RNG and Sampling 1D Distributions

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In [37]:
```
# Importing the required packages.
import numpy as np
import typing
import scipy
import matplotlib.pyplot as plt
from IPython.display import Markdown, display, Latex
```

---

1. LCG Random Number Generator

The RNG we will be using to sample uniform random numbers is,

$$n_{i+1} = (a n_i) \mod m$$

Where a is typically an odd integer and $m = 2^k$, where $k \approx 32 \; or \; 64$

In [38]:
```
# Defining the RNG function.
def RNG(n:int,a:int,m:int,seed:float = 0.):
    n_i = np.full((n),0.)
    n_i[0] = seed
    for i in range(1,len(n_i)):
        n_i[i] = (a*n_i[i-1]) % m
    return n_i
```

A. Even values of a

In this demonstration, We are using $m = 2^k$, where $k = 3$.

Generating 10 random numbers using $a = 2, 4$

In [39]:
```
k = 3
m = 2**k
a = 2
number_of_values = 10

random_values = RNG(number_of_values,a,m,1.0)
display(Latex(f"Random Values when $a = {a}$ are " + ''.join('$\ ' +(str(x) + ' ,  $') f
```

Random Values when $a = 2$ are $1.0, 2.0, 4.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,$

In [40]:
```
a = 4
random_values =  RNG(number_of_values,a,m,1.0)
display(Latex(f"Random Values when $a = {a}$ are " + ''.join('$\ ' +(str(x) + ' ,  $') f
```

Random Values when $a = 4$ are $1.0, 4.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,$

It can be observed from the above that whenever the values of a are even, the numbers generated increase upto a certain point and then become zero.

B. Odd values of a

Keeping the other parameters constant, we will use odd values of a.

Generating 10 random numbers using $a = 3, 5$.

In [41]:
```
a = 3
random_values = RNG(number_of_values,a,m,1.0)
display(Latex(f"Random Values when $a = {a}$ are " + ''.join('$\ ' +(str(x) + ' ,  $') f
```

Random Values when $a = 3$ are $1.0, 3.0, 1.0, 3.0, 1.0, 3.0, 1.0, 3.0, 1.0, 3.0,$

In [42]:
```
a = 5
random_values = RNG(number_of_values,a,m,1.0)
display(Latex(f"Random Values when $a = {a}$ are " + ''.join('$\ ' +(str(x) + ' ,  $') f
```

Random Values when $a = 5$ are $1.0, 5.0, 1.0, 5.0, 1.0, 5.0, 1.0, 5.0, 1.0, 5.0,$

To find the period of the generated values, we will use the following function, which has been created by Chat-GPT.

In [43]:
```
def find_period(arr):
    n = len(arr)
    for period in range(1, n // 2 + 1):
        is_periodic = True
        for i in range(period, n):
            if arr[i] != arr[i - period]:
                is_periodic = False
                break
        if is_periodic:
            return period
    return n
```

The conversation can be found at https://chat.openai.com/share/fdf88bd9-8891-45c7-8dae-db72a4a9c58b

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C. Period when $a = 3$ and $m = 2^4$, and $m = 2^5$

In [44]:
```
a = 3
k = 4
m = 2**k

period = find_period(RNG(100,a,m,1.0))

display(Latex(f"Period of a LCG RNG with $a = {a}$ and $m = 2^{k}$ is ${period}$"))
```

Period of a LCG RNG with $a = 3$ and $m = 2^4$ is 4

In [45]:
```
k = 5
m = 2**k
period = find_period(RNG(100,a,m,1.0))

display(Latex(f"Period of a LCG RNG with $a = {a}$ and $m = 2^{k}$ is ${period}$"))
```

Period of a LCG RNG with $a = 3$ and $m = 2^5$ is 8

D. Numerical Experiments for the given claim.

Claim : The period of the LCG RNG with $m = 2^k$ and odd a is 2^{k-2}.

Let us calculate periods for the odd value of a and $k = \{2, 3, 4, 5, 6\}$

We will be starting with $k = 2$ since having $k < 2$ will result in period which is 2^{-k}

In [46]:
```
a = 3
for k in range(2,7):
    m = 2**k
    period = (find_period(RNG(100,a,m,1.0)))
    display(Latex(f"For $ m = 2^{{{k}}} $ the period is {period} which is {('equal' if (
```

For $m = 2^2$ the period is 2 which is not equal to 2^0

For $m = 2^3$ the period is 2 which is equal to 2^1

For $m = 2^4$ the period is 4 which is equal to 2^2

For $m = 2^5$ the period is 8 which is equal to 2^3

For $m = 2^6$ the period is 16 which is equal to 2^4

Looking at the results above, we can see that the given claim is valid when $k \geq 2$

E. Period of RANDU RNG

Using the claim we can calculate the period for RANDU RNG.

Which will be,

$Period = 2^{31-2}$, where $k = 31$

The period of RANDU RNG is 536870912 which is less than 1 billion.

Sampling 1D distribution.

The distribution we are using is,

$$f(x) = 6x(1-x),\ where\ 0 \le x \le 1$$

A. Accept-reject method

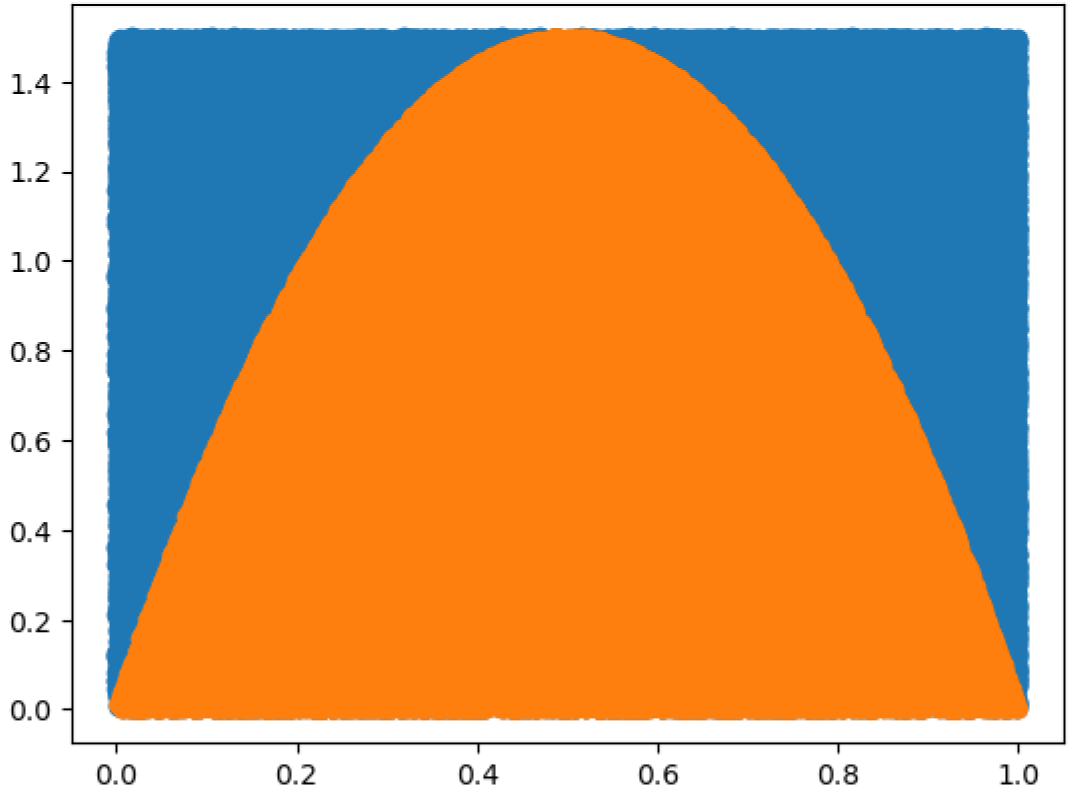
In [47]:
```
# Defining the f(x) function.
def f(x):
    return 6*x*(1-x)
```

We know that the area under the given curve is 1, and the domain we will be using for the accept-reject method will have an area of 1.5.

To get approx 10^5 samples from the given distribution, we would have to throw $1.5 * 10^5$ darts.

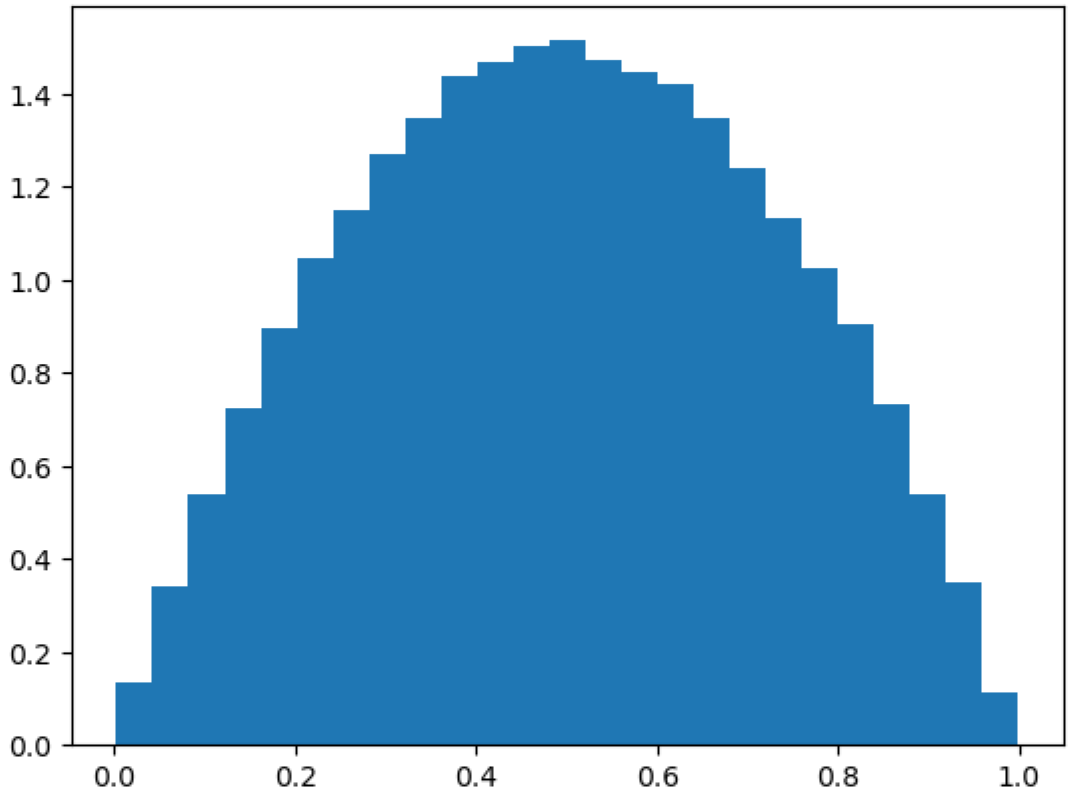
In [48]:
```
# Accept reject method.
fmax = 1.5
num = int(10**5 * 1.5)
x = np.random.uniform(0.0,1.0,num) #np.array([f(x) for x in np.random.uniform(0.0,1.0,10
y = np.random.uniform(0.0,1.5,num)

condition = (y <= f(x))

#Applying the distribution condition.
samples = x[condition]

# Visualizing the darts.
plt.scatter(x,y)
plt.scatter(x[condition],y[condition])
```

Out[48]:
```
<matplotlib.collections.PathCollection at 0x1c4709a0c10>
```



In [49]:
```
#Plotting the histogram.
plt.hist(samples, 25, density=True)
```

Out[49]:
```
(array([0.13605617, 0.34202659, 0.53793741, 0.7220282 , 0.89605937,
        1.04469005, 1.14930995, 1.26927074, 1.34773567, 1.43902659,
        1.46719348, 1.50114465, 1.51371916, 1.4719718 , 1.4463198 ,
        1.42242824, 1.34597524, 1.24009789, 1.13069967, 1.02507382,
        0.90335259, 0.73032737, 0.5381889 , 0.34906832, 0.11392503]),
 array([0.00243544, 0.04230208, 0.08216872, 0.12203537, 0.16190201,
        0.20176865, 0.2416353 , 0.28150194, 0.32136858, 0.36123523,
        0.40110187, 0.44096851, 0.48083516, 0.5207018 , 0.56056845,
        0.60043509, 0.64030173, 0.68016838, 0.72003502, 0.75990166,
        0.79976831, 0.83963495, 0.87950159, 0.91936824, 0.95923488,
        0.99910152]),
 <BarContainer object of 25 artists>)
```



0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

B. Transformation method

We will be using the inverse linear interpolation.

$$f(x) = 6x(1-x); 0 \leq x \leq 1$$

The CFD will be,

$$F(x) = \int_0^1 6x(1-x)dx = \int_0^1 6x - 6x^2 dx$$

$$F(x) = \int_0^1 6xdx - \int_0^1 6x^2 dx = 6\frac{x^2}{2} - 6\frac{x^3}{3}dx$$

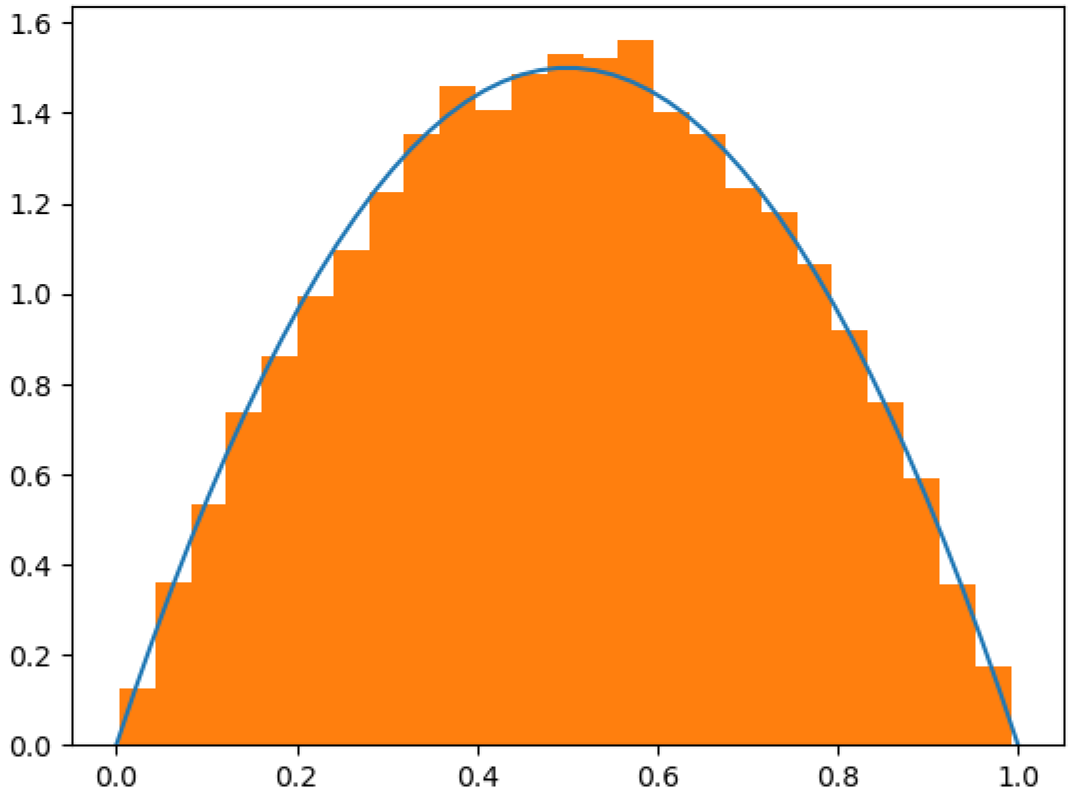
$$F(x) = 3x^2 - 2x^3$$

In [50]:
```
def F(x):
    return  3*(x**2) - 2*(x**3)
```

In [51]:
```
number_of_points = 10**5
x = np.linspace(0,1,number_of_points)

Fi = F(x)
```

In [52]:
```
Finv = scipy.interpolate.interp1d(Fi, x, 'cubic')
```

In [53]:
```
u = np.random.uniform(0,1, 10000)
x = Finv(u)

xi = np.linspace(0,1); fi = f(xi)

plt.plot(xi, fi)
_ = plt.hist(x, 25, density=True)
```



1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0