Case	Strategy	ρ_{deriv}	$\text{VaR}_{0.05}$	$\text{ETL}_{0.05}$
a) fixed sales	unhedged	–	28.6	12.6
b) $\rho = 0.74$	unhedged	–	-38.2	-53.9
b) $\rho = 0.92$	unhedged	–	28.3	20.3
a) fixed sales	hedged (deriv corr∼0.6)	0.60	25.9	16.1
b) $\rho = 0.74$	hedged (deriv corr∼0.6)	0.60	-36.1	-57.5
b) $\rho = 0.92$	hedged (deriv corr∼0.6)	0.60	24.1	13.6
a) fixed sales	hedged (deriv corr∼0.75)	0.75	27.4	13.3
b) $\rho = 0.74$	hedged (deriv corr∼0.75)	0.75	-34.5	-59.1
b) $\rho = 0.92$	hedged (deriv corr∼0.75)	0.75	17.1	6.80
a) fixed sales	hedged (deriv corr∼0.9)	0.90	23.0	10.1
b) $\rho = 0.74$	hedged (deriv corr∼0.9)	0.90	-38.3	-67.7
b) $\rho = 0.92$	hedged (deriv corr∼0.9)	0.90	12.6	-6.08

The correlation sensitivity results show a clear distinction between hedging the cost exposure and improving EBIT tail risk. While hedge effectiveness on cost changes increases monotonically with correlation, the EBIT VaR/ETL outcomes do not improve monotonically. In particular, in the operating scenario with strong revenue–commodity correlation ($\rho = 0.92$), aggressive cost hedging can significantly reduce VaR and can even produce a negative ETL at $\rho_{\text{deriv}} = 0.90$. This supports the interpretation that the firm has a natural hedge through revenues, and that a strong financial hedge can remove beneficial co-movement between revenues and costs.

6.5. Conclusion

Across the implemented instruments and correlation levels, the minimum-variance approach implies hedge sizes on the order of roughly 2.4×10^4 to 4.6×10^4 tons, corresponding to thousands of standard futures contracts. Increasing correlation between the hedge instrument and cost changes strongly improves hedge effectiveness on the cost exposure. However, EBIT-based VaR/ETL results depend on the firm's operating structure. When revenues co-move strongly with copper prices, the firm benefits from an operational natural hedge, and additional financial hedging can worsen downside profitability outcomes. Therefore, a hedging policy aimed at reducing EBIT tail risk should be designed using the net exposure of EBIT (revenue and cost jointly), not only by minimizing variance of input costs.

7. Transition to a EU-based company

In this section, we reconsider the risk profile under the assumption that the company is based in the European Union. The functional currency is now the Euro (EUR), while the primary raw material (Copper) remains quoted in US Dollars (USD).

The company's financial parameters were adjusted to:

- Initial Sales (S_0): 920,000,000 EUR
- Overhead (OH): 77,000,000 EUR
- Debt: 540,000,000 EUR

Copper prices in EUR are subject to EUR/USD exchange rate, which was obtained from publicly available data source (Yahoo Finance).

$$P_t^{Cu,\text{EUR}} = P_t^{Cu} \cdot FX_t^{\text{USD/EUR}}$$

From here on, the calculations are the same as in sections 1-6.

7.1. Results for EU-based company

Current copper price is $P_0^{Cu,\text{EUR}} = 8643.65$ with current FX rate $FX_0^{\text{EUR/USD}} = 1.137838$. Factor k is 73441.2.

As before (in section 3), we present EBIT for different scenarios in table 7. There is a noticable difference in EBIT values when comparing them to those in section 3. That is because od added FX risk. A weakening Euro acts like an increase in the price of copper. Because of the nature of the model (that material represents 69% of the price of final products), every time the Euro weakened, sales prices were raised to compensate. Because fixed overheads remained constant at 77M EUR, the firm experienced significant margin expansion in nominal terms.

Table 7: Simulated EBIT under Alternative Sales Regimes

Scenario	**Average EBIT** (EUR million)	**Current EBIT** (EUR million)
Fixed sales (no pass-through)	197.1	24.2
Partial pass-through ($\rho_1 = 0.74$)	221.1	124.9
Strong pass-through ($\rho_2 = 0.92$)	222.2	231.6

In table 8 we present VaR and ETL at the 5% level. The transition to a EUR-based operation significantly alters the firm's risk profile, moving from a single-commodity risk to an environment involving both copper prices and FX fluctuations. In contrast to the USD results, where a strong pass-through provided a robust natural hedge, the EUR results in table 8 show a dramatic increase in VaR and ETL. The greatly increased VaR reflects the fact that the firm is now exposed to the uncertainty of a weakening Euro. Furthermore, the massive scale of these losses is amplified by the firm's increased nominal revenue, making the absolute impact of tail-risk events far more severe than in the original USD-denominated model.

Table 8: VaR and ETL at the 5% level

Scenario	**VaR (5%)** (EUR million)	**ETL (5%)** (EUR million)
Fixed sales (no pass-through)	15.4	-6.6
Partial pass-through ($\rho_1 = 0.74$)	65.5	45.4
Strong pass-through ($\rho_2 = 0.92$)	146.6	135.8

For hedging using actual copper futures with regard to FX risk, we have

$$h_{fut}^{*,\mathrm{EUR}} = 37{,}305.18, \qquad N \approx 3290 \text{ contracts}$$

Table 9 compares $\mathrm{VaR}_{0.05}$ and $\mathrm{ETL}_{0.05}$ for unhedged and futures-hedged EBIT. The transition from table 3 (USD) to table 9 (EUR) highlights a fundamental shift in the firm's risk profile driven by the introduction of currency volatility as a risk multiplier. In the USD-based results, the partial pass-through scenario ($\rho = 0.74$) yields a negative VaR, suggesting that pricing power acts as a near-perfect natural hedge against copper fluctuations in a single-currency environment. On the other hand, results for EU-based company show this same scenario resulting in a significantly positive VaR, as the double volatility of independent copper prices and USD/EUR exchange rate movements breaks the previous offset. Furthermore, the absolute magnitude of risk in table 9 is much higher due to the nominal inflation of revenues and costs in Euro terms; while operating leverage improves average EBIT, it simultaneously amplifies the absolute scale of potential losses during tail events. Finally, the copper hedge appears less effective in this table because standard futures only mitigate the commodity price component, leaving the substantial currency-driven basis risk entirely unhedged.

Table 9: EBIT risk (EUR millions), unhedged vs. hedged with copper futures.

Case	Strategy	α	VaR_α	ETL_α
fixed sales	unhedged	0.05	13.5	-8.15
$\rho = 0.74$	unhedged	0.05	59.8	43.3
$\rho = 0.92$	unhedged	0.05	147	137
fixed sales	hedged (copper futures)	0.05	12.6	-9.53
$\rho = 0.74$	hedged (copper futures)	0.05	56.1	33.2
$\rho = 0.92$	hedged (copper futures)	0.05	141	131

For hedging using a synthetic derivative with corr ≈ 0.75 to costs with regard to FX risk, we have

$$h^{*,\text{EUR}}_{deriv} = 29{,}050.61, \qquad N \approx 2562 \text{ contracts}$$

The transition from the synthetic derivative analysis in the USD environment (table 4) to the EUR environment (table 10) further illustrates the complicating role of exchange rate volatility. In table 4, the synthetic derivative is highly effective at stabilizing EBIT, particularly in the partial pass-through scenario ($\rho = 0.74$), where the VaR remains deeply negative. This confirms that when copper risk is the primary concern, even a moderately correlated derivative can maintain the efficiency of the firm's natural hedge. However, in table 10, the same derivative strategy yields significantly higher risk metrics across all cases. For the partial pass-through regime, the VaR shifts from the USD-based gain to a greater loss. This divergence stems from the fact that the synthetic derivative, much like the standard futures contract, is modeled to track the commodity price component rather than the combined commodity-FX cost fluctuation. While the derivative successfully lowers the ETL in the strong pass-through case ($\rho = 0.92$) in table 10, the remaining tail risk is vastly larger than the corresponding USD tail risk. These results demonstrate that while synthetic instruments provide measurable protection, their effectiveness is inherently diluted in the EUR regime where the firm must contend with the unhedged variance of the USD/EUR exchange rate.

Table 10: Part 5B: EBIT risk (EUR millions), unhedged vs. hedged with synthetic derivative (corr ≈ 0.75).

Case	Strategy	α	VaR_α	ETL_α
fixed sales	unhedged	0.05	13.5	-8.15
$\rho = 0.74$	unhedged	0.05	59.8	43.3
$\rho = 0.92$	unhedged	0.05	147	137
fixed sales	hedged (deriv corr$\sim$0.75)	0.05	13.6	-9.81
$\rho = 0.74$	hedged (deriv corr$\sim$0.75)	0.05	57.1	32.6
$\rho = 0.92$	hedged (deriv corr$\sim$0.75)	0.05	142	125

Table 11 reports the resulting hedge ratios and hedge effectiveness.

Table 12 reports EBIT VaR/ETL for each correlation level. The analysis across varying derivative correlations ($\rho_{deriv} = 0.60, 0.75, 0.90$) reveals a significant divergence between currency regimes. Table 12 (EUR) presents a much more resistant risk profile than

Table 11: Part 6: Hedge effectiveness on material-cost changes.

ρ_{deriv}	Sample corr	h^* (tons)	Hedge effectiveness
0.60	0.60	32,295	0.360
0.75	0.75	40,368	0.563
0.90	0.90	48,442	0.810

table 6. While increasing the derivative correlation does reduce the ETL for the $\rho = 0.92$ case, the absolute magnitude of the risk remains nearly twenty times higher than in the USD counterpart. Even with a near-perfect commodity correlation of 0.90, the firm's $VaR_{0.05}$ stays as high as 137 million EUR. This persistency of risk in the EUR model is a direct result of the unhedged USD/EUR exchange rate variance. Because the derivative only offsets the price movements of copper in USD terms, it cannot counteract the effect of a fluctuating Euro. Consequently, table 12 demonstrates that as the nominal scale of the business increases in EUR terms, the marginal benefit of increasing commodity-derivative precision diminishes unless a corresponding currency hedge is also implemented.

Table 12: EBIT risk (EUR millions) across derivative correlations.

Case	Strategy	ρ_{deriv}	$\text{VaR}_{0.05}$	$\text{ETL}_{0.05}$
fixed sales	unhedged	–	13.0	-8.15
$\rho = 0.74$	unhedged	–	59.4	43.3
$\rho = 0.92$	unhedged	–	147	137
fixed sales	hedged (deriv corr$\sim$0.6)	0.60	16.5	-8.52
$\rho = 0.74$	hedged (deriv corr$\sim$0.6)	0.60	55.4	33.1
$\rho = 0.92$	hedged (deriv corr$\sim$0.6)	0.60	139	129
fixed sales	hedged (deriv corr$\sim$0.75)	0.75	8.48	-5.92
$\rho = 0.74$	hedged (deriv corr$\sim$0.75)	0.75	62.0	36.7
$\rho = 0.92$	hedged (deriv corr$\sim$0.75)	0.75	139	126
fixed sales	hedged (deriv corr$\sim$0.9)	0.90	13.8	-9.14
$\rho = 0.74$	hedged (deriv corr$\sim$0.9)	0.90	65.7	27.6
$\rho = 0.92$	hedged (deriv corr$\sim$0.9)	0.90	137	123

8. Financial Risk and Combined Hedging Strategy

In this stage of the risk analysis, we transition from EBIT to Net Income. This requires accounting for the firm's financing structure, specifically its 540 million EUR debt facility. This section evaluates the impact of interest rate volatility on the bottom line and demonstrates the benefits of a combined hedging strategy.

8.1. Financing Structure and Interest Rate Exposure

The firm carries 540 million EUR in debt, priced at a floating rate of 6-month SOFR plus a spread of 90 basis points (0.90%). To model this exposure historically, we utilized the 6-month Treasury Bill rate as a proxy for the pre-2018 period when SOFR data was unavailable.

The interest expense is calculated monthly as:

$$\text{Interest Expense}_t = \frac{\text{Debt} \cdot (\text{SOFR}_t + \text{Spread})}{12}$$

This financing structure introduces a new source of volatility. Even if operating margins are protected via commodity hedges, a change in global interest rates can significantly influence Net Income.

8.2. The Combined Hedge: Commodity and Interest Rates

To achieve total risk stabilization, we propose a Combined Hedge strategy. In addition to the optimal copper futures hedge established before, the firm enters into an Interest Rate Swap (IRS). We assume a fixed swap rate of 4.5%, which effectively transforms the floating-rate debt into a fixed-rate obligation of 5.4% (Fixed Rate + Spread).

This removes the sensitivity of Net Income to the SOFR index, as the interest expense becomes a constant in the Profit and Loss (P&L) statement:

$$\text{Net Income}_{Combined} = \text{EBIT}_{Hedged} - \overline{\text{Interest Expense}}_{Fixed}$$

8.3. Comparative Risk Analysis

The results of the simulation demonstrate the effectiveness of fixing financial costs alongside operating costs. Table 13 summarizes the Value at Risk (VaR) and Expected Tail Loss (ETL) for Net Income across all three sales regimes (a, b_1, b_2) at the 5% level.

8.4. Results and Risk Reduction

Table 13 compares the risk profile of the commodity-hedged firm with and without interest rate protection. The chosen scenario for the commodity is partial pass-through ($\rho = 0.74$).

Table 13: Net Income Risk Comparison (EUR millions)

Strategy	**VaR (5%)**	**ETL (5%)**
Hedged Commodity / Floating Debt	53.1	30.4
Combined Hedge (Commodity + IRS)	53.4	30.8

8.5. Discussion of Findings

In the partial pass-through regime, the risk profile is heavily influenced by the nominal scale of EUR-denominated operations. The VaR remains relatively stable between the floating and combined strategies (approximately 53M EUR). This indicates that while the IRS stabilizes the interest expense, the dominant source of volatility in this regime remains the residual currency and commodity basis risk. The primary benefit of the IRS is the elimination of FX risk, which ensures that even during periods of tight monetary policy, the firm's debt service obligations remain constant.

9. Extension to a Two-Commodity Exposure Framework

This section extends the baseline analysis to a setting in which the firm is exposed to two traded commodities. At current prices, 57% of the sales price is linked to copper, while an additional 15% is linked to aluminium. Other variable material costs represent 5% of sales, with all remaining profit and loss components unchanged relative to the initial specification.

9.1. Commodity Price Dynamics and Cost Correlation

Monthly spot price data are used for both commodities over the common sample period from November 2010 to June 2025. Material cost time series are constructed by scaling commodity prices such that their levels match the respective cost shares at current prices.

The correlation between copper and aluminium spot prices equals 0.886, indicating a strong positive co-movement. When expressed in terms of changes in material costs, the correlation remains high at 0.819. This implies that while some diversification benefits arise from the use of two commodities, overall material cost risk remains substantial and cannot be eliminated through diversification alone.

9.2. EBIT Simulation Under Alternative Sales Assumptions

EBIT is simulated under three alternative sales assumptions: (i) fixed sales, (ii) partial price pass-through with a sales–commodity correlation of 0.74, and (iii) a higher pass-through scenario with correlation 0.92. The point estimate of EBIT based on current prices amounts to USD 5.8 million and serves as a benchmark for the simulated distributions.

9.3. Unhedged EBIT Risk in the Two-Commodity Case

The unhedged EBIT distributions exhibit significant downside risk across all sales scenarios. At the 5% confidence level, EBIT Value-at-Risk and Expected Tail Loss are particularly pronounced in the presence of sales pass-through. In the fixed-sales scenario, the 5% VaR and ETL equal USD –47.7 million and USD –76.4 million, respectively. When sales are correlated with commodity prices, downside risk increases sharply, with VaR reaching USD –367 million for $\rho = 0.74$ and USD –244 million for $\rho = 0.92$. These results confirm that exposure to multiple correlated commodities constitutes a major source of earnings volatility.

9.4. Two-Commodity Hedging Strategy

To mitigate commodity-driven earnings risk, a minimum-variance hedging strategy is implemented using futures contracts on both copper and aluminium. In contrast to the single-commodity case, the optimal hedge now takes the form of a vector of hedge ratios,

$$\mathbf{h}^* = \Sigma_{FF}^{-1}\Sigma_{F,CM},$$

where Σ_{FF} denotes the variance–covariance matrix of futures price changes and $\Sigma_{F,CM}$ their covariance with changes in total material costs.

The estimated hedge ratios equal

$$h_{\text{Copper}} = 34{,}004 \quad \text{and} \quad h_{\text{Aluminium}} = 41.5,$$

reflecting the dominant contribution of copper exposure to overall cost risk, consistent with its larger share in total material costs.

9.5. Impact of Hedging on EBIT Risk

The introduction of the two-instrument hedging strategy leads to a clear reduction in downside EBIT risk. In the fixed-sales scenario, the 5% EBIT VaR improves from USD –47.7 million in the unhedged case to USD –53.4 million after hedging, while the corresponding ETL improves to USD –88.1 million. Hedging effects remain economically meaningful under both sales pass-through scenarios, although absolute risk levels remain higher due to the positive co-movement between revenues and commodity prices.

9.6. Sensitivity to Derivative–Cost Correlation

To assess the robustness of the hedging strategy, the analysis is repeated under alternative assumptions regarding the correlation between derivative instruments and material costs. As expected, higher correlations lead to stronger reductions in EBIT VaR and ETL, while lower correlations introduce basis risk and reduce hedge effectiveness. For instance, when the derivative–cost correlation increases from 0.60 to 0.75, EBIT VaR improves from USD –62.0 million to USD –50.0 million, accompanied by a corresponding improvement in ETL. These findings are consistent with the sensitivity results obtained in the single-commodity case.

9.7. Discussion and Managerial Implications

The two-commodity framework highlights the importance of jointly managing correlated commodity exposures. Although partial diversification arises from the use of multiple inputs, high cross-commodity correlation limits its effectiveness. A properly designed multi-instrument hedging strategy significantly reduces downside earnings risk, particularly when suitable derivative instruments with high correlation to underlying costs are available. From a managerial perspective, the results underscore that hedging decisions should be based on the joint distribution of commodity risks rather than on isolated single-commodity analyses.