

**Exercise 1** Let  $a$  and  $b$  be two points in  $\mathbb{R}^d$ . We say that  $a$  dominates  $b$  if  $b_i \leq a_i$  for all  $i = 1, \dots, d$  and  $b_j < a_j$  for at least a  $j$ ,  $j \in \{1, \dots, d\}$ . Given a set  $P = \{p^1, \dots, p^n\}$  of points in  $\mathbb{R}^d$ , the subset of points that are not dominated by any other point in  $P$  is called the *nondominated subset*.

- a) Given a set  $P = \{p^1, \dots, p^n\}$  of  $n$  points in  $\mathbb{R}^2$ , sketch the pseudo-code of an algorithm that returns the nondominated subset by performing pairwise comparisons. Assume that all points in  $P$  are distinct. Discuss its time complexity.
- b) Improve the previous algorithm by using a sweeping approach. Discuss its time complexity.
- c) Consider that the complete input is not known in advance. Every time a new point arrives, the algorithm should return the updated nondominated subset. Sketch the pseudo-code of a procedure that receives the current nondominated subset and the new point and returns the updated nondominated subset. Discuss its time complexity for  $n$  points.

**Exercise 2** Given a set of cubes, compute the volume of their intersection. Assume that each cube is described by its size and by the vertice with the smallest (x,y,z)-coordinate value.