Data Mining
Classification: Alternative Techniques

Imbalanced Class Problem

Introduction to Data Mining, 2nd Edition
by
Tan, Steinbach, Karpatne, Kumar

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Class Imbalance Problem

- Lots of classification problems where the classes are skewed (more records from one class than another)
  - Credit card fraud
  - Intrusion detection
  - Defective products in manufacturing assembly line
  - COVID-19 test results on a random sample

- **Key Challenge**:
  - Evaluation measures such as accuracy are not well-suited for imbalanced class

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Confusion Matrix

- Confusion Matrix:

	PREDICTED CLASS		
		Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	a	b
	Class=No	c	d

a: TP (true positive)
b: FN (false negative)
c: FP (false positive)
d: TN (true negative)

Accuracy

	PREDICTED CLASS		
		Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	a (TP)	b (FN)
	Class=No	c (FP)	d (TN)

- Most widely-used metric:

$$\text{Accuracy} = \frac{a+d}{a+b+c+d} = \frac{TP+TN}{TP+TN+FP+FN}$$

works for symmetric
binary variable
(if class is binary)

Problem with Accuracy

- Consider a 2-class problem
  - Number of Class NO examples = 990
  - Number of Class YES examples = 10
- If a model predicts everything to be class NO, accuracy is 990/1000 = 99 %
  - This is misleading because this trivial model does not detect any class YES example
  - Detecting the rare class is usually more interesting (e.g., frauds, intrusions, defects, etc)

	PREDICTED CLASS		
		Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	0	10
	Class=No	0	990

Which model is better?

A

	PREDICTED		
		Class=Yes	Class=No
ACTUAL	Class=Yes	0	10
	Class=No	0	990

10 yes

1000 No

Accuracy: 99%

B

	PREDICTED		
		Class=Yes	Class=No
ACTUAL	Class=Yes	10	0
	Class=No	500	490

10 yes

1000 no

Accuracy: 50%

I'd say that B is better since it provides some informatiom on the Yes class.

Which model is better?

misclassifies 50% of yes. Is that important or not? Yes is much rarer than no

accuracy: 0.995

A

	PREDICTED	
	Class=Yes	Class=No
ACTUAL Class=Yes	5	5
ACTUAL Class=No	0	990

10 samples

990 samples

accuracy; 0.5

B

	PREDICTED	
	Class=Yes	Class=No
ACTUAL Class=Yes	10	0
ACTUAL Class=No	500	490

10 samples

990 samples

Alternative Measures

	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	a	b
ACTUAL CLASS Class=No	c	d

$$\text{Precision (p)} = \frac{a}{a+c}$$

Jaccard: a / (a + b + c + d)

$$\text{Recall (r)} = \frac{a}{a+b}$$

$$\text{F - measure (F)} = \frac{2rp}{r+p} = \frac{2a}{2a+b+c}$$

if r == p,

F = 1

F = 2 / (1/ r + 1/ p)

If r << p, F = 2r

if p << r, F = 2p

if p == r, F = r

High F implies that both r and p are high!

Alternative Measures

	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	10	0
Class=No	10	980

$$\text{Precision (p)} = \frac{10}{10+10} = 0.5$$

$$\text{Recall (r)} = \frac{10}{10+0} = 1$$

$$\text{F - measure (F)} = \frac{2*1*0.5}{1+0.5} = 0.62$$

$$\text{Accuracy} = \frac{990}{1000} = 0.99$$

Recall: 1 : all actual “yes” predicted correctly

Precision: 1 : all “yes” predictions are correct

Alternative Measures

	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	10	0
Class=No	10	980

$$\text{Precision (p)} = \frac{10}{10+10} = 0.5$$

$$\text{Recall (r)} = \frac{10}{10+0} = 1$$

$$\text{F - measure (F)} = \frac{2*1*0.5}{1+0.5} = 0.62$$

$$\text{Accuracy} = \frac{990}{1000} = 0.99$$

	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	1	9
Class=No	0	990

$$\text{Precision (p)} = \frac{1}{1+0} = 1$$

$$\text{Recall (r)} = \frac{1}{1+9} = 0.1$$

$$\text{F - measure (F)} = \frac{2*0.1*1}{1+0.1} = 0.18$$

$$\text{Accuracy} = \frac{991}{1000} = 0.991$$

either r=p=0.18
or
(r << p or p << r
and min(r, p)=0.09)

Same accuracy for both models

Which of these classifiers is better?

A

		PREDICTED CLASS	
		Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	40	10
	Class=No	10	40

Precision (p) = 0.8
Recall (r) = 0.8
F - measure (F) = 0.8
Accuracy = 0.8

Balanced class.

B

		PREDICTED CLASS	
		Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	40	10
	Class=No	1000	4000

Precision (p) =~ 0.04
Recall (r) = 0.8
F - measure (F) =~ 0.08
Accuracy =~ 0.8

Imbalanced.

Concentrate on smaller class

4% of 'yes' predictions correct.

80% of 'yes' predicted correctly

Measures of Classification Performance

		PREDICTED CLASS	
		Yes	No
ACTUAL CLASS	Yes	TP	FN
	No	FP	TN

α is the probability that we reject the null hypothesis when it is true. This is a Type I error or a false positive (FP).

β is the probability that we accept the null hypothesis when it is false. This is a Type II error or a false negative (FN).

$$Accuracy = \frac{TP + TN}{TP + FN + FP + TN}$$

$$ErrorRate = 1 - accuracy$$

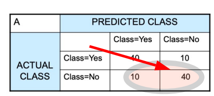
$$Precision = Positive\ Predictive\ Value = \frac{TP}{TP + FP}$$

TPR

$$Recall = Sensitivity = TP\ Rate = \frac{TP}{TP + FN}$$

TNR

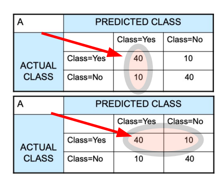
$$Specificity = TN\ Rate = \frac{TN}{TN + FP}$$

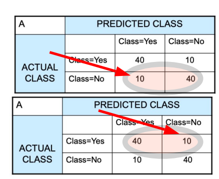


$$FP\ Rate = \alpha = \frac{FP}{TN + FP} = 1 - specificity$$ (type I error)

$$FN\ Rate = \beta = \frac{FN}{FN + TP} = 1 - sensitivity$$ (type II error)

$$Power = sensitivity = 1 - \beta$$





Recall = 1 : all actual “yes” predicted correctly

Precision = 1 : all “yes” predictions that are correct

Recall: fraction of actual “yes” predicted correctly

Precision: fraction of “yes” predictions that are correct

Alternative Measures

A	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	40	10
Class=No	10	40

Precision (p) = 0.8
TPR = Recall (r) = 0.8
FPR = 0.2
F−measure (F) = 0.8
Accuracy = 0.8

$$\frac{TPR}{FPR} = 4$$

B	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	40	10
Class=No	1000	4000

Precision (p) = 0.038
TPR = Recall (r) = 0.8
FPR = 0.2
F−measure (F) = 0.07
Accuracy = 0.8

$$\frac{TPR}{FPR} = 4$$

Recall: TP / (TP + FN)

A	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	40	10
Class=No	10	40

Precision: TP / (TP + TN)

A	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	40	10
Class=No	10	40

Recall: fraction of actual “yes” predicted correctly

Precision: fraction of “yes” predictions that are correct

Which of these classifiers is better?

Classes are balanced:
each class has 50 samples

A	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	10	40
Class=No	10	40

Precision (p) = 0.5
TPR = Recall (r) = 0.2
FPR = 0.2
F − measure = 0.28

B	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	25	25
Class=No	25	25

Precision (p) = 0.5
TPR = Recall (r) = 0.5
FPR = 0.5
F − measure = 0.5

No better than random

C	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	40	10
Class=No	40	10

Precision (p) = 0.5
TPR = Recall (r) = 0.8
FPR = 0.8
F − measure = 0.61

Highest FPR.

False Positive Rate is high
(false positive predictions)

Recall: TP / (TP + FN)

A	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	40	10
Class=No	10	40

Precision: TP / (TP + TN)

A	PREDICTED CLASS	
ACTUAL CLASS	Class=Yes	Class=No
Class=Yes	40	10
Class=No	10	40

$$Accuracy = \frac{TP + TN}{TP + FN + FP + TN}$$

$$ErrorRate = 1 - accuracy$$

$$Precision = Positive\ Predictive\ Value = \frac{TP}{TP + FP}$$

TPR

$$Recall = Sensitivity = TP\ Rate = \frac{TP}{TP + FN}$$

TNR

$$Specificity = TN\ Rate = \frac{TN}{TN + FP}$$

A	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	40	10
Class=No	10	40

$$FP\ Rate = \alpha = \frac{FP}{TN + FP} = 1 - specificity$$ (type I error)

$$FN\ Rate = \beta = \frac{FN}{FN + TP} = 1 - sensitivity$$ (type II error)

$$Power = sensitivity = 1 - \beta$$

ROC (Receiver Operating Characteristic)

- A graphical approach for displaying trade-off between detection rate and false alarm rate
- Developed in 1950s for signal detection theory to analyze noisy signals
- ROC curve plots TPR against FPR
  - Performance of a model represented as a point in an ROC curve

- Recall vs Type I error

FPR

A	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	40	10
Class=No	10	40

FP / (FP + TN)

Recall: TP / (TP + FN)

A	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	40	10
Class=No	10	40

Recall: fraction of actual "yes" predicted correctly

ROC Curve

FPR: fraction of actual "no" predicted incorrectly
(the prediction is a "yes", which is wrong)

(0,0): all predictions are "no"

(1,1): all predictions are "yes"

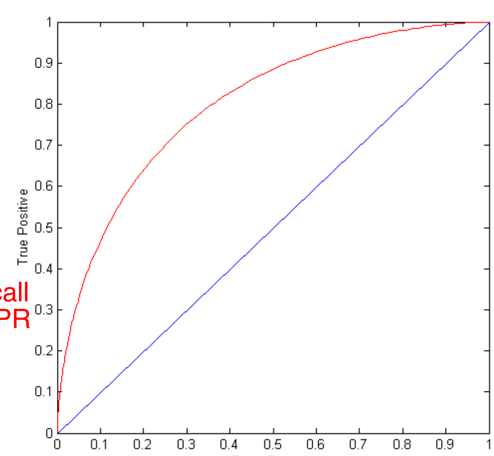
(1,0): FN = 0
FP = 0
(the confusion matrix is diagonal)

(TPR,FPR):

- (0,0): declare everything to be negative class
- (1,1): declare everything to be positive class
- (1,0): ideal

- Diagonal line:
  - Random guessing
  - Below diagonal line:
    - prediction is opposite of the true class

Recall = TPR



FPR

Recall: TP / (TP + FN)

A	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	40	10
Class=No	10	40

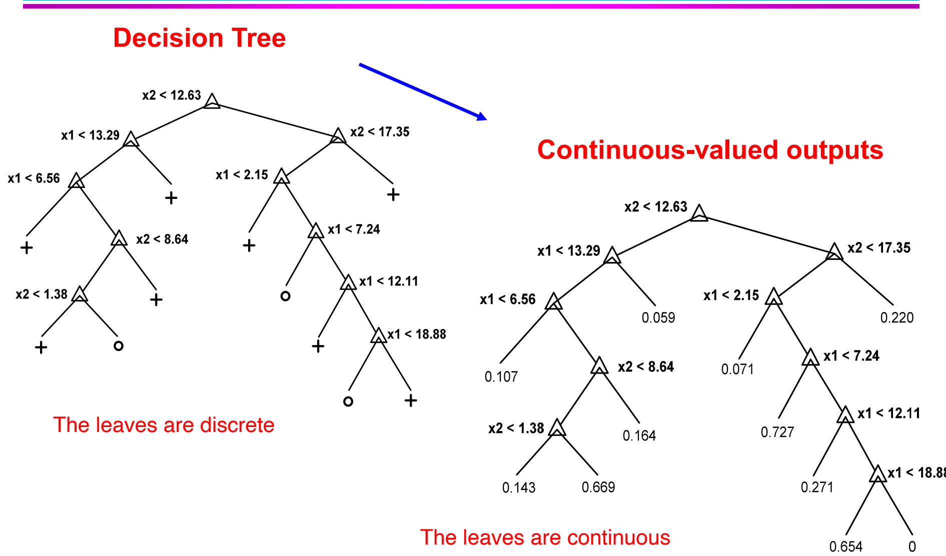
FPR: FP / (FP + TN)

A	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	40	10
Class=No	10	40

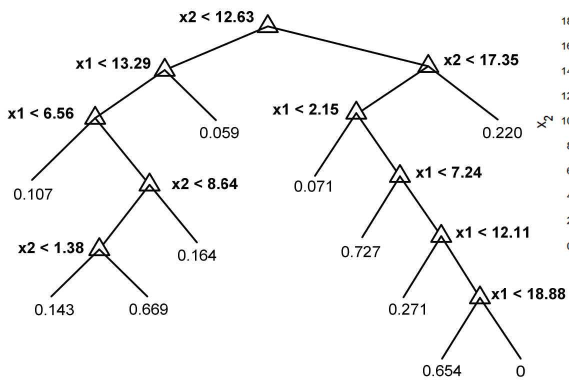
ROC (Receiver Operating Characteristic)

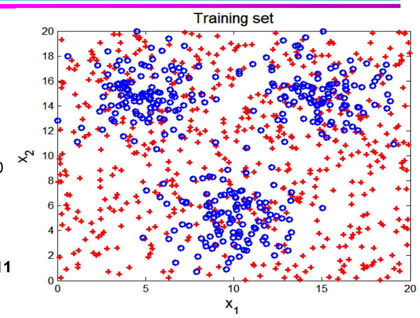
- To draw ROC curve, classifier must produce continuous-valued output
  - Outputs are used to rank test records, from the most likely positive class record to the least likely positive class record
  - By using different thresholds on this value, we can create different variations of the classifier with TPR/FPR tradeoffs
- Many classifiers produce only discrete outputs (i.e., predicted class)
  - How to get continuous-valued outputs?
    - Decision trees, rule-based classifiers, neural networks, Bayesian classifiers, k-nearest neighbors, SVM

Example: Decision Trees



ROC Curve Example





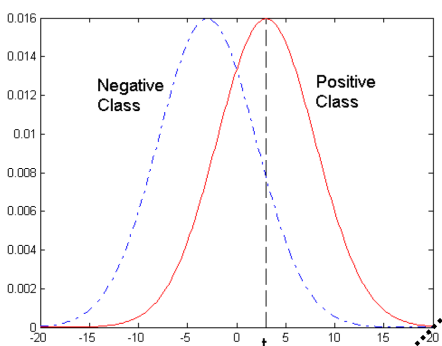
$\alpha = 0.3$		Predicted Class	
		Class o	Class +
Actual Class	Class o	645	209
	Class +	298	948

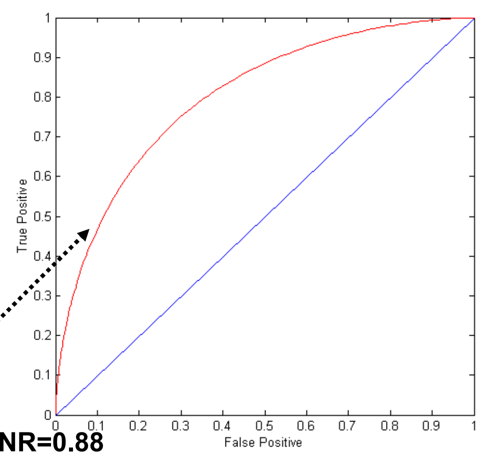
$\alpha = 0.7$		Predicted Class	
		Class o	Class +
Actual Class	Class o	181	673
	Class +	78	1168

ROC Curve Example

- 1-dimensional data set containing 2 classes (positive and negative)

- Any points located at x > t is classified as positive





At threshold t:

TPR=0.5, FNR=0.5, FPR=0.12, TNR=0.88

How to Construct an ROC curve

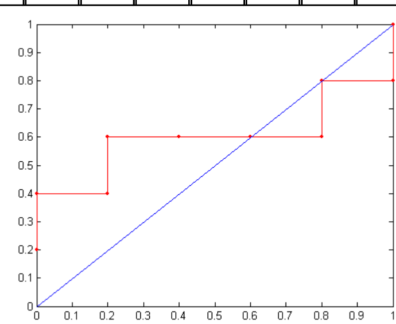
Instance	Score	True Class
1	0.95	+
2	0.93	+
3	0.87	-
4	0.85	-
5	0.85	-
6	0.85	+
7	0.76	-
8	0.53	+
9	0.43	-
10	0.25	+

- Use a classifier that produces a continuous-valued score for each instance
  - The more likely it is for the instance to be in the + class, the higher the score
- Sort the instances in decreasing order according to the score
- Apply a threshold at each unique value of the score
- Count the number of TP, FP, TN, FN at each threshold
  - TPR = TP/(TP+FN)
  - FPR = FP/(FP + TN)

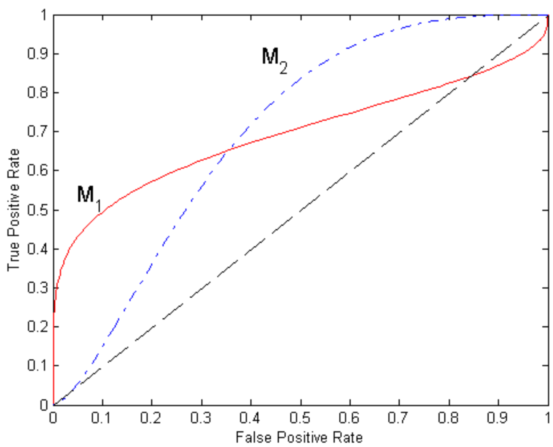
How to construct an ROC curve

	Class	+	-	+	-	-	-	+	-	+	+	
Threshold >=		0.25	0.43	0.53	0.76	0.85	0.85	0.85	0.87	0.93	0.95	1.00
	TP	5	4	4	3	3	3	3	2	2	1	0
	FP	5	5	4	4	3	2	1	1	0	0	0
	TN	0	0	1	1	2	3	4	4	5	5	5
	FN	0	1	1	2	2	2	2	3	3	4	5
→	TPR	1	0.8	0.8	0.6	0.6	0.6	0.6	0.4	0.4	0.2	0
→	FPR	1	1	0.8	0.8	0.6	0.4	0.2	0.2	0	0	0

ROC Curve:



Using ROC for Model Comparison



- No model consistently outperforms the other
  - M_1 is better for small FPR
  - M_2 is better for large FPR
- Area Under the ROC curve (AUC)
  - Ideal:
    - Area = 1
  - Random guess:
    - Area = 0.5

Dealing with Imbalanced Classes - Summary

- Many measures exists, but none of them may be ideal in all situations
  - Random classifiers can have high value for many of these measures
  - TPR/FPR provides important information but may not be sufficient by itself in many practical scenarios
  - Given two classifiers, sometimes you can tell that one of them is strictly better than the other
    - C1 is strictly better than C2 if C1 has strictly better TPR and FPR relative to C2 (or same TPR and better FPR, and vice versa)
  - Even if C1 is strictly better than C2, C1's F-value can be worse than C2's if they are evaluated on data sets with different imbalances
  - Classifier C1 can be better or worse than C2 depending on the scenario at hand (class imbalance, importance of TP vs FP, cost/time tradeoffs)

Which Classifer is better?

T1	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	50	50
ACTUAL CLASS Class=No	1	99

$$\text{Precision} \quad (p) = 0.98$$
$$\text{TPR} = \text{Recall} \quad (r) = 0.5$$
$$\text{FPR} = 0.01$$
$$\text{TPR/FPR} = 50$$
$$F-\text{measure} = 0.66$$

T2	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	99	1
ACTUAL CLASS Class=No	10	90

$$\text{Precision} \quad (p) = 0.9$$
$$\text{TPR} = \text{Recall} \quad (r) = 0.99$$
$$\text{FPR} = 0.1$$
$$\text{TPR/FPR} = 9.9$$
$$F-\text{measure} = 0.94$$

T3	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	99	1
ACTUAL CLASS Class=No	1	99

$$\text{Precision} \quad (p) = 0.99$$
$$\text{TPR} = \text{Recall} \quad (r) = 0.99$$
$$\text{FPR} = 0.01$$
$$\text{TPR/FPR} = 99$$
$$F-\text{measure} = 0.99$$

Which Classifer is better? Medium Skew case

T1	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	50	50
ACTUAL CLASS Class=No	10	990

$$\text{Precision} \quad (p) = 0.83$$
$$\text{TPR} = \text{Recall} \quad (r) = 0.5$$
$$\text{FPR} = 0.01$$
$$\text{TPR/FPR} = 50$$
$$F-\text{measure} = 0.62$$

T2	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	99	1
ACTUAL CLASS Class=No	100	900

$$\text{Precision} \quad (p) = 0.5$$
$$\text{TPR} = \text{Recall} \quad (r) = 0.99$$
$$\text{FPR} = 0.1$$
$$\text{TPR/FPR} = 9.9$$
$$F-\text{measure} = 0.66$$

T3	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS Class=Yes	99	1
ACTUAL CLASS Class=No	10	990

$$\text{Precision} \quad (p) = 0.9$$
$$\text{TPR} = \text{Recall} \quad (r) = 0.99$$
$$\text{FPR} = 0.01$$
$$\text{TPR/FPR} = 99$$
$$F-\text{measure} = 0.94$$

Which Classifer is better? High Skew case

T1	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS: Class=Yes	50	50
ACTUAL CLASS: Class=No	100	9900

$$\text{Precision } (p) = 0.3$$
$$\text{TPR} = \text{Recall } (r) = 0.5$$
$$\text{FPR} = 0.01$$
$$\text{TPR/FPR} = 50$$
$$\text{F} - \text{measure} = 0.375$$

T2	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS: Class=Yes	99	1
ACTUAL CLASS: Class=No	1000	9000

$$\text{Precision } (p) = 0.09$$
$$\text{TPR} = \text{Recall } (r) = 0.99$$
$$\text{FPR} = 0.1$$
$$\text{TPR/FPR} = 9.9$$
$$\text{F} - \text{measure} = 0.165$$

T3	PREDICTED CLASS	
	Class=Yes	Class=No
ACTUAL CLASS: Class=Yes	99	1
ACTUAL CLASS: Class=No	100	9900

$$\text{Precision } (p) = 0.5$$
$$\text{TPR} = \text{Recall } (r) = 0.99$$
$$\text{FPR} = 0.01$$
$$\text{TPR/FPR} = 99$$
$$\text{F} - \text{measure} = 0.66$$

Building Classifiers with Imbalanced Training Set

- Modify the distribution of training data so that rare class is well-represented in training set
  - Undersample the majority class
  - Oversample the rare class