NUMBER SYSTEM

Number System:-

Q. What is number system?

→ Representation of a digit or number in different way is called number system.

→ A number system defines a set of value used to represent quantity.

Radix or Base:- The number of values that a digit can assume is equal to the base of system.

Ex:- A decimal no contain 10 digit i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ... 10, its base or radix is 10.

→ largest value of a digit is always one less than the base

→ M.S.D : Most significant dight.

The leftmost digit having the highest weight is called as the most significant digit of a number.

→ L.S.D : Least significant digit.

The rightmost digit having the lowest weight is called as the least significant digit of a number.

Types of Number System

There are four types of number system:
Binary number system
Octal number system
Decimal number system
Hexadecimal number system

Binary Number System:- A number system in which only two digit present they are 0, 1 and its base/radix is 2 is called binary no. system.

$(101)_2$, $(101110)_2$, $(111111)_2$, $(1110.11)_2$
Binary point

$(121)_2 \rightarrow$ It is not a binary number.

Octal Number System:- A number system in which only eight digit present they are 0, 1, 2, 3, 4, 5, 6, 7 and its base/radix is 8 is called octal number system.

$(101)_8$, $(123)_8$, $(214.6)_8$
Octal point

$(0118)_8 \rightarrow$ It is not a octal number.

iii) Decimal Number System:- A number system in which only ten digit present they are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and its base/radix is 10 is called decimal number system.

$(101)_{10}$, $(121)_{10}$, $(1921)_{10}$, $(124.61)_{10}$

~~Radix~~ Decimal point

iv) Hexadecimal Number System:- A number system in which only sixteen digit present they are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F are called hexadecimal number system.

(A = 10, B = 11, C = 12, D = 13, E = 14, F = 15)

$(101)_{16}$, $(126.AB)_{16}$, $(1246)_{16}$, $(ABDE.CF)_{16}$

Hexadecimal point

Q. Conversion from decimal to binary, octal and hexadecimal.

i) $(153.26)_{10} = (10001001.010)_2 = (231.205)_8 = (99.428)_{16}$

2	153	1
2	76	0
2	38	0
2	17	1
2	8	0
2	4	0
2	2	0
	1	

8	153	1
8	19	3
	2	

16	153	9
	9	

$0.26 \times 8 = 2.08 \rightarrow 2$
$0.08 \times 8 = 0.64 \rightarrow 0$
$0.64 \times 8 = 5.12 \rightarrow 5$

$0.26 \times 2 = 0.52 \rightarrow 0$
$0.52 \times 2 = 1.04 \rightarrow 1$
$0.04 \times 2 = 0.08 \rightarrow 0$

$0.26 \times 16 = 4.16 \rightarrow 4$
$0.16 \times 16 = 2.56 \rightarrow 2$
$0.56 \times 16 = 8.96 \rightarrow 8$

ii) $(326.329)_{10} = (101000110.010)_2 = (506.250)_8 = (146.543)_{16}$

2	326	0
2	163	1
2	81	1
2	40	0
2	20	0
2	10	0
2	5	1
2	2	0
	1	

8	326	6
8	40	0
	5	

16	326	6
16	20	4
	1	

$0.329 \times 8 = 2.632 \rightarrow 2$
$0.632 \times 8 = 5.056 \rightarrow 5$
$0.056 \times 8 = 0.448 \rightarrow 0$

$0.329 \times 16 = 5.264 \rightarrow 5$
$0.264 \times 16 = 4.224 \rightarrow 4$
$0.224 \times 16 = 3.584 \rightarrow 3$

$0.329 \times 2 = 0.658 \rightarrow 0$
$0.658 \times 2 = 1.316 \rightarrow 1$
$0.316 \times 2 = 0.632 \rightarrow 0$

B. Conversion from any base to decimal.

i) $(99)_{16} = (153)_{10}$

$\overleftarrow{\underset{1\ 0}{99}} \Rightarrow 16^0 \times 9 + 16^1 \times 9$

$\quad = 1 \times 9 + 144 = 153$

2nd Method :-

$$\sum_{i=0}^{n} b^i \cdot c = b^0 \cdot C_0 + b^1 \cdot C_1 + \dots + b^n \cdot C_n$$

Ex:- $(69)_{16}$

$b^0 = 16^0$	$C_0 = 9$
$b^1 = 16^1$	$C_1 = 6$

$$\sum_0^1 b^i \cdot C_i = b^0 \cdot C_0 + b^1 \cdot C_1$$

$= 16^0 \times 9 + 16^1 \times 6$

$= 1 \times 9 + 96$

$= 105$

$(69)_{16} = (105)_{10}$

iii) $(236.25)_8 = (158.328)_{10}$

$8^2 \times 2 + 8^1 \times 3 + 8^0 \times 6 + 8^{-1} \times 2 + 8^{-2} \times 5$

$64 \times 2 + 8 \times 3 + 1 \times 6 + \frac{1}{8} \times 2 + \frac{1}{8^2} \times 5$

$= 128 + 24 + 6 + \frac{2}{8} + \frac{5}{64}$

$= 158 + \frac{16+5}{64}$

$= 158 + \frac{21}{64} = 158 + 0.328 = 158.328$

Conversion from binary to octal and hexadecimal number system.

~~$(24)_{16}$~~ $\underbrace{011}_{3}\,\underbrace{000}_{0} = (30)_8$

$\underbrace{0001}_{1}\,\underbrace{1000}_{8} = (18)_{16}$

* **1's complement :-**

1's complement is nothing but it is the conversion of 0 to 1 or 1 to 0 of any given binary number.

Ex:- Binary = $(1101011)_2$
1's complement = (0010100)

* **2's complement :-**

2's complement is nothing but it is the addition of 1 with 1's complement of any given binary number.

2's complement = 1's complement + 1

Ex:- Binary = $(1101011)_2$
1's complement = 0010100
+ 1
0010101

* **7's complement :-**

7's complement is nothing but it is the substraction of the given octal no. from set of seven.

Ex:- $(62)_8$

77
62
7's comp. = 15

* 8's complement :- 7's complement + 1

Ex:- $(62)_8$

$77 - 62 = 15 \rightarrow$ 7's complement

$+1$

8's complement $= 16$

* 9's complement :- 9's complement is nothing but it is the subtraction of given decimal no. from set of nines.

Ex:- $(862)_{10}$

$$\begin{array}{r} 999 \\ -862 \\ \hline \end{array}$$

9's complement $= 137$

* 10's complement :- 9's complement + 1

$(862)_{10}$

$$\begin{array}{r} 999 \\ -862 \\ \hline \end{array}$$

9's complement $= 137$

$+1$

10's complement $= 138$

* 15's complement :— 15's complement is nothing but it is subtraction of given hexadecimal no. from the set of 15 (that is F).

Ex: $(A9C)_{16}$

$$\begin{array}{r} FFF \\ -A9C \\ \hline 563 \end{array}$$

15's com. = 563

* 16's complement :— 15's complement + 1

Ex:

$$\begin{array}{r} FFF \\ A9C \\ \hline 563 \\ +1 \\ \hline 564 \end{array}$$

15's comp = 563

564 ⟶ 16's complement.

* BCD (Binary Coded Decimal) :—
BCD is nothing but it is conversion of decimal no. into 4 bytes of binary of individual digits of a number.

Ex: $(11)_{10} = (\quad)_{BCD}$

$\Rightarrow (10001)_{BCD}$

Ex:- $(12)_{10} = (\quad)_{BCD}$

12

0001 0010 ⇒ $(10010)_{BCD}$

Ex:- 962

1001 0110 0010 ⇒ $(100101100010)_{BCD}$

Decimal	Binary	BCD
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	1000
9	1001	1001
10	1010	00010000

* Excess-3

It is an un-weighted code. To get excess-3 code just add 3 into decimal equivalent and convert it into binary. This code is also known as self-complementing code.

BCD $b_3 b_2 b_1 b_0$	Ex-3 $e_3 e_2 e_1 e_0$
0000	0011
0001	0100
0010	0101
0011	0110
0100	0111
0101	1000
0110	1001
0111	1010
1000	1011
1001	1100

Excess-3 code of A

Complement (Ex-3 code of A)
= Ex-3 code of 9's complement of A

Ex:

Let $A = (3)_{10}$

Ex-3 code $= 3 + 3 = 6 \rightarrow 0110$

complement (Ex-3 code of A) $= 1001$

Now,

$A = (3)_{10}$

9's complement of $A = 9 - 3 = 6$

Hence, Ex-3 code of 9's complement of

$$A = \begin{array}{r} 6 \\ +3 \\ \hline 9 \end{array} \longrightarrow 1001$$

* Gray Code

→ It is a single bit change code.

Binary $B_3B_2B_1B_0$	Gray $G_3G_2G_1G_0$
0000	0000
0001	0001
0010	0011
0011	0010
0100	0110
0101	0111
0110	0101
0111	0100
1000	1100
1001	1101
1010	1111
1011	1110
1100	1010
1101	1011
1110	1001
1111	1000

General expression for 4-bit conversion from Binary to Gray code.

$G_3 = b_3$	$A \odot B = AB + \bar{A}\bar{B}$
$G_2 = b_3 \oplus b_2$	
$G_1 = b_2 \oplus b_1$	$A \oplus B = \bar{A}B + A\bar{B}$
$G_0 = b_1 \oplus b_0$	

Ex: Find the gray code of $(8)_{10}$ using general expression

$\overset{b_3 b_2 b_1 b_0}{(1000)_2} \rightarrow 1100$

$G_3 = b_3 = 1$

$G_2 = b_3 \oplus b_2 = \bar{b_3} \cdot b_2 + b_3 \cdot \bar{b_2}$
$= 0 \cdot 0 + 1 \cdot 1$
$= 0 + 1 = 1$

$G_1 = b_2 \oplus b_1 = \bar{b_2} \cdot b_1 + b_2 \cdot \bar{b_1}$
$= 1 \cdot 0 + 0 \cdot 1$
$= 0 + 0 = 0$

$G_0 = b_1 \oplus b_0 = \bar{b_1} \cdot b_0 + b_1 \cdot \bar{b_0}$
$= 1 \cdot 0 + 0 \cdot 1$
$= 0 + 0 = 0$

* Easy trick to find the gray code from binary code.

{left most same, next bit add adjacent one (discard carry)}

(i) 1 1 0 1 1 0
↓ ↓ ↓ ↓ ↓
1 1 1 0 1

(ii) 0 + 0 + 1 + 1 + 0 + 1 → binary
↓ ↓ ↓ ↓ ↓ ↓
0 0 1 0 1 1 → gray

* Easy trick to find the binary code from gray code.

(i) 1 1 0 1 1 1 → Gray
↓ ↓ ↓ ↓ ↓ ↓
1 0 0 1 0 1 → Binary

{left most same, add ans + next bit (discard carry)}

(ii) 0 0 1 0 1 1 → Gray
↓ ↓ ↓ ↓ ↓ ↓
0 0 1 1 0 1 → Binary

(iii) 1 0 1 1 0 → Gray
↓ ↓ ↓ ↓ ↓
1 1 0 1 1 → Binary

* Binary arithmetic :-

i) Binary addition :-

	sum	carry
$0 + 0 =$	0	0
$0 + 1 =$	1	0
$1 + 0 =$	1	0
$1 + 1 =$	0	1
$1 + 1 + 1 =$	1	1
$0 + 1 + 1 =$	0	1

Q. Find the addition of $(52)_{10}$ and $(72)_{10}$ using binary addition or boolean algebra.

64 32 16 8 4 2 1

$(52)_{10} = (0110100)_2$

$(72)_{10} = (1001000)_2$

$$\begin{array}{r} 0110100 \\ +\ 1001000 \\ \hline 1111100 \end{array} = (124)_{10}$$

ii) Binary subtraction :—

	diff	B_{out}
0 − 0 =	0	0
0 − 1 =	1	1
1 − 0 =	1	0
1 − 1 =	0	0
0 − 1 − 1 =	0	1
1 − 1 − 1 =	1	1

Q. Subtraction by using 1's complement :—

(i) $(13)_{10} = (001101)_2$

$(63)_{10} = (111111)_2$

Now,

1's complement of $(13)_{10} = 110010$

Now,

$$\begin{array}{r} 111111 \\ +\ 110010 \\ \hline 1\,110001 \\ +1 \\ \hline 110010 \end{array}$$

$110010 \rightarrow (50)_{10}$ A

(ii) $(53)_{10} - (14)_{10}$

$(53)_{10} = (110101)_2$

$(14)_{10} = (001110)_2$

Now,

1's complement of $(14)_{10} = 110001$

Now,

```
  110101
+ 110001
 1100110
 └──────→ +1
  100111  → (39)10
```

(iii) $(94)_{10} - (44)_{10}$

$(94)_{10} = (1011110)_2$

$(44)_{10} = (0101100)_2$

↓ 1's complement

1010011

```
  1011110
+ 1010011
 10110001
 └──────→ +1
  0110010  → (50)10
```