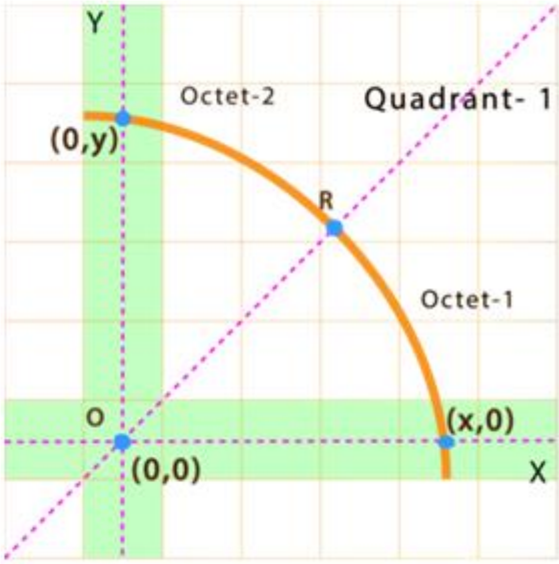
Bresenham's circle drawing algorithm.

Bresenham's algorithm is also used for circle drawing. It is known as *Bresenham's circle drawing algorithm*. It helps us to draw a circle. In this algorithm, we will select the closest pixel position to complete the arc.



Let's say if current pixel coordinates are (x_k, y_k). Then, we need to find out our next pixel (x_{k+1}, y_{k+1}).

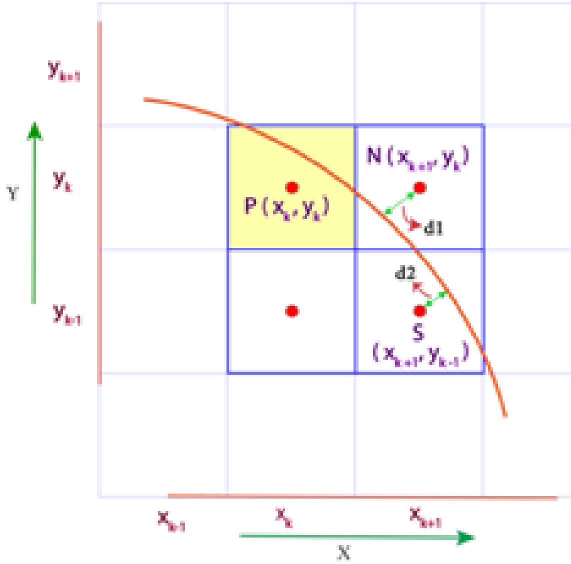
Further, choices are the pixels at positions (x_k+1, y_k) and (x_k+1, y_k-1) .

Equation of Circle with Radius r

$$(x-h)^2 + (y-k)^2 = r^2$$

If origin is the center of circle put (h=0, k=0)

$x^2 + y^2 = r^2$ **(Pythagoras theorem)**



Here it needs to be decided whether to select N or S.

As $x^2 + y^2 = r^2$ (Pythagoras theorem)

For pixel N this distance is

$D(N) = (x_k+1)^2 + (y_k)^2$

For pixel S this distance is

$D(S) = (x_k+1)^2 + (y_k-1)^2$

However, The distance between actual circle and the centre is r.

Actually squaring is done just to avoid square root calculations.

$E(N) = \{D(N)\}^2 - r^2$

Also, $E(S) = \{D(S)\}^2 - r^2$

The value of E(N) will always be positive because N is out-side the circle and the value of E(S) will always be negative because S is inside the circle.

Now, lets calculate decision parameter **D_k in a simpler way,**

$D_k = E(N)+E(S)$

CASE 1 :

If $D_k < 0$, it implies E(S)>E(N), since E(S) is negative, that again implies that point N is closer to the circle than point S.
So pixel N is our next pixel. (x_k+1, y_k)

CASE 2 :

If $D_k > 0$ means E(N)>E(S) and since E(N) is positive, that again implies that point S is closer to the circle than point N.
So pixel S is our next pixel. (x_k+1, y_k-1).

Calculating Decision parameter

Now lets derive D_k

$D_k = (x_k+1)^2 + (y_k)^2 – r^2 + (x_k+1)^2 + (y_k-1)^2 – r^2$ which can also be written as,

$$= 2(x_k + 1)^2 + (y_k)^2 + (y_k -1)^2 – 2r^2 \qquad \text{—– } (i)$$

Now lets find D_{k+1} also,
So, Replacing every k with k+1

$$D_{k+1} = 2(x_{k+1} + 1)^2 +(y_{k+1})^2 + (y_{k+1} -1)^2 – 2r^2$$

(Replacing x_{k+1} with $x_k + 1$ but now we can’t replace y_{k+1} because we don’t know the exact value of y_k)

$= 2(x_k+1+ 1)^2 + (y_{k+1})^2 + (y_{k+1} -1)^2 – 2r^2$ which can also be written as

$$= 2(x_k+2)^2 + (y_{k+1})^2 + (y_{k+1} -1)^2 – 2r^2 \qquad \text{—– } (ii)$$

Calculating Decision parameter for next pixel

Now to find out the decision parameter of next pixel i.e. D_{k+1}
We need to find

$$\mathbf{D_{k+1} – D_k = (ii) – (i)}$$

$= 2(x_k+2)^2 + (y_{k+1})^2 + (y_{k+1} -1)^2 – 2r^2$

$– [\ 2(x_k + 1)^2 + (y_k)^2 + (y_k -1)^2 – 2r^2]$ Simplify further,

$= \ 2(x_k)^2 + 8x_k + 8 + (y_{k+1})^2 + (y_{k+1})^2 – 2y_{k+1} + 1 – 2r^2$

$– 2(x_k)^2 – 4x_k – 2 – (y_k)^2 – (y_k)^2 + 2y_k – 1 + 2r^2$ Then we get,

$= 4x_k + 2(y_{k+1})^2 – 2y_{k+1} – 2(y_k)^2 + 2y_k + 6$

$\mathbf{D_{k+1} = D_k + 4x_k + 2(y_{k+1})^2 – 2y_{k+1} – 2(y_k)^2 + 2y_k + 6}$ **—– *(iii)***

If ($D_k < 0$) then we will choose N point as discussed i.e. (x_k+1, y_k)

Then, $y_{k+1} = y_k$, putting y_k in *(iii)* in place of y_{k+1}

So, we get $D_{k+1} = D_k + 4x_k + 2(y_k)^2 – 2y_k – 2(y_k)^2 + 2y_k + 6$

$\mathbf{= D_k + 4x_k + 6}$

If ($D_k > 0$) then we will choose S point i.e. (x_k+1, y_k-1)

$y_{k+1} = y_k-1$putting y_k-1in *(iii)* in place of y_{k+1}

Hence, $D_{k+1} = D_k + 4x_k + 2(y_k -1)^2 – 2(y_k -1)- 2(y_k)^2 + 2y_k + 6$

$= D_k + 4x_k + 2(y_k)^2 + 2 – 4y_k – 2y_k +2- 2(y_k)^2 + 2y_k + 6$ So now we get,

$= D_k + 4x_k – 4y_k + 10$

$\mathbf{=D_k + 4(x_k – y_k)+ 10}$

Initial decision Parameter

Take starting point (0,r) ,value of y is r

Putting (0,r) in eq (*i*)

$D_k = 2(x_k + 1)^2 + (y_k)^2 + (y_k -1)^2 – 2r^2$

So, $D_0 = 2(0+ 1)^2 + r^2 + (r-1)^2 – 2r^2$

$= 2 + r^2 + r^2 + 1 – 2r – 2r^2$ Simplifying further we get,

$\mathbf{= 3-2r}$

Question

Given the centre point coordinates (0, 0) and radius as 8, generate all the points to form a circle.

Solution-

Given-

- Centre Coordinates of Circle (X_0, Y_0) = (0, 0)
- Radius of Circle = 8

Step-01:

Assign the starting point coordinates (X_0, Y_0) as-

- $X_0 = 0$
- $Y_0 = R = 8$

Step-02:

Calculate the value of initial decision parameter P_0 as-

$P_0 = 3 – 2 \times R$

$P_0 = 3 – 2 \times 8$

$P_0 = -13$

Step-03:

As $P_{initial} < 0$, so case-01 is satisfied.

Thus,

- $X_{k+1} = X_k + 1 = 0 + 1 = 1$
- $Y_{k+1} = Y_k = 8$
- $P_{k+1} = P_k + 4 \times X_{k+1} + 6 = -13 + (4 \times 1) + 6 = -3$

Step-04:

This step is not applicable here as the given centre point coordinates is (0, 0).

Step-05:

Step-03 is executed similarly until $X_{k+1} >= Y_{k+1}$ as follows-

P_k	P_{k+1}	(X_{k+1}, Y_{k+1})
		(0, 8)
-13	-3	(1, 8)
-3	11	(2, 8)
11	5	(3, 7)
5	7	(4, 6)
7		(5, 5)