Question 1

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1 Introduction

We will be writing a program to integrate a function $\sin(x)$ using Trapezoidal and Simpson Rules. Limits for this integration will be from 0 to π.

$$I = \int_0^{\pi} f(x)\,dx = \int_0^{\pi} sin(x)\,dx$$

2 Program

The program is written in the programming language 'C' and compiled using the compiler G++.

2.1 Code Formatting

Below are the formatting conventions used for this code.
Casing: Camel-Case
Indentation: 4 spaces
Line break: LF

2.2 Version Control

Git has been used to manage the code. You can track all the changes made in the files while developing the program.

2.3 Integrating $sin(x)$ from 0 to π

We are integrating the function $f(x)$ using Trapezoidal and Simpsons Rules. These functions are named as *TrapezoidalIntegrate* and *SimpsonIntegrate* respectively and can be found in the file named 'main.c' on line 76 and line 42.

2.4 Calculating the absolute error

For both of the Trapezoidal and Simpsons rules, the absolute error can be calculated by finding the difference between the true value of the integral and the calculated value. In our case the true value of $f(x)$ is 2. Therefore, the error will be,

$$error = |2 - calculated\ value|$$

Implementation of this function can be found in the file 'main.c' on lines 146 and 147.

2.5 Calculating Order of Convergence

The order of convergence is calculated by

$$Order of Convergence = \frac{log(E_1/E_2)}{ln(2)}$$

Where E_1 and E_2 are the absolute values of the errors of two consecutive values of N. The code for this function can be found in 'main.c' file at line 121.

2.6 Executing the program

To execute the program written in the file 'main.c' we will need the compiler called 'gcc'. After installing the compiler please run the following commands to build and run the executable file.

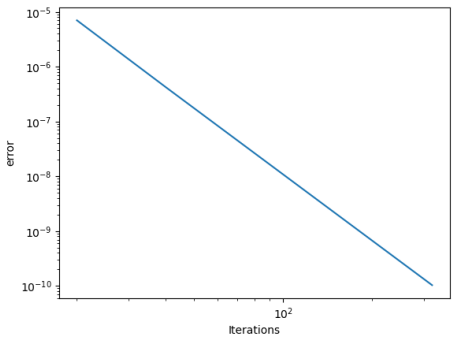
```
gcc main.c
./a.out
```

3 Results

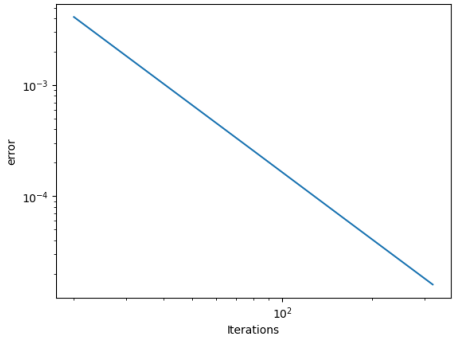
Number of Intervals	Trapezoidal Rule Result	Error	Convergence Order	Simpson Rule Result	Error	Convergence Order
20	1.995886	0.004114		2.000007	0.000007	
40	1.998972	0.001028	2.000445	2.000000	0.000000	4.003183
80	1.999743	0.000257	2.000111	2.000000	0.000000	4.000795
160	1.999936	0.000064	2.000028	2.000000	0.000000	4.000197
320	1.999984	0.000016	2.000007	2.000000	0.000000	4.000062

If we plot a logarithmic graph with error on the Y-axis and iterations on X-axis. We get the following graphs.

Simpsons rule



Trapezoidal rule



References

[1] ChatGPT conversation:
`https://chat.openai.com/share/169bc425-d81f-4985-99b1-47a0c8c127b3`.