Logic Family and Circuits :-

* Combinational Circuit :-

Adder :-

⇒ It is a digital circuit which add two or more binary digit or number.
⇒ Adder contain two output they are 'sum' & carry.

Types of Adder :-

Half Adder :-

⇒ It is a digital circuit in which only two binary digit added.
⇒ Half Adder has two output → sum & carry.
⇒ Half Adder is a two input two output device.

Truth Table :-

Input		output	
A	B	sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

K- map always design for output.

K- map of Sum : { No boundation for POS or SOP }

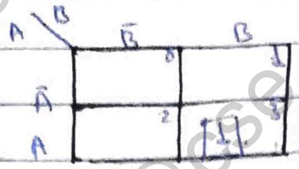
A \ B	$\bar{B}$	B
$\bar{A}$	(0)	$\boxed{1}$ (1)
A	$\boxed{1}$ (2)	(3)

$\rightarrow f(A,B) = A \cdot \bar{B} + \bar{A} \cdot B$

$Sum = A\bar{B} + \bar{A}B$

$= A \oplus B$

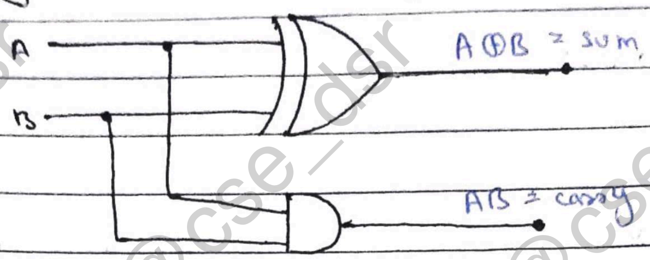
→ K-map of Carry :-



→ $f(A,B) = A \cdot B$

Carry $= A \cdot B$

Logic Circuit Diagram :-

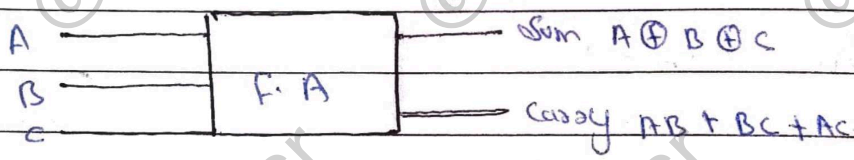


Full Adder :-

⇒ It is a digital in circuit in which only three binary digit added.

⇒ Full adder has two output sum and carry.

⇒ Full adder is a three input, two output device.



Truth Table :-

Input			Output	
A	B	C	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

K- map of sum

A \ BC	$\bar B\bar C$	$\bar B C$	BC	$B\bar C$
$\bar A$		1		1
A	1		1	

$A\bar B\bar C$ $\quad \bar A\bar B C$ $\quad ABC$ $\quad \bar A B\bar C$

sum $= A\bar B\bar C + \bar A\bar B C + ABC + \bar A B\bar C$

$= \bar B(A\bar C + \bar A C) + B(AC + \bar A\bar C)$ —— (1)

Sol :-

$A\bar C + \bar A C = X = A \oplus C$

Adding complement both sides:-

$\overline{A\bar C + \bar A C} = \bar X$

$\overline{A\bar C} \cdot \overline{\bar A C} = \bar X$

$(\bar A + \bar{\bar C}) \cdot (\bar{\bar A} + \bar C) = \bar X$

$(\bar A + C)(A + \bar C) = \bar X$

$\bar A + C \cdot A + \bar C = \bar X$

$\cancel{\bar A A} + \bar A\bar C + AC + \cancel{C\bar C} = \bar X$

$\bar A\bar C + AC = \bar X$

Putting the value of in eqⁿ (i)

$\Rightarrow \bar B(X) + B(\bar X)$

$\Rightarrow \bar B X + B\bar X$

$\Rightarrow B \oplus X$

$\Rightarrow B \oplus (A\bar C + \bar A C)$

$\Rightarrow B \oplus A \oplus C$

K - map of carry :-

A \ BC	$\bar B\bar C$	$\bar B C$	BC	$B\bar C$
$\bar A$			1	
A	φ	1	1	1

$f(A,B,C) = (AC + BC + AB)$

Carry $= AC + BC + AB$

Diagram

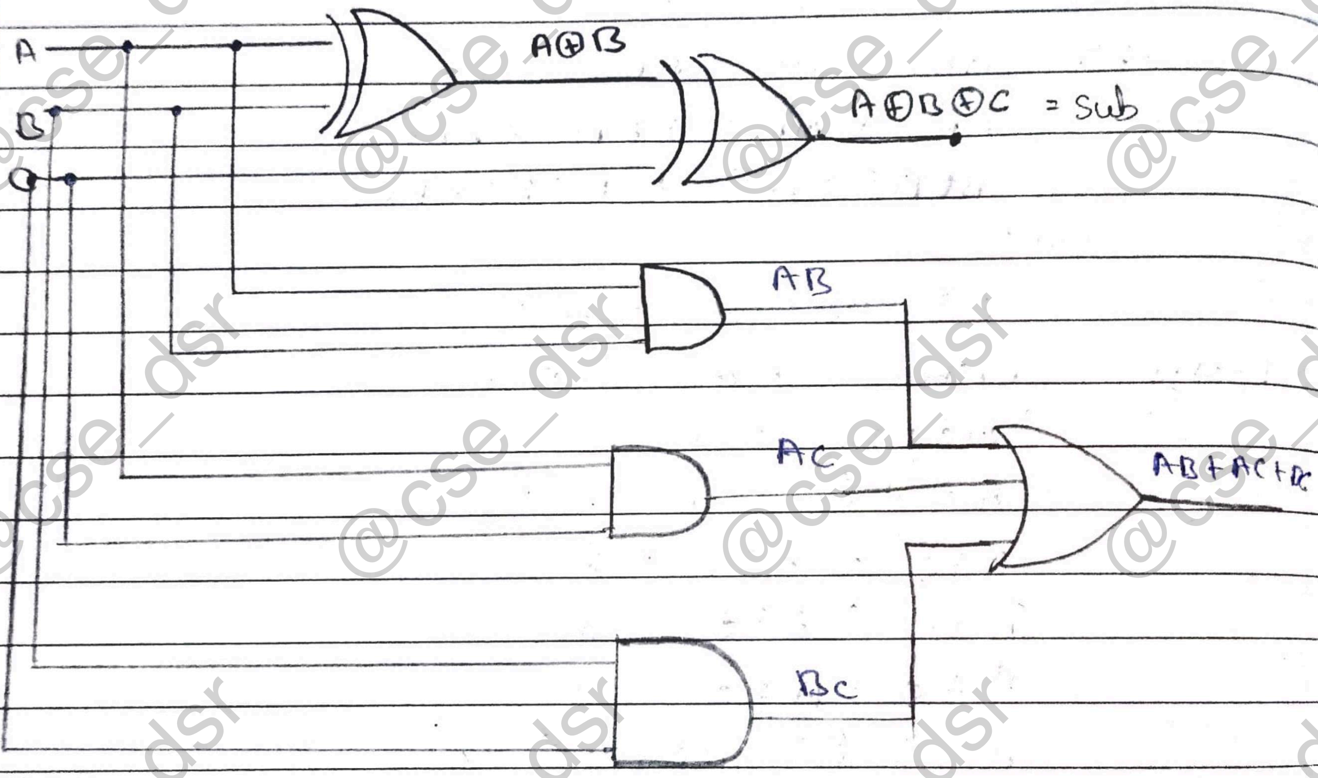


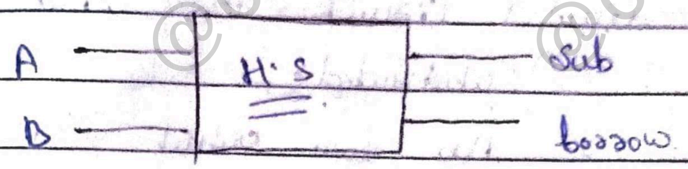
fig : full adder

Substractor :-

⇒ It is a digital ckt which substract two or more binary digit or number.
⇒ Substraction has two output sub & borrow.

Half substractor :-

⇒ It is a digital ckt which substracts two binary digit.
⇒ It has 2 output :- sub and borrow.
⇒ Half substractor is a two input & two output device.



Truth Table :-

Input		Output	
A	B	Sub	borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

• K-map of sub :-

A \ B	$\bar{B}$	B
$\bar{A}$	(0)	$\boxed{1}$ (1)
A	$\boxed{1}$ (2)	(3)

$A\bar{B}$ $\bar{A}B$

Sub = $A\bar{B} + \bar{A}B$
⇒ $A \oplus B$.

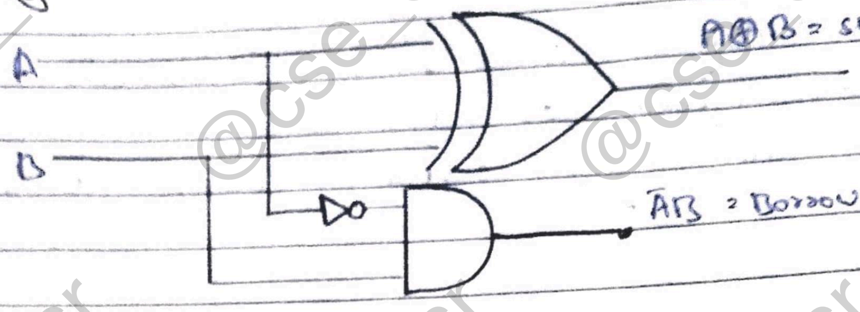
• K map of borrow :-

	$\bar{B}$	B
$\bar{A}$	(0)	$\boxed{1}$ (1)
A	(2)	(3)

$\bar{A}B$

⇒ Borrow =
⇒ $\bar{A}B$

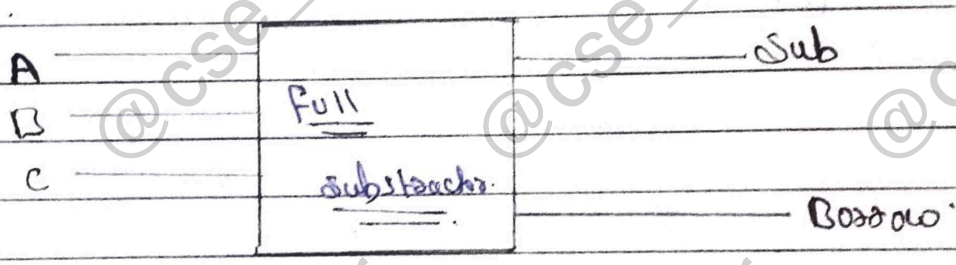
Logic ckt Diagram:-



fig

Full Substraction:-

→ It is a digital Circuit in which only three binary digit Substracted.
→ Full Substractor has two output Sub and borrow.
→ Full Substractor is a three input two output device.



Truth Table:-

Input			Output	
A	B	C	sub	borrow
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

K - map of Sub:-

A \ BC	$\bar B\bar C$	$\bar B C$	BC	$B\bar C$
$\bar A$	(0)	1 (1)	(3)	1 (2)
A	1 (4)	(5)	1 (7)	(6)

$A\bar B\bar C$ $\quad \bar A\bar B C$ $\quad ABC$ $\quad \bar A B\bar C$

$$Sub = A\bar B\bar C + \bar A\bar B C + ABC + \bar A B\bar C$$
$$\bar B(A\bar C + \bar A C) + B(AC + \bar A\bar C)$$

Let $A\bar C + \bar A C = X = A \oplus C$

Taking compliment both Side:-

$$\overline{A\bar C + \bar A C} = \bar X$$
$$\overline{A\bar C}\cdot\overline{\bar A C} = \bar X$$
$$(\bar A + C)(A + \bar C) = \bar X$$
$$\bar A A + \bar A\bar C + AC + C\bar C = \bar X$$
$$\bar A\bar C + AC = \bar X$$

$$Sub = \bar B X + B\bar X = B \oplus X = B \oplus A \oplus C.$$
$$Sub = A \oplus B \oplus C.$$

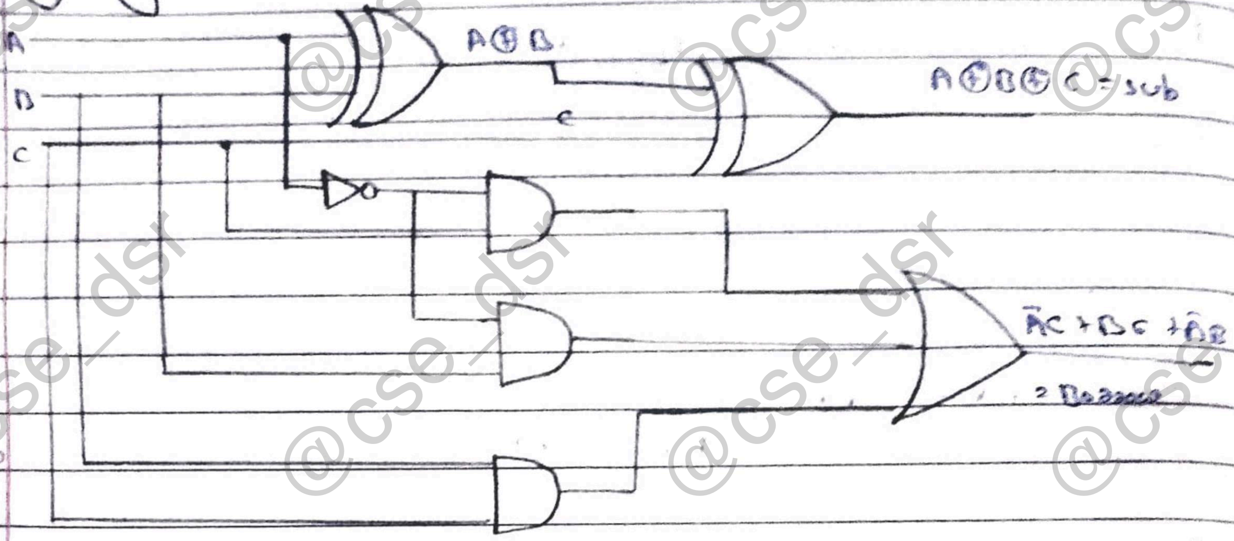
K - map of Borrow:-

A \ BC	$\bar B\bar C$	$\bar B C$	BC	$B\bar C$
$\bar A$	(0)	1 (1)	1 (3)	1 (2)
A	(4)	(5)	1 (7)	(6)

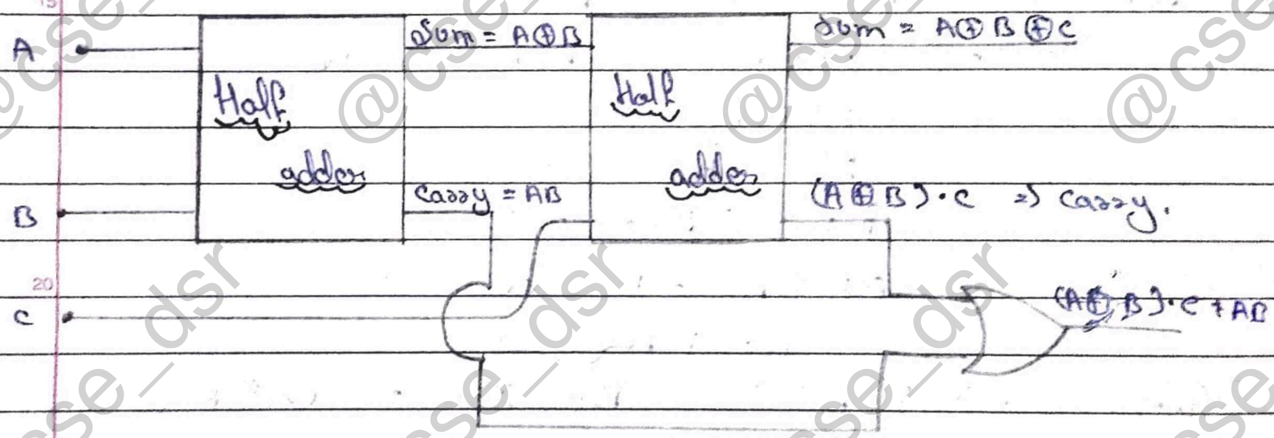
$\bar A C$ $\quad BC$ $\quad \bar A B$

$\therefore$ Borrow $= \bar A C + BC + \bar A B$

Logic Circuit Diagram:-



Making full adder with the help of half adder



Solving:-

$(A \oplus B)\cdot C + AB$

$\Rightarrow (\bar{A}B + A\bar{B})C + AB$

$\Rightarrow \bar{A}BC + A\bar{B}C + AB$

$\Rightarrow \bar{A}BC + A(B + \bar{B}C)$

$\Rightarrow \bar{A}BC + A(B+\bar{B})\ A(B+C)$

$\Rightarrow \bar{A}BC + AB + AC$

$\Rightarrow B(A + \bar{A}C) + AC$

$\Rightarrow B(A+\bar{A})(A+C) + AC$

$\Rightarrow AB + BC + AC.$