# Probing Michelson Interferometer: Measuring the Wavelength of Helium-Neon Laser and Refractive Index of Glass Slide

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In this experiment, the Michelson interferometer was used to examine the interference of light, which is a wave-like property, to measure the wavelength of the Helium-Neon (He-Ne) Laser, which turned out to be $640 \pm 20$ nm, and the refractive index of a glass slide, which turned out to be $1.3 \pm 0.1$. We observed a circular, concentric fringe pattern by optically aligning our laser beams and counted the number of fringes disappearing from the center by both varying the distance moved by the mirror for the calculation of the wavelength and the angle by which the glass slide was rotated for the calculation of the refractive index.

## I. INTRODUCTION

Interferometers exploit the wave property of the interference of light, which itself is a consequence of the superposition of two or more light waves in space. Such interference can either be constructive or destructive, causing the bright and dark regions in the interference pattern to form respectively. There are different types of interferometers used for different purposes: the wavefront splitting interferometer and the amplitude-splitting interferometer. The wavefront splitting interferometer splits the wavefront that emerges through a point, makes them go through different paths, and later superimposes. The famous Young's Double Slit Experiment used such an interferometer to create the interference pattern on the screen [2].

The amplitude-splitting interferometer splits the amplitude of the incident light beam into two separate beams and later recombines it in such a way that it gives an interference pattern [2]. By far the most famous and historically significant interferometer is the Michelson interferometer, which we shall also use for our experiment. It played an instrumental role in disproving the luminiferous aether and paved the way for Einstein's Theory of Special Relativity [1]. Its applications are numerous, ranging from the spectral imaging of the atmosphere to the detection of gravitational waves [6]. We shall explore further two of its applications: the measuring of the Helium-Neon Laser and the refractive index of glass.

## II. THEORY

The Michelson interferometer works on the principle of superposition, which results from the linearity of the wave equation. Moreover, it also relies on the coherence of waves, where the frequency and the phase difference between the two separated beams are the same.

To calculate the wavelength of the He-Ne laser, we first comprehend the schematic diagram of the interferometer, as shown in Figure (1).

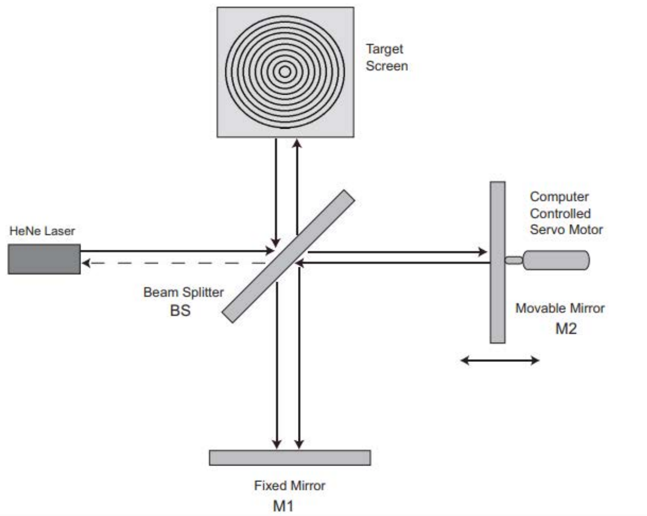


FIG. 1: Schematic Diagram of the Michelson Interferometer. Here, "BS" stands for beam splitter.

As the amplitude of the beam is split by the beam splitter (BS), they follow different optical paths such that the phase difference between the two beams is $\pi$. Therefore, the optical path difference, where destructive interference occurs, turns out to be

$$n\lambda = 2d\cos\theta_n, \tag{1}$$

where $n$ is an integer, $d$ is the distance between the BS and Movable Mirror (M2), $\theta_n$ is the angle between the center of the fringes and the $n$th circle, and $\lambda$ is the wavelength of the He-Ne laser [3]. Moving M2 varies $d$, which linearly depends upon $n$. Therefore, rearranging Equation (1) for $\lambda$ and taking the central reference point (i.e. $\theta_n = 0$), we get

$$\lambda = \frac{2\Delta d}{\Delta N}, \tag{2}$$

where $\Delta d$ is simply the difference in distance moved by M2 and $\Delta N$ is the number of fringes disappearing from the center.

Furthermore, to calculate the refractive index of a glass slide, there must be a change in the optical path length, which is defined as the distance traveled by light in air to create the same phase difference as it were to travel through some other homogenous medium [4]. There is an increase in the optical path length as light travels from air to the glass, which is

$$\begin{aligned} 2n_g t - 2n_a t &= N\lambda, \\ 2(n_g - 1)t &= N\lambda, \end{aligned} \tag{3}$$

where $n_g$ is the refractive index of the glass slide, $t$ is the thickness of the glass slide, and $N$ is the number of fringes disappearing when the glass slide is inserted. The refractive index of air $n_a$ is taken to be 1. The factor of 2 comes as light traverses the plate twice.

When we rotate the glass slide and calculate the change in optical path lengths as shown in Figure (2), we must consider two scenarios: before rotations when the light is parallel to the normal of the plate and after rotations when the light is at an angle $i$ to the normal of the plate.

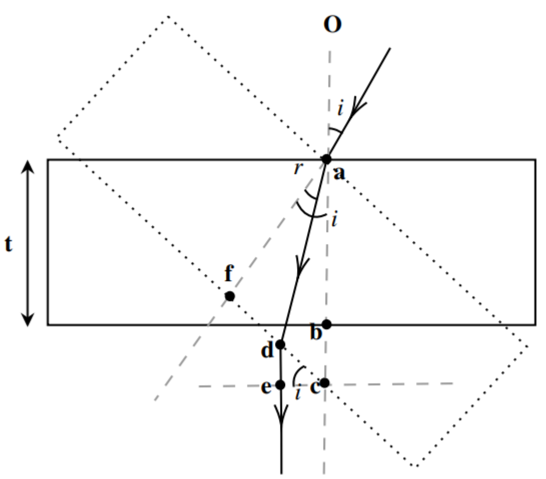


FIG. 2: Schematic diagram of light passing through the glass slide as it is being rotated.

For the former, the total optical path length between a and c is

$$\Longrightarrow (n_g)t + (1)bc = n_g t + bc.$$

Similarly, for the latter

$$\Longrightarrow (n_g)ad + (1)de = n_g ad + de.$$

Thus,

$$\begin{aligned} \text{Total change in path length } &= 2(n_g ad + de) - 2(n_g t + bc), \\ &= 2(n_g ad + de - n_g t - bc). \end{aligned} \tag{4}$$

Looking at the geometry described in Figure (2), we can conclude certain expressions:

$$\begin{aligned} ad &= \frac{t}{\cos r}, \qquad de = dc \sin i = (fc - fd)\sin i, \\ bc &= \frac{t}{\cos i} - t, \qquad de = t\tan i \sin i - t \tan r \sin i. \end{aligned}$$

Inputting these expressions into Equation (4), we get

$$\frac{nt}{\cos r} + t\tan i \sin i - t\tan r \sin i - nt - \frac{t}{\cos i} + t = \frac{N\lambda}{2}. \tag{5}$$

Using Snell's Law ($n \sin r = \sin i$), we make appropriate substitutions in Equation (5),

$$\frac{nt}{\cos r} + t\left(\frac{\sin^2 i}{\cos i}\right) - t\left(\frac{\sin r}{\cos r}\right)\sin i - nt - \frac{t}{\cos i} + t = \frac{N\lambda}{2},$$

and after some algebraic manipulations, we get the expression for the refractive index of glass [5]:

$$n_g = \frac{(2t - N\lambda)(1 - \cos i)}{2t(1 - \cos i) - N\lambda}. \tag{6}$$

## III. EXPERIMENTAL SETUP AND PROCEDURE

This experiment was divided into two parts: first, we measured the wavelength of the monochromatic He-Ne laser, and second, we measure the refractive index of a glass slide. For both the parts, the general experimental setup was the same, with slight differences that will be discussed onwards, as shown in Figure (3).

All the components labeled in Figure (3) from Thor Labs, and were screwed to the breadboard, which was horizontally aligned with the table. The beam splitter (BS) is an uncoated glass plate [3] and was placed in front of the He-Ne laser. It was tilted by 45° in such a way that one of its sides faced the fixed mirror M1 and the other faced the movable mirror M2. Both distances between M1 and BS and M2 and BS are roughly the same at the start.

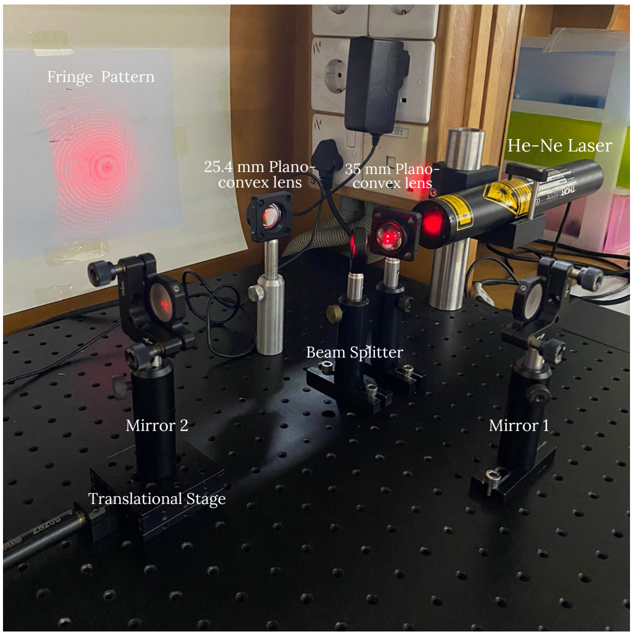


FIG. 3: Laboratory setup of the Michelson Interferometer after optical alignment.

Before the experiment formally started, we needed to optically align the laser beams in a way that there was only a bright dot on the screen. This was made sure by ensuring that all the laser beams were incident on the middle of the components. Then, we placed in the two plano-convex lens: L1 of focal length 35 mm was placed between the laser and the BS with its convex side facing the BS, and L2 of focal length 25.4 mm on the opposite side of M1, in between the BS and the target screen with its convex side facing the target screen. We then observe the concentric fringe pattern, with regions of darkness and brightness. L2 in particular magnified the interference pattern so that it was easier for us to count the disappearing fringes.

For some Michelson interferometers, a compensator plate is used, which is a duplicate of the BS, but with a thin silver film coating. It is required to minimize the effect of dispersion of white light [3]. However, because of our monochromatic laser, we did not need it.

For the first part, we connected M2 to the DC Servo controlled motor which varies the distance between it and the BS. We controlled the motor using the computer software "Kinesis", where we set it to move M2 by a fixed distance of 0.010 mm, with an acceleration of 0.03 mms$^{-2}$ and a maximum velocity of 0.0003 mms$^{-1}$. During this motion, we counted the number of fringes disappearing $N$ from the center. This procedure was repeated five times to reduce random error, and thus, the value wavelength of the He-Ne laser was calculated using Equation (2) along with its uncertainty.

For the second part, we first took the reading of the thickness $t$ of the glass slide we used using the micrometer screw gauge, which turned out to be $1.04 \pm 0.01$ mm. Then, we fixed the same distance between M2 and BS and the distance between M1 and BS. We placed a precision rotation platform on the breadboard, with a stand on top of it. A thin glass slide was carefully inserted into the stand, and the was rotated roughly by $\theta = (20 \pm 2)^\circ$. The number of fringes disappearing $N$ were subsequently counted from the center. This procedure was repeated five times to reduce the random error, and thus, the value of the refractive index of the glass slide was calculated using Equation (6), where $\theta = i$, along with its uncertainty.

## IV. RESULTS

We produced the concentric fringe pattern after the alignment of our laser beams and placing the two plano-convex lenses, as shown in Figure (4).

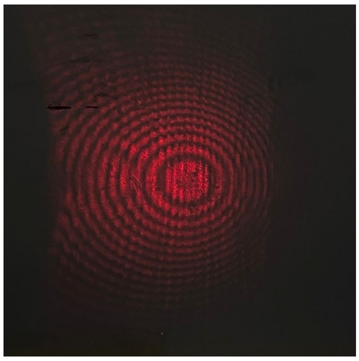

FIG. 4: Fringe pattern formed.

The tabular results of the first part of the experiment where we moved the distance $\Delta d = 0.010$ mm between M2 and BS and counted the disappearing fringes are as follows:

| S.No. | $\Delta d$/mm | Disappearing Fringes ($N$) |
|---|---|---|
| 1 | 0.010 | 30 |
| 2 | 0.010 | 32 |
| 3 | 0.010 | 31 |
| 4 | 0.010 | 32 |
| 5 | 0.010 | 32 |

TABLE I: Number of disappearing fringes $N$ as M2 is moved a distance $\Delta d = 0.010$ mm.

Furthermore, for the second part of the experiment, the tabular results for disappearing fringes at a fixed angle $\theta = (20 \pm 2)^\circ$ of rotation are as follows:

| S.No. | Rotational Angle ($\theta$) | Disappearing Fringes ($N$) |
|---|---|---|
| 1 | 20 | 43 |
| 2 | 20 | 48 |
| 3 | 20 | 53 |
| 4 | 20 | 49 |
| 5 | 20 | 55 |

TABLE II: Number of disappearing fringes as glass slide is rotated by $\theta = (20 \pm 2)^\circ$.

## V. DISCUSSION

For the calculation of the wavelength of the He-Ne laser, the average value of $N$ comes out to be 31.4 from Table I. Using this value, we input it into Equation (2) to get $\lambda_{av} = 633$ nm. To calculate its uncertainty, we use the technique of propagation of uncertainties using the formula

$$\delta\lambda = \sqrt{\left(\frac{\partial\lambda}{\partial N}\delta N\right)^2 + \left(\frac{\partial\lambda}{\partial(\Delta d))}\delta(\Delta d)\right)^2},$$

where $\delta N$ and $\delta(\Delta d)$ are the uncertainties in $N$ and $\Delta d$. Therefore, our calculated value for the wavelength of the He-Ne laser is $630 \pm 20$ nm. According to Thor Labs, the actual value is 632.8 nm. This means that the error is a mere 0.44%, implying that our value is in great accordance with the manufacturer's value.

Similarly, for the calculation of the refractive index of the glass slide, the average value of $N$ comes out to be 49.6 from Table II. Using this value, we input it into Equation (6) to get the average refractive index $n_g = 1.32$. For its uncertainty, we use the technique of propagation of uncertainties using the formula

$$\delta n_g = \sqrt{\left(\frac{\partial n_g}{\partial N}\delta N\right)^2 + \left(\frac{\partial n_g}{\partial\theta}\delta\theta\right)^2 + \left(\frac{\partial n_g}{\partial\lambda}\delta\lambda\right)^2},$$

where $\delta N$, $\delta\theta$, and $\delta\lambda$ are the uncertainties in $N$, $\theta$, and $\lambda$. Therefore, our calculated value for the refractive index of glass is $1.3 \pm 0.1$. The actual value is 1.52, which means that the error is 14.47%, implying that our calculated value is slightly out of bounds for the original value. One reason for this error is the sensitivity of the apparatus. While manually rotating the platform by hand, even a slight pressure on the breadboard caused many disappearing fringes from the center which might have contributed to this sizable error.

## VI. CONCLUSION

By probing into two of its insightful implementations, the Michelson interferometer is as important currently as it was in the late nineteenth century. While we did get a significant error for the refractive index, the experimental setup can be improved upon with the use of an automatic mechanism that reduces the human involvement in rotating the precision rotation platform. Nevertheless, its utilization in determining the wavelength of light with remarkable accuracy and the refractive index of a medium by simply analyzing the interference pattern opens doors to many other experiments and engineering applications where examining these characteristics is of utmost importance.


[1] R. S. Shankland, *Michelson and his interferometer*, Physics Today 27 (1974).

[2] Wikipedia, *Interferometry* (2023).

[3] E. Hecht, *Optics*, Chapter. 9, Page 407 (2002).

[4] Wikipedia, *Optical Path Length.*

[5] G. S. Monk, *Light Principles and Experiments* (McGraw-Hill book company, 1937).

[6] Wikipedia, *Michelson Interferometer.*