Discrete mathematics

Logic

a, b, c, d, e – Alphabets

Ram, Book – Word

This is a boy – Sentence

Sentence

- Delcarative sentence — We can Declare is T/F.
- Imperative "
- Introgative "
- Exclamatory "

We can't Declare in T/F

p : Deepak is a good boy

q : $2 \times 3 = 6$

Propositions

- atomic
  - This is a boy
- Compound
  - atomic + atomic
  - India's capital is Delhi and Delhi capitals is New Delhi

Basic logical operations

- Conjunction ($\wedge$) — and — AND
- Disjunction ($\vee$) — or — OR
- Negation ($\sim$) — not — NOT

In math term | In DE

Conjunction ($\wedge$)

P : Deepak is a good boy

q : Deepak is an intelligent boy

$\boxed{P+q = \text{Combination}}$

$\Rightarrow$ Deepak is a good and an intelligent boy

$$\boxed{\begin{array}{c} 2^1 = 2 \\ 2^2 = 4 \\ 2^3 = 8 \end{array}}$$

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction ($\vee$)

P : Deepak is going patna

q : Deepak is going Delhi

$\Rightarrow$ Deepak is going patna or delhi.

P	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation operation ($\sim$)

P : Deepak is a good Boy

$\sim p$:- Deepak is not a good boy

p	$\sim p$
T	F
F	T

$\boxed{\sim p : \text{It is not the case that } p}$

P: $2+3>1$

$\sim p \Rightarrow 2+3 \leq 1$

P: It is cold

$\sim p \Rightarrow$ It is not cold

$\sim p \Rightarrow$ It is not the case that it is cold.

Derived Logical operators

- Exclusive OR (XOR) $p \oplus q$

XOR $\longrightarrow p \oplus q$ "p or q but not both"

"p or q and not both p and q"

$p \oplus q \rightarrow (p \vee q) \wedge \sim (p \wedge q)$

If p and q have different value T/F the XOR will be True

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p	q	p∨q	p∧q	~(p∧q)	XOR
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

★ Negation of Conjuction, (NAND), (~(p∧q))
It will be false if p and q both are true other wise True

NAND, ~(p∧q), And's Not, And's negation

p	q	p∧q	~(p∧q)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

NAND

★ Negation of Disjunction (NOR), (~(p∨q))
It will be true if p and q are ~~false~~ false other wise false

(NOR), ~(p∨q), And's or, And's negation

Logical Equivalence ($\equiv$)

Two different Compound propositions are said to be logically equivalence if they have the same truth value or truth tables.

Q $(p \wedge \sim q) \vee (q \wedge \sim p) \equiv (p \vee q) \wedge \sim (p \wedge q)$

Prove that

$\Rightarrow$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \vee q$	$p \wedge \sim q$	$q \wedge \sim p$	R.H.S
T	T	F	F	T	T	F	F	F
T	F	F	T	F	T	T	F	T
F	T	T	F	F	T	F	T	T
F	F	T	T	F	F	F	F	F

$\sim (p \wedge q)$	L.H.S
F	F
T	T
T	T
T	F

R.H.S = L.H.S

proved

~~Predicates and quantifiers~~

Tautologies and Contradictions

Tautologies :-

A tautologies is an statement always true.

Ex –

P	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

This Example of Tautologies.

Contradictions

A tautologies is an statement always false.

Ex –

P	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

This is the example of Contradictions

Q $(p \land \sim q) \land (\sim p \lor q)$

p	q	$\sim p$	$\sim q$	$p \land \sim q$	$\sim p \lor q$	$(p \land \sim q) \land (\sim p \lor q)$
T	F	F	T	F	F	F
T	T	F	F	F	T	F
F	T	T	T	F	T	F
F	F	T	F	F	T	F

Contradictions

De Morgan's Law

$$\sim(p \land q) \equiv \sim p \lor \sim q$$
$$\sim(p \lor q) \equiv \sim p \land \sim q$$

p	q	$\sim p$	$\sim q$	$p \land q$	$p \lor q$	$\sim(p \land q)$	$\sim(p \lor q)$
T	T	F	F	T	T	F	F
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T

$\sim p \lor \sim q$	$\sim p \land \sim q$
F	F
T	F
T	F
T	T

We can see that $\sim(p \land q) \equiv \sim p \lor \sim q$

$\sim(p \lor q) \equiv \sim p \land \sim q$

logically proved

Example

Q

$0 \cancel{<} x \cancel{\leq} 3$

$\Rightarrow 0 \cancel{<} x$ and $x \geq 3$

$0 \cancel{<} x \cancel{\leq} 3$

$p \qquad q \qquad \Rightarrow p$ and q $(p \wedge q)$

Now

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim p$ or $\sim q$

$0 \geq \cancel{x}$ or $x > 3$

$\boxed{0 \geq x \text{ or } x > 3}$ **Ans**

Q

$-9 < x < 2$

$\Rightarrow -9 < x$ and $x < 2$

$p \qquad q$

$p \wedge q$

Now

$\sim(p \wedge q) = \sim p \vee \sim q$

$\sim p$ or $\sim q$

$\boxed{-9 \geq x \text{ or } x \geq 2}$

Logical Equivalence laws

Commutative laws: →	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws: →	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r = p \vee (q \vee r)$
Distributive laws: →	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$
Identity laws: →	$p \wedge t \equiv p$	$p \vee c \equiv p$
Negation laws: →	$p \vee \sim p \equiv t$	$p \wedge \sim p \equiv c$
Double negative laws: →	$\sim(\sim p) \equiv p$	
Idempotent laws: →	$p \wedge p \equiv p$	$p \vee p \equiv p$
universal bound laws: →	$p \vee t \equiv t$	$p \wedge c \equiv c$
Demorgen's laws: →	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
Absorpotion laws: →	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations laws:	$\sim t \equiv c$	$\sim c \equiv t$

Conditional statements ($\longrightarrow$)

(If .. — then)

p: hard work
q: get paid

if you work hard then you get paid.
(p: hypothesis, q: Conclusion)

if _p_ then q.

P	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

If you work hard then you not paid) This is

a) $\sim p \vee q \longrightarrow \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim p \vee q \longrightarrow \sim q$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Q2 $p \vee \sim q \longrightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$p \vee \sim q \longrightarrow \sim p$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Negation of Conditiol statement.

If my dad is at home then he Can't pick me up

⇒ My dad is at home and he Can pick me up.

$$\boxed{\sim (p \rightarrow q) \equiv p \wedge \sim q}$$

Quantified statements - ALL

Ex - ①

$A = \{1, 2, 3, 4, 5\}$

Quantifiers = All $\forall$ (universal quantifiers)

$\forall_x \in A$, $x > 0$ (universal statement)

All the elements Elements That are in a Such that the elements is greater than 0

$\forall_x \in A$, $x^2 \geq x$ (universal statement)

②

$A = \{1, 2, 3, 4, 5, 0.1\}$

$\forall_x \in A$, $x^2 \geq x$ ✗

Quantified statements - THREE EXISTS

Existential quantifier ($\exists$)

m = any integers

$\mathbb{Z}$ = set of all integers

Existential statement

$\exists m \in \mathbb{Z} \mid m^2 = m$

There exists an integer That the set of all the integers

EX = ① $A : \{5, 6, 7\}$

m : any integer

$\exists m \in A$ such that $m^2 = m$

$5^2 = 25 \neq 5$
$6^2 = 36 \neq 6$
$7^2 = 49 \neq 7$

② $\boxed{\forall x \in R, \; x > \frac{1}{x}}$ ✗ false

$5 > \frac{1}{5}$

but

$0.1 > \frac{1}{0.1}$.

③ $\boxed{\exists m \in R \text{ such that } \frac{1}{m} \text{ is undifiend}}$ ✓ True

$m = 1$

$\frac{1}{1}$ = defined

$m = 5$

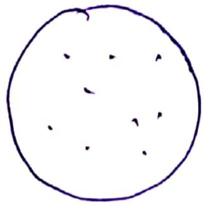
$\frac{1}{5}$ = defined

$m = 0$

$\boxed{\frac{1}{0} = \text{undefined}}$

Negation of quantified statements

- All mens wear hats
  $\forall_x$ $Q(x)$



$$\boxed{\forall_x \in A,\ Q(x)}$$

$\downarrow$

Negation of this

$$\boxed{\exists_x \in A,\ \sim Q(x)}$$

Ex = $A = \{1, 2, 3\}$

$$\boxed{\forall_x \in A,\ x > 0}$$

$\downarrow$ ⟵ Negation

$$\boxed{\exists_x \in A,\ x \leq 0}$$

Ex - $A = \{1, 2, 5, 3.8, -5.5\}$

$\forall_x \in A,\ x \in N$ → false

$\downarrow$ Negation

$\exists_x \in A,\ x \notin N$

$\exists x \in A$, x is multiple of 10 → false

$\downarrow$ Negation

$\forall x \in A$, x is not multiple of 10