Arc Length

$$y = \frac{2}{3}(x^2+1)^{3/2} \quad , (0,2)$$

Formula: $s = \int_a^b \sqrt{1+(f'(x))^2}\, dx$

$$y' = \frac{2}{3}\left(\frac{3}{2}\right)(x^2+1)^{\frac{1}{2}}(2x)$$

$$s = \int_0^2 \sqrt{1+4x^2(x^2+1)}\, dx = \int_0^2 \sqrt{(2x^2+1)^2}\, dx = \int_0^2 (2x^2+1)\, dx = \ldots$$

Area of Revolution

$$y = x^2 - \frac{1}{8}\ln x \;,\quad 1 \leq x \leq 2 \;,\quad y\text{-axis}$$

$$A = \int_a^b 2\pi x \sqrt{1+[f'(x)]^2}\, dx$$

$$f(x) = x^2 - \frac{1}{8}\ln x$$

$$[f'(x)]^2 = 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$$

$$A = \int_1^2 2\pi x \sqrt{4x^2 + \frac{1}{64x^2} + \frac{1}{2}}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{\frac{256x^4+32x^2+1}{64x^2}}\, dx = 2\pi \int_1^2 (16x^2+1)\, dx$$

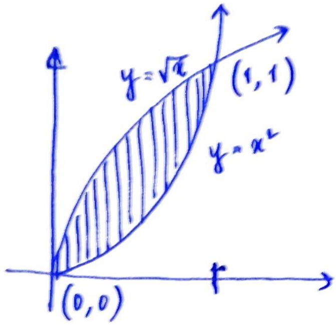
Volume

$y = x^2$

$x = y^2$

about x-axis

$$V = \int_a^b \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$$



$y = x^2$

$x = y^2 \Rightarrow \sqrt{x} = y$

$$V = \int_0^1 \{\pi(\sqrt{x})^2 - \pi(x^2)^2\} dx$$

$$= \int_0^1 \{\pi(x) - \pi(x^4)\} dx$$

$$= \pi \int_0^1 (x^2 - x^4)\, dx \quad \ldots$$

Arc length

① $y = \sqrt{x+2}$, $1 \leq x \leq 7$

② $x = \cos y$, $0 \leq x \leq \frac{1}{2}$

③ $y = 7(6+x)^{\frac{3}{2}}$, $189 \leq y \leq 875$

④ $y = \ln(\cos x)$, $0 \leq x \leq \frac{\pi}{3}$

⑤ $x = \frac{y^4}{8} + \frac{1}{4y^2}$, $1 \leq y \leq 2$

Area of revolution/surface

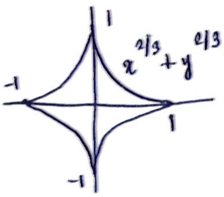
① $y = \frac{1}{6}x^3 + \frac{1}{2x^2}$, $1 \leq x \leq 3$.

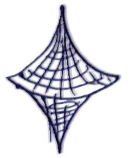
Compute the area of the surface of revolution formed by revolving this graph about the x-axis.

② $x = \frac{1}{8}y^4 + \frac{1}{4y^2}$, $1 \leq y \leq 2$.

Compute the area of the surface of revoln formed by revolving this graph about the y-axis.

(3) $x^{2/3} + y^{2/3} = 1$ astroid. Compute the area of the surface of revolution formed by revolving this graph about the y-axis.





(4) $y = (2x - x^2)^{1/2}$, $0 \leq x \leq 2$, about x-axis

Volume

(1) The flat base of a solid sits in the xy-plane in the region bounded by the x-axis, the y-axis and the line $x + 2y = 6$. Set up an integral which represents the volume of this solid if cross-sections taken perpendicular to the x-axis at x are

(a) squares (b) semi-circles (c) rectangles of height 5.

(2) The flat base of a solid sits in the xy-plane in the region bounded by the x-axis, the line $y = 8$ and $y = x^3$. Set up an integral which represents the volume of this solid if cross sections taken perpendicular to the x-axis at x are

(a) squares (b) equilateral triangles (c) rectangles of perimeter 16

(Method of slice) (3) Consider the region bounded by the graphs of $y = \ln x$, $y = 0$ and $x = e$. Find the volume of the solid formed by revolving this region about

(a) x-axis (b) $y = -1$ (c) $y = 3$.

(4) Consider the region bounded by the graphs of $y = e^x$, $y = 1$ and $x = \ln 3$. Find

the volume of the solid formed by revolving this region about

(a) the x-axis (b) the line y-axis (c) $x=4$ (d) $y=1$ (e) $x=-3$ (f) $y=3$.

(Method of cylinders) (5) Consider the region bounded by the graphs of $y=x^2$ and $x=8\sqrt{x}$. Find the volume of the solid formed by revolving this region about

(a) y-axis (b) x-axis (c) $x=-2$ (d) $x=6$, (e) $y=20$ (f) $y=-1$

(6) Consider the circle of radius R centered at the origin. Form a torus (dough-nut) by revolving this circle about the vertical line $x=-6$. Find the volume of the torus.