

Make sure your name is on all reports and code files.

The use of AI is permitted to solve the following two questions. Please document all your functions (explain what each argument does). Each question should have its own corresponding code and results, each in its own folder (labelled Question 1 and question 2). These two folders live in a folder labeled with your name. For example, if I was submitting this homework, my folder structure would be:

Top_folder: Gordon_Erlebacher_Aug28_2023/ (use the date of submission)
contains the folders:

Question1/ and Question2/

You then zip the top folder:

`zip -r Gordon_Erlebacher_Aug28_2023.zip Gordon_Erlebacher_Aug28_2023`

The -r option will recursively collect all data in all folders and subfolders. The Question folders will contain source code (.c), results files (the requested table), any conversations generated by AI (provide a link to these conversations, or copy/paste the conversations into your report). The reports should be a .pdf file. Before zipping the files, please remove all executables (typically a.out), and object files (files ending in .o). The zipped file should be uploaded to Canvas. Not following these instructions will cost you 10 points. ;

You are only allowed to use the following AI tools: ChatGPT (<https://chat.openai.com>), Claude-2 (<https://claude.ai>), and Perplexity (<https://perplexity.ai>). All three are free. Each AI has its own peculiarities, which you can research yourself. Consider this a personal exploration. All conversations with AI must be reported via sharing links, or via Copy/Paste.

Please solve the following two questions:

Question 1: (70%)

Learn the material: "Trapezoidal and Simpson Rules.pdf." Using the Trapezoidal and Simpson Rules, write a program to integrate $\sin(x)$, from 0 to π .

Please report your results in a table:

Number of Intervals	Trapezoidal Rule Result	Error	Convergence Order	Simpson Rule Result	Error	Convergence Order
N = 20						
N = 40						
N = 80						
N = 160						
N = 320						

Convergence order (a positive number) can be calculated by

$$\text{Order of convergence is } \log(E_1/E_2)/\ln(2), \quad (1)$$

where \log is the natural logarithm. E_1 and E_2 are the absolute values of the errors of two consecutive values of N . In more detail, let E scale as h^p where p is the order of convergence and $h = 1/N$ is a measure of the spacing between consecutive points along the x axis. Run the simulation with two values of h : h_1 and h_2 . Then,

$$E_1 = C_1 (h_1)^p, \quad E_2 = C_2 (h_2)^p$$

Divide one equation by the other and solve for p :

$$\frac{E_1}{E_2} = \frac{C_1}{C_2} \left(\frac{h_1}{h_2} \right)^p$$

Taking the logarithm of both sides,

$$\log \frac{E_1}{E_2} = \log \frac{C_1}{C_2} + p \log \left(\frac{h_1}{h_2} \right)$$

Simply draw a plot of $\log \frac{E_1}{E_2}$ (Y axis) versus $\log \left(\frac{h_1}{h_2} \right)$ (X axis). The order of convergence is the slope of the line.

$$p = \log(E_1 / E_2) / \log(h_1 / h_2)$$

The formula (1) above holds when $h_1 = 2 h_2$

Question 2: (30%)

What is the average distance between two random points within a square?

Please watch the following YouTube video: <https://www.youtube.com/watch?v=i4VqXRRXi68> .

Write a program using Trapezoidal OR Simpson's rule to do the numerical integration in the 4D space to compute the answer. Compare the error between your answer and the exact solution for different choices of the number of intervals. Calculate the order of convergence of the algorithm using functions from Question 1.