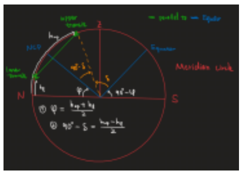
Summary

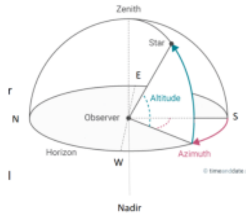
17 December 2019 13:47

- Diurnal motion
  - Rotation of the earth: east to west
    - Equator $v_{eq} = \frac{2\pi R_{eq}}{P_{earth}} \approx 465$ m/s
    - At latitude φ: $v = \cos\varphi\, v_{eq}$
  - Apparant rotation of the celestial sphere: west to east
    to observer at different latitude: -> figures
  - Visibility of stars
    - Star culminates at local meridian (⊥ plane of horizon, oriented S-N)
    - Condition of visibility $h_{*,max} = (90° - \varphi) + \delta > 0°$
    - Circumpolar star: $90° - \delta < \varphi$

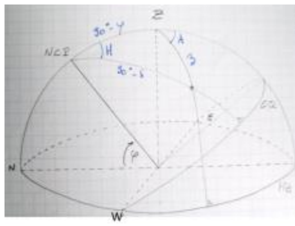
    -> estimation of φ, δ (declination of the star wrt celestial equator) from upper and lower transit:
    - If upper transit between NCP and zenith: $\varphi = \frac{h_{upper}+h_{lower}}{2}$, $\delta = 90° - \frac{h_{upper}-h_{lower}}{2}$
    - If upper transit south of zenith: $\delta = \frac{h_{upper}+h_{lower}}{2}$, $\varphi = 90° - \frac{h_{upper}-h_{lower}}{2}$



  -
  - Coordinate systems



System Name	Center	Fundamental circle	Zero point	"Longitude"	"Latitude"	Example
Horizontal (Alt-Azimuth)	Observer	Horizon of observer	South cardinal point	Azimuth A S: $A = 0°$, W: $A = 90°$	Altitude/Elevation/Height h $h > 0°$: above the horizon Zenith distance $z = 90° - h$	At star-rise, a star with $\delta = 0° \Rightarrow$ $A = 270°, h = 0°$
Equatorial (Hour)	Observer	Celestial equator	Intersection between local meridian & celestial equator	Hour angle H S: $H = 0°$, W: $H = 90°$	Declination δ (constant for a star) $\delta > 0°$: above the cel. Eq.	A culminating star $H = 0°$



Spherical triangle: $180° < A + B + C < 540°$
- Cosinus rule: $\cos a = \cos b \cos c + \sin b \sin c \cos A$
- Sine rule: $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$
- Less used: $\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$

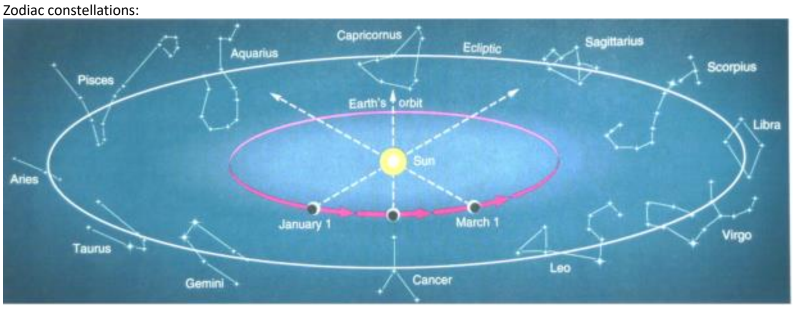
$\cos z = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos H$ -> how high in the sky an object ~ where (location of observer φ & location of star δ) & when (time of observation H)

Duration of visibility of stars $= 2H_{star\ set}$. $\cos(H_{set}) = \cos(z = 90°) = -\tan\varphi \tan\delta$
- $\delta = 0°$ → Duration of visibility = 12 h, always
- $\delta = 90° - \varphi$ → circumpolarity limit

Coordinate transformation: $\sin\delta = \sin\varphi \cos z - \cos\varphi \sin z \cos A$, $\cot H = \frac{\cos\varphi \cot z + \cos A \sin\varphi}{\sin A}$

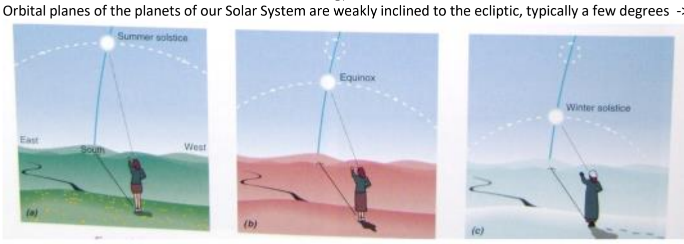
Star rise/set: $z = 90° \Rightarrow \cos A_{set} = -\frac{\sin\delta}{\cos\varphi} \Rightarrow$ independent of sign of the latitude

- Annual motion
  - Summer: brightest section of the Milky Way visible (-> direction of Sagittarius); Winter: faintest section
    Earth moves around the Sun counterclockwise $v \cong 30$ km/s
    Zodiac constellations:



Sidereal day (23h56m04s) < solar day (24h00m00s); stars move westwards wrt the Sun
e.g. constellations which are high in the sky in the middle of a summer night will be located, at the same time of the day, a long the west horizon in autumn and will have disappeared under the plane of horizon in winter

  - Obliquity (tilt): tropic of cancer -> Summer solstice $\delta_{\odot,max} = \epsilon \cong 23.5°$, tropic of Capricorn -> winter solstice; Spring & Autumn equinoxes $\delta_{\odot} = 0°$
    Orbital planes of the planets of our Solar System are weakly inclined to the ecliptic, typically a few degrees -> on celestial sphere, planets wander close to the ecliptic

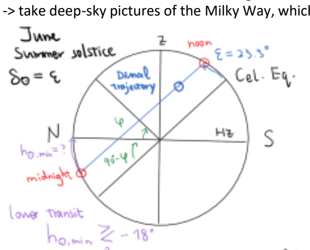


Position of the Sun at noon at (from left to right) the summer solstice, the fall equinox (and spring equinox), and winter solstice

At the summer solstice, the daily path of the Sun - as observed in the northern atmosphere - is higher and longer ('wider')

Twilight:
- Civil twilight $-6° < h_{\odot} < 0°$
- Nautical twilight $-6° < h_{\odot} < -12°$
- Astronomical twilight $-12° < h_{\odot} < -18°$

-> take deep-sky pictures of the Milky Way, which is high in the sky in June -> better take it in August in "astronomically dark"



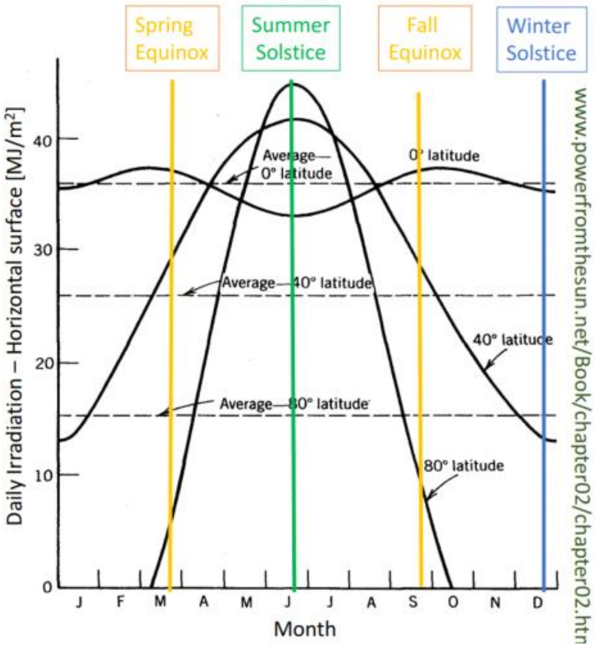
$h_{\odot,min} \gtrsim -18°$

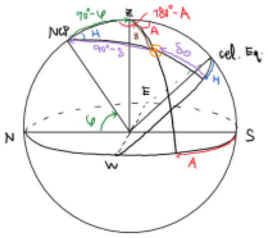
No astronomical night

$h_{\odot,min} = -(90° - \varphi - \delta_{\odot})$
$= -(90° - 50° - 23.5°) = \underline{-16.5°}$

HD

  - Solar constant (amount of solar radiation received by a 1m^2-surface (⊥ sunlight) on Earth: $C_{\odot} = 1365$ W/m^2
    Aphelion July: 1320 W/m^2, Perihelion Janurary: 1410 W/m^2





$\cos z = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos H$

$$W_\odot = \int_{\text{rising}}^{\text{setting}} F_\odot \, dt = \int_{-H_s}^{+H_s} F_\odot \frac{dt}{dH} dH = \frac{1}{\omega}\int_{-H_s}^{+H_s} C_\odot \cos z_\odot \, dH = \frac{2C_\odot}{\omega}\left(\sin\varphi \sin\delta_\odot \cdot H_s + \cos\varphi \cos\delta_\odot \sin H_s\right)$$

$H_s \leftarrow H_\odot$ at setting

$\cos H_s = -\tan\varphi \tan\delta_\odot$

$\frac{dt}{dH} = \frac{24h}{360°} = \frac{1}{\omega}$

$\underline{F_\odot = C_\odot \cos z_\odot}$

① $\varphi = +40°$, $\delta_\odot = +23.5°$ (Summer Solstice)

$H_s = 111.4° = 1.94$ rad

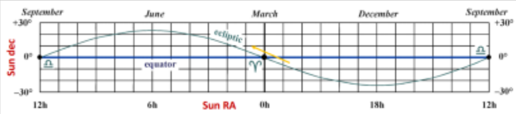
$W_\odot = \frac{2 \cdot 1320 \text{ W/m}^2}{\frac{2\pi}{24 \cdot 3600 s}} (0.49 + 0.65) = 41.8$ MJ (1m²)

② $\varphi = 40°$, $\delta_\odot = -23.5°$ (Winter Solstice)

$H_s = 68.6° = 1.20$ rad

$W_\odot = 13.4$ MJ (1410 W/m²)

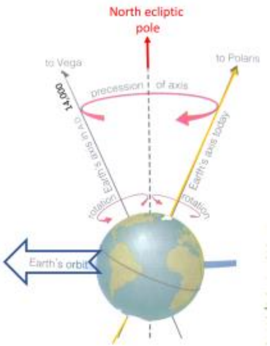
- More coordinate systems

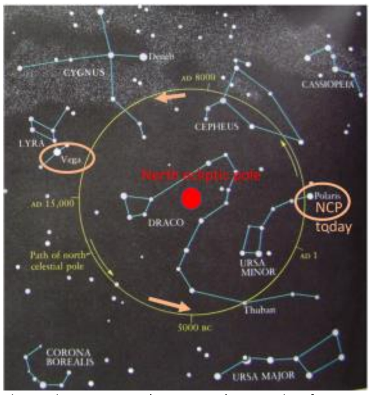
System Name	Center	Fundamental circle	Zero point	"Longitude"	"Latitude"	Example
Equatorial	Earth	Celestial equator	Vernal equinox	Right ascension α Counted +ly counterclockwise (eastwards)	Declination δ	
Ecliptic	Earth	Ecliptic	Vernal euinox	Ecliptic longitude λ Counted +ly counterclockwise	Ecliptic latitude β $\beta > 0°$: above ecliptic	A culminating star $H = 0°$

Transform: $\sin\beta = \cos\epsilon \sin\delta - \sin\epsilon \cos\delta \sin\alpha$, $\tan\lambda = \frac{\tan\delta \sin\epsilon + \sin\alpha \cos\epsilon}{\cos\alpha}$

** see application at the end

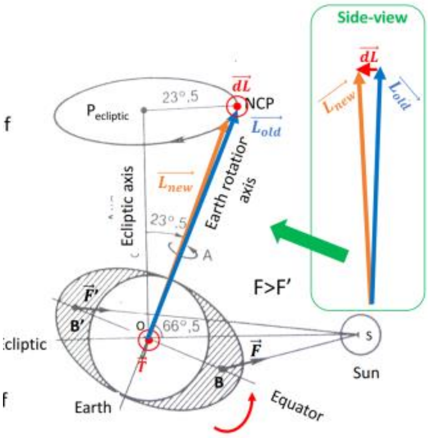
- Precession





The Earth rotation axis 'sweeps out' a conical surface

because of gravitational pull (Sun and Moon), 'tilt' between its rotation axis and the ecliptic, non -spherical shape (equatorial bulge), is rotating



$\vec{T} = \overrightarrow{OB} \times \vec{F} + \overrightarrow{OB'} \times \vec{F}$ The torque attempts to rotate the Earth s.t. the equatorial plane coincides with the ecliptic plane

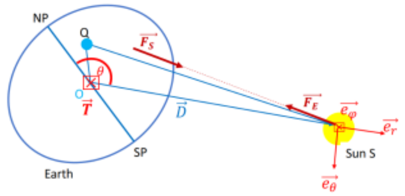
$\vec{L} = I\vec{\omega}, \frac{d\vec{L}}{dt} = \vec{T} \Rightarrow \vec{L}_{new} = \vec{L}_{old} + \vec{T} \cdot dt$

In 13000 years, orientations of $\vec{T}$ and of $d\vec{L}$ reversed

Calculation of the torque

$\vec{T}$ wrt an origin: Earth center.

Hypotheses: the Sun is the sole perturbuing body, the Earth is treated as a rigid and homogenous body (no oceans, no liquid c ore)



$d\vec{T} = \overrightarrow{OQ} \times \overrightarrow{dF_S} = \left(\vec{D} - \overrightarrow{QS}\right) \times \overrightarrow{dF_S} = \vec{D} \times \overrightarrow{dF_S} = -\vec{D} \times \overrightarrow{dF_E}$ (change subject => $\vec{T}$ as a function of the shape of the Earth)

Earth equatorial bulge, non-zero obliquity -> $\overrightarrow{F_E}$ not a central force -> $\vec{T} \neq 0$

The gravitational potential for an oblate body, per unit mass $V_E(D) = -\frac{GM_E}{D}\left[1 - \frac{C-A}{M_E D^2}\frac{1}{2}\left(3\cos^2\theta - 1\right)\right]$

$C = \frac{2}{5} M_E R_{Eq}^2 > A = \frac{1}{5} M_E \left(R_{Eq}^2 + R_P^2\right)$ (moment of inertia of the Earth **wrt its polar axis** > **wrt one of its equatorial axis** ($R_{Eq} = 6378.1\ km, R_p = 6356.7\ km$ **(**polar raidus))

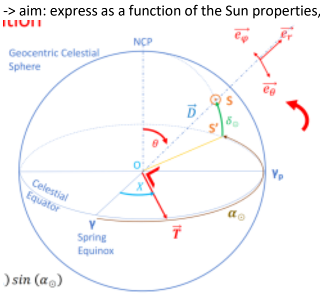
$\overrightarrow{F_E} = -M_S \cdot \overrightarrow{grad\, V_E} \Rightarrow \vec{T} = -\vec{D} \times \overrightarrow{F_E} = -3M_S \frac{G(C-A)}{D^3} \sin\theta \cos\theta\, \overrightarrow{e_\varphi}$

note: figure can depict any configuration in which the Sun is south of the equator, i.e. the equinox axis not necessarily $\perp$ the plane of the sheet

- $\theta = 90° \rightarrow$ equinox: $\vec{T} = 0$
- $0° < \theta < 90° \rightarrow$ Spring and summer: $\vec{T}$ anti-parallel to $\overrightarrow{e_\varphi}$
- $90° < \theta < 180° \rightarrow$ Fall and winter (this figure): $\vec{T}$ parallel to $\overrightarrow{e_\varphi}$

The region of the equatorial bulge facing the Sun tends to "fall" towards the Sun -> one can say, the Sun attempts to tug at the equatorial plane to make it contain the perturbing body

-> aim: express as a function of the Sun properties, e.g. inclination $\epsilon, \lambda_\odot$ Sun ecilptic longitude



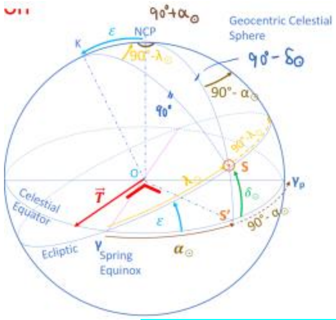
$)\, sin\,(\alpha_\odot)$

To get $\vec{T}$ as a function of the Sun equatorial coordinates $(\alpha_\odot, \delta_\odot)$, we now turn to a coordinate system $(O\gamma, O\gamma_p, O\text{-}NCP)$

- $\vec{T} = T\,(\cos(X), \sin(X), 0)$
- $0°<\theta<90°$: $\vec{e_\varphi} = (-\cos(X), -\sin(X), 0)$
- $X = \alpha_\odot - 90°$
- $\theta = 90° - \delta_\odot$

- $T_{O\gamma} \quad =. \quad 3GM_S\frac{(C-A)}{D^3}\cos(\delta_\odot)\, sin(\delta_\odot)\, sin\,(\alpha_\odot)$
- $T_{O\gamma,perp} = -3GM_S\frac{(C-A)}{D^3}\cos(\delta_\odot)\, sin(\delta_\odot)\, cos(\alpha_\odot)$
- $T_{O-NCP} = 0$

Genevieve Parmentier - Celestial M

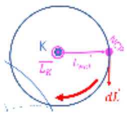


Triangular formula ⇒ $T_{O\gamma} = 3\frac{GM_S(C-A)}{4D^3}\sin 2\epsilon\left(1-\cos 2\lambda_\odot\right)$ = precession ($\lambda_\odot$-indep; steady) + longitude nutation ($\lambda_\odot$-dep; periodic) => both move the NCP along a circle centered on K

$T_{O\gamma,perp} = -3\frac{GM_S(C-A)}{2D^3}\sin\epsilon\sin 2\lambda_\odot$ = obliquity nutation ($\lambda_\odot$-dependent; periodic) => move the NCP raidally, i.e. in-/de-crease ϵ between NCP and K

$\vec{T} = T_{prec}\vec{e}_{O\gamma} + \left(T_{LN}\vec{e}_{O\gamma} + T_{ON}\vec{e}_{O\gamma,perp}\right)$

Equinoxes: $\lambda_\odot = 0°$ or $180° \Rightarrow T_{O\gamma} = T_P + T_{LN} = 0, T_{O\gamma,perp} = T_{ON} = 0$; solstices: $\lambda_\odot = 90°$ or $270° \Rightarrow T_{O\gamma} = T_{O\gamma,max} = 3\sin 2\epsilon\frac{GM_S(C-A)}{4D^3}\times 2, T_{O\gamma,perp} = 0$

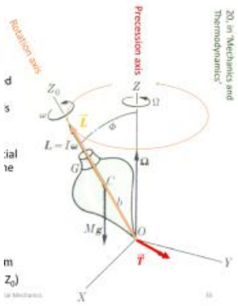


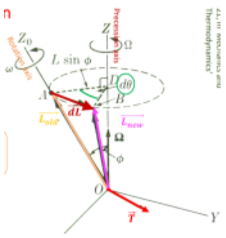
$\vec{T}\perp\vec{L} \Rightarrow \left|\vec{L}\right|$ = const. $\overrightarrow{L_K}$ = const. $\left|\vec{L}_{ecl}\right|$ = const. In figure: $d\vec{L} = dt\cdot\vec{T}_{O\gamma,precession}$

Estimate the precession period

For the spinning top: $\vec{L} = I\vec{\omega}, I$ = moment of inertia wrt axis of rotation, **valid if $\Omega \ll \omega$**, i.e. contribution of $\vec{\Omega}$ to $\vec{L}$ negligible

$\vec{T} = \overrightarrow{OC}\times M\vec{g} \Rightarrow \vec{T}\perp$ plane $OZZ_0 \Rightarrow \vec{L}$ change direction, not amplitude





$\Rightarrow \Omega = \frac{d\theta}{dt} = \frac{dL}{DA\cdot dt} = \frac{T}{DA} = \frac{Mg\,b\sin\phi}{I\omega\sin\phi} = \frac{Mg\,b}{I\omega}$ (T = const. $= T_{precession}$ => no nutation, because of the simplifying hypotheses we have made)

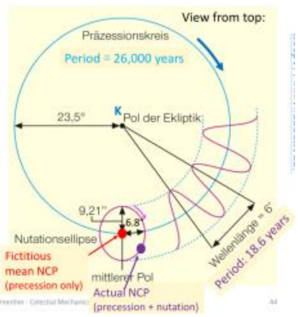
For the precessing Earth: $\Omega = \frac{T}{I\omega\sin\epsilon}, I = C$ = moment of inertia of the Earth wrt polar axis, $T = T_{precession} = 3\frac{GM_S(C-A)}{4D^3}\sin 2\epsilon$

$\Rightarrow \Omega = 3\frac{GM_S}{2D^3}\frac{(C-A)}{C}\frac{\cos\epsilon}{\omega} = 2.444\cdot 10^{-12}$ rad/s $= 15.9''$/year

$\frac{\left[\frac{M_M}{D_M^3}\right]}{\left[\frac{M_S}{D^3}\right]} = 2.178 \Rightarrow \Omega_{Moon-corrected} = -(1+2.17)15.9'' = -50.4''$/year, westwards (minus sign) => precession period ≅ 25700 years

Nutation

Motion of the NCP around K consists of: a steady circular motion caused by T_{prec} -> **precssion** motion of the mean NCP, a **nutation** ellipse caused by T_{LN}, T_{ON} (periodic terms)

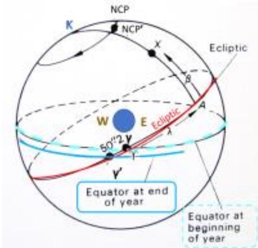


Nutation period: half a year; observed period: 18.6 years. => contribution from the Moon greatest, the plane of the Moon orb it about the Earth is itself precessing with a period of 18.6 years. The line of intersection of the Moon orbital plane with the ecliptic makes thus one complete revolution in 18.6 years.

The precession motion is not a perfect circle , because the ecliptic axis moves, too -> **planetary precession** cased by other planetsd of the Solar System

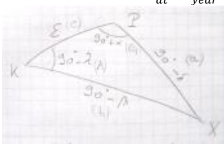
Consequence of precession

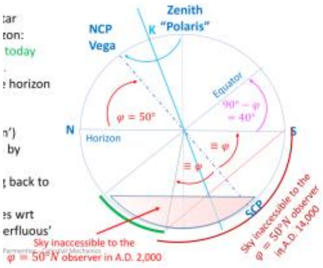
NCP & SCP move -> celestial equator moves, the equinoxes move "precession of the equinoxes" -> location of the Sun among the constellations at any particular time of the year changes with time. -> The γ-point **retrograde** -> ecliptic longitude λ increases; ecliptic latitude β does not change, assuming that the ecliptic plane is fixed in space; α increases, δ changes



-> necessary to quote the date (the "epoch") for which a particular set of coordinates is given, e.g. the year 2000

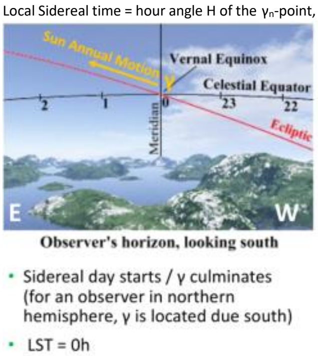
Assuming ϵ, β constant. $\frac{d\lambda}{dt} = \frac{50.2''}{year} \Rightarrow \frac{d\delta}{dt} = \sin\epsilon\cos\alpha\frac{d\lambda}{dt} = \frac{20.1''\cos\alpha}{year}$; $\frac{d\alpha}{dt} = [\tan\delta\sin\epsilon\sin\alpha + \cos\epsilon]\frac{d\lambda}{dt} = (20.1''\tan\delta\sin\alpha + 46.2'')/year$

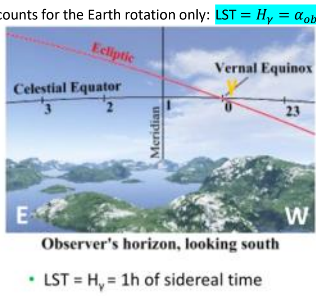


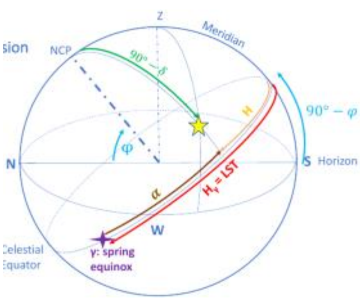


TODO: applications

- Time
  - Local Sidereal time = hour angle H of the γ_n-point, accounts for the Earth rotation only: $LST = H_\gamma = \alpha_{obj.} + H_{obj.}$.







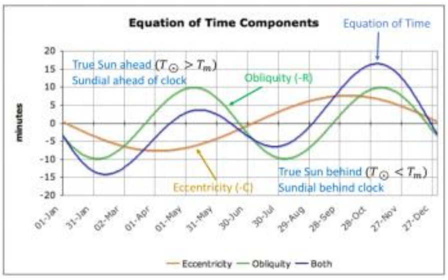
-> application: $HST\&(\alpha,\delta) \rightarrow$ visibility of the star, which stars are crossing the local meridian ($H_{object} = 0$)
Conversely, the RA of culminating objects gives the LST. e.g. $\alpha_{vega} = 18h36m56s$

Sidereal day = time-span between two consecutive culminations of mean γ-point across the observer's meridian; $LST = \alpha_\gamma + H_\gamma = 0 + 0 = 0$

Rigorously: Earth 360-rotation = 23 h 56 m 04, 098 s, sidereal time = 23 h 56 m 04, 090 s (shorter since γ_m move to the west $\left(\frac{50''}{365.25d}\right)\cos\epsilon = \frac{0.125''}{day} = 0.008s$

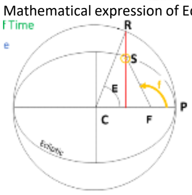
  - Solar time $T_\odot = H_\odot$, accounts for both the rotation and revolution of the Earth
But the Sun is a poor time keeper, because
    - the Earth orbital speed not constant (quickest at perihelion (on 1st April), slowest at aphelion (on 5th July)). $e \neq 0 \Rightarrow \frac{d\lambda_\odot}{dt} \neq const.$
    - the speed of the motion projected onto the equator is time-varying. $\delta_\odot = \delta_\odot(t) \Rightarrow \frac{d\alpha}{dt} = \frac{\cos\epsilon}{(\cos\delta_\odot)^2}\frac{d\lambda_\odot}{dt}$

=> mean Sun = moves along the equator with $\frac{d\alpha}{dt} = const.$ => our clock
$EqT = T_\odot - T_m \Rightarrow$ accounts for the irregularities of the motion of the true Sun projected onto the equator



E.g. end of July, EqT = -7 min: $T_\odot < T_m$ → the mean Sun is located westward of the true Sun
→ a sundial shows a true solar time $T_\odot$ that is 7 minutes behind the mean solar time T_m
(e.g. when the true Sun culminates: $T_\odot$ = 0h00m and T_m = 0h07m)

- Mathematical expression of EqT



E = Eccentric anomaly: Kepler => $E - e\sin E = nt$ (e = orbit eccentricity, for earth $e = 0.0167$; $n = \frac{2\pi}{P} \sim \leq 3600'' = 1°$ mean motion of the Sun about the Earth)

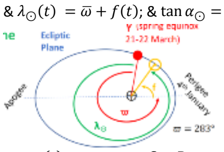
f = True anomaly; $\tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \cong \left(1 + e + \frac{1}{2}e^2\right)\tan\frac{E}{2} \Rightarrow f \cong E + esin\ E + \frac{e^2}{4}\sin 2E\ + o\left(e^3\right)$ using Taylor series of arctan

One can solve Eq.1 iteratively: $E = nt + e\sin(E)$
- 1st iter.: $\boxed{E\ = nt} + o(e)$
  This is the exact result for a circular orbit (e=0)
- 2nd iter.: $E\ = nt + e\sin(nt) + o(e^2)$
- 3rd iter.: $E\ = nt + e\sin[nt + e\sin(nt) + o(e^2)]$

and: $E(t)\ = nt + e\sin(nt) + \frac{e^2}{2}\sin(2nt) + o(e^3)$

-> $f\ = nt + 2e\sin nt + \frac{5}{4}e^2\sin 2nt + o\left(e^3\right) =: nt + C$

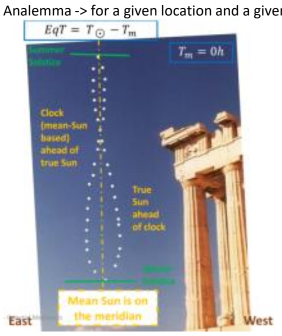
& $\lambda_\odot(t)\ = \bar{\varpi} + f(t)$; & $\tan\alpha_\odot = \cos\epsilon\tan\lambda_\odot \Rightarrow \alpha_\odot = \lambda_\odot - \tan^2\frac{\epsilon}{2}\sin 2\lambda_\odot = \lambda_\odot + R$



=> $\alpha_\odot(t) \cong \bar{\varpi} + nt + C + R$ gives the motion of the true Sun projected on the Equator
- $\alpha_\odot$: RA of Sun, measured from γ-point
- ϖ: longitude of Sun perigee, measured from γ-point in the ecliptic plane
- n: mean motion/angular speed of the Sun apparent motion
- C: correction for eccentricity (time-varying orbital speed) $C = 2esin(nt) + \frac{5}{4}e^2\sin(2nt) + o(e^3)$
- R: correction for obliquity (projection effect) $R = -tg^2(\frac{\epsilon}{2})\ sin(2\lambda_\odot)$

EqT $= \alpha_m - \alpha_\odot = -(C + R)$, where for mean Sun $e = 0 \Rightarrow C = 0; \epsilon = 0 \Rightarrow R = 0$

- Analemma -> for a given location and a given T_m, it gives the height of the true Sun and its variabtions all through the year, and EqT



- LCT = Local Civil Time = $T_m + 12h$ (Starts as the mean Sun does its lower transit of the local meridian)
  -> longitude dependent
  => standard time and time zones, $1h = 15°$
- UT = Universal Time = $LCT_{Greenwich}$ = $T_{m,Greenwich}$ + 12 h