Name	Assumptions	Method / mass eq. (continuity eq)	Momentum eq (+ Energy eq)	Eqs/Quantities	Implies/characteristics
HD	Scale; bulk velocity constant in time; short timescale	Eulerian description; $\frac{\partial\rho}{\partial t}+\nabla(\rho\vec{v})=0$; Langrangian -> $\frac{\partial}{\partial t}+v\cdot\nabla\rightarrow\frac{D}{Dt}$ comoving derivative	$\rho\left(\frac{\partial\vec{v}}{\partial t}+\vec{v}\nabla\vec{v}\right)=-\nabla p$ $\left\{+\mu\nabla^2\vec{v}+\left[\beta+\frac{1}{3}\mu\right]\nabla(\nabla\cdot\vec{v})+\rho\vec{g}\right\}$ {Navier-Stokes}; $\frac{\partial\epsilon}{\partial t}+\vec{\nabla}\cdot(\epsilon\vec{v})=-p\nabla\cdot\vec{v}-\nabla\cdot\vec{F}_{cond}+\Psi+\Gamma-\Lambda$	Energy eq. + 1.law of thermodyn. -> heat eq. $\rho T\left[\frac{\partial s}{\partial t}+\nabla\cdot(s\vec{v})\right]=-\nabla\cdot\vec{F}_{cond}+\psi+\Gamma-\Lambda$, s entropy, Radiative heating rate per unit volume Γ, Radiative cooling rate per unit volume Λ, Viscous dissipation rate $\psi:=\pi_{ik}\frac{\partial v_i}{\partial x_k}$	$\mathrm{Re}=\frac{\text{adv.}}{\text{vis.}}=\frac{vL}{\nu},\nu=\frac{\mu}{\rho}$ kinematic viscosity. High Re -> often turbulent; low Re -> dissipational effects damp out turbulence before it can be established
MHD	Time variation in the fields relatively slow; gas elec. neutral; drift velocity << bulk motion; ignore inertial of e^-; field does not vary too rapidly on small scales; non-relativ.	HD + B-field (Maxwell); $\frac{\partial\rho}{\partial t}+\nabla(\rho\vec{v})=0$	N-S + $\vec{F}_L=q\left(\vec{E}+\frac{\vec{v}}{c}\times\vec{B}\right)=\frac{1}{c}\vec{j}_e\times\vec{B}=\frac{1}{4\pi}\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B}=$ $\frac{1}{4\pi}\left(\vec{B}\cdot\vec{\nabla}\right)\vec{B}-\frac{1}{8\pi}\vec{\nabla}\left(\lvert\vec{B}\rvert^2\right)$ magnetic tension (resist being bent) + ∇(magnetic pressure)	Energy eq. + Ohmic dissipation(work done by $\vec{j}_e$) $=\vec{j}_e'\cdot\vec{E}'=\frac{1}{\sigma}\lvert\vec{j}_e\rvert^2=\frac{\eta}{4\pi}\lvert\vec{\nabla}\times\vec{B}\rvert^2>0$; $\frac{\partial\vec{B}}{\partial t}+\vec{\nabla}\times\left(\vec{B}\times\vec{v}\right)=-\vec{\nabla}\times(\eta\vec{\nabla}\times\vec{B})$ evolutionary equ. for $\vec{B}$, elec. resistivity $\eta:=\frac{c^2}{4\pi\sigma}$; **Ideal MHD**: $\eta=0\rightarrow$ flux ($\phi=\int_A\vec{B}\cdot\vec{n}\,dA$) freezing $\frac{d\phi}{dt}=0$	
Bondi accretion ($v_\infty\ll c_\infty$) Gas initial velocity v_∞	Uniform density, no viscosity, steady flow; spher. symm. coord.; accreting object point mass M at rest wrt gas distribtution far from the object	Dim. Ana. -> $r_G=\frac{GM}{c_\infty^2},t_{acc}=\frac{M}{\dot{M}}\propto\frac{1}{M}$, $\dot{M}_{Bondi}\sim\rho_\infty c_\infty r_G^2\sim\frac{(GM)^2\rho_\infty}{c_\infty^3}$-> in reality acc. luminosity & ρ_∞ drops; const. $=4\pi\rho r^2v=\dot{M}$	Euler eq. + ideal gas $p=K\rho^\gamma$: $\nabla\left(\frac{v^2}{2}\right)-\vec{v}\times\left(\vec{\nabla}\times\vec{v}\right)=(\vec{v}\cdot\nabla)\vec{v}=-\nabla h-\nabla\Phi,h=\frac{\gamma}{\gamma-1}\frac{p}{\rho}$ enthalpy	Bernoulli $\frac{v^2}{2}+h+\phi=$ const; $\dot{M}=4\pi\rho_\infty\frac{(GM)^2}{c_\infty^3}q_s,q_s(\gamma)=\frac{1}{4}\left(\frac{2}{5-3\gamma}\right)^{(5-3\gamma)/(2\gamma-2)}$	assuming unperturbed gas at rest $-\frac{GM}{r}+\frac{v^2}{2}+\frac{\gamma}{\gamma-1}\frac{p}{\rho}=C=\frac{\gamma}{\gamma-1}\frac{p_\infty}{\rho_\infty}=\frac{c_\infty^2}{\gamma-1}$ with $c_s=\left(\frac{\gamma p}{\rho}\right)^{\frac{1}{2}}$ for adiabatic gas => v,c_s ellipse $\frac{v^2}{2}+\frac{c_s^2}{\gamma-1}=\frac{c_\infty^2}{\gamma-1}+\frac{GM}{r}$ => Classify solutions: supersonic, subsonic, Parker wind, Bondi accretion; Sonic point $v=c_s\Rightarrow r_{sonic}=\frac{GM}{2c_s^2}$
Hoyle-Lyttleton acc. ($v_\infty\gg c_\infty$)	Accreting object moving wrt the gas with $v\gg c_s\Rightarrow c_s\sim0$ No pressure effects, gas ballistic trajactory	Dim. Ana. -> $\dot{M}_{LH}\sim\frac{(GM)^2\rho_\infty}{v_\infty^3}$	$u=\frac{GM}{h^2}(1+\cos\theta)-\frac{v_\infty}{h}\sin\theta$, $u=\frac{1}{r}$	$\dot{M}_{LH}=4\pi\frac{(GM)^2\rho_\infty}{v_\infty^3}=\pi b_{crit}^2\rho_\infty v_\infty$	$\theta=0\Rightarrow r=\frac{h^2}{2GM}$ collision leads to gas losing v_θ but remain its v_r => $v=v_\infty$. If $E_{kin}<E_{pot,G}\Rightarrow$ gas gravit. bound to the star, assume in this case that it will all be accreted.Gas accretion when $b<b_{crit}=\frac{2GM}{v_\infty^2}$ impact param.
B-H acc. ($v_\infty\sim c_\infty$) <u>Eddington limit</u>	accreting object has a well-defined surface at which almost all of the accretion energy is radiated	Assume $L_{acc}=L_{Edd}\Rightarrow\dot{M}_{Edd}$ sustainable max. accr. rate without terminating the inflow (i.e. L > L_Edd)	Euler & $-\nabla p_{rad}=\frac{\rho\kappa}{c}\vec{F}_{rad}$ with κ mean opacity, radiation pressure stops the inflow of gas $\vec{F}_{rad}=\frac{c}{\kappa}\nabla\Phi$	$\dot{M}_{Edd}=\frac{4\pi R_sc}{\eta_{eff}\kappa}$ (often good approx. to assume κ dominated by Thomson scattering of protons off free electrons, i.e. $\kappa=\sigma_T/m_p$); General B-H: $\dot{M}\approx4\pi\frac{(GM)^2\rho_\infty}{(c_\infty^2+v_\infty^2)^{3/2}}$	$L_{acc}=\eta_{eff}\frac{GM\dot{M}}{R_s},L_{Edd}=\int_S\vec{F}_{rad}\cdot\vec{n}dS=\frac{4\pi GMc}{\kappa}$ (mostly $\eta_{eff}\sim1$, exception BH) In practice, $L_{acc}=\epsilon\dot{M}c^2$ with $\epsilon=\frac{radiated}{inflow}$
Accretion disk (non-0 ang. Mom.): Radial	Axissymmetric; $h\ll R_{disk}$ (valid if disk cold, rot. supersonic); M_gas in the disk << M_central obj.; motion of gas ~ Kepler, diff. rot. (shear visco.); no self-grav.	$\frac{\partial\Sigma}{\partial t}+\frac{1}{R}\frac{\partial}{\partial R}(R\Sigma v_R)=0$, mass surface density $\Sigma(R,t)=\int_{-\infty}^{\infty}\rho(R,z,t)\,dz$	$R\frac{\partial}{\partial t}(\Sigma R^2\Omega)+\frac{\partial}{\partial R}(\Sigma R^3\Omega v_R)=\frac{1}{2\pi}\frac{\partial T}{\partial R}$ (net torque per unit area $T=2\pi\nu\Sigma R^3\frac{d\Omega}{dR}$)	Left: deviates from Kep_rot small -> $\nabla\cdot\vec{v}$ small -> dominant source of viscosity ~ shear viscosity; $\frac{d\Omega}{dR}<0\Rightarrow$ viscosity slow down inner part, speed up outer part	Assume $t_{acc}=\frac{M}{\dot{M}}\gg T_{one\ orbit},\Omega=\Omega_{\mathrm{Kep}}=\left(\frac{GM}{R^3}\right)^{\frac{1}{2}}\Rightarrow\frac{\partial\Sigma}{\partial t}=\frac{3}{R}\frac{\partial}{\partial R}\left[R^{\frac{1}{2}}\frac{\partial}{\partial R}\left(\nu\Sigma R^{\frac{1}{2}}\right)\right]=\frac{1}{2\pi R}\frac{\partial\dot{M}}{\partial R}$
Vertical (tbc)	Vertical hydrostatic $\Rightarrow\frac{d\phi}{dz}=-\frac{1}{\rho}\frac{dp}{dz}$; $h\ll R_{disk}$;(valid if disk cold, rotation supersonic) M_gas in disk << M_cent_obj.;	$\rho\propto\exp\left(-\frac{z^2}{2h^2}\right)$; Kep_rot, $c_s\approx h\Omega$; $t_{acc,\nu}\sim\frac{R}{v_R}\sim\frac{R^2}{\nu}$, std molecular viscosity, $\nu\sim v_T\lambda$ -> Need for anomale viscosity		$\alpha-$disk: $\nu=\alpha hc_s,\alpha\le1$ uncertainty (turbul. approx isotropic -> $L_{turb}\sim h$) -> from MRI: grows rapidly, once non-linear drives turbl. (turbl. visco.) outward transp of ang_mom: turbl. diffusion.	If ang_mom ↑ with r -> distance bt A(inner), B(outer) ↑ -> mag. tension force ↑ -> init tiny dist. diff amplify rapidly -> instab.; If ↓-> MRI not operat. However, disk violate the Rayleigh stability criterion $\frac{d(r^2\Omega)}{dr}>0$ -> unstable to purely hydrody perturb.; in practice disk suscp to MRI only if $\vec{B}$ weak, i.e. $\frac{B_z^2}{8\pi}<\frac{3}{\pi^2}\rho c_s^2,\frac{d}{dr}(\Omega^2)<0$
Parker(thermal press. driven) wind	Solar wind ~ Isothermal => $c_s=\sqrt{\frac{GM}{2r_{sonic}}}=$ const.	From v -continuity equ.-> ρ. $\dot{M}=$ const. => $r^2\rho v=$ const.	$\frac{1}{v}\frac{\partial v}{\partial r}(v^2-c_s^2)=\frac{2c_s^2}{r}-\frac{GM}{r^2}$	$\frac{v^2}{c_s^2}-\ln\left(\frac{v^2}{c_s^2}\right)=\frac{4r_{sonic}}{r}+4\ln\left(\frac{r}{r_{sonic}}\right)-3$ $r\ll r_{sonic}$: $v\approx c_se^{\frac{3}{2}}\left(\frac{r_{sonic}}{r}\right)^2\exp\left(-\frac{2r_{sonic}}{r}\right)$ minimal at $r=r_*$	$r\gg r_{sonic}$: right-hand side dominated by ln(.), at sufficiently large r, left-hand side dominated by ^2 -> $v=2c_s\left[\ln\left(\frac{r}{r_{sonic}}\right)\right]^{\frac{1}{2}}\rightarrow$ unphysical as $r\rightarrow\infty$, no more isothermal in reality
Radiation-driven wind	Spher. Symm.(e.g. Parker), freq-indep opacity; Most cases, line opacity ν (freq-dep) domin, here, wind steady accel as it away from the star-> Doopler shift so that absorb-/scatterable photon ~ $\nu_0\left(1+\frac{v(r)}{c}\right)$	$\rightarrow\frac{GM}{r^2}(1-\Gamma),\Gamma=\frac{L}{L_{edd}}\Rightarrow r_{sonic}=\frac{GM}{2c_s^2}(1-\Gamma)$ closer to stellar surface; a single strong line captures only a small fraction, $\frac{v_\infty}{c}$, of ava. photon mom. -> strong wind from many strong lines; Sobolev approx. (large velocity gradient)	$\nabla p_{rad}=-\frac{\rho}{c}\int\kappa_\nu\vec{F}_\nu d\nu$ with opacity κ_ν	Radiative accel. prod. by absorp in a single line at r from the star $g_L(r)=g_{thin}\int_{-\infty}^{+\infty}dx\phi\left[x-\frac{v(r)}{v_{th}}\right]e^{-\tau(x,r)}$, $g_{thin}=\frac{\kappa v_{th}\nu_0L_\nu}{4\pi r^2c^2}$, $v_{th}=\sqrt{\frac{kT}{m}}$, $x:=\frac{c}{v_{th}}\left(\frac{\nu}{\nu_0}-1\right)$, $\phi(x)$ line profile, $\tau(x,r)=\int_{r_*}^{r}dr'\kappa\rho(r')\phi\left[x-\frac{v(r')}{v_{th}}\right]$, $\kappa_\nu=\kappa\phi(\nu)$ optical depth between r_* and r.	Sobolev length $L_{sob}:=\frac{v_{th}}{\lvert\frac{dv}{dr}\rvert}\ll l$ scale where ρ or $v(v\gg v_{th}$, supersonic flow)changes significantly -> $\rho,v_{th},\frac{dv}{dr}$ const over time. $\Rightarrow g_L(r)=g_{thin}\frac{(1-e^{-\tau_S(r)})}{\tau_S(r)},\tau_S(r)=\frac{\kappa\rho v_{th}}{\lvert dv/dr\rvert}$ Sobolev optical depth.; $\tau_S\ll1\Rightarrow g_L(r)\sim g_{thin}$. $\tau\ll1,g_L\sim\frac{\kappa v_{th}}{c^2}\frac{\nu_0L_\nu}{4\pi r^2}$; $\tau\gg1,g_L\sim\frac{\lvert\frac{dv}{dr}\rvert}{\rho c^2}\frac{\nu_0L_\nu}{4\pi r^2}$ In cool stars, another source of opacity -> dust
Sound wave	$\rho=$ const., unmagnetised gas initially at rest, neglect effects of gravity, in pressure equillibrium throughout; evolution adiabatic $p=K\rho^\gamma$;	Perturb. Theory $\rho=\rho_0+\rho_1(x,t),\vec{v}(x,t)=\vec{v}_1(x,t)$; $\frac{\partial\rho_1}{\partial t}=-\rho_0\nabla\cdot\vec{v}_1$	ρ_0,p_0 spatial, timely constant => $\frac{\partial\rho_1}{\partial t}=-\rho_0\nabla\cdot\vec{v}_1,\rho_0\frac{\partial\vec{v}_1}{\partial t}=-\frac{\gamma p_0}{\rho_0}\nabla\rho_1$	$\Rightarrow\frac{\partial^2\rho_1}{\partial t^2}=\frac{\gamma p_0}{\rho_0}\nabla^2\rho_1$ ->Longit. wave $\omega^2={c_s}^2k^2,c_s=\sqrt{\frac{\gamma p_0}{\rho_0}}$. ω real for all k, perturb. osci. (not grow nor decay with time)	Perturbations propagate with $c_s=\left(\frac{\partial p}{\partial\rho}\right)^{1/2}\cong\frac{\gamma p_0}{\rho_0}$. In general, ideal gas $p=K\rho^\gamma$ ⇒sound waves propagate faster in denser gas
Sound wave + viscosity	1d	Ansatz plane wave $e^{-i(kx+\omega t)}$; $\frac{\partial\rho_1}{\partial t}=-\rho_0\frac{\partial v_1}{\partial x}$	$\rho_0\frac{\partial v_1}{\partial t}=-\frac{\gamma p_0}{\rho_0}\frac{\partial\rho_1}{\partial x}+\left(\beta+\frac{4}{3}\mu\right)\frac{\partial^2v_1}{\partial x^2}$	$\omega=-\frac{1}{2}\tilde{\nu}k^2i+\left(c_s^2k^2-\frac{1}{4}\tilde{\nu}^2k^4\right)^{\frac{1}{2}},\tilde{\nu}:=\frac{\beta}{\rho_0}+\frac{4}{3}\frac{\mu}{\rho_0}$; $k^2>\frac{4c_s^2}{\tilde{\nu}^2}\Rightarrow$ sound waves not osci., simply decay(ω negative imaginary);	k^2 smaller $\Rightarrow e^{-i\{kx+Re(\omega)t\}}e^{-\frac{1}{2}\tilde{\nu}k^2t}\Rightarrow$ sound wave osci., $v_{ph}=\frac{Re(\omega)}{k}\Rightarrow$ dispersive wave (waves with diff. ω prop. at slightly diff. v) & decays as $e^{-t/\tau_{dec}}\Rightarrow$ dissipative wave(amp. decays with t)
MHD wave	No viscosity, ideal MHD	like above with const $\vec{B}=\vec{B}_0,\nabla\times\vec{B}_0=0$ then perturb; $\frac{\partial\rho_1}{\partial t}=-\rho_0\frac{\partial v_1}{\partial x}$	$\rho_0\frac{\partial\vec{v}_1}{\partial t}=-\frac{\gamma p_0}{\rho_0}\nabla\rho_1+\frac{1}{4\pi}\left(\nabla\times\vec{B}_1\right)\times\vec{B}_0$	$\frac{\partial^2\vec{v}_1}{\partial t^2}=c_s^2\nabla(\nabla\cdot\vec{v}_1)-(\nabla\times\{\nabla\times[\vec{v}_A\times\vec{v}_1]\})\times\vec{v}_A,\vec{v}_A:=\frac{\vec{B}_0}{\sqrt{4\pi\rho_0}}$ Alfven velocity => $\omega^2\vec{v}_1=(c_s^2+v_A^2)\left(\vec{k}\cdot\vec{v}_1\right)\vec{k}+\vec{v}_A\cdot\vec{k}\left[\left(\vec{v}_A\cdot\vec{k}\right)\vec{v}_1-(\vec{v}_A\cdot\vec{v}_1)\vec{k}-\left(\vec{k}\cdot\vec{v}_1\right)\vec{v}_A\right]$	transverse displacement($\perp\vec{k},\perp\vec{B}_0$): **Alfven waves** $\omega=\pm kv_A\cos\theta,\theta\sim<\vec{k},\vec{B}_0>$, transverse wave propagates $v_g=v_A\cos\theta$; other cases: fast & slow mode (ass. with changes in ρ&p) **magnetosonic waves;** $\vec{k}\perp\vec{B}_0\Rightarrow$ fast mode propogates with c_s+v_A, slow mode not exist
Steepening of sound wave	no assumption that perturbation small or the nature of the background flow	1d continuity + Euler + $\frac{d\rho}{\rho}=\frac{2}{\gamma-1}\frac{dc_s}{c_s}$	Consider sinusodinal density perturbation -> perturbation steepens as it propagates -> show forms	$\left[\frac{\partial}{\partial t}+(v\pm c_s)\frac{\partial}{\partial x}\right]\left(v\pm\frac{2}{\gamma-1}c_s\right)=0$	2 quantities conserve along trajecotries $\frac{dx}{dt}=v\pm c_s$ (characteristics: along which the perturbations move in the (x,t) plane)
Non-radiative shock	A shock wave is a region of small thickness over which the properties of the flow change rapidly. ignore what's going on within the shock wave itself	treat it as a math. disconti. in the flow; inside the shock Re ~ M, $s_2-s_1=c_p\ln\left(\frac{T_2}{T_1}\right)-\frac{k}{m}\ln\left(\frac{p_2}{p_1}\right)$; Oblique shock: shock front not $\perp$ inflow gas -> decompose in $\parallel$& $\perp$	Shock jump conditions(Rankine - Hugoniot): upstream $v\perp$ a station. shock. $\rho_1v_1=\rho_2v_2,p_1+\rho_1v_1^2=p_2+\rho_2v_2^2,\frac{1}{2}v_1^2+\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}=\frac{1}{2}v_2^2+\frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2}$	$\frac{\rho_2}{\rho_1}=\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$ Mach number $M_1:=\frac{v_1}{c_{s,1}}$; $M_1<1\Rightarrow\rho_2<\rho_1\Rightarrow v_2>v_1$ rarefaction shock, <u>physically not exist;</u> $M_1>1\Rightarrow\rho_2>\rho_1\Rightarrow v_2<v_1$ shock compresses the gas, slows it down, raises gas pressure, raises T	inside the shock (N-S instead of Euler): $M_1>1\Rightarrow$ look at weak shock ($M_1\sim1$)... => entropy increases, monoton increasing function of M_1 => rarefaction shocks involve a decrease in the entropy, forbidden by the second law of thermodynamics. Oblique shock: for a compressive flow, flow turns towards the plane of the shock
Radiative	radiative heat losses non-negligible; Extremely narrow shock region, width ~ mean free path, followed by radiative relaxation layer: narrow cooling-domi zone.	If only interested in scales >> width of RRL -> useful to write down jump conditions that relate pre-shock conditions to conditions at the end of this layer (i.e. once the gas T has reached equil); $\rho_1v_1=\rho_3v_3$	$p_1+\rho_1v_1^2=p_3+\rho_3v_3^2$ 1d: $\rho u\frac{d\epsilon}{dx}=-p\frac{dv}{dx}-\rho\Lambda(\rho,T)$ with gas optically thin $\Lambda=\Lambda(\rho,T)$	For ideal, chemically inert gas $\epsilon=\frac{1}{\gamma-1}\frac{p}{\rho}$ & $\rho v=$ const within RRL & $\frac{dp}{dx}=-\rho v\frac{dv}{dx}\Rightarrow\frac{1}{\gamma-1}(c_s^2-v^2)\frac{dv}{dx}=-\Lambda(\rho,T)$; if $\Lambda>0$ => $\frac{dT}{dx}<0$ substantially because of EoS and fractional increase of p << that of ρ	In some cases(e.g. within dense molecular clouds), the equil. T rel insensitive to ρ => isothermal approx appropriate $T_1=T_3$ ==with jump conditions==> $v_1v_3=c_1^2\Rightarrow\frac{\rho_3}{\rho_1}=\left(\frac{v_1}{c_1}\right)^2\Rightarrow$ much larger ρ contrasts than non-radiative shocks; radiative precursor: strong shock, pre-shock gas largely neutral ->typically high optical depth to ionized (by RRL) photons -> post-shock emission ionize and heat the pre-shock gas
MHD: Single-fluid shock (tbc)	ideal MHD applies everywhere but within the shock-front itself, v_ions = v_neutral	$\rho_1v_{1,\perp}=\rho_2v_{2,\perp}$	$\frac{\partial}{\partial t}(\rho v_i)+\partial_j\left(\rho v_jv_i+p\delta_{ij}-T_{ij}\right)=0,T_{ij}=\frac{1}{4\pi}\left(B_iB_j-\frac{1}{2}\lvert\vec{B}\rvert^2\delta_{ij}\right)$ Maxwell stress tensor		Multi-fluid: fractional ionization small -> v_ions ≠ v_neutral -> $v_{\mathrm{Alfven,ion}}\ll v_{\mathrm{A,neutral}}$; J(ump)-shocks: shock region very narrow -> neutrals ~ in a single fluid shock or purely hydrodyn shock, with an almost disconti change in properties; C(ontinuous)-shocks: energy dissip spread over an extended region upstream of the shock (possible if $v_{A,i}\gg v_{flow,upstream}$) -> smooth change in fluid quantities in ions and neutrals
Gravitational instabilities	Perturbation + self-gravity of the gas $\nabla^2(\phi_0+\phi_1)=4\pi G(\rho_0+\rho_1)$	Jeans swindle (disregard ϕ_0 term. justified for e.g. rotating body in equi); As sound wave + $-\rho_0\nabla\phi_1$	$\frac{\partial^2\rho_1}{\partial t^2}=\frac{\gamma p_0}{\rho_0}\nabla^2\rho_1+4\pi G\rho_0\rho_1$	$\omega^2=c_s^2k^2-4\pi G\rho_0$. large k (perturb. small) -> sound wave; Grav. unstab. for perturb. large, timescale $t_{grav}=(4\pi G\rho_0)^{-1/2}$ perturb amplify exp-ly.	$k_J^2=\frac{4\pi G\rho_0}{c_s^2}$, Jeans length $\lambda_J=\frac{2\pi}{k_J}$, Jeans mass $=\frac{4\pi}{3}\rho_0\left(\frac{\lambda_J}{2}\right)^3$ ~ crit. mass for E_grav of a perturb. > its E_therm; Dimless growth rate $\Omega=i\omega t_{\mathrm{grav}}$
	Infinite thin sheet of matter with initially uniform Σ_0 (Similar reslut for more realistic case of a self-graviting isothermal sheet)	replace ρ with Σ &$\nabla^2\Phi=4\pi G\Sigma\delta(z)$, 1. order perturb + Jeans swindle		$\Phi_1=\Phi_ae^{-\lvert kz\rvert}e^{-i(\vec{k}\cdot\vec{x}+\omega t)},\Phi_a=\frac{2\pi G\Sigma_a}{\lvert k\rvert}\Rightarrow\omega^2=c_s^2k^2-2\pi G\Sigma k$. k small: grav. unstable; k large: thermal pressure domin, sheet stable;	$H=\frac{c_s^2}{\pi G\Sigma}$, Perturbation grow -> $\Omega^2=\left(\frac{i\omega H}{c_s}\right)^2>1\Rightarrow k_{crit}=\frac{2}{H}$, unlike 3d, fastest growing mode in 2d occurs when $k=\frac{k_{crit}}{2}$
Toomre	differentially rotating, gaseous accretion disc with constant Σ,Ω_0; small perturbation, in co-rotating frame	1. order perturb, no need for Jeans swindle; $\frac{\partial\Sigma}{\partial t}+\nabla\cdot(\Sigma\vec{v})=0\Rightarrow\frac{\partial\Sigma_1}{\partial t}=-\Sigma_0\nabla\cdot\vec{v}_1$	$\frac{\partial\vec{v}}{\partial t}+\vec{v}\cdot\nabla\vec{v}=-\frac{\nabla p}{\Sigma}-\nabla\phi-2\vec{\Omega}\times\vec{v}+\Omega^2\left(x\hat{\vec{e}}_x+y\hat{\vec{e}}_y\right)\Rightarrow\frac{\partial\vec{v}_1}{\partial t}=-\frac{c_s^2}{\Sigma_0}\nabla\Sigma_1-\nabla\phi_1-2\vec{\Omega}\times\vec{v}_1,\nabla^2\phi_1=4\pi G\Sigma_1\delta(z)$	$\omega^2=c_s^2k^2+4\Omega^2-2\pi G\Sigma_0\lvert k\rvert$ Remain stable: $\omega^2>0\rightarrow$ thermal p and rot. stablize disk, "-" destablizes it; $k<k_{rot}=\frac{2\Omega^2}{\pi G\Sigma_0}$ stable; $k_{rot}>k_J\Leftrightarrow Q:=\frac{\Omega c_s}{\pi G\Sigma_0}>1$ stable on all scales (Toomre stability parameter Q); $k_{rot}<k<k_J$ disk unstable to the growth of small perturb.	When $\Omega=\Omega(r)\Rightarrow$ stable for $Q:=\frac{\kappa c_s}{\pi G\Sigma_0}<1$, κ epicyclic frequency of the disk
Kelvin-Helmholtz (vortex-like)	No gravity, incompressible $\nabla\cdot\vec{v}=0$, irrotational $\vec{\omega}=\nabla\times\vec{v}=0$; $\vec{v}_{1,2}=\nabla\Phi_{1,2}$; perturb $\Phi_{1,2}=v_{1,2}x+\phi_{1,2}$	Potential flow $\nabla^2\Phi=0$, $\xi(x,t)=$ z position of the interface; $\frac{D\xi}{Dt}=\frac{\partial\Phi_1}{\partial z}$	perturb small -> $\frac{\partial\xi}{\partial x}$ small -> consider $\frac{\partial\phi_1}{\partial x}\frac{\partial\xi}{\partial x}$ 2 order -> $\frac{\partial\xi}{\partial t}+v_1\frac{\partial\xi}{\partial x}=\frac{\partial\phi_1}{\partial z},\frac{\partial\xi}{\partial t}+v_2\frac{\partial\xi}{\partial x}=\frac{\partial\phi_2}{\partial z};\frac{\partial\phi}{\partial t}+\frac{1}{2}v^2+\frac{P}{\rho}=$ const. ~ Bernoulli -1. order-> $P_{1,2}=P_{init}-\rho_{1,2}\left(\frac{\partial\phi_1}{\partial t}+U_{1,2}\frac{\partial\phi_{1,2}}{\partial x}\right)$ & p equil.	$w=\frac{k(\rho_1v_1+\rho_2v_2)}{\rho_1+\rho_2}\pm i\frac{\sqrt{\rho_1\rho_2}k}{\rho_1+\rho_2}\lvert v_1-v_2\rvert$. interface always displays oscillatory behaviour (unless fluids at rest), but also both growing and decaying modes whenever $\lvert v_1-v_2\rvert\neq0$; $t_{grow}=\frac{\rho_1+\rho_2}{\sqrt{\rho_1\rho_2}}\frac{1}{k\lvert v_1-v_2\rvert}$ in reality, effects of viscosity reduce $\lvert v_1-v_2\rvert$ and instability occur more slowly	
Rayleigh-Taylor	K-H + gravity, on small scales close to the interface can still assume $P_1=P_2$	$\frac{\partial\Phi}{\partial t}+\frac{1}{2}v^2+\frac{P}{\rho}+gz=$ const.	$\frac{\partial\phi_1}{\partial t}+\frac{1}{2}\left(\left(U_1+\frac{\partial\phi_1}{\partial x}\right)^2+\left(\frac{\partial\phi_1}{\partial z}\right)^2\right)+\frac{P_1}{\rho_1}+g\zeta=\frac{1}{2}U_1^2+\frac{P_{init}}{\rho_1}$ -1.order -> $P_{1,2}=P_{init}-\rho_{1,2}\left(\frac{\partial\phi_1}{\partial t}+U_{1,2}\frac{\partial\phi_{1,2}}{\partial x}+g\zeta\right)$	Fluids at rest $U_1=U_2=0\Rightarrow\omega^2=\frac{(\rho_1-\rho_2)kg}{(\rho_1+\rho_2)}$; Heavier on bottom => only osci., stable for all k; Heavier on top => unstable for all k, driven by buoyancy of the lighter fluid, R-T instab.	
Parker (Can occur on large scales within galactic disks)	Init $\vec{B}\parallel$ mid-plane of galactic disk; p due to non-thermal particles (cosmic rays) = 0, gas at rest, p&ρ uniform in x-y plane; Plasma β parameter $\beta:=\frac{8\pi P_0}{B_0^2}=$ const	analyze equi. config. of the field $\left(1+\frac{1}{\beta}\right)\frac{dP_0}{dz}=-\rho_0\frac{d\Phi}{dz},\rho_0=\frac{P_0}{c_s^2}$, Φ grav. potential		sol $B_0(z)=B_0(0)\exp[-\frac{\Phi(z)}{2(1+\beta^{-1})c_s^2}]$, same $P_0(z)$; In this equil only P_mag important, now perturb -> magn tension suppresses growth of short-λ perturb; Approx $\Phi(z)=g_0\lvert z\rvert$ -> scale height $H=\frac{(1+\beta^{-1})c_s^2}{g_0}$=> fastest growing mode $\lambda\sim2\pi H$	