

**Exercise 1** Consider the two following problems:

- Sorting problem: Given a set  $S = \{s_1, \dots, s_n\}$  of  $n$  non-negative integer values, order them in non-decreasing order.
  - Median problem: Given a set  $R = \{r_1, \dots, r_m\}$  of  $m$  non-negative integer values, find its median.
- a) Reduce the median problem to the sorting problem. What is the time complexity of this transformation?

**Exercise 2** Consider the following problem:

- Graph coloring: Given a graph  $G = (V, E)$ , and a positive integer  $k$ , color the nodes with at most  $k$  colors such that no two adjacent nodes have the same color.
- a) Sketch an algorithm that, given a graph that is arbitrarily coloured with  $k$  colors, and a function  $color(u)$  that returns the color of node  $u$ , it outputs the number of pairs of nodes that violate the constraint above. Discuss its time complexity.
- b) Assume that you have an algorithm  $\mathcal{A}$  to solve the Graph Coloring problem as stated above. Sketch an algorithm to solve the optimization version of this problem (to minimize the number of colors) that uses  $\mathcal{A}$ . Give the number of calls to  $\mathcal{A}$ .

**Exercise 3** Consider the two following problems:

- 3-Sum: Given a set of  $n$  distinct integers, find whether it contains three elements that sum to 0.
  - 3-Collinear: Given a set of  $m$  distinct points in the plane, find whether it contains three collinear points.
- a) A set of points is collinear if the slopes between any pair of points of that set match. Explore this property to solve the 3-Collinear problem. Discuss its time complexity.
- b) Show that the 3-Sum problem can be reduced to the 3-Collinear problem. Hint: Consider the property above.
- c) A widely known conjecture claims that it is not possible to solve the 3-Sum problem in less than  $O(n^2)$  amount of time. If this conjecture is true, what can you state about the time complexity of solving the 3-Collinear problem?

**Programming Exercise**

- Read the problem A - *Party Icebreaker* in EA2024\_PL at mooshak. Relate it with the 3-Sum problem and solve it. Hint: An  $O(n^3)$ -time algorithm may not be enough to solve all test cases.