

INTRODUCTION

Problem Description

Given a Lake-Reservoir system, which supplies water from the lake to a small farm.

There is a retaining wall located at the upstream face of a lake in the shape of parabola. Water stands up to 'd' m above base.

So, our objective is to find the resultant force acting on the retaining wall and respective center of pressure of both horizontal and vertical forces.

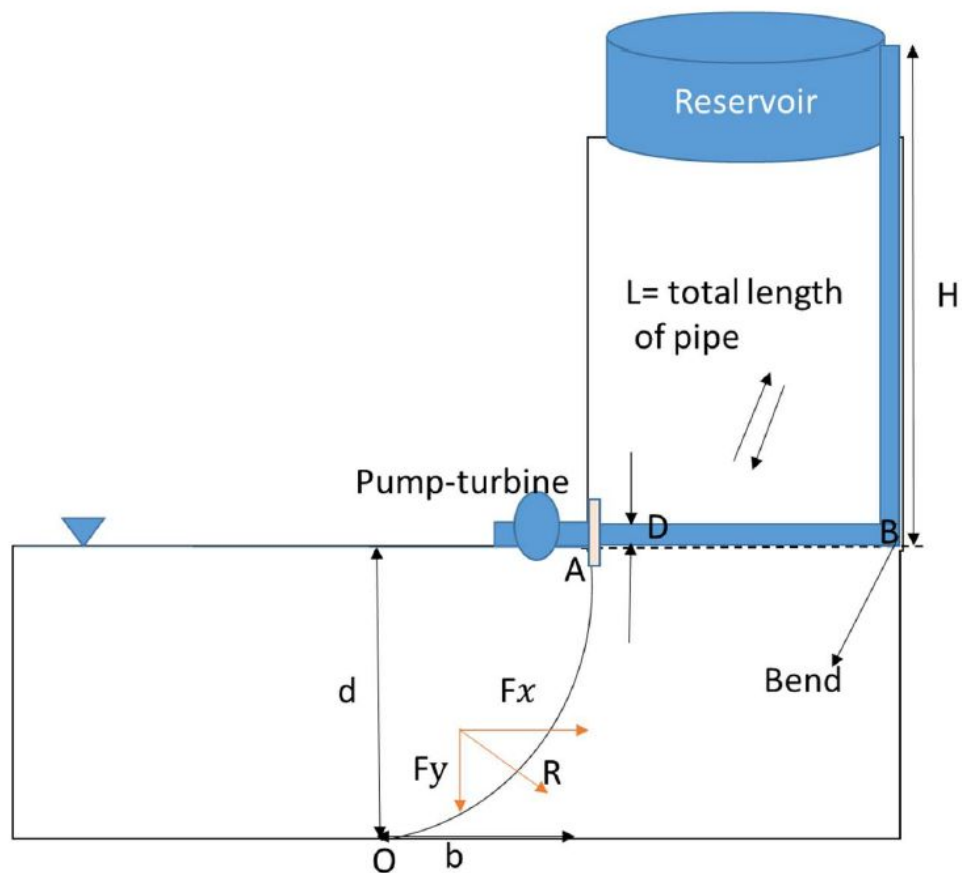


Figure 1 Lake and Reservoir system

Input Parameters:

b(m)	d(m)	Q(m ³ /s)	H(m)	L(m)	D(m)
5	25	2	95	200	1.7

ASSUMPTIONS

Acceleration due to gravity = 9.81 m/s²

The density and viscosity of water are taken to be 1000 kg/m³ and 0.00131 Kg/m · s, respectively.

PROPERTIES

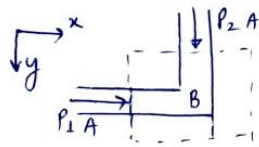
Bernoulli's Equation: For a steady, irrotational flow of non-viscous, incompressible fluids

$$P/\rho g + v^2/2g + h = H$$

Values from Previous Assignment:

- Velocity of water moving inside the pipe = $Q / A = 0.88$ m/s
- Area of the pipe = $\pi * D^2 / 4 = \pi * 1.7^2 / 4 = 2.27$ m²
- The pressure head required by pump to lift water from lake to reservoir = 95.07m

ANALYSIS & CALCULATION



In x-direction:

$$\sum F_x = P_1 A_1 + (\rho Q V \frac{dV}{dx} - 0)$$

$$\Rightarrow F_x = P_1 A_1 + \rho Q V \quad \text{--- (I)}$$

In y-direction:

$$F_y = P_2 A + (\rho Q V - 0) + W = P_2 A + \rho Q V + W \quad \text{--- (II)}$$

↖ weight of water
in y-direction

Using Bernoulli's Theorem:

$$\frac{P_{atm}}{\rho g} + \frac{P_{pump}}{\rho g} = \frac{P_1}{\rho g} + \frac{V^2}{2g}$$

$$\Rightarrow \frac{(1.01 \times 10^5)}{9810} + (95.07) = \frac{P_1}{9810} + \frac{(0.88)^2}{2 \times 9.81}$$

$$\Rightarrow P_1 = 10.33 \times 10^5 \text{ Pa}$$

$$\text{Now, } P_2 = P_{atm} = 1.01 \times 10^5 \text{ Pa}$$

From (I): $F_x = (10.33 \times 10^5)(2.27) + (1000)(2)(0.88) \text{ N}$

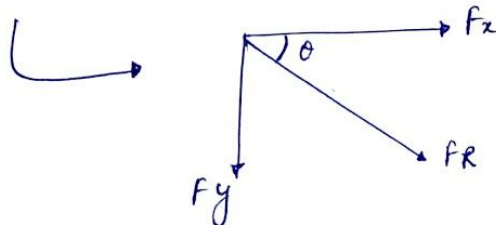
$$F_x = 2346.67 \text{ kN}$$

From (II): $F_y = (1.01 \times 10^5)(2.27) + (1000)(2)(0.88) + (9810)(2.27)(95)$

$$F_y = 2346.55 \text{ kN}$$

∴ Resultant force, $F = \sqrt{F_x^2 + F_y^2} = 3318.67 \text{ kN}$

$$\theta = \tan^{-1} \left(\frac{2346.55}{2346.67} \right) = 44.998^\circ$$



(4) The rated power of the pump required to lift water from lake to reservoir, P

$$= \rho g Q H_{\text{pump}} / \eta_{\text{pump-motor}} \quad (\eta_{\text{pump-motor}} = 0.85, \text{ given})$$

$$= 1000 * 9.81 * 2 * 95.07 / 0.85 \text{ W} = 21.94 * 10^5 \text{ W}$$

(5) Given :

- The farmland converted to industrial area and the power requirement to industry is increased during day compared to night.
- The selling electricity power during day and night are 10 rupees per kWh and 5 rupees per kWh.
- Assume the same flow rate of Q m³/sec can be used in either direction.
- The system operates for 11 hour each in the pump and turbine modes during a typical day (24hr).
- Assume the combined turbine-generator efficiency as 0.75.
- Electric power output from the turbine (P) = $\rho g Q * H_{\text{Turbine}} * \eta_{\text{Turbine}} * \eta_{\text{generator}}$

Calculations:

At point A and R(just at top of reservoir), Pressure = $P_A = P_R = P_{\text{atm}} = 1.01 * 10^5$

Velocity at A, $v_A = 0 \text{ m/s}$; Velocity at R, $v_R = 0.88 \text{ m/s}$

Bernoulli theorem between A and R:

$$P_{\text{atm}} / \rho g + 0^2/2g + 0 + P_{\text{pump}} / \rho g = P_{\text{atm}} / \rho g + v^2/2g + \text{Losses(Major \& Minor)} \quad (1)$$

During Daytime, water flows from reservoir to lake:

$$P_{\text{atm}} / \rho g + 0^2/2g + H = P_{\text{atm}} / \rho g + v^2/2g + \text{Losses(Major \& Minor)} + P_{\text{turbine}} / \rho g \quad (2)$$

Equating, eqn. (1) & (2):

$$2H = P_{\text{turbine}} / \rho g + P_{\text{pump}} / \rho g$$

From previous assignments, we have $P_{\text{pump}} / \rho g = 95.07 \text{ m}$

Therefore, $P_{\text{turbine}} / \rho g = 2(95) - 95.07 = 94.93 \text{ m}$

$$\text{Electric power output from the turbine (P)} = \rho g Q * H_{\text{Turbine}} * \eta_{\text{Turbine}} * \eta_{\text{generator}}$$

$$= 1000 * 9.81 * 2 * 94.93 * 0.75 = 1396.895 \text{ kW}$$

As Turbine works in day time, Total revenue generated in day-time on a typical day

$$= 1396.895 * 11 * 10 \text{ INR} = 153658.45 \text{ INR}$$

During Night, there is Power required to pump water from lake to reservoir, P

$$= \rho g Q H_{\text{pump}} / \eta_{\text{pump-motor}} \quad (\eta_{\text{pump-motor}} = 0.85, \text{ given})$$

$$= 1000 * 9.81 * 2 * 95.07 / 0.85 \text{ W} = 2194.439 \text{ kW}$$

Total expenses in night on a typical day = $2194.439 * 11 * 5 \text{ INR} = 120694.14 \text{ INR}$

∴ Net revenue generated on a single-day = $153658.45 - 120694.14 \text{ INR} = 32964.31 \text{ INR}$

∴ The potential revenue this pump-turbine system can generate per year
= $32964.31 \text{ INR} * 365 = 12031973.15 \text{ INR}$

CONCLUSION

1. The resultant force acting on bend = **3318.61 kN**, at an angle of **44.998 degrees** with horizontal.
2. The rated power of the pump required to lift water from lake to reservoir, $P = 21.94 * 10^5 \text{ W}$
3. The potential revenue this pump-turbine system can generate per year = $32964.31 \text{ INR} * 365 = 12031973.15 \text{ INR}$

24th September, 2020

CE 223

1. INTRODUCTION

A. Problem Description:

Given a Lake-Reservoir system, which supplies water from the lake to a small farm.

There is a retaining wall located at upstream face of a lake in the shape of parabola. Water stand upto 'd' m above base.

So, our objective is to find the resultant force acting on the retaining wall and respective center of pressure of both horizontal and vertical forces.

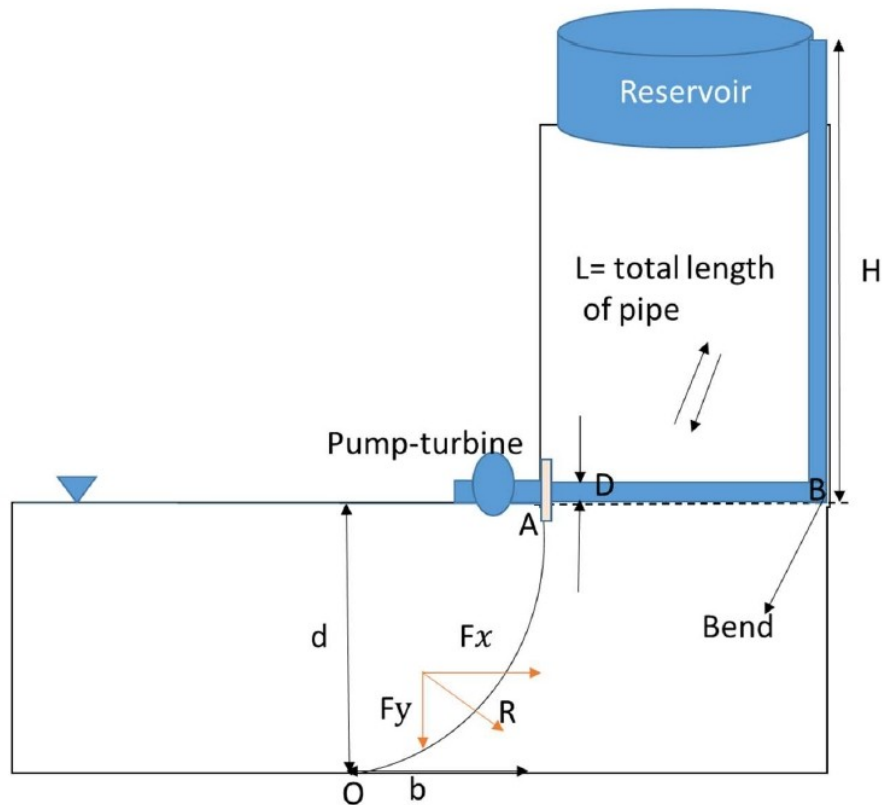


Figure 1 Lake and Reservoir system

B. Input Parameters:

b(m)	d(m)	Q(m ³ /s)	H(m)	L(m)	D(m)
5	25	2	95	200	1.7

2. ASSUMPTIONS

Retaining wall is friction less. So, in calculation of resultant forces on retaining wall we will neglect frictional forces.

Width (w) of retaining wall is assumed to be 1 meter.

Acceleration due to gravity = $9.81 \text{ m}^2/\text{s}$

The density and viscosity of water are taken to be $1000 \text{ kg}/\text{m}^3$ and $0.00131 \text{ Kg}/\text{m}\cdot\text{s}$, respectively.

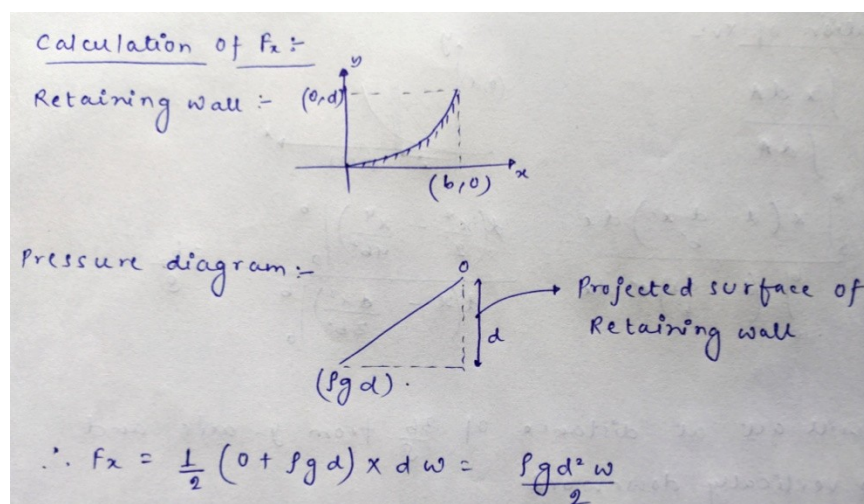
3. PROPERTIES

The horizontal component of the hydrostatic force on submerged curved surface is equal to the hydrostatic force acting on the projected plane surface in that direction and it acts through the center of pressure of projected area.

The vertical component of the hydrostatic force on a submerged curved surface is equal to the weight of the liquid volume vertically the solid surface of the liquid and acts through the center of gravity of the liquid in that volume.

4. ANALYSIS

A. Calculation of F_x :

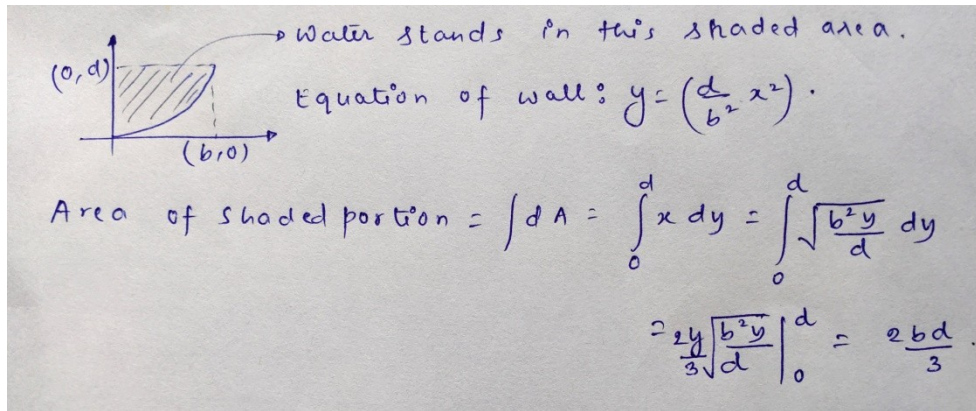


$$F_x = (\rho g d + 0) * d * w / 2 = \rho g d^2 w / 2 = (1000 * 9.81 * 25 * 25 * 0.5) \text{ N} = 3062.5 \text{ kN}$$

$$\text{Center of pressure of projected area} = d / 3 \text{ (above base)} = 25 / 3 \text{ m} = 8.33 \text{ m}$$

$\therefore F_x$ will act at $d/3$ distance from base.

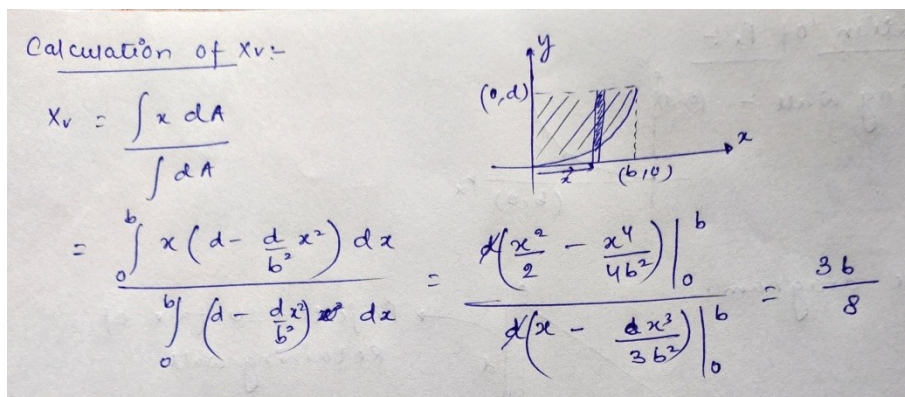
B. Calculation of F_y :



$$\text{Weight of water above retaining wall} = 2bdw\rho g / 3 = (2 * 5 * 25 * 1 * 1000 * 9.81 / 3) \text{ kN}$$

$$\therefore F_v = \text{Weight of water above retaining wall} = 2bdw\rho g / 3 = 817.5 \text{ kN}$$

i. Calculation of X_v :

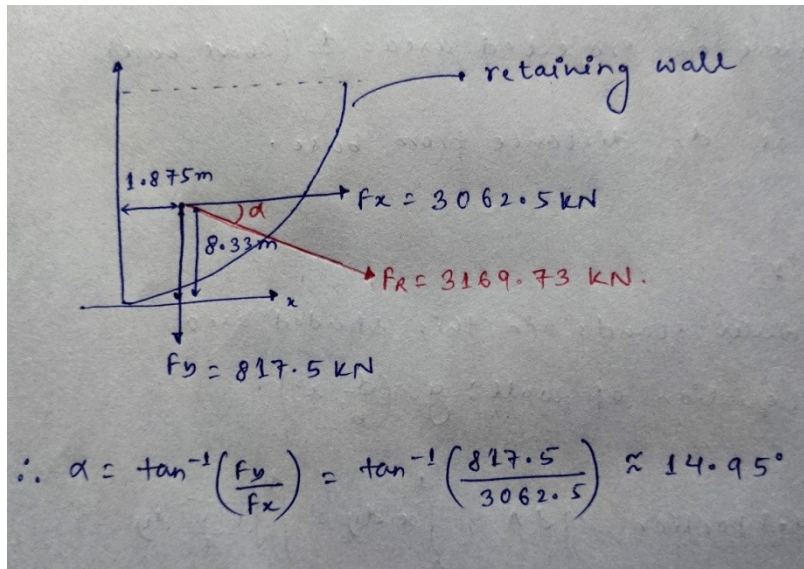


$$X_v = \frac{\int x dA}{\int dA} = \frac{\int_0^b x \left(d - \frac{d x^2}{b^2}\right) dx}{\int_0^b \left(d - \frac{d x^2}{b^2}\right) dx} = 3b / 8 = (3 * 5 / 8) \text{ m} = 1.875 \text{ m}$$

$\therefore F_y$ will act at a distance of $3b/8$ from y-axis and acts in a vertically downward direction.

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{3062.5^2 + 817.5^2} \text{ kN} = 3169.73 \text{ kN}$$

Fig: Diagram showing hydrostatic forces acting on retaining wall



5. CONCLUSION

Value of $F_x = 3062.5 \text{ kN}$ and center of pressure for horizontal direction is 8.33 m above the base.

Value of $F_y = 817.5 \text{ kN}$ and center of pressure for vertical direction is 1.875 m away from y-axis.

Value of $F_R = 3169.73 \text{ kN}$ at an angle of 14.95° with F_x .

Here, the value of resultant forces on retaining wall is not as accurate as in real life, because there will be frictional forces and atmospheric pressure which we have neglected.