Project Assignment 2 EQ1220 Signal Theory

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I. INTRODUCTION

As part of this project, we will need to decipher a picture in order to learn the name of the person James Bond is pursuing as part of his current mission. In order to properly retrieve the initial message, we need to equalize the received key, which has been distorted due to the unknown channel impulse response. In order to do this, we will make use of the given training sequence and will create a linear MMSE estimator by using a FIR filter. Using the mean squared error (MSE), we will compare the accuracy of filters with varying lengths. The influence that random bit mistakes in the key have on the decoding of the picture is another aspect that we investigate. This report starts by giving a brief idea about how the FIR filter was designed and how the equalizer was evaluated. Later, we discuss about image recovery, random bit errors and the conclusions we derived from this project.

II. PROBLEM FORMULATION AND SOLUTION

A. Step 1 – Linear MMSE Estimation

MMSE stands for Minimum Mean Squared Error. Firstly, we will build a linear MMSE estimator (FIR Wiener Filter) which will basically be a filter of order L. Considering X as the process to be estimated, Y as the observations and h as the filter of order L we can write the MSE function that needs to be minimized as (1).

$$\overline{MSE}(\hat{X}) = \mathbb{E}\{(X - Y^T h)^2\} \qquad (1)$$

We differentiate the above equation with respect to h and equate it to zero to obtain the minimum. Now, the optimal filter or h_{opt} that satisfies the orthogonality principle is given by the Wiener-Hopf equation (2). Hence, the optimal filter can be computed if we have the autocorrelation matrix (needs to be a square matrix) and the cross-correlation vector.

$$h_{opt} = R_Y^{-1} r_{XY} \qquad (2)$$

B. Step 2 – Parameter Estimation

According to equation (2), to obtain the optimal filter we need to compute the cross-correlation vector and the autocorrelation matrix. The cross-correlation vector can be found out by the following equation:

$$r_{XY} = \mathbb{E}\left\{ \mathrm{X(n)} \begin{pmatrix} Y(n) \\ Y(n-1) \\ \vdots \\ Y(n-L) \end{pmatrix} \right\} = \begin{pmatrix} r_{XY}(0) \\ r_{XY}(1) \\ \vdots \\ r_{XY}(L) \end{pmatrix} \qquad (3)$$

To get the autocorrelation matrix (R_Y) we use

$$R_Y = \begin{pmatrix} r_Y(0) & r_Y(1) & \cdots & r_Y(L) \\ r_Y(1) & r_Y(0) & \cdots & r_Y(L-1) \\ \vdots & \vdots & \cdots & \vdots \\ r_Y(L) & r_Y(L-1) & \cdots & r_Y(0) \end{pmatrix} \qquad (4)$$

Using the 32 samples of the training sequence as well as the symbols that were received, we are going to calculate an estimate for each component of the cross-correlation vector and the autocorrelation matrix. From this, we can find the optimal value of L which will be the case of FIR filter h_{opt}. Linear MMSE estimator was used, and its formula is given by equation (2).

C. Step 3 – Equalizer Evaluation & Decoding

The MSE is a useful metric for determining how well the equalization is working. Because we do not have access to the whole key, we are only able to calculate the MSE ($MSE = E\{(\hat{b}(k) - b(k))^2\}$) in reference to the values of the training sequence. From plotting the graph of L values against their corresponding MSE values (Figure 1) and from visual inspection (Figure 2) we can say that the optimal value of L is 14 for a MSE value of 0.35. It is possible to implement the decoder using a sign function considering its highest likelihood. Figure 1 shows the decoded images using the key that is recovered from the several equalizer filters (i.e., for different value of filter order L).

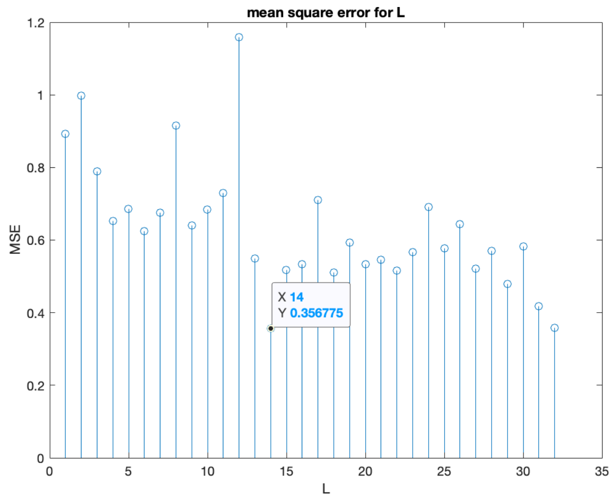

Figure 1: Equalizer Performance – L vs MSE plot

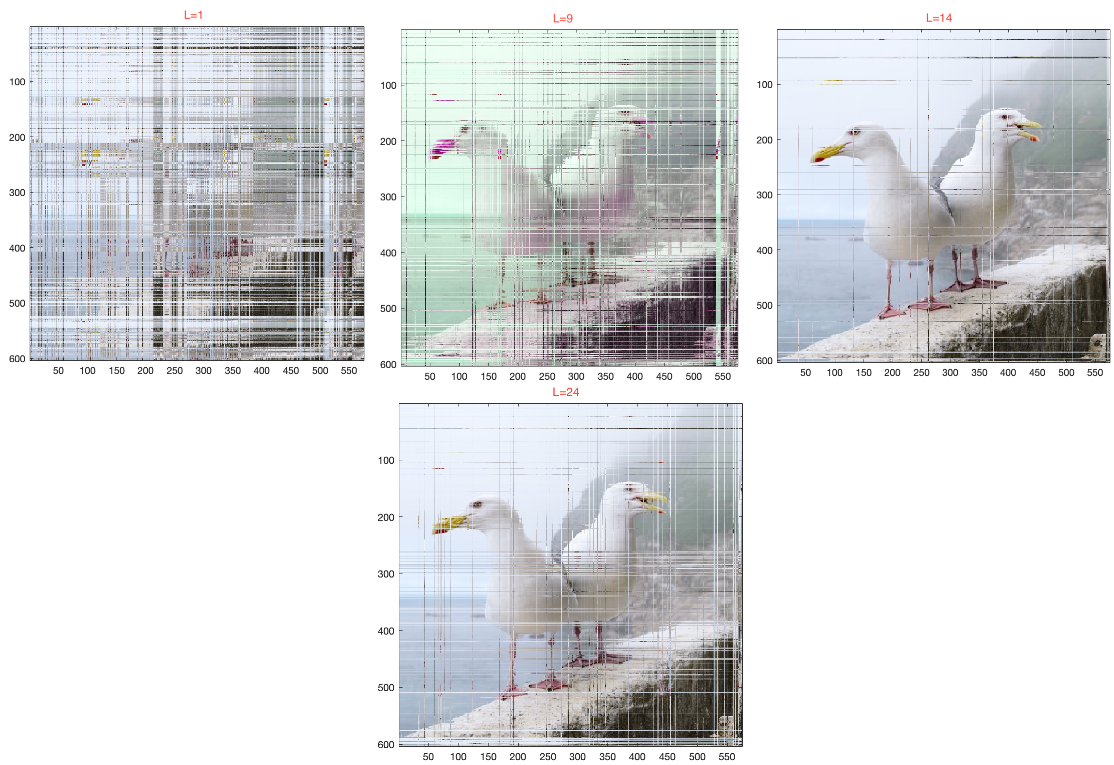

Figure 2: Decoded images obtained from filters of different L values

D. Step 4 – Random Bit Errors

We now introduce random bit errors to the reconstructed key with the optimal filter order 14. Our aim here is to get the value of the bit error after which it is impossible for the decoder to decode the image. The decoded images that were observed with different bit error values are shown in Figure 3.

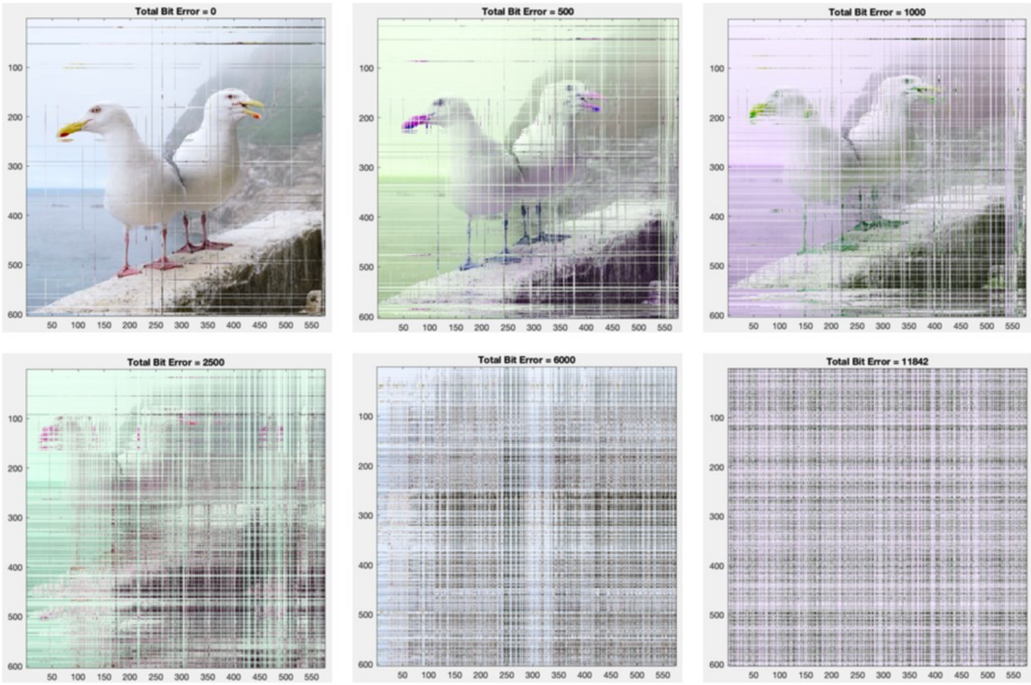

Figure 3: Effect of introducing random bit errors

From Figure 3, we can observe that even at a bit error of around 10000 the decoder serves its purpose, but it is impossible to recognize the image. As a matter of fact, after adding a bit error of around 3000 we were not able recognize what the contents of the image was due to high distortion and corruption in the decoded image. However, after adding an exact bit error of 11843 it becomes impossible for the decoder to anymore decode the image and it throws an error.

III. CONCLUSIONS

In this project, we designed and optimized a FIR Wiener Filter by using autocorrelation and cross-correlation matrices estimated from the training sequence that was provided to us. Once having the filter coefficients from the Wiener-Hopf equation we also obtained an optimal filter order which resulted in a minimum MSE value which gave us the optimal filter to proceed in this project. In order to decode the encrypted image with the reconstructed key we used an equalizer on the received signal, followed by passing it through a sign function and thereby successfully decoding the image. In the last section of this project, we introduced additional bit errors in the filtered key that we obtained from the previous sections to test for the threshold of our designed system and find out after what point does our decoder stops working.

REFERENCES

[1] P. Handel, R. Ottoson, H. Hjalmarsson, Signal Theory, KTH, 2012

[2] Find coefficients of optimal Wiener filter of length 2 n.d., accessed on 1 October 2022, < https://dsp.stackexchange.com/questions/76154/find-coefficients-of-optimal-wiener-filter-of-length-2>