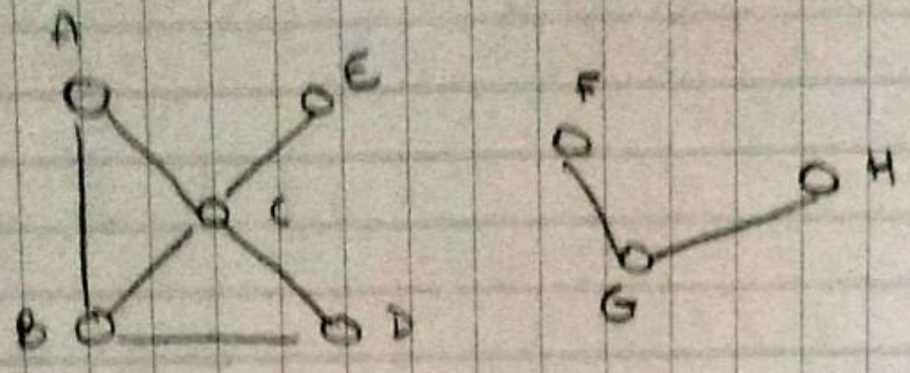
Homework session 1

1) Small network $N = [5, 10]$ with a giant component of at least 5 nodes.



a) Average path length:

$d(A,B) = 1$	$d(B,C) = 1$	$d(C,D) = 1$	$d(D,E) = 2$
$d(A,C) = 1$	$d(B,D) = 1$	$d(C,E) = 1$	$d(D,F) = \infty$
$d(A,D) = 2$	$d(B,E) = 2$	$d(C,F) = \infty$	$d(D,G) = \infty$
$d(A,E) = 2$	$d(B,F) = \infty$	$d(C,G) = \infty$	$d(D,H) = \infty$
$d(A,F) = \infty$	$d(B,G) = \infty$	$d(C,H) = \infty$	
$d(A,G) = \infty$	$d(B,H) = \infty$		
$d(A,H) = \infty$			
$d(E,F) = \infty$	$d(F,G) = 1$		
$d(E,G) = \infty$	$d(F,H) = 2$		
$d(E,H) = \infty$			

Frost

$$\text{average path length} = \ell = \frac{1}{N(N-1)} \sum_{i \neq j} d(i,j) = \infty$$

if we calculate the average path length for the whole network, it tends to infinity since there are disconnected components.

$$\text{average path length of the giant comp.} = \frac{1}{5(5-1)} \sum_{i \neq j} d(i,j) = \frac{1}{20} \cdot 28 = \underline{\underline{1'4}}$$

- diameter = 2
- effective diameter = $2 \cdot 0'9 = 1'8$

b) Average & individual clustering coefficient

i = node

E_i = # links between neighbors of i

k_i = degree of i

$$C_i = \frac{2E_i}{k_i(k_i-1)}$$

$$C_A = \frac{2\cdot 1}{2(2-1)} = \frac{2}{4} = \frac{1}{2} \qquad C_B = \frac{2\cdot 2}{3\cdot 2} = \frac{2}{3}$$

$$C_C = \frac{2\cdot 2}{4\cdot 3} = \frac{1}{3} \qquad C_D = \frac{2\cdot 1}{2} = 1$$

$$C_E = \frac{0}{1\cdot 0} = 0 \qquad C_F = 0$$

$$C_G = \frac{2\cdot 0}{2\cdot 1} = 0 \qquad C_H = 0$$

$$\tilde{C} = \frac{1}{n}\sum_{i=1}^{n} C_i = \frac{1}{8}\left(\frac{1}{2}+\frac{1}{3}+\frac{2}{3}+1\right) = \frac{5}{16}$$

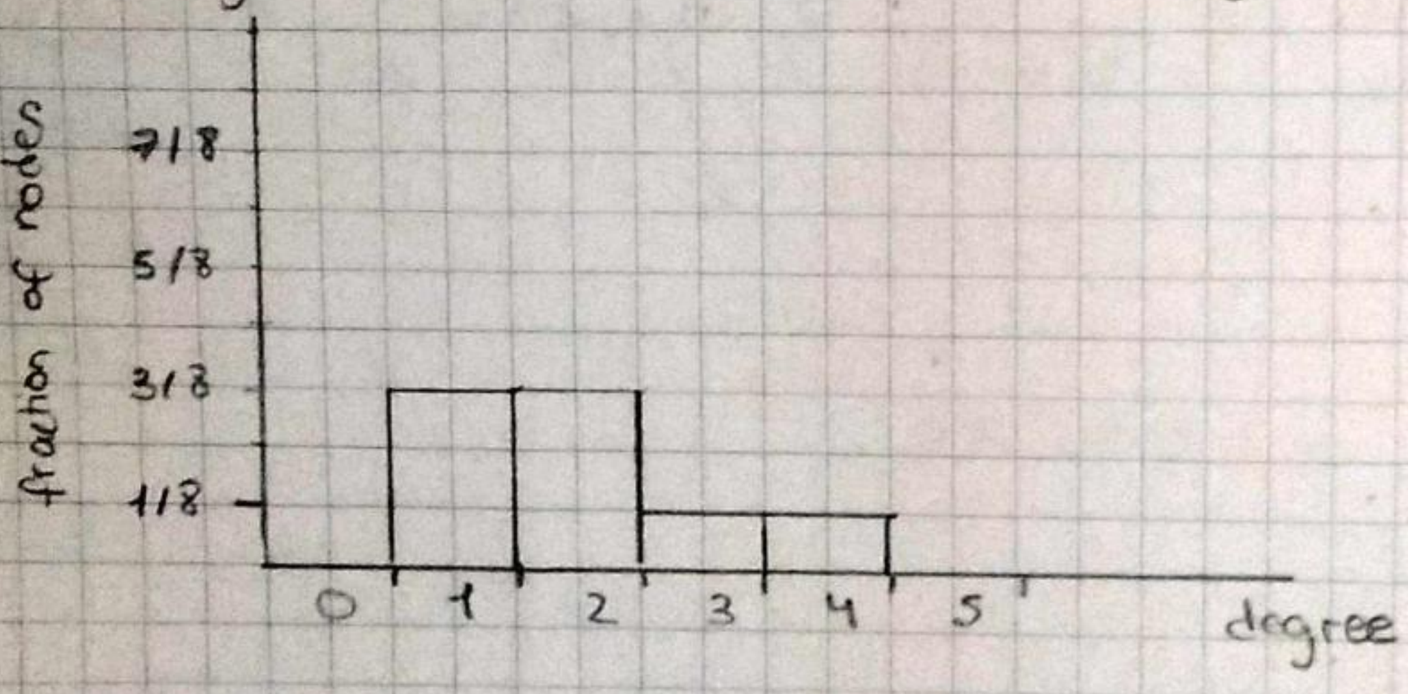
c) Histogram of the degrees.

$k_A = 2$ $\quad k_C = 4$ $\quad k_E = 1$ $\quad k_G = 2$

$k_B = 3$ $\quad k_D = 2$ $\quad k_F = 1$ $\quad k_H = 1$

$P_{deg}(1) = 3/8 = 0'375$ $\quad P_{deg}(3) = 1/8 = 0'125$

$P_{deg}(2) = 3/8 = 0'375$ $\quad P_{deg}(4) = 1/8 = 0'125$



d) Betweenness centrality $C_B(i) = \sum \frac{\sigma_{st}(i)}{\sigma st}$

$C_B(A) = 0$

$C_B(B) = \frac{1}{2}$

$C_B(E) = 0$

$C_B(G) = 1$

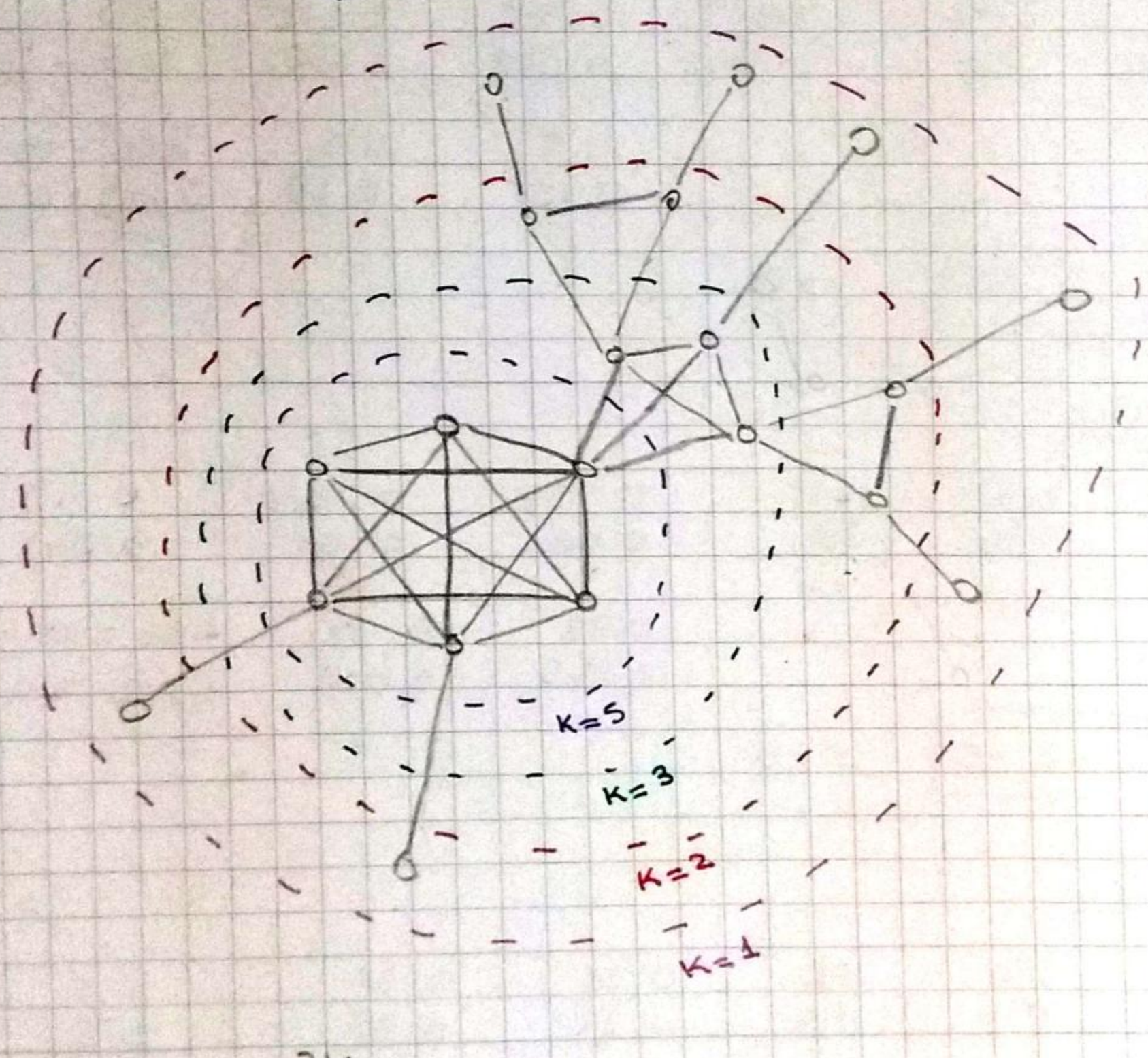
$C_B(C) = \frac{1}{2} + 1 + 1 + 1 = 3'5$

$C_B(D) = 0$

$C_B(F) = 0$

$C_B(H) = 0$

2) Small dense network $N = [15, 20)$ & $K > 30$ & report its K-shell decomposition



$N = 20$, $K = 34$

3) Mixing matrix (males / females) ⇒ Assortative, dissortative and neutral mixing.

• Original ⇒

	Males	Females
Males	2272	296
Females	296	53

$$r = \frac{\sum_i e_{ii} - \sum_i a_i b_i}{1 - \sum_i a_i b_i}$$

$r = 0.037$

• Assortative :

	Male	Female	Male	Female	a_i
Male	2272	296	0.442	0.058	0.5
Female	296	2272	0.058	0.442	0.5
b_i			0.5	0.5	$\sum e_{ii} = 0.884$
b_i^2			0.25	0.25	$\sum b_i^2 = 0.5$

NEW

⇒ $\boxed{r = 0.768}$ $r > 0$

• Dissortative :

Male	2272	2272	0.464	0.464	0.93
Female	296	53	0.06	0.01	0.07
b_i			0.93	0.07	$\sum e_{ii} = 0.564$
b_i^2			0.865	0.005	$\sum b_i^2 = 0.87$

⇒ $\boxed{r = -2.35}$ $r < 0$

• Neutral :

Male	296	296	0.25	0.25	0.5
Female	296	296	0.25	0.25	0.5
b_i			0.5	0.5	$\sum e_{ii} = 0.5$
b_i^2			0.25	0.25	$\sum b_i^2 = 0.5$

⇒ $\boxed{r = 0}$