EP2200 Course Project
Project I - Error Control in Relay Networks

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1 AF RELAYING WITH END-TO-END ARQ

The queuing model for the RS and MS nodes - M/M/1.
1) The end-to-end error probability of packet transmission ($p_{e,e2e}$):

$$p_{e,e2e} = 1 - \prod_{k=1}^{r+1}(1 - p_{(e,k)}) \tag{1}$$

The packet arrival rate at the first relay (λ_1) :

$$\lambda_1 = \lambda_{r+1}.p_{e,e2e} + \lambda$$

$$\lambda_2 = \lambda_1$$

$$\lambda_3 = \lambda_3$$

$$...$$

$$\lambda_{r+1} = \lambda_r$$

We can write,

$$\lambda r + 1 = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{(e,k)})}$$

Therefore,

$$\lambda_1 = \lambda_{r+1}.p_{e,e2e} + \lambda = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{(e,k)})} \tag{2}$$

2) For RS:

$$\rho_i^{AF} = \frac{\lambda_i^{AF}}{\mu_i^{AF}} = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{(e,k)}).\mu_{AF}}$$

For M/M/1 systems, the waiting time is $W = \frac{1}{\mu - \lambda} - \frac{1}{\mu}$. The average queuing delay is:

$$W_{RS} = \frac{\lambda}{\mu_{AF}^2 \prod_{k=1}^{r+1}(1 - p_{(e,k)}) - \lambda.\mu_{AF}} \tag{3}$$

For M/M/1 systems, the average number in the system is $N = \frac{\rho}{1-\rho}$. The average number of packets (waiting or under processing) is:

$$N_{RS} = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{(e,k)}).\mu_{AF} - \lambda} \tag{4}$$

For MS:

$$\rho_{MS} = \frac{\lambda_{r+1}}{\mu_{MS}} = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{(e,k)}).\mu_{MS}}$$

Average queuing delay is:

$$W_{MS} = \frac{\lambda}{\mu_{MS}^2 \prod_{k=1}^{r+1}(1 - p_{(e,k)}) - \lambda.\mu_{MS}} \tag{5}$$

Average number of packets (waiting or under processing) is:

$$N_{MS} = \frac{\lambda}{\prod_{k=1}^{r+1}(1 - p_{(e,k)}).\mu_{MS} - \lambda} \tag{6}$$

3) The average amount of time between the production of a packet and its successful reception at the MS is equal to the total of all the time spent in the RS and the MS.

$$\bar{T} = \Sigma_{i=1}^{r}(W_{RS} + \frac{1}{\mu_i^{AF}}) + W_{MS} + \frac{1}{\mu_{MS}}$$

The time spent in each RS is also the same because a packet is only decoded and examined on the final hop, when the MS gets it. The average end-to-end delay is:

$$\bar{T} = \frac{r.\prod_{k=1}^{r+1}(1 - p_{(e,k)})}{\mu_{AF}.\prod_{k=1}^{r+1}(1 - p_{(e,k)}) - \lambda} + \frac{\prod_{k=1}^{r+1}(1 - p_{(e,k)})}{\mu_{MS}.\prod_{k=1}^{r+1}(1 - p_{(e,k)}) - \lambda} \tag{7}$$

2 DF RELAYING WITH END-TO-END OR HOP-BY-HOP ARQ

A. DF Relaying With End-To-End ARQ
1) The packet arrival rate at each RS and MS (λ_j):

$$\lambda_1 = \lambda + \lambda_1.p_{e,1} + \lambda_2.p_{e,2} + ... + \lambda_{r+1}.p_{e,r+1}$$

$$= \lambda + \lambda.\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{k}^{j=1}(1 - p_{(e,k)})]$$

$$= \frac{\lambda}{1 - \Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{k}^{i=1}(1 - p_{(e,i)})]}$$

Therefore,

$$\lambda_i = \lambda_1 \prod_{i-1}^{k=0}(1-p_{(e,k)})$$

$$= \frac{\prod_{i-1}^{k=1}(1-p_{(e,k)})}{1-\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{k}^{j=1}(1-p_{(e,j)})]} \quad (8)$$

2) For RS:
Average queuing delay $W_{RS}^i = \frac{1}{\mu_{DF}-\lambda_i} - \frac{1}{\mu_{DF}}$:

$$\lambda.\prod_{i-1}^{k=1}(1-p_{(e,k)})\div$$

$$\mu_{DF}^2[1-\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{k}^{j=1}(1-p_{(e,j)})]-\lambda\mu_{DF}\prod_{i-1}^{k=1}(1-p_{(e,k)}) \quad (9)$$

As the utilization here is:

$$\rho_{RS}^i = \frac{\lambda_i}{\mu_{DF}}$$

$$= \frac{\lambda.\prod_{k=1}^{i-1}(1-p_{(e,k)})}{\mu_{DF}-\mu_{DF}\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{j=1}^{k}(1-p_{(e,j)})]}$$

Hence, average number of packets at every RS is $N_{RS}^i = \frac{\rho_{RS}^i}{1-\rho_{RS}^i}$:

$$\lambda.\prod_{k=1}^{i-1}(1-p_{(e,k)})\div$$

$$\mu_{DF}-\mu_{DF}\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{j=1}^{k}(1-p_{(e,j)})]-\lambda\prod_{k=1}^{i-1}(1-p_{(e,k)}) \quad (10)$$

For MS:
Average queuing delay $W_{MS} = \frac{1}{\mu_{MS}-\lambda_{r+1}} - \frac{1}{\mu_{MS}}$:

$$\lambda.\prod_{k=1}^{r}(1-p_{(e,k)})\div$$

$$\mu_{MS}^2[1-\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{j=1}^{k}(1-p_{(e,j)})]]-\lambda\mu_{MS}\prod_{k=1}^{r}(1-p_{(e,k)}) \quad (11)$$

As the utilization here is:

$$\rho_{MS} = \frac{\lambda_{r+1}}{\mu_{MS}}$$

$$= \frac{\lambda.\prod_{k=1}^{r}(1-p_{(e,k)})}{\mu_{MS}-\mu_{MS}\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{j=1}^{k}(1-p_{(e,j)})]}$$

Hence, average number of packets at every MS is $N_{MS} = \frac{\rho_{MS}}{1-\rho_{MS}}$:

$$\lambda.\prod_{k=1}^{r}(1-p_{(e,k)})\div$$

$$\mu_{MS}-\mu_{MS}\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{j=1}^{k}(1-p_{(e,j)})]-\lambda\prod_{k=1}^{r}(1-p_{(e,k)}) \quad (12)$$

3) The average end-to-end delay, that is, the average time from the generation of a packet until it is received successfully at the MS ($\bar{T}$).
Here, we know that,

$$\bar{T} = \Sigma_{i=1}^{r}(W_{RS}^i + \frac{1}{\mu_i^{DF}}) + W_{MS} + \frac{1}{\mu_{MS}}$$

The average end-to-end delay is $\bar{T}$:

$$\Sigma_{i=1}^{r}\frac{1-\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{j=1}^{k}(1-p_{e,j})]}{\mu_{DF}-\mu_{DF}\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{j=1}^{k}(1-p_{e,j})]-\lambda\prod_{k=1}^{i-1}(1-p_{e,k})}$$
$$+\frac{1-\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{j=1}^{k}(1-p_{e,j})]}{\mu_{MS}-\mu_{MS}\Sigma_{k=0}^{r}[p_{e,k+1}.\prod_{j=1}^{k}(1-p_{e,j})]-\lambda\prod_{k=1}^{r}(1-p_{e,k})} \quad (13)$$

B. DF Relaying With Hop-By-Hop ARQ
1) Packet arrival rate at each RS and MS (λ_j):

$$\lambda_1 = \lambda + \lambda_1.p_{e,1}$$
$$\lambda_2 = \lambda_1.(1-p_{e,1}) + \lambda_2.p_{e,2}$$
$$...$$
$$\lambda_r = \lambda_{r-1}.(1-p_{e,r-1}) + \lambda_r.p_{e,r}$$
$$\lambda_{r+1} = \lambda_r.(1-p_{e,r}) + \lambda_{r+1}.p_{e,r+1}$$

From above, we get equations for λ_i and λ_{r+1}:

$$\lambda_i = \frac{\lambda}{1-p_{e,i}}$$

$$\lambda_{r+1} = \frac{\lambda}{1-p_{e,r+1}} \quad (14)$$

2) For RS: Average queuing delay $W_{RS}^i = \frac{1}{\mu_{RS}-\lambda_i} - \frac{1}{\mu_{RS}}$:

$$W_{RS}^i = \frac{\lambda}{\mu_{DF}.[\mu_{DF}.(1-p_{e,i})-\lambda]} \quad (15)$$

As the utilization here is:

$$\rho_{RS}^i = \frac{\lambda_i}{\mu_{DF}} = \frac{\lambda}{\mu_{DF}.(1-p_{e,i})}$$

Hence, average number of packets at every RS is $N_{RS}^i = \frac{\rho_{RS}^i}{1-\rho_{RS}^i}$:

$$N_{RS}^i = \frac{\lambda}{\mu_{DF}(1-p_{e,i})-\lambda} \quad (16)$$

For MS: Average queuing delay $W_{MS} = \frac{1}{\mu_{MS}-\lambda_i} - \frac{1}{\mu_{MS}}$:

$$W_{MS} = \frac{\lambda}{\mu_{MS}.[\mu_{MS}.(1-p_{e,r+1})-\lambda]} \quad (17)$$

As the utilization here is:

$$\rho_{MS} = \frac{\lambda_{r+1}}{\mu_{MS}} = \frac{\lambda}{\mu_{MS}.(1-p_{e,r+1})}$$

Hence, average number of packets at MS, $N_{MS} = \frac{\rho_{MS}}{1-\rho_{MS}}$:

$$N_{MS} = \frac{\lambda}{\mu_{MS}.(1-p_{e,r+1}-\lambda)} \quad (18)$$

3) Here, we can write that

$$\bar{T} = \Sigma_{i=1}^{r}\frac{1-p_{e,i}}{\mu_{DF}(1-p_{e,i})-\lambda} + \frac{1-p_{e,r+1}}{\mu_{MS}(1-p_{e,r+1})-\lambda}$$

Average end-to-end delay $\bar{T}$:

$$\bar{T} = \Sigma_{i=1}^{r}\frac{1-p_{e,i}}{\mu_{DF}(1-p_{e,i})-\lambda} + \frac{1-p_{e,r+1}}{\mu_{MS}(1-p_{e,r+1})-\lambda} \quad (19)$$

3 STABILITY REGION

A. For AF relaying with end-to-end ARQ

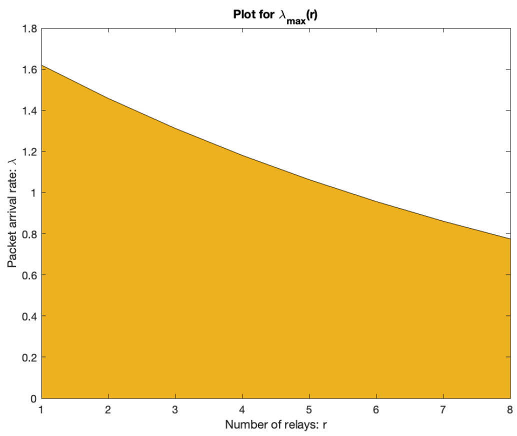

Figure 1: Plot of $\lambda_{max}(r)$ for AF relaying with end-to-end ARQ

In this case, $\lambda \leq 2 * 0.9^{r+1}$, so as r tends to infinity $2 * 0.9^{r+1}$ will tend to 0. As a result, $\lambda_{max}(r)$ will tend to 0 as well.

B. For DF relaying with end-to-end ARQ

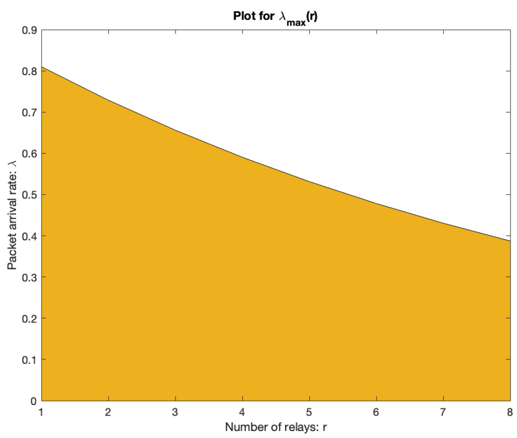

Figure 2: Plot of $\lambda_{max}(r)$ for DF relaying with end-to-end ARQ

In this case, $(\lambda \leq 0.9^{r+1}) \cap (\lambda \leq 1.8)$, so when r tends to infinity $2 * 0.9^{r+1}$ tends to 0. As a result, $\lambda_{max}(r)$ will tend to 0 as well.

C. For DF relaying with hop-by-hop ARQ

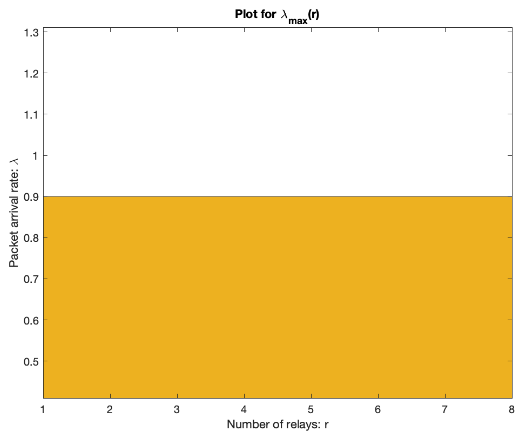

Figure 3: Plot of $\lambda_{max}(r)$ for DF relaying with hop-by-hop ARQ

In this case, $(\lambda \leq 0.9) \cap (\lambda \leq 1.8)$, as r tends to infinity, there is no influence on $\lambda_{max}(r)$

4 ARRIVAL RATE-DELAY CHARACTERISTICS

A. For AF relaying with end-to-end ARQ

$$T = \frac{5 * 0.9^5}{2 * 0.9^5 - \lambda}$$

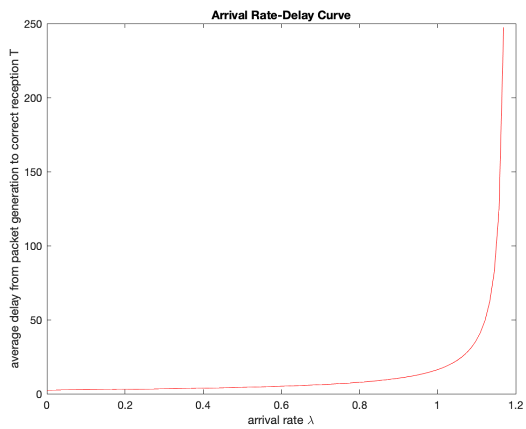

Figure 4: Plot of arrival-rate delay for AF relaying with end-to-end ARQ

From this plot we can observe that, as $p_{e,j}$ increases, the curve shifts to the left.

B. For DF relaying with end-to-end ARQ

$$T = \sum_{i=1}^{4} \frac{0.9^5}{0.9^5 - \lambda * 0.9^{i-1}} + \frac{0.9^5}{2 * 0.9^5 - 0.9^4 * \lambda}$$

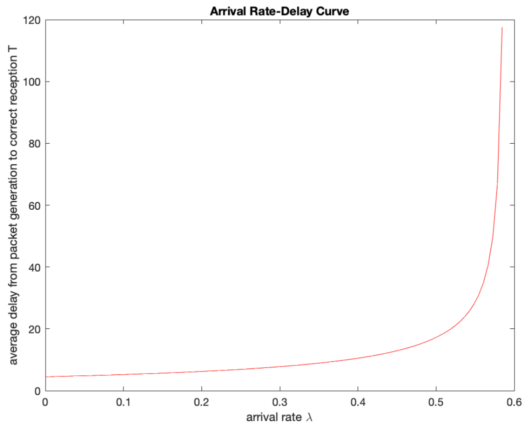

Figure 5: Plot of arrival-rate delay for DF relaying with end-to-end ARQ

From this plot we can observe that, as $p_{e,j}$ increases, the curve shifts to the left.

C. For DF relaying with hop-by-hop ARQ

$$T = \frac{3.6}{0.9 - \lambda} + \frac{0.9}{1.8 - \lambda}$$

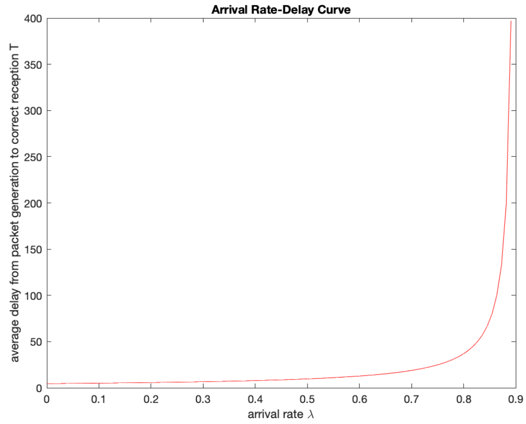

Figure 6: Plot of arrival-rate delay for DF relaying with hop-by-hop ARQ

From this plot we can observe that, as $p_{e,j}$ increases, the curve shifts to the left.

5 AF OR DF ?

A. Part I

We are to consider $p_{e,j} = 1, 2, ..., r+1$ for this part. In this case, we can formulate the average system delays as follows:

For AF relaying with end-to-end ARQ:

$$T_1 = 5$$

For DF relaying with end-to-end ARQ:

$$T_2 = \sum_{i=1}^{4} \frac{(1-p)^5}{(1-p)^5 - 0.5 * (1-p)^{i+4}}$$

$$+ \frac{(1-p)^5}{2 * (1-p)^5 - 0.5 * (1-p)^9}$$

For DF relaying with hop-by-hop ARQ:

$$T_3 = \frac{26}{3}$$

From this, we can plot a graph for $0.05 \leq j \leq 0.95$.

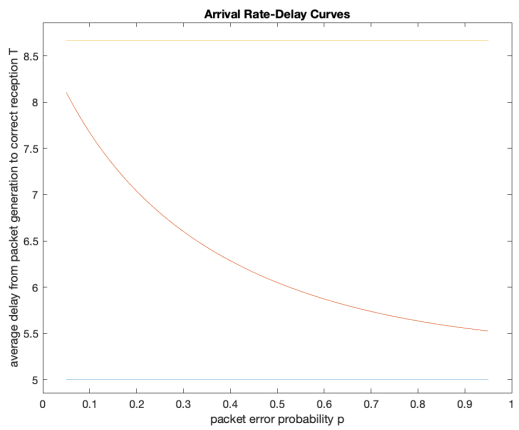

Figure 7: Plots of arrival-rate delay for the three systems

B. Part II

In this case, we can formulate the average system delays as follows:

For AF relaying with end-to-end ARQ:

$$T = \frac{4}{k-1} + 1$$

For DF relaying with end-to-end ARQ:

$$T = \sum_{i=1}^{4} \frac{(0.9)^5}{(0.9)^5 - 0.5 * (0.9)^{i+4}} + \frac{(0.9)^5}{2 * (0.9)^5 - 0.5 * (0.9)^9}$$

For DF relaying with hop-by-hop ARQ:

$$T = \frac{26}{3}$$

From this, we can plot a graph where k is ranging from 1 to 4.

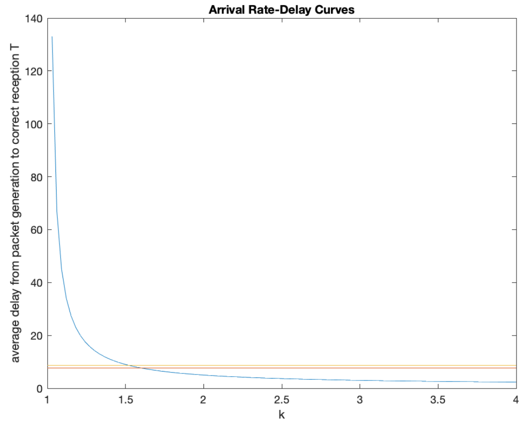

Figure 8: Plots of arrival-rate delay for the three systems

From the above graphs we can observe and therefore conclude that in the interval $1 \leq k \leq 1.6$, the most favorable choice would be DF relaying with end-to-end ARQ and in the interval of $1.6 \leq k \leq 4$, the system with AF relaying with end-to-end ARQ shows an advantage.

6 PROBABILISTIC ARQ

A. The P-ARQ queuing network model

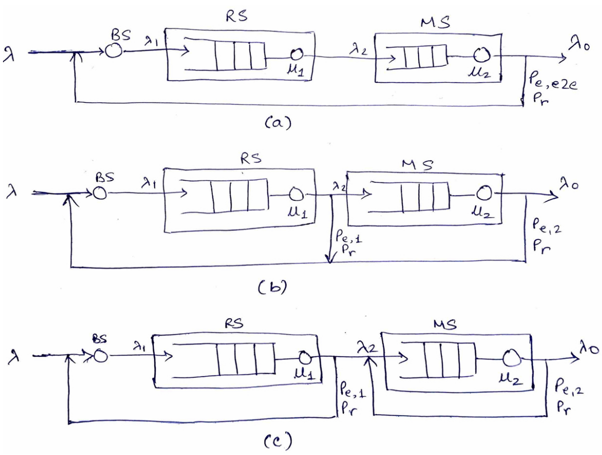

Figure 9: (a)For AF relaying with end-to-end ARQ, (b)For DF relaying with end-to-end ARQ, (c)For DF relaying with hop-by-hop ARQ

B. Performance evaluation

1) P-ARQ - AF relaying with end-to-end ARQ, T^P_{AF-ETE}:

$$\frac{1-(1-p_{e,1})[1-(1-p_{e,1})(1-p_{e,2})]p_r}{\mu_{AF}-\mu_{AF}p_r[1-(1-p_{e,1})(1-p_{e,2})]-\lambda} + \frac{1-(1-p_{e,1})[1-(1-p_{e,1})(1-p_{e,2})]p_r}{\mu_{MS}-\mu_{MS}p_r[1-(1-p_{e,1})(1-p_{e,2})]-\lambda} \tag{20}$$

2) P-ARQ - DF relaying with end-to-end ARQ, T^P_{DF-ETE}:

$$\frac{1-p_{e,1}p_r-(1-p_{e,1})p_{e,2}p_r}{\mu_{DF}-\mu_{DF}p_{e,1}p_r-\mu_{DF}(1-p_{e,1})p_{e,2}p_r-\lambda} + \frac{(1-p_{e,1})(1-p_{e,1}p_r-(1-p_{e,1})p_{e,2}p_r)}{\mu_{MS}-\mu_{MS}p_{e,1}p_r-\mu_{MS}(1-p_{e,1})p_{e,2}p_r-\lambda(1-p_{e,1})} \tag{21}$$

3) P-ARQ - DF relaying with hop-by-hop ARQ, T^P_{DF-HBH}:

$$\frac{1-p_{e,1}p_r}{\mu_{DF}-\mu_{DF}p_{e,1}p_r-\lambda} + \frac{(1-p_{e,2}p_r)(1-p_{e,1}p_r)}{\mu_{MS}(1-p_{e,2}p_r)(1-p_{e,1}p_r)-(1-p_{e,1})\lambda} \tag{22}$$

4) With the given information we can plot a graph and rewrite the above equations as follows:

$$T^P_{AF-ETE} = \frac{2-0.342*p_r}{1.8-0.38*p_r}$$

$$T^P_{DF-ETE} = \frac{1-0.19*p_r}{0.8-0.19*p_r} + \frac{0.9-0.171*p_r}{1.82-0.38*p_r}$$

$$T^P_{DF-HBH} = \frac{1-0.1*p_r}{0.8-0.1*p_r} + \frac{(1-0.1*p_r)^2}{2*(1-0.1*p_r)^2-0.18}$$

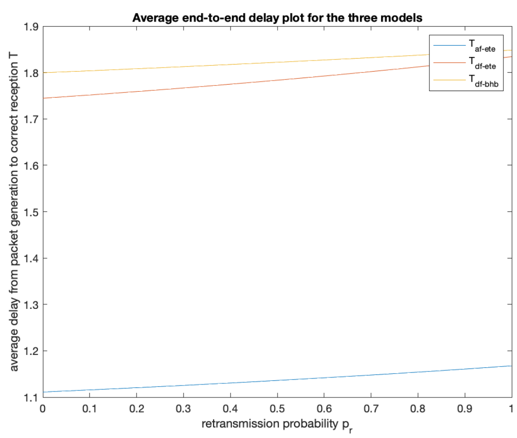

Figure 10: End-to-end Delays for the three models