Summary

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Item	Newtonian	SR	GR
Metric	Euclidean $(\delta_{ij}) = \begin{pmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{pmatrix}$	Minkowski $(\eta_{\mu\nu}) = \begin{pmatrix} -1\,0\,0\,0 \\ 0\;\;1\,0\,0 \\ 0\;\;0\,1\,0 \\ 0\;\;0\,0\,1 \end{pmatrix}$	General (g_{ij}) e.g. in spherical $(g_{\mu\nu}) = \begin{pmatrix} -1\,0\,0 & 0 \\ 0\;\;1\,0 & 0 \\ 0\;\;0\,r^2 & 0 \\ 0\;\;0\,0\,r^2\sin^2\theta \end{pmatrix}$
Lagrangian L	$L = T - V = \frac{1}{2}m\dot{\boldsymbol{x}}^2 - m\Phi$	$L = -mc^2 + \frac{1}{2}m\dot{\boldsymbol{x}}^2 - m\Phi$ (non-rel.)	
A free, point-like (non-relativistic) particle	$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$	$L = -mc^2\sqrt{1 - \frac{\dot{\boldsymbol{x}}^2}{c^2}} \approx -mc^2 + \frac{1}{2}m\dot{\boldsymbol{x}}^2$	$L = -mc\sqrt{g_{ij}\dot{x}^i\dot{x}^j}$
Action $S = \int_{t_1}^{t_2} L[\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t]dt$	$S = \frac{1}{2}m\int_{t_1}^{t_2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)dt$		$S = \frac{1}{2}m\int_{t_1}^{t_2} g_{ij}\dot{x}^i\dot{x}^j dt$
Principle of least action $\delta S = \int_{t_1}^{t_2}\left(\frac{\partial L}{\partial q^i}\delta q^i + \frac{\partial L}{\partial \dot{q}^i}\delta\dot{q}^i\right)dt = 0$	Euler-Lagrange eq. $\frac{\partial L}{\partial q^i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}^i} = 0$	$\frac{dp_\mu}{d\tau} = 0$ with $p_\mu = m\dot{x}_\mu = \left(-\frac{E}{c}, \boldsymbol{p}\right)$ where $\dot{x}^\mu = \frac{dx^\mu}{d\tau} = (\gamma c, \gamma\boldsymbol{v})$ with $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -c^2$	Geodesic eq. $\ddot{x} + \Gamma^i_{jk}\dot{x}^j\dot{x}^k = 0$ (=> way to calculate Γ^i_{jk})
Length of the trajectory curve $l = \int_\Gamma dl$	$l = \int_{t_1}^{t_2}\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}dt$	$l = \int_\Gamma \sqrt{-ds^2}$	$l = \int_\Gamma \sqrt{g_{ij}\dot{x}^i\dot{x}^j}dt$
Equation of motion	$m\ddot{\boldsymbol{x}} = -\nabla V$	Space-like/time-like/light-like trajectory	
			Christoffel symbols $\Gamma^i_{jk} = \frac{1}{2}g^{il}\left(g_{lk,j} + g_{jl,k} - g_{jk,l}\right)$ Covariant derivative $\nabla_\nu u^\mu = \frac{\partial u^\mu}{\partial x^\nu} + \Gamma^\mu_{\nu\rho}u^\rho$ Geodesic $\Leftrightarrow \frac{(\nabla_\nu u^\mu)dx^\nu}{d\tau} = 0$
A point-like particle in gravitational field	$L = \frac{1}{2}m\dot{\boldsymbol{x}}^2 + \frac{G_N Mm}{r}$	$L \approx -mc\sqrt{c^2 - \boldsymbol{v}^2 + 2\Phi}$	
		$S = -mc\int\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}dt$ with $(g_{\mu\nu}) = \begin{pmatrix} -(1 + \frac{2\Phi}{c^2})\,0\,0\,0 \\ 0\;\;1\,0\,0 \\ 0\;\;0\,1\,0 \\ 0\;\;0\,0\,1 \end{pmatrix}$ => metric "absorb" gravitational field	
Equation of motion	$L_z = mr^2\dot{\phi} = const.$ $E = T + V = \frac{1}{2}m\dot{r}^2 + \frac{L_z^2}{2mr^2} - \frac{G_N Mm}{r} = const.$		