# Markov Chain Monte Carlo

ISC 5225/4933 Midterm

Oct 24, 2023

### Multidimensional Distributions and Importance Sampling

Consider a joint probability distribution $\pi_{\text{joint}}(x_1, x_2)$ corresponding to independent random variables $x_1$ and $x_2$.

Both $x_1$ and $x_2$ come from the *same* exponential distribution $\pi(x) = e^{-x}$, for $x \geq 0$. Thus,

$$\pi_{\text{joint}}(x_1, x_2) = \pi(x_1)\pi(x_2) = e^{-x_1} \cdot e^{-x_2} = e^{-(x_1+x_2)}. \tag{1}$$

In class we learned that random variables from an exponential distribution $\pi(x) = e^{-x}$ can be sampled using standard uniform random numbers $u \sim U[0, 1]$ by setting

$$x = -\log(1 - u). \tag{2}$$

(i) Use this to generate $N = 5000$ samples of $(x_1, x_2)$ from $\pi_{\text{joint}}(x_1, x_2)$. Report the average $(\bar{x}_1, \bar{x}_2)$ and the standard deviation $(\sigma_{x_1}, \sigma_{x_2})$ of these samples.

(ii) What is the conditional PDF $\pi(x_1|x_2)$?[1]

(iii) Suppose we are interested in evaluating the double integral,

$$I = \int_0^\infty \int_0^\infty e^{-(x_1+x_2)^2} x_1^2 \, dx_1 dx_2 \tag{3}$$

using importance sampling. Inspecting the form of the integrand, we guess that $\pi_{\text{joint}}(x_1, x_2)$ is be a good candidate for importance sampling.[2]

Using $N = 500$, estimate the integral and associated error using importance sampling.[3]

Show your work on the answer sheet, and upload your code on Canvas.

(iv) Someone suggests using a two-dimensional normal or Gaussian distribution instead of $\pi_{\text{joint}}(x_1, x_2)$ from eqn 1. Is this a good suggestion? Comment.

Points: (i) 20, (ii) 10, (iii) 55, (iv) 15.

[1] You should not need to perform derivations for this part, but may if that suits you.

[2] We already figured out how to sample this distribution in part (i).

[3] The true value of the integral is $I = 1/6$.