# COmbinatorial Number Theory And Connected Topics II

**February 04-05, 2023**

**Titles and Abstracts**

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**Archit Agarwal** (IIT Indore)
**Schedule:** Sunday, 3:00 PM
**Title:** Bressoud-Subbarao type weighted partition identities for a generalized divisor function
**Abstract:** In 1984, Bressoud and Subbarao obtained an interesting weighted partition identity for a generalized divisor function, by means of combinatorial arguments. Recently, S.C. Bhoria, P. Eyyunni, and B. Maji found an analytic proof of the aforementioned identity of Bressoud and Subbarao starting from a $q$-series identity of Ramanujan. In this talk, we shall discuss the combinatorial arguments of Bressoud and Subbarao, and derive a more general weighted partition identity. Furthermore, with the help of a fractional differential operator, we establish a few more Bressoud-Subbarao type weighted partition identities beginning from an identity of Andrews, Garvan and Liang. We will also discuss a one-variable generalization of an identity of Uchimura related to Bell polynomials. This is joint work with S.C. Bhoria, P. Eyyunni, and B. Maji.

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**Ali Alsetri** (University of Kentucky USA)
**Schedule:** Sunday, 6:00 PM
**Title:** Upper bound for dimension of Hilbert cubes contained in the quadratic residues of $F_p$
**Abstract:** We consider the problem of bounding the dimension of Hilbert cubes in $F_p$ which do not contain any primitive roots. We show the dimension of such Hilbert cubes is $O_\epsilon(p^{1/8} + \epsilon)$ for any $\epsilon > 0$, matching what can be deduced from the classical Burgess estimate in the special case when the Hilbert cube is an arithmetic progression.

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**Katie Ansaldi** (Wabash College USA)
**Schedule:** Sunday, 5:30 PM
**Title:** Rainbow numbers of equations over $\mathbb{Z}_n$
**Abstract:** An exact $r$-coloring of a set S is a surjective function $c : S \to [r]$ and a solution is a rainbow if each of its element has a distinct color. In this presentation, we discuss the rainbow numbers, the fewest number of colors needed to guarantee a rainbow solution, of $\mathbb{Z}_p$ for the equation $x - y = z^k$ for $k > 1$ where $p$ is prime.

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**Anamitro Biswas** (NIT Silchar)
**Schedule:** Saturday, 3:30 PM
**Title:** On $r$-wise Davenport constant for finite abelian groups
**Abstract:** For a finite abelian group $G$ and $r \in \mathbb{N}$, *$r$-wise Davenport Constant*, denoted by $D_r(G)$, is defined to be the least positive integer $k$ such that every sequence of length at least $k$ has $r$ disjoint zero-sum subsequences. For $r = 1$, this invariant is known as Davenport constant, first mentioned by K. Rogers in 1963 in the context of it being an important invariant of the ideal class group in the ring of integers of an algebraic number field. Davenport constant is also applicable in cryptology. In this talk we examine the behavior

of the generalized version $D_r(G)$ for finite abelian groups and related problems, in particular its exact value for $p$-groups with sufficiently large primes $p$. This is joint work with Dr. Eshita Mazumdar. The talk shall also highlight some future problems.

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**Mrityunjoy Charan** (NISER Bhubaneswar)
**Schedule:** Sunday, 12:00 PM
**Title:** Poincaré series associated to any $q$-series
**Abstract:** Recently Brandon William study Poincaré series associated to any $q$-series whose coefficients grow slowly enough. In this talk we briefly discuss some properties of this type of Poincaré series. As an application we discuss the adjoint of higher order Serre derivative map.

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**Mithun Das** (NISER Bhubaneswar)
**Schedule:** Saturday, 10:30 AM
**Title:** Higher dimensional Poissonian pair correlation
**Abstract:** We shall discuss the pair correlation statistics for higher dimensional sequences. Conditionally on the size of the joint additive energy of higher dimensional sequence, we establish the metric Poissonian pair correlation with respect to sup-norm. Further, in two dimensions, we obtain an analogous result with respect to 2-norm. This is joint work with T. Bera and A. Mukhopadhyay.

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**Pranabesh Das** (Xavier University of Louisiana)
**Schedule:** Sunday, 10:30 AM
**Title:** Application of Multi-Frey-Hellegouarch approach to a Diophantine equation
**Abstract:** Let $k \geq 1, n \geq 2$ be integers. A power sum is a sum of the form $x_1^k + x_2^k + \cdots + x_n^k$ where $x_1, x_2, \cdots, x_n$ are all integers. Perfect powers appearing in power sums have been well studied in the literature and are an active field of research. In this talk, we consider the Diophantine equation

$$(x-r)^5 + x^5 + (x+r)^5 = y^n, \qquad n \geq 2 \tag{1}$$

where $r, x, y \in \mathbb{Z}$ and $r$ is composed of certain fixed primes. For each, fixed integer tuples $(n, r)$, the curve (1) is a superelliptic curve of genus greater than 1. We determine all the integral points on the infinite family of curves (1) as an application of modularity using the multi-Frey-Hellegouarch method. This talk is based on a joint work with Dey, Koutsianas, and Tzanakis.

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**Meghali Garg** (IIT Indore)
**Schedule:** Saturday, 11:30 AM
**Title:** Hardy-Littlewood-Riesz type criteria for the generalized Riemann Hypothesis
**Abstract:** In 1916, Riesz proved that the Riemann hypothesis is equivalent to the bound $\sum_{n=1}^{\infty} \frac{\mu(n)}{n^2} \exp\left(-\frac{x}{n^2}\right) = O_\epsilon\left(x^{-\frac{3}{4}+\epsilon}\right)$, as $x \to \infty$, for any $\epsilon > 0$. Around the same time, Hardy and Littlewood gave another equivalent criteria for the Riemann hypothesis while correcting an identity of Ramanujan. In this talk, we shall discuss a one-variable generalization of the identity of Hardy and Littlewood and as an application, we provide a Riesz-type criteria for the Riemann hypothesis. In particular, we obtain the bound given by Riesz as well as the bound of Hardy and Littlewood. We have further given a bound equivalent to GRH for Dirichlet-L functions.

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**Vinodkumar Ghale** (BITS-Pilani Hyderabad Campus)
**Schedule:** Sunday, 3:30 PM
**Title:** On the 2-Selmer ranks of Heronian elliptic curves
**Abstract:** Determining integers which occur as the area of Heron triangles (rational triangles) is a generalization of the Congruent number problem. The elliptic curves associated with Heron triangles are called Heronian elliptic curves. Selmer groups help to understand the rank of elliptic curves. We shall discuss the 2-Selmer ranks of such elliptic curves. This is joint work with Debopam Chakraborty.

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**Shivani Goel** (IIIT Delhi)
**Schedule:** Saturday, 1:00 AM
**Title:** On the pair correlation of real valued vector sequences
**Abstract:** In this talk, we generalize the work of Boca and Zaharescu on local statistics of integer valued vector sequences and investigate the pair correlation function for real valued vector sequences. We show Poissonian behavior of the pair correlation function for certain classes of real valued vector sequences.

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**Mohd. Harun** (IIT Kanpur)
**Schedule:** Saturday, 10:00 AM
**Title:** Shifted convolution sum with weighted average: $GL(3) \times GL(3)$ setup
**Abstract:** Shifted convolution sums are the central objects to study many prominent problems in analytic number theory, such as Subconvexity, Moments of $L$-functions, binary additive divisor problem, and many more. In my talk, I will prove the non-trivial estimates for the average and weighted average version of general $GL(3) \times GL(3)$ shifted convolution sums using the circle method. I will first give brief literature and preliminary background related to my problem. Then I will briefly explain the key steps involved in the proof..

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**Habiba Kadiri** (University of Lethbridge Canada)
**Schedule:** Saturday, 8:30 AM
**Title:** An introduction to explicit number theory
**Abstract:** This talk will be an overview of explicit results in number theory, starting with Rosser and Schoenfeld who, between 1939 and 1976, proved a series of theorems about the zeros of the Riemann zeta function and the error term in the prime number theorem. The rise of computational tools has allowed us to partially verify conjectures, such as the Riemann Hypothesis, and to establish or refine statements of conjectures, and has also helped to explicitly confirm some statements known to be true only asymptotically, such as the Odd Goldbach Conjecture. The most recent years have seen an exponential increase in results of an explicit nature, with various "schools" essentially throughout Canada, US, Europe, and Australia, and with various objects of study, from primes and the Riemann zeta functions, to arithmetic progressions and Dirichlet $L$-functions, and to primes in number fields and Hecke $L$- or Dedekind zeta-functions. Explicit results are interesting on their own as they quantitatively measure the state of our understanding and the efficiency of our techniques. Their nature also allows a wide array of applications in Diophantine approximation, arithmetic, cryptography, and other fields of mathematics.

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**Khyati Khurana** (IIIT Delhi)
**Schedule:** Sunday, 11:30 AM

**Title:** Distribution of values of general Euler Totient function
**Abstract:** In this paper, we study the values of distribution of a general Euler totient function introduced by Calderon et al, thereby generalizing previous works on the special case $k = 1$ of the Euler totient function. Moreover, we also determine its maximal and minimal order. This is a joint work with Prof Sneha Chaubey , her PhD student Bittu and Prof Debika Banerjee.

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**Florian Luca** (University of Witwatersrand Johannesburg South Africa)
**Schedule:** Saturday, 6:30 PM
**Title:** Universal Skolem sets
**Abstract:** The celebrated Skolem–Mahler–Lech theorem asserts that if $\mathbf{u} := (u_n)_{n\geq 0}$ is a linearly recurrent sequence of integers then the set of its zeros, that is the set of positive integers $n$ such $u_n = 0$, form a union of finitely many infinite arithmetic progressions together with a (possibly empty) finite set. Except for some special cases, is not known how to bound effectively all the zeros of $\mathbf{u}$. This is called *the Skolem problem*. In this talk we present the notion of a *universal Skolem set*, which an infinite set of positive integers $\mathcal{S}$ such that for every linearly recurrent sequence $\mathbf{u}$, the solutions $u_n = 0$ with $n \in \mathcal{S}$ are effectively computable. We present a couple of examples of universal Skolem sets, one of which has positive lower density as a subset of all the positive integers.

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**Bibekananda Maji** (IIT Indore)
**Schedule:** Sunday, 12:30 PM
**Title:** An infinite series associated to a generalized divisor function and modified Bessel function
**Abstract:** In his lost notebook, Ramanujan noted down many elegant identities involving divisor functions and the modified $K$-Bessel function, and some of them are connected with the Fourier series expansion of the non-holomorphic Eisenstein series. Recently, Cohen established interesting generalizations of some of the identities of Ramanujan. In this talk, we will discuss a few Ramanujan and Cohen-type identities associated to a generalized divisor function and the modified $K$-Bessel function. This is joint with Debika Banerjee.

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**Joshua Males** (University of Manitoba Canada)
**Schedule:** Sunday, 6:30 PM
**Title:** Forgotten conjectures of Andrews for Nahm-type sums
**Abstract:** In his famous '86 paper, Andrews made several conjectures on the function $\sigma(q)$ of Ramanujan, including that it has coefficients (which count certain partition-theoretic objects) whose sup grows in absolute value, and that it has infinitely many Fourier coefficients that vanish. These conjectures were famously proved by Andrews-Dyson-Hickerson in their '88 Invent. paper, and the function $\sigma$ has been related to the arithmetic of $\mathbb{Z}[\sqrt{6}]$ by Cohen (and extensions by Zwegers), and is an important first example of quantum modular forms introduced by Zagier.
A closer inspection of Andrews' '86 paper reveals several more functions that have been a little left in the shadow of their sibling $\sigma$, but which also exhibit extraordinary behaviour. In an ongoing project with Folsom, Rolen, and Storzer, we study the function $v_1(q)$ which is given by a Nahm-type sum and whose coefficients count certain differences of partition-theoretic objects. We give explanations of four conjectures made by Andrews on $v_1$, which

require a blend of novel and well-known techniques, and reveal that $v_1$ should be intimately linked to the arithmetic of the imaginary quadratic field $\mathbb{Z}[\sqrt{-3}]$.

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**Ritabrata Munshi** (ISI Kolkata)
**Schedule:** Sunday, 9:00 AM
**Title:** $GL(3) \times GL(2)$ Rankin-Selberg $L$-function in the level aspect
**Abstract:** Bounding the size of the $GL(3) \times GL(2)$ Rankin-Selberg $L$-function is an important problem in Number Theory. Recently delta method has been used to obtain non-trivial (sub-convex) bound for such $L$-functions in the $t$-aspect and the spectral aspect. In this talk I will explain how delta method can be used to prove sub-convex bounds in the level aspect.

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**Vishnu Prakash P** (Arunai Engineering College Tiruvannamalai)
**Schedule:** Sunday, 4:00 PM
**Title:** Cordial Labeling in graphs
**Abstract:** New families of E- Cordial and Prime Cordial Labeling of graphs.

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**Imre Z. Ruzsa** (Alfréd Rényi Institute of Mathematics Hungary)
**Schedule:** Sunday, 7:30 PM
**Title:** Signed primes as multiple subset
**Abstract:** It is conjectured that the set of primes is not asymptotically a sumset, and it is known (Elsholtz) that it is not a triple sumset. I proved that assuming the prime-tuple hypothesis the set of positive and negative primes together is a sumset. Here I outline the proof of the following result for more than two summands. Assuming the prime-tuple hypothesis there are infinite sets $A_1, \dots, A_k$ such that every element of $A_1 + \dots + A_k$ is a prime, and for every sufficiently large prime $p$ exactly one of $p$ and $-p$ is in this sumset.

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**Manjil Saikia** (IIIT Manipur)
**Schedule:** Saturday, 5:30 PM
**Title:** Partitions with fixed differences between largest and smallest parts with fixed multiplicity of the smallest part
**Abstract:** We look at extensions of formulas given by Vladeta and recently proved by Dhar on integer partitions where the smallest part occurs at least m times and on integer partitions with fixed differences between the largest and smallest parts where the smallest part occurs at least k times. Our results extend Dhar's results for the m = 2 and k = 1 cases to the general cases for arbitrary m and k. We also look at analogous results for overpartitions and -regular partitions. This is joint work with Pankaj J. Mahanta.

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**Neelam Saikia** (IIT Bhubaneswar)
**Schedule:** Saturday, 12:30 PM
**Title:** Sato-Tate distribution of $p$-adic hypergeometric functions
**Abstract:** Sato-Tate distribution for Frobenius trace of elliptic curves is one of the celebrated result in arithmetic statistics. Motivated by this result recently with Ken Ono and Hasan Saad we have initiated a study of value distribution of two and three param- eters families of Gaussian hypergeometric functions over large finite fields. Then
it is a natural question to study similar results for other hypergeometric functions.
In this talk we will consider two and six parameters families of $p$-adic hypergeo- metric functions and will discuss that their limiting distributions are Sato-Tate or

semicircular over large finite fields. This is a joint work with Sudhir Pujahari.

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**Aditi Savalia** (IIT Gandhinagar)
**Schedule:** Saturday, 12:00 PM
**Title:** An induction principle for the Bombieri-Vinogradov theorem over $\mathbb{F}_q[t]$ and a variant of the Titchmarsh divisor problem
**Abstract:** The Bombieri-Vinogradov theorem establishes that the primes are equidistributed in arithmetic progressions "on average" for moduli $q$ in the range $q \leq x^{1/2-\epsilon}$ for any $\epsilon > 0$. Let $\mathbb{F}_q[t]$ be the polynomial ring over the finite field $\mathbb{F}_q$. For arithmetic functions $\psi_1; \psi_2 : \mathbb{F}_q[t] \to \mathbb{C}$, we establish that if a Bombieri-Vinogradov type equidistribution result holds for $\psi_1$ and $\psi_2$, then it also holds for their Dirichlet convolution $\psi_1 * \psi_2$. As an application, we obtain an asymptotic for the average behavior of the divisor function over shifted products of two primes in $\mathbb{F}_q[t]$. This is joint work with Sampa Dey.

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**Gorekh Prasad Sena** (NISER Bhubaneswar)
**Schedule:** Sunday, 2:30 PM
**Title:** Lehmer's problem under various conditions
**Abstract:** For an algebraic integer $\alpha$, let $M(\alpha)$ be its Mahler measure. One of the long-standing open problems related to the Mahler measure is Lehmer's problem, which asks for an absolute constant $c > 1$ such that either $M(\alpha) = 1$ or $M(\alpha) > c$ for all algebraic integers $\alpha$. Though this problem remains open, it has been solved for various classes of algebraic integers. In this talk, we shall first give an overview of Lehmer's problem and then discuss our results on Lehmer's problem, which is a joint work with Dr. K. Senthil Kumar.

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**Alok Shukla** (Ahmedabad University)
**Schedule:** Saturday, 5:00 PM
**Title:** Tiling proofs of some celebrated $q$-series identities
**Abstract:** We will describe an elementary combinatorial approach (using the method of tilings) to obtain proofs of some celebrated q-series identities, such as Jacobi triple product identity, Rogers-Ramanujan identities, and some identities of Rogers. We will also discuss some new q-series identities obtained using this method.

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**Manjit Singh** (Deenbandhu Chhotu Ram University of Science and Technology)
**Schedule:** Saturday, 4:00 PM
**Title:** A role of combinatorial number theory in algebraic coding theory
**Abstract:** In order to construct cyclic, negacyclic, and constacyclic codes over finite fields, we require the irreducible factorization of polynomials of the form $x^n - \lambda$ over finite fields. In this talk, we present some results arising from the combinatorial number theory that are used to deal with the problem of irreducible factorization of the binomials over a finite field $\mathbb{F}_q$.

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**Manoj Upreti** (Shiv Nadar University)
**Schedule:** Sunday, 5:00 PM
**Title:** Berkovich-Uncu type partition inequalities concerning impermissible sets and perfect power frequencies
**Abstract:** In a previous work, Rattan and the Damanvir Singh Binner (first author) proved a conjectured inequality of Berkovich and Uncu concerning partitions with an impermissible part. In this article, we generalize this inequality upon considering t impermissible

parts. We also study partitions whose certain parts appear with a frequency which is a perfect $t$-th power. To prove these inequalities, our methods involve constructing injective maps between the relevant sets of partitions. Surprisingly, our methods also crucially involve concepts from analysis and calculus, such as explicit maps used to prove countability of $N^t$, and Jensen's inequality for convex functions, and then merge them with techniques from number theory such as Frobenius numbers, congruence classes, binary numbers and quadratic residues. Finally, we pose an open problem which seems to be related to the almost universality of diagonal ternary quadratic forms.

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**Akshaa Vatwani** (IIT Gandhinagar)
**Schedule:** Saturday, 2:30 PM
**Title:** Limitations to equidistribution in arithmetic progressions
**Abstract:** It is well known that the prime numbers are equidistributed in arithmetic progressions. Such a phenomenon is also observed more generally for a class of arithmetic functions. A key result in this context is the Bombieri-Vinogradov theorem which establishes that the primes are equidistributed in arithmetic progressions "on average" for moduli $q$ in the range $q \leq x^{1/2-\epsilon}$ for any $\epsilon > 0$. In 1989, building on an idea of Maier, Friedlander and Granville showed that such equidistribution results fail if the range of the moduli $q$ is extended to $q \leq x/(\log x)^B$ for any $B > 1$. We discuss variants of this result and give some applications. This is joint work with Aditi Savalia.

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**Chi Hoi Yip** (University of British Columbia Canada)
**Schedule:** Sunday, 11:00 AM
**Title:** Sperner systems with restricted differences modulo $q$
**Abstract:** Let $\mathcal{F}$ be a family of subsets of $[n]$ and $L$ be a subset of $[n]$. We say $\mathcal{F}$ is an $L$-differencing Sperner system if $|A \setminus B| \in L$ for any distinct $A, B \in \mathcal{F}$. Let $p$ be a prime and $q$ be a power of $p$. Frankl first studied $p$-modular $L$-differencing Sperner systems and showed an upper bound of the form $\sum_{i=0}^{|L|} \binom{n}{i}$. In this talk, I will explain how ideas from combinatorial number theory and linear algebra lead to new upper bounds on $q$-modular $L$-differencing Sperner systems. In particular, we provide a significant improvement on a result of Felszeghy, Hegedűs, and Rónyai. This is a joint work with Zixiang Xu.

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