Risk Management Assignment – Parts 5 & 6: Hedging Strategy and Results

1 Scope and objective

This report presents the results for Parts 5 and 6 of the assignment, where the firm is exposed to copper price fluctuations through its material costs. The goal is to design a hedge and evaluate how hedging affects the distribution of EBIT under three operating cases: (a) fixed sales, and (b) sales correlated with the commodity with correlation parameters $\rho = 0.74$ and $\rho = 0.92$. Downside risk is measured by the 5% Value-at-Risk ($\mathrm{VaR}_{0.05}$) and the Expected Tail Loss ($\mathrm{ETL}_{0.05}$). All risk numbers below are reported in USD millions, so a larger VaR/ETL corresponds to less severe downside outcomes for profitability.

2 Methodology

Hedges are constructed using the minimum-variance hedge ratio applied to first differences. Let ΔCM_t denote the change in the firm's total material costs (USD), and let ΔX_t denote the change in the hedge instrument price expressed in USD per ton. The hedge quantity (in tons) is estimated as

$$h^* = \frac{\mathrm{Cov}(\Delta CM, \Delta X)}{\mathrm{Var}(\Delta X)}.$$

Hedge P&L is then computed as $\mathrm{PnL}_t = h^* \Delta X_t$ and added to each EBIT series. Economically, the firm is harmed by rising copper prices through higher input costs, so taking a *long* position in the hedge instrument provides positive hedge P&L when copper prices rise.

3 Part 5: Hedging with copper futures and with a synthetic derivative

3.1 Unit conversion and contract sizing

Copper futures (HG=F) are quoted in USD per pound. Prices are converted to USD per metric ton using

$$1 \text{ metric ton} = 2204.6226 \text{ lb}, \qquad F_t^{\text{ton}} = F_t^{\text{lb}} \cdot 2204.6226.$$

A standard contract covers 25,000 lb, corresponding to approximately 11.3398 metric tons.

3.2 Part 5a: Hedge using actual copper futures

The estimated hedge quantity using copper futures is

$$h^*_{\text{fut}} = 32{,}193.71 \text{ tons}, \qquad N \approx \frac{32{,}193.71}{11.3398} \approx 2{,}839 \text{ contracts.}$$

Table 1 compares $\text{VaR}_{0.05}$ and $\text{ETL}_{0.05}$ for unhedged and futures-hedged EBIT.

Table 1: Part 5A: EBIT risk (USD millions), unhedged vs. hedged with copper futures.

Case	Strategy	α	VaR_α	ETL_α
a) fixed sales	unhedged	0.05	28.6	12.6
b) $\rho = 0.74$	unhedged	0.05	-38.2	-53.9
b) $\rho = 0.92$	unhedged	0.05	28.3	20.3
a) fixed sales	hedged (copper futures)	0.05	24.1	12.0
b) $\rho = 0.74$	hedged (copper futures)	0.05	-36.8	-61.7
b) $\rho = 0.92$	hedged (copper futures)	0.05	21.3	10.4

The futures hedge does not lead to uniform risk reduction in EBIT. In case (a) fixed sales, both VaR and ETL decrease after hedging, indicating worse left-tail profitability. In case (b) with $\rho = 0.74$, VaR improves slightly (from -38.2 to -36.8), but ETL becomes more negative (from -53.9 to -61.7), meaning the most adverse outcomes in the far tail deteriorate. In case (b) with $\rho = 0.92$, both VaR and ETL fall strongly, which is consistent with an operational natural hedge: when revenues co-move strongly with copper, the unhedged EBIT already benefits from offsetting movements, and a financial hedge can remove part of this beneficial co-movement.

3.3 Part 5b: Hedge using a synthetic derivative with corr ≈ 0.75 to costs

A synthetic derivative instrument is constructed so that its changes have sample correlation 0.75 with ΔCM. The estimated hedge quantity is

$$h^*_{\text{deriv}} = 37{,}971.23 \text{ tons}, \qquad N \approx \frac{37{,}971.23}{11.3398} \approx 3{,}348 \text{ contracts (futures-equivalent).}$$

Table 2 reports the corresponding VaR/ETL results.

Table 2: Part 5B: EBIT risk (USD millions), unhedged vs. hedged with synthetic derivative (corr ≈ 0.75).

Case	Strategy	α	VaR_α	ETL_α
a) fixed sales	unhedged	0.05	28.6	12.6
b) $\rho = 0.74$	unhedged	0.05	-38.2	-53.9
b) $\rho = 0.92$	unhedged	0.05	28.3	20.3
a) fixed sales	hedged (deriv corr$\sim$0.75)	0.05	27.4	13.3
b) $\rho = 0.74$	hedged (deriv corr$\sim$0.75)	0.05	-34.5	-59.1
b) $\rho = 0.92$	hedged (deriv corr$\sim$0.75)	0.05	17.1	6.80

Compared to the futures hedge, the synthetic derivative hedge improves VaR more noticeably in the $\rho = 0.74$ operating case (from -38.2 to -34.5), but ETL remains worse than unhedged. This indicates that correlation below one can reduce moderately adverse outcomes around the 5% quantile while still leaving very severe tail outcomes largely unchanged or slightly worse. In the high revenue–commodity correlation case ($\rho = 0.92$), hedging again lowers both VaR and ETL, reinforcing the interpretation that the firm already has a strong natural hedge through revenues.

3.4 Implementation note: how the synthetic derivative was constructed

To create a derivative series with a *controlled* correlation to material-cost changes, the code starts from standardized cost changes and adds a noise term that is explicitly made orthogonal to those standardized changes. In practice, this is done by projecting random noise onto the cost-change direction and subtracting that projection, so the remaining noise has (approximately) zero sample correlation with the cost changes. The final derivative change is then formed as a weighted mixture of the standardized cost changes and the orthogonal noise, which guarantees the desired sample correlation level (e.g. 0.60, 0.75, 0.90) while keeping the variability comparable by scaling to a chosen target standard deviation.

4 Part 6: Correlation sensitivity analysis (synthetic derivative)

4.1 Construction and hedge effectiveness metric

Part 6 repeats the derivative hedge for correlations $\rho_{\text{deriv}} \in \{0.60, 0.75, 0.90\}$. Derivative changes are constructed so that their *sample* correlation with ΔCM equals the target correlation, and the volatility is scaled to be comparable across correlation levels.

To evaluate hedge quality on the cost exposure alone, hedge effectiveness is computed as

$$\text{HE} = 1 - \frac{\text{Var}(\Delta CM - h^* \Delta D)}{\text{Var}(\Delta CM)}.$$

Table 3 reports the resulting hedge ratios and hedge effectiveness.

Table 3: Part 6: Hedge effectiveness on material-cost changes.

ρ_{deriv}	Sample corr	h^* (tons)	Hedge effectiveness
0.60	0.60	30,377	0.360
0.75	0.75	37,971	0.562
0.90	0.90	45,565	0.810

As expected, hedge effectiveness increases strongly with correlation: moving from correlation 0.60 to 0.90 increases the fraction of cost-change variance removed by the hedge from 36% to 81%.

4.2 EBIT VaR/ETL across correlations

Table 4 reports EBIT VaR/ETL for each correlation level.

Table 4: Part 6: EBIT risk (USD millions) across derivative correlations.

Case	Strategy	ρ_{deriv}	$\text{VaR}_{0.05}$	$\text{ETL}_{0.05}$
a) fixed sales	unhedged	–	28.6	12.6
b) $\rho = 0.74$	unhedged	–	-38.2	-53.9
b) $\rho = 0.92$	unhedged	–	28.3	20.3
a) fixed sales	hedged (deriv corr∼0.6)	0.60	25.9	16.1
b) $\rho = 0.74$	hedged (deriv corr∼0.6)	0.60	-36.1	-57.5
b) $\rho = 0.92$	hedged (deriv corr∼0.6)	0.60	24.1	13.6
a) fixed sales	hedged (deriv corr∼0.75)	0.75	27.4	13.3
b) $\rho = 0.74$	hedged (deriv corr∼0.75)	0.75	-34.5	-59.1
b) $\rho = 0.92$	hedged (deriv corr∼0.75)	0.75	17.1	6.80
a) fixed sales	hedged (deriv corr∼0.9)	0.90	23.0	10.1
b) $\rho = 0.74$	hedged (deriv corr∼0.9)	0.90	-38.3	-67.7
b) $\rho = 0.92$	hedged (deriv corr∼0.9)	0.90	12.6	-6.08

The correlation sensitivity results show a clear distinction between hedging the cost exposure and improving EBIT tail risk. While hedge effectiveness on cost changes increases monotonically with correlation, the EBIT VaR/ETL outcomes do not improve monotonically. In particular, in the operating scenario with strong revenue–commodity correlation ($\rho = 0.92$), aggressive cost hedging can significantly reduce VaR and even produce a negative ETL at $\rho_{\text{deriv}} = 0.90$. This supports the interpretation that the firm has a natural hedge through revenues, and that a strong financial hedge can remove beneficial co-movement between revenues and costs.

5 Conclusion

The minimum-variance approach implies hedge sizes on the order of 3×10^4 to 4.6×10^4 tons, corresponding to thousands of standard futures contracts. Increasing correlation between the hedge instrument and cost changes strongly improves hedge effectiveness on the cost exposure. However, EBIT-based VaR/ETL results depend on the firm's operating structure. When revenues co-move strongly with copper prices, the firm benefits from an operational natural hedge, and additional financial hedging can worsen downside profitability outcomes. Therefore, a hedging policy aimed at reducing EBIT tail risk should be designed using the net exposure of EBIT (revenue and cost jointly), not only by minimizing variance of input costs.