Applied Computational Science

Homework: Linear Algebra.

Anand Kamble

amk23j@fsu.edu

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1.

We have,

$$A = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, v^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using Gauss-Seidel method,

Iteration 1:

$$[v_1^{(1)} = \frac{b_1}{a_{11}} = \frac{1}{-2} = -0.5]$$

$$[v_2^{(1)} = \frac{b_2 - a_{12}v_1^{(1)}}{a_{22}} = \frac{2 - 1 \cdot (-0.5)}{-2} = 0.75]$$

$$[v_3^{(1)} = \frac{b_3}{a_{33}} = \frac{3}{-2} = -1.5]$$

$$[v_4^{(1)} = \frac{b_4 - a_{34}v_3^{(1)}}{a_{44}} = \frac{4 - 1 \cdot (-1.5)}{-2} = 2.25]$$

Iteration 2:

$$[v_1^{(2)} = \frac{b_1 - a_{12}v_2^{(1)}}{a_{11}} = \frac{-0.5 - 1 \cdot 0.75}{-2} = 0.125]$$

$$[v_2^{(2)} = \frac{b_2 - a_{21}v_1^{(2)} - a_{23}v_3^{(1)}}{a_{22}} = \frac{2 - 1 \cdot 0.125 - 0 \cdot (-1.5)}{-2} = 0.59375]$$

$$[v_3^{(2)} = \frac{b_3 - a_{32}v_2^{(2)} - a_{34}v_4^{(1)}}{a_{33}} = \frac{3 - 0 \cdot 0.59375 - 1 \cdot 2.25}{-2} = 1.1875]$$

$$[v_4^{(2)} = \frac{b_4 - a_{43}v_3^{(2)}}{a_{44}} = \frac{4 - 1 \cdot 1.1875}{-2} = 1.90625]$$

2.

Let, A be an n×n symmetric, positive definite tridiagonal matrix:

$$L = \begin{bmatrix} I_1 & m_1 & \dots & \dots & 0 \\ m_1 & I_2 & m_2 & \dots & 0 \\ \vdots & m_2 & \ddots & \dots & \vdots \\ 0 & \dots & \ddots & I_{n-1} & m_{n-1} \\ 0 & \dots & \dots & m_{n-1} & I_n \end{bmatrix}$$

Equating (i, i) entries of LL^T to A gives:

$I_1^2 = a_1$

$I_2^2 = a_2 - m_1^2$

$I_n^2 = a_n - m_{n-1}^2$

Equating the $(i, i+1)$ entries of LL^T to A gives:

$$l_1 m_1 \; = \; b_1 \; l_2 m_2 \; = \; b_2 \; l_3 m_3 \; = \; b_3 \; \dots \; l_{n-1} m_{n-1} \; = \; b_{n-1}$$

Therefore, the Cholesky factorization equations for the tridiagonal matrix A are:

$$l_1 = \sqrt{(a_1)} m_1 = b_1/l_1$$
$$l_2 = \sqrt{(a_2 - m_1^2)} m_2 = b_2/l_2$$
$$l_3 = \sqrt{(a_3 - m_2^2)} m_3 = b_3/l_3$$
$$\ldots$$
$$l_{n-1} = \sqrt{(a_{n-1} - m_{n-2}^2)} m_{n-1} = b_{n-1}/l_{n-1}$$
$$l_n = \sqrt{(a_n - m_{n-1}^2)}$$

3.

Python code for the implementation of power method is saved in the Jupyter Notebook named 'HW 3.ipynb'.

4.

Condition number is also calculated in the Jupyter notebook using,

$$cond(A) = \frac{\lambda_{min}}{\lambda_{max}}$$