Scientific Visualization

Homework: Transformation

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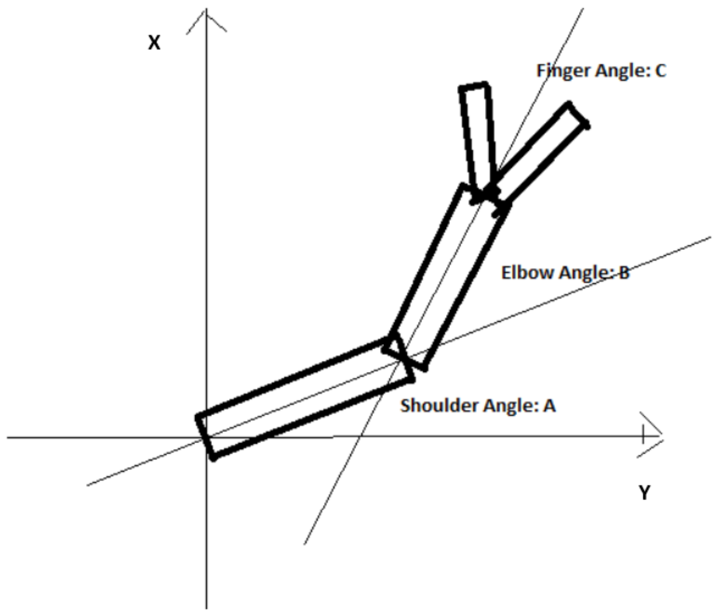
1 Introduction

In this assignment, we aim to transform a unit cube into a simple robot with a front arm, lower arm, and two fingers. The unit cube, centered at the origin, has a side size of 1. Each component—upper arm (2x0.4x0.4), lower arm (2x0.4x0.4), and fingers (1x0.2x0.2)—is subject to rotation around the z-axis. The upper arm rotates by angle A, the lower arm by B related to the upper arm, and the fingers open by angle C. Our task is to calculate the transformation matrices for these rotations, unraveling the steps involved in achieving the desired robot configuration.



Note: The orientation of axes is assumed since it is not explicitly mentioned in the original diagram.

2 Lower Arm

We will start with a unit cube placed at the origin. This cube first needs to be scaled to $2 \times 0.4 \times 0.4$ and also needs to be rotated with an angle A. We will use the following matrices to transform the cube.

2.1 Scaling Matrix

$$S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.2 Translation Matrix

After scaling we need to align the bottom face of the cube with the origin, for that, we will use the following translation matrix.

$$T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.3 Rotation Matrix

To complete the transformation of the unit cube, we will apply a rotation about the Z-axis. Let's denote the rotation angle by A. The rotation matrix for a rotation angle A will be:

$$R = \begin{bmatrix} \cos(A) & -\sin(A) & 0 & 0 \\ \sin(A) & \cos(A) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

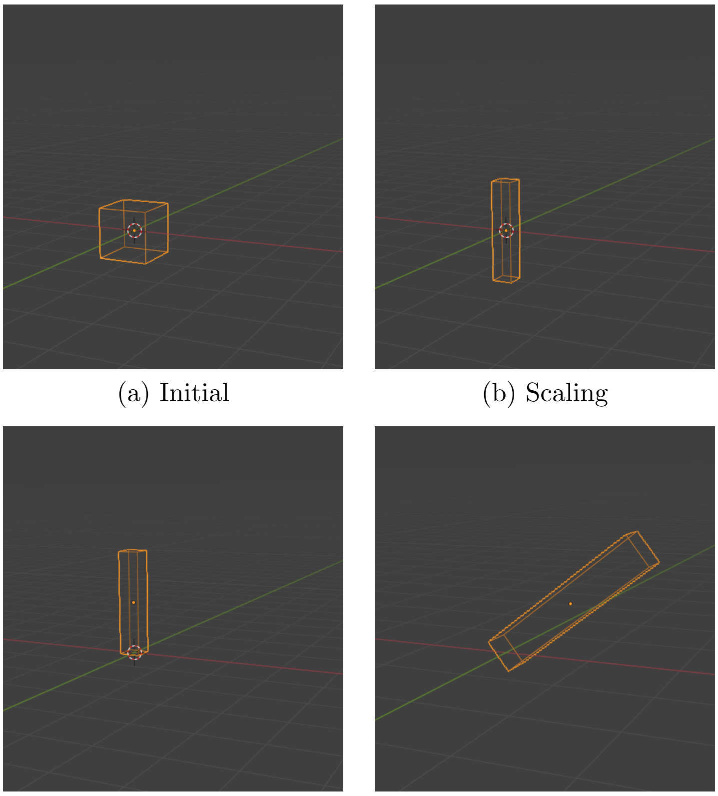
2.4 Final Transformation Matrix

The transformation that we will apply will look like this,

$$M_U = R(0,0,A)T(1,0,0)S(2,0.4,0.4)$$

Our final transformation matrix for creating the lower arm from the unit cube will be:

$$M_U = \begin{bmatrix} \cos(A) & -\sin(A) & 0 & 0 \\ \sin(A) & \cos(A) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



(a) Initial (b) Scaling

(c) Translation (d) Rotation

Figure 1: Lower Arm

3 Upper Arm

For the upper arm, we will start with a unit cube at origin and apply the same scaling and translation as the lower arm. However, we must apply a different rotation to rotate it around the Z-axis. This matrix will look like:

$$R = \begin{bmatrix} \cos(B) & -\sin(B) & 0 & 0 \\ \sin(B) & \cos(B) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

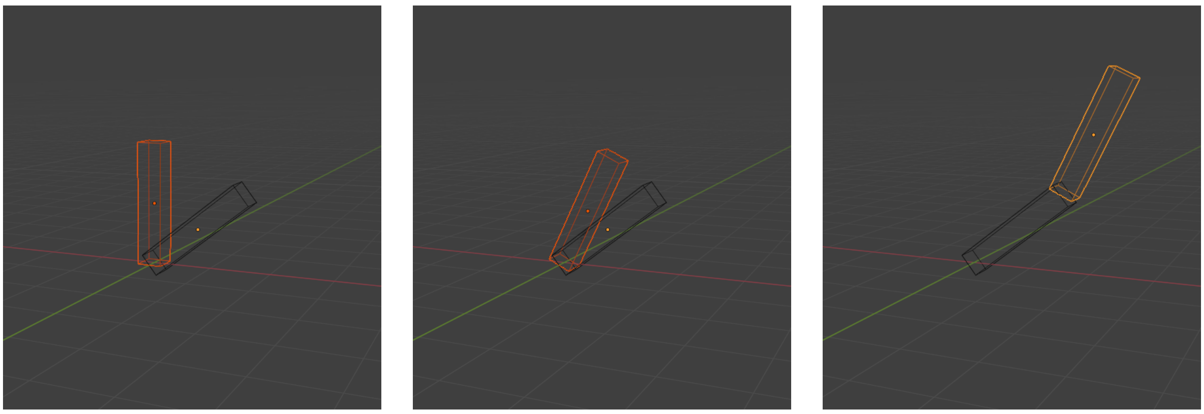
Also, the upper arm is connected to the lower arm, which means the top face of the lower arm and the bottom face of the upper arm intersect at $X = 2 \times sin(A)$ and $Y = 2 \times cos(A)$ (Multiplying by 2, since it is the arm length), which means we have to transform the upper arm using following matrix.

$$T = \begin{bmatrix} 1 & 0 & 0 & 2sin(A) \\ 0 & 1 & 0 & 2cos(A) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The final transformation matrix will be,

$$M_L = T(2sin(A), 2cos(A))[T(1,0,0)S(2,0.4,0.4)R(0,0,B)]$$

$$M_L = \begin{bmatrix} 1 & 0 & 0 & 2sin(A) \\ 0 & 1 & 0 & 2cos(A) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(B) & -\sin(B) & 0 & 0 \\ \sin(B) & \cos(B) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)$$



(a) Scale & Translate (b) Rotate (c) Translate

Figure 2: Upper Arm

4 Fingers

Both the figures have the same size, which means we can use the same scaling matrix for both fingers, which will be,

$$S_F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Also, both of the fingers are connected to the top face of the upper arm, which means we can apply the same translation matrix to both,

$$T_F = \begin{bmatrix} 1 & 0 & 0 & 2sin(A) + 2sin(B) \\ 0 & 1 & 0 & 2cos(A) + 2cos(B) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since the given angle C is between two fingers, we will assume that the angle between the two fingers is symmetric to the line passing through the upper arm and those are aligned to the upper arm, which means each finger is relatively rotated to the upper arm and needs to be rotated by $C/2$.

For the first finger,

$$R_{F1} = \begin{bmatrix} \cos(B + (C/2)) & -\sin(B + (C/2)) & 0 & 0 \\ \sin(B + (C/2)) & \cos(B + (C/2)) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And for the second finger,

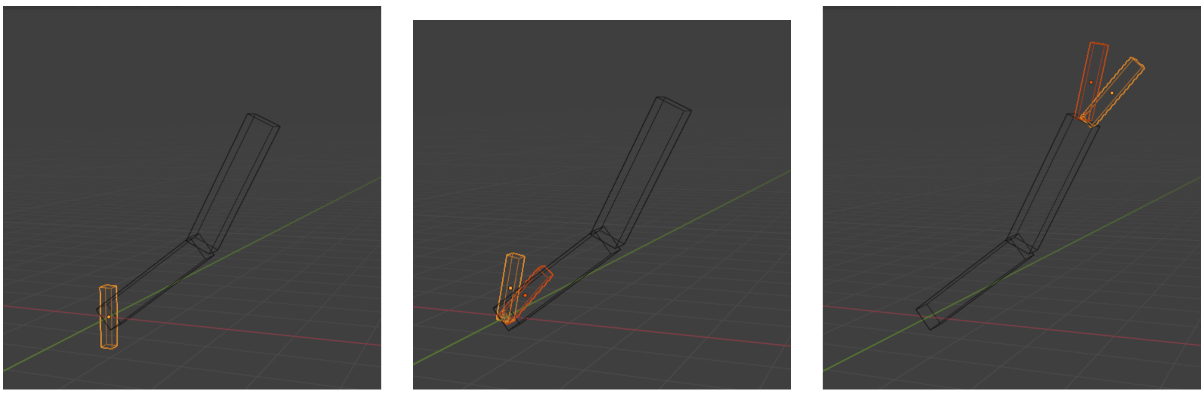
$$R_{F2} = \begin{bmatrix} \cos(B - (C/2)) & -\sin(B - (C/2)) & 0 & 0 \\ \sin(B - (C/2)) & \cos(B - (C/2)) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The final transformation for the first finger will be,

$$M_{F1} = T_F S_F R_{F1}$$

The final transformation for the second finger will be,

$$M_{F2} = T_F S_F R_{F2}$$



(a) Scale (b) Rotate (c) Translate

Figure 3: Fingers