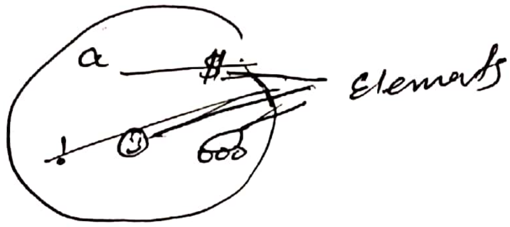
SETS

A set is a Collection of things (objects) that are referred to as the elements of the set.



$A = \{a, \$, !, ☺, [illegible]\}$

Name of set

Set brackets or braces

Elements

$A = \{ a, \$, !, --- \{$

ellipsis

$a \in A$ "a is element of 'A'"

$\$ \in A$ "$ is element of 'A'"

$! \in A$ "! is element of 'A'"

$b \notin A$ "b is not Element of 'A'"

Number sets

- Natural number (N) : 1, 2, 3, 4, ———
  0, 1, 2, 3, 4, ———

- Integers (Z) : --- 4, -3, -2, -1, 0, 1, 2, 3, 4 ———

- Rational numbers (Q) : $4/2$, $1/2$, $\frac{10}{4} = 2.5$
- Irrational numbers ($\overline{Q}$) : $\pi \simeq e.1415...$, e, $\sqrt{2}$
- Real number (R) : both rational and irrational numbers
- Imaginary Number (I) : $i^2 = -1$

$$i = \sqrt{-1}$$

$$ex: \sqrt{-9} = \sqrt{9} \times \sqrt{-1} - \textcircled{i}$$

$$= \underline{\underline{3i}}$$

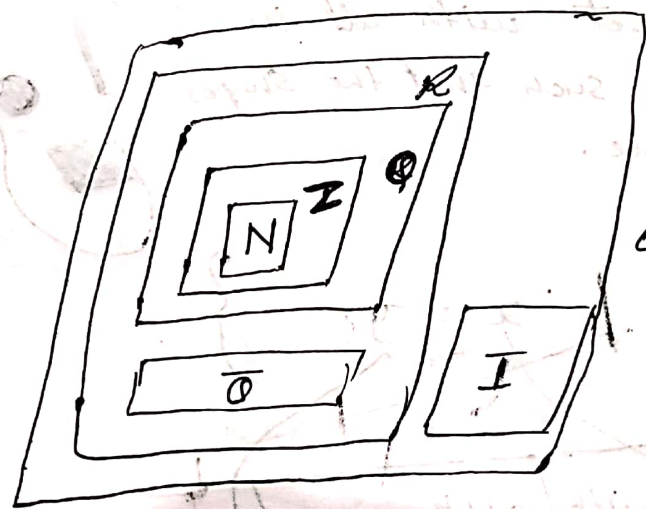
- Complex Number : $a+bi$

$$\underbrace{2}_{\text{Real number}} + \underbrace{3i}_{\text{Imaginary number}}$$

i

ex - 3

$$\underbrace{3}_{\text{Real}} + \underbrace{0i}_{\text{imginary}}$$



complex number

SET Equality

Axiom of Extension :- A set is determined by what its elements are - not the order in which they might be listed, or the fact that some elements might be listed more than once.

$A = \{1, 3, 5, 5, 3, 1, 1\}$

$B = \{1, 3, 5\}$

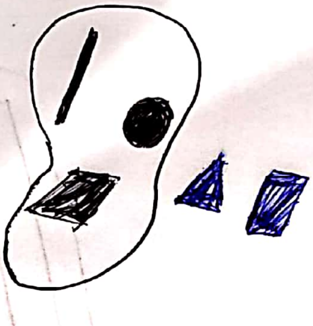
$C = \{1, 5, 3\}$

* They contains same elements so,

$A = B = C$

SET-Builder Notatio

The set with all the shapes. such that the shapes are Blue.



$\{\, x \mid x = y \,\}$

The sets

with

all the shaps

Such that (s.t, ∋)

shapes

are Blue

Ex 1 **Postive**

$\{x \mid x > 0\}$ $\{4, 3, 2\}$

$-5, 4, 3, -10, -5, 2$

Negative

$\{x \mid x < 0\}$ $\{-5, -10, 2\}$

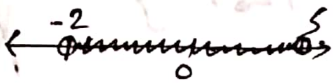
Ex 2 $\{x \in A \mid x > 0\}$ $\{4, 3, 2, 1\}$

A

$-5, 4, 3, -10$

$-5, 2, 1$

Ex - 3 $\{x \in R \mid -2 < x < 5\}$



Ex - 4 $\{x \in R \mid x^2 = 4\}$ Excluding - $-2, 5$

$\{-2, 2\}$

Types of Sets

1. Universal set (U)

A: $\{1, 3, 4\}$

B: $\{2, 6, 7, 8\}$

U: $\{1, 2, 3, 4, 5, 6, 7, 8\}$

(2) Empty set // Null set ϕ ph

$\underbrace{\{\ \}}_{\text{Blank}} \neq 0$

Blank container

Ex— $\{x \in N / 3 < x < 4\}$

Ay, $\{\ \}$ empty set

(3) Singelton set

Ex — $\{1\}$, ~~$\{12345\}$~~ only one element

(4) Finite set

Ex— $\{1, 2, 4, 5, 6\}$, $\{-1, -5, -3, -4\}$

(5) Infinite set

Ex — $\{1, 2, 3, 4 \ldots\}$

$\{\ldots, -5, -4, -2, -1, 0, 1, 2 \ldots\}$

(6) Sub set

$A = \{1, 5, 7, 8, -5, -4\}$

$B = \{1, -5, 7\}$

B is a subset of A

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Cardinal Number of set

A : set

$n(A) =$

Ex – $A = \{1, 2, 4, 6, 45\}$

$n(A) = 5$

$B = \{143, 56, 1001\}$

$n(B) = 3$

$C = \{\}$

$n(C) = 0$

Equivalent set

$A = \{1, !, \#\}$

$B = \{10, 11, 12\}$

$n(A) = 3$, $n(B) = 3$

$\boxed{A \sim B}$

$A = \{1, !, \#\}$

$B = \{10, 12, 11, 10, 12, 11\}$

$n(A) = 3$, $n(B) = 3$

$\boxed{A \sim B}$

SUB Sets

If A and B are sets, then A is called a subset of B, written $A \subseteq B$, if, and only if every element of A is also an Element

Ex — $A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4, 5\}$

$A \subseteq B$

Ex —

$A = \{1, 2, 3\}$

$\cancel{A}\ B = \{1, 2, 3\}$

* fact
Every set is a subset of it ~~self~~ self

$A \subseteq \cancel{B}\ A$

Ex —

$A = \{1, 2, 3\}$

$B = \{\}$ → Empty set

* fact
Empty set is subset of all set.

$\underline{B \subseteq A}$

• **Proper subset** :- at least one more element in bigger set

$x = \{1, 5, 6\}$

$y = \{1, 5, m, n, 6\}$

$x \subset y$

if
$x = \{1, 5, 6\}$
$y = \{1, 5, 6\}$
we can't write
$\cancel{x \subset y}$
$\boxed{x \subseteq y}$ ✓

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ex

V.V.I

① $1 \in \{1, 2, 3\}$

: True

② $1 \subseteq \{1, 2, 3\}$

: false

③ $\{1\} \subseteq \{1, 2, 3\}$

: True

ex :

① $\{2\} \in \{1, 2, 3\}$

: False

② $2 \in \{1, 2, 3\}$

: True

③ $\{2\} \subset \{1, 2, 3\}$

: True

④ $\{2\} \subseteq \{\{1\}, \{2\}\}$

$\downarrow$ Element 2

$\downarrow$ Eleme $\{1\}, \{2\}$

= False

⑤ $\{2\} \subseteq \{\{1\}, \{2\}, 2\}$

$\downarrow$ Element 2

$\downarrow$ Element $\{1\}, \{2\}, 2$

= True

Power sets

Power set of A, denoted P(A), is the set of all subsets of A

① f = {L, M, R} E - 3 S - 8

P(f) = {{}, {L}, {M}, {R}, {L, M}, {L, R}, {R, M}, {L, M, R}}

Power set

It is set of all subsets of F.

② B = {0, 1} E - 2 S - 4

P(B) = {{}, {0}, {1}, {0, 1}}

Calculating no of subset

use formula

2^n → n → no of Elements in set

2^3 — 8 subset

2^2 — 4 ~~set~~ sub set

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Ordered pairs

$A = \{1, 2, 3, 4\}$

$B = \{1, 4, 3, 2\}$

Set A = Set B

Ordered n_tuples

$(1, 2, 3, 4) \neq (1, 2, 4, 3)$

$(1, 2) \neq (2, 1)$

$(x_1, x_2, x_3, x_4 \ldots x_n) = (y_1, y_2, y_3, y_4 \ldots y_n)$

so,

$x_1 = y_1$

(a, b) ordered pair / ordered 2-Tuples

(a, b, c) ordered triple / ordered 3-Tuples

Cartesian product

Given sets A and B, the ~~cartesian~~ Cartesian product of A and B, denoted $A \times B$ and read "A cross B", is the of all ordered paires (a, b) where a is in A am b is in B.

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

Ex — ① $A = \{1, 2\}$ $B = \{c, d\}$

$$A \times B = \{(1,c), (1,d), (2,c), (2,d)\}$$

$$B \times A = \{(c,1), (c,2), (d,1), (d,2)\}$$

$\boxed{A \times B \neq B \times A}$

Ex — ② $A = \{1, 2, 3, 4\}$ $B = \{c, d, e\}$

$A \times B = \{(1,c), (1,d)\ (1,e), (2,c), (2,d), (2,e),$
$(3,c), (3,d), (3,e), (4,c), (4,d), (4,e)\}$

$B \times A$	1	2	3	4
c	(c,1)	(c,2)		
d	(d,1)	(d,2)		
e	(e,1)	(e,2)		

$\neq$

$(A \times B)$ =	1	2	3	4
c	(1,c)	(2,c)	-	-
d	(1,d)	(2,d)	-	-
e	(1,e)	(2,e)	-	-

Ex — ③ $A = \{1, 2\}$ $B = \{\$, !\}$ $C = \{x, y\}$

$A \times B \times C = \{(1, \$, x), (1, \$, y), (1, !, x), (1, !, y)$
$, (2, \$, x), (2, \$, y), (2, !, x)$
$, (2, !, y)\}$

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Ex - 4 $(A \times B) \times C$

$A = \{1, 2\}$ $B = \{\$, !\}$ $C = \{x, y\}$

$A \times B = \{(1, \$), (1, !), (2, \$), (2, !)\}$

$(A \times B) \times C = \{((1, \$), x), ((1, \$), y), ((1, !), x), ((1, !), y), ((2, \$), x), ((2, \$), y), ((2, !), x), ((2, !), y)\}$

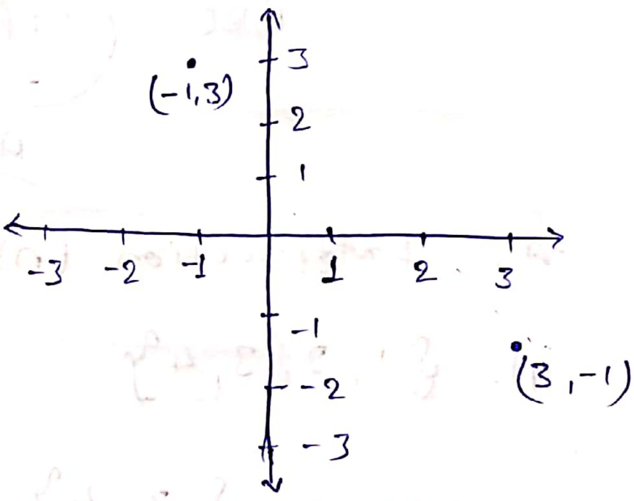
Cartesian plane

$A = \{-1\}$

$B = \{3\}$

$A \times B = \{(-1, 3)\}$

$B \times A = \{(3, -1)\}$



So, $(A \times B) \neq (B \times A)$

Venn Diagrams

$A = \{1, 2, 3, 4, 5\}$

$B = \{1, 2, 3\}$

$B \subseteq A$

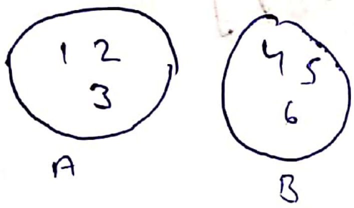
Set Operations (union (∪) & Intersection (∩))

☆ union (∪)

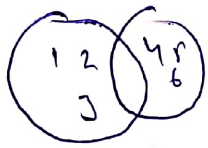
$A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$.

$$\boxed{A \cup B = \{x \mid x \in A \text{ or } x \in B\}}$$



~~AUB~~



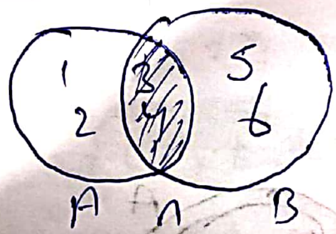
A∪B

☆ Intersection (∩)

$A = \{1, 2, 3, 4\}$ $B = \{4, 5, 6, 3\}$

$A \cap B = \{3, 4\}$

$$\boxed{A \cap B = \{x \mid x \in A \text{ and } x \in B\}}$$



Ex-1 $A = \{1, 3, 5, 7, 9\}$

$B = \{3, 8, 1\}$

$(A \cup B) \cap (A \cap B)$

$A \cup B = \{1, 3, 5, 7, 9, 8\}$

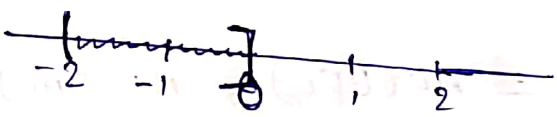
$A \cap B = \{1, 3\}$

$= \{1, 3\}$

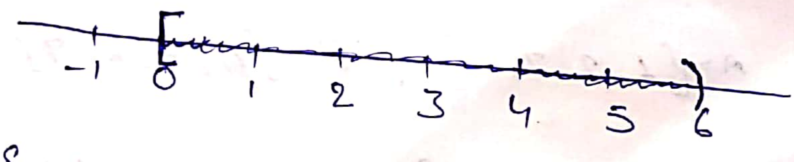
(2) $A = \{x \in R \mid -2 < x \leq 0\}$

$B = \{x \in R \mid 0 \leq x < 6\}$

A =



B =



$A \cup B = \{x \in R \mid -2 < x < 6\}$

$= \{-1, 0, 1, 2, 3, 4, 5\}$

$A \cap B = \{x \in R \mid x = 0\}$

$= \{0\}$

~~De Morgan's Laws~~

Properties of union & intersection

① **Commutative law**

→ $\underbrace{A \cup B}_{\text{operands}} = B \underset{\substack{\downarrow \\ \text{operator}}}{\cup} A$, $A \cap B = B \cap A$

Ex— $A = \{1, 2, 3, 4\}$
$B = \{4, 5, 6, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$, $B \cup A = \{4, 5, 6, 7, 1, 2, 3\}$

$A \cap B = \{4\}$, $B \cap A = \{4\}$

② **Associative Law**

→ $(A \cup B) \cup C = A \cup (B \cup C)$, $A \cap (B \cap C) = (A \cap B) \cap C$

Ex — $A = \{1, 2, 3, 4\}$, $B = \{4, 5, 6, 7\}$, $C = \{0, 5\}$

$(A \cup B) = \{1, 2, 3, 4, 5, 6, 7\}$ $(B \cup C) = \{4, 5, 6, 7, 0\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7, 0\} = A \cup (B \cup C) = \{4, 5, 6, 7, 0, 1, 2, 3\}$

Ex— $A \cap (B \cap C)$ $(A \cap B) \cap C$

$(B \cap C) = \{5\}$ $(A \cap B) = \{4\}$

$A \cap (B \cap) = \{\}$ $(A \cap B) \cap C = \{\}$

3 Distributive Law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A = \{1, 2, 3, 4\}$

$B = \{4, 5, 6, 7\}$

$C = \{0, 5\}$

Ex-① $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$\{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5, 6, 7\} \cap \{1, 2, 3, 4, 0, 5\}$

$\{1, 2, 3, 4, 5\} = \{1, 2, 3, 4, 5\}$

Ex-② $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\{4\} = \{4\} \cup \{\}$

$\{4\} = \{4\}$

4. $A \cup \{\} = A$, $A \cap \{\} = \{\}$

$A = \{1, 2\}$

$A \cup \{\} = \{1, 2\}$, $A \cap \{\} = \{\}$

⑤ $A \cup U = U$

$A = \{1, 2\}$

$U = \{1, 2, 3, 4, 5\}$

Set Operations (Difference & Complement)

Difference (−)

$A = \{1, 2, 3, 4\}$, $B = \{4, 5, 6\}$, $C = \{2\}$

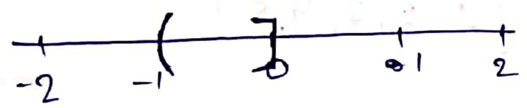
Ex – (i) $A - B = \{1, 2, 3\}$

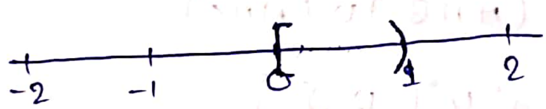
(ii) $(A-B)-C$ = $(A-B) = \{1, 2, 3\}$

$(A-B) - C = \{1, 3\}$

$A = (x \in R \mid -1 < x \leq 0)$

$B = (x \in R \mid 0 \leq x < 1)$





B − A → $(x \in R \mid 0 < x < 1)$

A − B → $(x \in R \mid -1 < x < 0)$

Complement (A^c)

$U = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 2, 3\}$

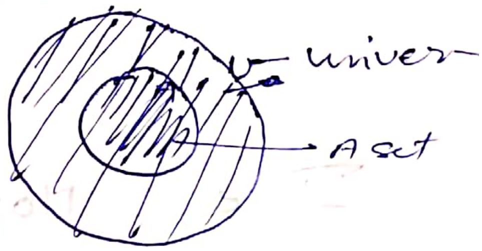
$A^c = \{4, 5, 6, 7\}$

$A = (-1, 0]$

$A^c = \{x \in \mathbb{R} \mid x \le -1 \text{ or } x > 0\}$

Properties of Difference & Complement

① $A \cup A^c = U$



Ex - $A = \{1, 2\}$

$U = \{1, 2, 3, 4, 5\}$

$A^c = \{3, 4, 5\}$

$\boxed{A \cup A^c = U}$

② $(A^c)^c = A$

$A = \{1, 2, 3\}$

$U = \{1, 2, 3, 4, 5\}$

$A^c = \{4, 5\}$

$(A^c)^c = \{1, 2, 3\}$

③ $U^c = \{\}$, $\{\}^c = U$

$U = \{1, 2, 3, 4, 5\}$, $\{\}^c$

$U^c = \{\}$ $= U$

(4) $A - B = A \cap B^c$

$A = \{1, 2, 3, 4, 5\}$

$B = \{4, 5, 6\}$

$A - B = \{1, 2, 3\}$

$B^c = \{1, 2, 3\}$

$A \cap B^c = \{1, 2, 3\}$

De Morgan's Law

* $(A \cup B)^c = A^c \cap B^c$

$A = \{1, 2, 3, 4, 5\}$

$B = \{4, 5, 6, 7\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$\boxed{(A \cup B)^c = \{8, 9, 10\}}$

$A^c = \{6, 7, 8, 9, 10\}$

$B^c = \{1, 2, 3, 8, 9, 10\}$

$\boxed{A^c \cap B^c = \{8, 9, 10\}}$

$(A \cap B)^c = A^c \cup B^c$

$A = \{1, 2, 3, 4, 5\}$

$B = \{4, 5, 6, 7, 8, 9\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A \cap B = \{4, 5\}$

$\boxed{(A \cap B)^c = \{1, 2, 3, 6, 7, 8, 9, 10\}}$

$A^c = \{6, 7, 8, 9, 10\}$

$B^c = \{1, 2, 3, 10\}$

$\boxed{(A^c \cup B^c) = \{1, 2, 3, 6, 7, 8, 9, 10\}}$

Parition of Sets

Disjoint sets :— Two sets are called disjoint if they have no element in common.

Ex:— $A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

$\boxed{A \cap B = \phi}$ — This is disjoint set

Ex(2) — $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

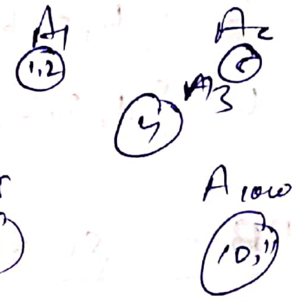
$A \cap B = \{3\}$ — This is not disjoint set.

- Matually Disjoint sets :— A_1, A_2, A_3, A_4 ——
if and only if no two sets have an element in Common

$A_1 \cap A_2 = \{\}$

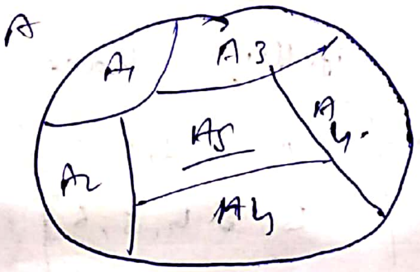
$A_2 \cap A_5 = \{\}$

$A_2 \cap A_5 \cap A_{1000} = \{\}$



- A finite or infinite collection of none empty sets $\{A_1, A_2, A_3, \ldots\}$ is a partition of a set A if and only if

1. A is the union of all of the sets
2. The sets are mutually disjoint.



Ex-1 $A = \{1,2,3,4,5,6,7,8\}$

$A_1 = \{1,2\}$, $A_2 = \{3,4\}$, $A_3 = \{5,6\}$,

$A_4 = \{6,7\}$, $A_5 = \{8\}$

~~= {{1,2}, {8}, {4,5,6}}~~

$A = \{\{1,2\}, \{3,4\}, \{5,6\}, \{6,7\}\}$

Ex ② $A_i = \{i, i^2\}$ for all integers $i = 1, 2, 3$

$A = \{1, 2, 4, 3, 9\}$ Is $\{A_1, A_2, A_3\}$ a partition of A?

$A_1 = \{1, 1\}$, $A_2 = \{2, 4\}$, $A_3 = \{3, 9\}$

① 1st condition = $A_1 \cup A_2 \cup A_3 = \{1, 2, 4, 3, 9\}$

2nd Condition = $\cancel{A_1 \cap A_2 \cap A_3 = \{\}}$

$A_1 \cap A_2 = \{\}$

$A_2 \cap A_3 = \{\}$

$A_3 \cap A_1 = \{\}$