NETWORK FORMATION MODELS II

⇒ Show that two star components have lower utility than a single one with same number of nodes.

1 STAR COMPONENT'S UTILITY:

$$2(K+K'-1)(\delta-C) + \delta^2 (K+K'-1)(K+K'-2) \equiv$$

$$2(K+K'-1)(\delta-C) + \delta^2 (K^2+KK'-2K+K'K+K'^2-2K'-K-K'+2) \equiv$$

$$2(K+K'-1)(\delta-C) + \delta^2 (K^2+K'^2+2KK'-3K-3K'+2)$$

2 STAR COMPONENT'S UTILITY

$$2\left[(K-1)+(K'-1)\right](\delta-C) + \delta^2\left[(K-1)(K-2)+(K'-1)(K'-2)\right]$$

$$2(K+K'-2)(\delta-C) + \delta^2\left[(K^2-3K+2)+(K'^2-3K'+2)\right]$$

$$\text{''} \quad + \delta^2\left[K^2+K'^2-3K-3K'+4\right]$$

- We want to prove that two star components have lower utility than a single one ⟹ 1STAR - 2STAR > 0

$$\begin{array}{rl} & 2(K+K'-1)(\delta-C) + \delta^2(K^2+K'^2+2KK'-3K-3K'+2) \\ - & 2(K+K'-2)(\delta-C) + \delta^2(K^2+K'^2-3K-3K'+4) \\ \hline & \underbrace{2(\delta-C)} + \delta^2(2KK'-2) > 0 \end{array}$$

The efficiency of a STAR is proven when $C > \delta - \delta^2$, therefore the worst case scenario would be $c = \delta - \delta^2$ ⟹ we are going to substitute c's value ⟹ $c = \delta - \delta^2$

$$2(\delta-\delta+\delta^2) + \delta^2(2KK'-2) > 0 \equiv$$

$$2\delta^2 + \delta^2(2KK'-2) > 0 \equiv 2\delta^2 + 2\delta^2(KK'-1) > 0 \equiv$$

$$1 + KK' - 1 > 0 \equiv KK' > 0 \Leftarrow \underline{\text{PROVEN!}}$$