ISC 5228
Markov Chain Monte Carlo
In-class Assignment

Due: In 20 minutes

The (Euclidean) distance between two points $\boldsymbol{R}_i$ and $\boldsymbol{R}_j$ in 3D is[1]

$$R_{ij} = |\boldsymbol{R}_i - \boldsymbol{R}_j| = \sqrt{(\boldsymbol{R}_{i,x} - \boldsymbol{R}_{j,x})^2 + (\boldsymbol{R}_{i,y} - \boldsymbol{R}_{j,y})^2 + (\boldsymbol{R}_{i,z} - \boldsymbol{R}_{j,z})^2}$$

I want to find the *average* distance between two points inside a sphere of unit radius.

(i) Describe how you would sample $N \approx 1000$ random points inside the sphere.

(ii) What are the theoretical maximum and minimum values that R_{ij} can take?

(iii) Imagine what the PDF of R_{ij} might look like?

(iv) Consider every pair of points in the sample and compute the corresponding distance. What are the maximum, minimum, and average values of R_{ij} in the sample?[2]

(v) One way to characterize the error in the estimated average is to compute the standard error. This is given by the standard deviation divided by the square-root of the number of samples.

[1] In numpy this can be computed using `np.linalg.norm(Ri, Rj)`, where `Ri` and `Rj` are arrays with three elements.

[2] This part is optional; The true answer is 36/35.