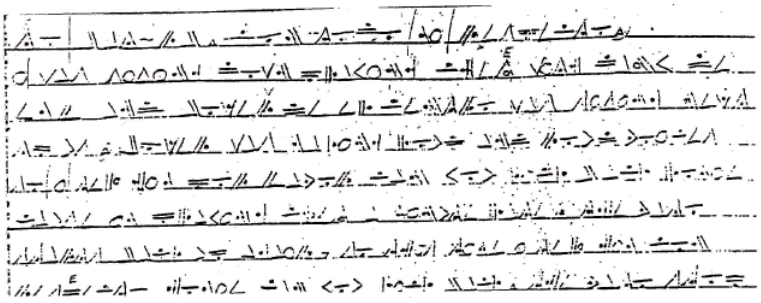
ISC 5228
Markov Chain Monte Carlo
Cryptography and Substitution Ciphers

1 Introduction

Perci Diaconis[1] narrates a fascinating incident:

> Stanford's Statistics Department has a drop-in consulting service. One day, a psychologist from the state prison system showed up with a collection of coded messages.



Assuming a simple substitution cipher, where each letter (symbol) is mapped to a unique letter of the alphabet (example: A $\rightarrow$ S, B $\rightarrow$ Z, etc.), and Monte Carlo simulation, they were able to translate the message into:

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  to bat-rb. con todo mi respeto. i was sitting down playing chess with
danny de emf and boxer de el centro was sitting next to us. boxer was
making loud and loud voices so i tell him por favor can you kick back
homie cause im playing chess a minute later the vato starts back up again
so this time i tell him con respecto homie can you kick back.  the vato
stop for a minute and he starts up again so i tell him check this out shut
the f**k up cause im tired of your voice and if you got a problem with it
we can go to celda and handle it. i really felt disrespected thats why i
told him. anyways after i tell him that the next thing I know that vato
slashes me and leaves. dy the time i figure im hit i try to get away but
the c.o. is walking in my direction and he gets me right dy a celda. so i
go to the hole. when im in the hole my home boys hit doxer so now "b" is
also in the hole. while im in the hole im getting schoold wrong and
```

[1] "Markov Chain Monte Carlo revolution", *Bull. Amer. Math. Soc.* 46 (**2009**), 179-205

We will try to mimic this kind of exercise in the project. For simplicity we will consider a unique 1-1 "substitution" mapping among the 26 letters of the alphabet. For example:

$$f : \{\text{'A', 'B', ..., 'Z', ' '}\} \rightarrow \{\text{'S','Z', ..., 'R', ' '}\}$$

We will ignore all punctuation, numbers, etc., except "space", which we will assume always maps into itself.

1.1 Goal

The goal of this lab is to decode the "secret" phrase:[2]

F RZXN JNVINOTFXN AS TRN LTZSS TRFT Q TNOW TA PN FZTAK-
FTQIFUUB INVTFQO AS QL QT TZVOL AZT TATFUUB HVAOX FOW
WNUZWNW RNVNL AON NGFKJUN AS TRN ZTTNV HVAOXONLL
AS LAKNTRQOX Q TNOW TA PN FZTAKFTQIFUUB LZVN AS ND-
NVBTRQOX QO KB AHO QKKNWQFTN NGJNVQNOIN LZJJAVTL KB
WNNJ PNUQNS TRFT Q FK TRN FPLAUZTN INOTNV AS TRN ZOQD-
NVLN TRN VNFUNLT KALT DQDQW FOW QKJAVTFOT JNVLAO QO
NGQLTNOIN HN VFVNUB TFUM FPAZT TRQL LAVT AS OFTZVFU
PFLQI LNUS INOTNVNWONLL PNIFZLN QTL LA LAIQFUUB VN-
JZULQDN PZT QTL JVNTTB KZIR TRN LFKN SAV FUU AS ZL WNNJ
WAHO QT QL AZV WNSFZUTLNTTQOX RFVWHQVNW QOTA AZV
PAFVWL FT PQVTR TRQOM FPAZT QT TRNVN QL OA NGJN-
VQNOIN BAZDN RFW TRFT BAZ HNVN OAT FT TRN FPLAUZTN IN-
OTNV AS TRN HAVUW FL BAZ NGJNVQNOIN QT QL VQXRT TRNVN
QO SVAOT AS BAZ AV PNRQOW BAZ TA TRN UNST AV VQXRT AS
BAZ AO BAZV TD AV BAZV KAOQTAV AV HRFTNDNV ATRNV JNA-
JUNL TRAZXRTL FOW SNNUQOXL RFDN TA PN IAKKZOQIFTNW
TA BAZ LAKNRAH PZT BAZV AHO FVN LA QKKNWQFTN ZVXNOT
VNFUBAZ XNT TRN QWNF PZT JUNFLN WAOT HAVVB TRFT QK
XNTTQOX VNFWB TA JVNFIR TA BAZ FPAZT IAKJFLLQAO AV
ATRNVWQVNITNWONLL AV TRN LAIFUUNW DQVTZNL TRQL QL

[2]There is a space after each word that ends a line in the text-block below.

OAT F KFTTNV AS DQVTZNQTL F KFTTNV AS KB IRAALQOX TA WA TRN HAVM AS LAKNRAH FUTNVQOX AV XNTTQOX SVNN AS KB OFTZVFU RFVWHQVNW WNSFZUTLNTTQOX HRQIR QL TA PN WNNJUB FOW UQTNVFUUB LNUSINOTNVNW FOW TA LNN FOW QOTNVJVNT NDNVBTRQOX TRVAZXR TRQL UNOL AS LNUS

In other words we seek the mapping f that helps us make sense of this phrase.

2 Approach

Before we consider the approach, let us try to understand why a brute force search is a bad idea. There are 26 letters. It can be shown that the number of different maps possible is $26! > 10^{26}$, which would take too long to explore exhaustively.

One idea is to consider a significant corpus of the literature, and record *first order transitions* from a particular letter "x" to another letter "y". Thus, we can try to estimate for example, how frequently the letter "U" follows letter "Q" etc.

For this lab, I downloaded "War and Peace", "Oliver Twist", and "King James Bible" from Project Gutenberg to compute this metric.[3] This yields a 27×27 table or matrix which is available as the file `pairFreq.dat`. In this matrix M_{ij} represents the transition frequency from the ith letter of the alphabet to the jth. Thus M_{13} represents the frequency with which the letter "A" is followed by the letter "C" in this corpus. For clarity, I will sometimes refer to M_{13} as $M(A, C)$.

We start with a randomly guessed mapping f. If N is the number of letters in the message, then we compute the *plausibility* or likelihood function,

$$\mathrm{Pl}(f) = \prod_{i=1}^{N-1} M(f(s_i), f(s_{i+1})), \tag{1}$$

where s_i is the ith letter of the encrypted message. One can think of this as the probability distribution of the different mappings f. To protect from

[3] https://www.gutenberg.org/

underflow and overflow, we will work with the logarithm of $\mathrm{Pl}(f)$.

3 Strategy

We can use Metropolis-Hastings to sample the probability distribution.

1. Guess a mapping f
2. Compute $\log \mathrm{Pl}(f)$ from eqn. 1.
3. Propose a new mapping f' by swapping two random letters.
4. Compute $\log \mathrm{Pl}(f')$.
5. Decide whether to accept or reject the mapping by comparing $\mathrm{Pl}(f')/\mathrm{Pl}(f)$
6. Repeat steps 3-6 until the phrase makes sense, or there is no subsequent change in the "deciphered" text.
7. If the message is still garbled, try a different initial guess f.

You will need to write functions to do the following tasks:

- Generate a random mapping f
- Compute $\log \mathrm{Pl}(f)$ given the message, f, and matrix $\mathbf{M}$
- Swap two letters to generate a proposed mapping f' from f.
- Compute Hastings ratio, and accept or reject f'

Your MCMC simulation does not have to precisely decrypt the message. If it gets close enough, so that you can use your human skills to "read" the message, you should be good!

4 Deliverables

Write a detailed, and well-organized report (PDF format) with descriptions, figures, sample runs etc. Attach well-documented code separately.

Organize the material, so that it flows well. Use symbols consistently. This is one place in the course, where I will explicitly reward good presentation (30% of grade).

There is no page-limit, but I prefer succinct reports.

The report is due by 11:59PM on the last day of class.