UKSta18_Liang_Ex02

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1 Fathers and Sons on the Career Ladder

Occupation in group of upper(u), middle(m) and lower(l). Subscripts index the generation.

The following table gives conditional probabilities, for example $P(U_2|U_1) = 0.45$

	U_2	M_2	L_2
U_1	0.45	0.48	0.07
M_1	0.05	0.70	0.25
L_1	0.01	0.50	0.49

Suppose that of the father's generation 10% are in U, 40% in M, and 50% in L.

- Q1. What is the probability that a son in the next generation is in U? Analoguously in M, or L?

Using the law of total probability we get:

$P(U_2) = P(U_2|U_1) * P(U_1) + P(U_2|M_1) * P(M_1) + P(U_2|L_1) * P(L_1) = 0.45 * 0.1 + 0.05 * 0.4 + 0.01 * 0.5 = 0.07$

Similarly we get $P(M_2) = 0.48 * 0.1 + 0.70 * 0.4 + 0.50 * 0.5 = 0.578$ and $P(L_2) = 0.352$ (Alternatively $P(L_2) = 1 - P(U_2) - P(M_2)$)

- Q2. If a son has occupational status U2, what is the probability that his father had occupational status U1? Analoguously M1, or L1?

Using the formular of joint probabilty we get $P(U_1|U_2) = \frac{P(U_1)*P(U_2|U_1)}{P(U_2)} = \frac{0.10*0.45}{0.07} \approx 0.643$

Similarly we get $P(M_1|U_2) = \frac{0.40*0.05}{0.07} \approx 0.286$ and $P(L_1|U_2) \approx 0.071$(Alternatively $P(L_1|U_2) = 1 - P(U_1|U_2) - P(M_1|U_2)$)

2 The Unavoidable Drawing from a Box (or urn)

There's a box with N marbles, of which M are black(b) and the rest are white(w).

- Q1. What is the probability that you draw a black marble on the rst draw?

The probabilty equals to the proportion of black marble in all marbles: $P(b) = \frac{\#(b)}{\#(total)} = \frac{M}{N}$

- Q2. What is the probability that you draw a black marble on the second draw, if you know that a black marble was drawn on the rst draw and not replaced?

After drawing out a black marble we have one less: $P(b|b) = \frac{\#(b)-1}{\#(total)-1} = \frac{M-1}{N-1}$

- Q3. What is now the probability that you draw a black marble on the second draw if you are not told what colour was picked on the rst draw (and that rst marble was also not replaced)? Hint: The result is counter-intuitive. Apply the law of total probability!

$P'(b) = P(b) * P(b|b) + P(w) * P(b|w) = \frac{M*(M-1)}{N*(N-1)} + \frac{(N-M)*M}{N*(N-1)} = \frac{M}{N}$

3 A Small Pictoral Atlas of Propability Distributions

We investigate various (univariate) probability distributions: Poisson, Beta, χ^2

- General information of these distributions:

Distribution	PDF	contin./disc.	domain of definition	expectation value	variance
Poisson	$\frac{\lambda^k e^{-\lambda}}{k!}$	discrete	$k \in \mathbb{N}$	λ	λ
Beta	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$	continuous	$x \in [0,1]$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
χ^2	$\frac{1}{2^{k/2}\Gamma(k/2)}x^{k/2-1}e^{-x/2}$	continuous	$x \in [0,+\infty]$	k	$2k$

where $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$. The Gamma Function is defined as $\Gamma(n) = (n-1)!$ if n is a positive integer, generally $\Gamma(x) = \int_0^\infty x^{z-1}e^{-x}dx$. For χ^2 if $k = 1, 0$ is not allowed in the domain.

- The poisson distribution gives the probability of k events occurring in a fixed interval of time or space. Each event occurs with a known constant rate and they are mutually independent.

The beta distribtution is a family of continuous probability distribution defined on [0, 1] with two parameters α and β. It can be used to model the behavio of random variables that has a limited range.

The χ^2 distribution gives the distribution of sum of k independent standard normal random variables.

- Then we create gures illustrating their dependence on parameters

Try to write a code suciently general being able to handle all cases.

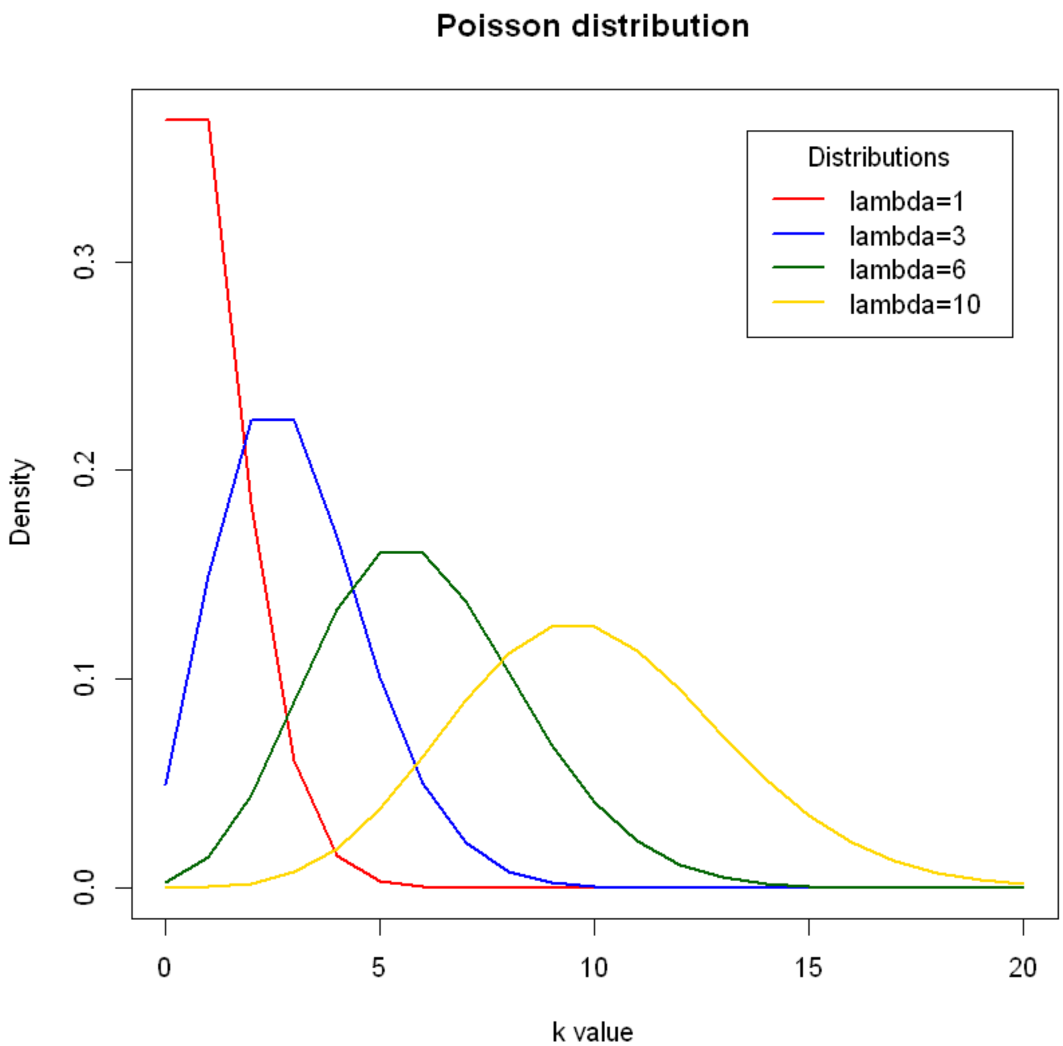
In [1]:
```
# Poisson distribution
k <- seq(0, 20, 1)
lambda <- c(1, 3, 6, 10)
```

```
colors <- c("red", "blue", "darkgreen", "gold")

plot(k, dpois(k, 1), type="l", lty=2, xlab="k value", ylab="Density", main="Poisson dis

for (i in 1:4){
  lines(k, dpois(k,lambda[i]), lwd=2, col=colors[i])
}

legend("topright", inset=.05, title="Distributions",
  c("lambda=1", "lambda=3", "lambda=6", "lambda=10"),  lwd=2, lty=c(1, 1, 1, 1), col=co
```



In [2]:
```
# Beta distribution
x <- seq(0, 1, length=50)
```

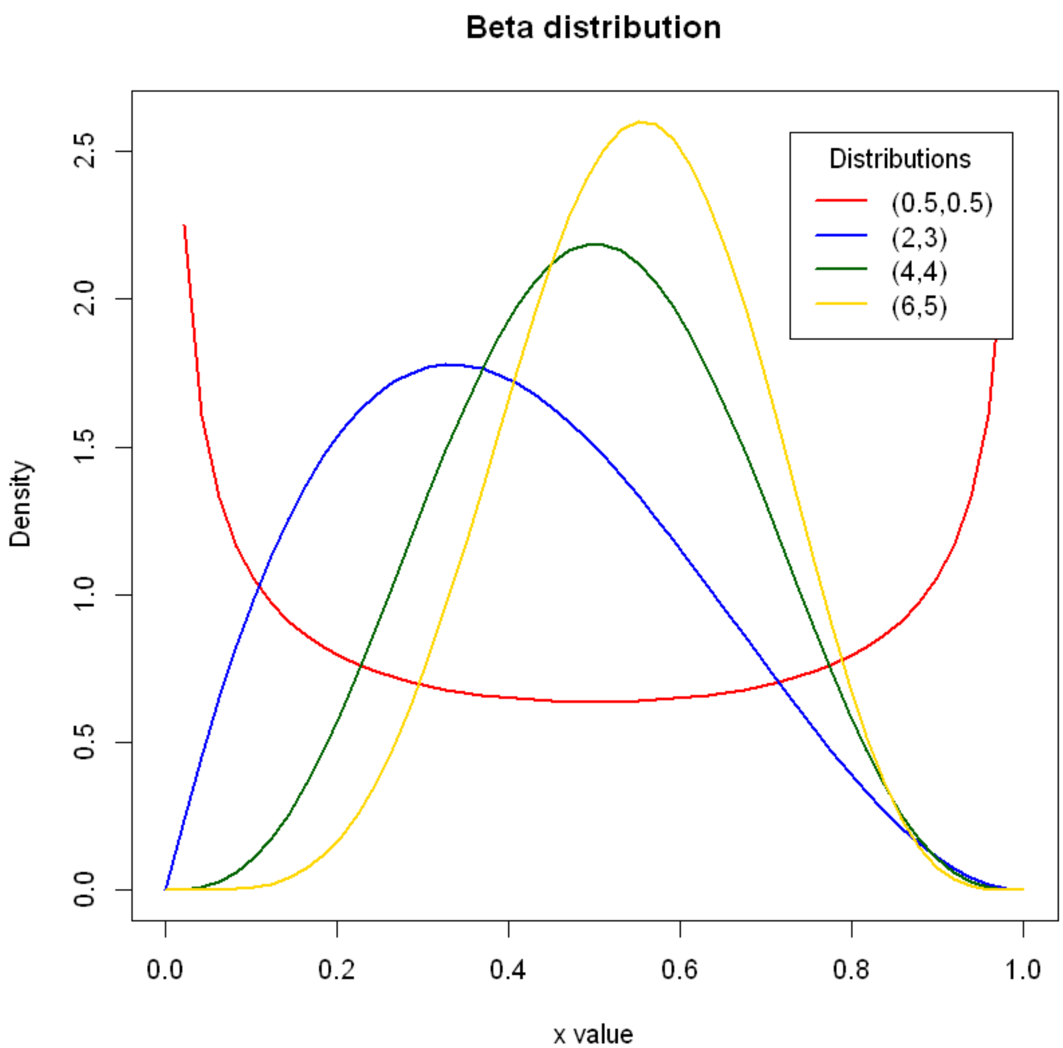
```r
a <- c(0.5, 2, 4, 6)
b <- c(0.5, 3, 4, 5)

colors <- c("red", "blue", "darkgreen", "gold")

plot(x, dbeta(x,6,5), type="l", lty=2, xlab="x value", ylab="Density", main="Beta dist

for (i in 1:4){
  lines(x, dbeta(x,a[i],b[i]), lwd=2, col=colors[i])
}

legend("topright", inset=.05, title="Distributions",
  c("(0.5,0.5)", "(2,3)", "(4,4)", "(6,5)"),  lwd=2, lty=c(1, 1, 1, 1), col=colors)
```



In [11]:
```
# chi^2 distribution
y <- seq(0, 8, length=50)
df <- c(2,3,4,6)
colors <- c("red", "blue", "darkgreen", "gold")

plot(y, dchisq(y, df=2), type="l", lty=2, xlab="x value", ylab="Density", main="chi^2

for (i in 1:4){
  lines(y, dchisq(y,df[i]), lwd=2, col=colors[i])
}

legend("topright", inset=.05, title="Distributions",
  c("df=2","df=3","df=4","df=6"),  lwd=2, lty=c(1, 1, 1, 1), col=colors)
```