Functions

A function is a relation that uniquely associates each member of one set with exactly on member of another set.

output Range

4, 25, 100, 4, 0

Domain

Input

2, 5, 10, -2, 0

Square each number

$f(x)$

name of function

~~used~~ what goes inside the function

$f(x) = x^2$

$f(x) = x+1$

$f(x) = x^2+1$

$f(y) = y^3+5$

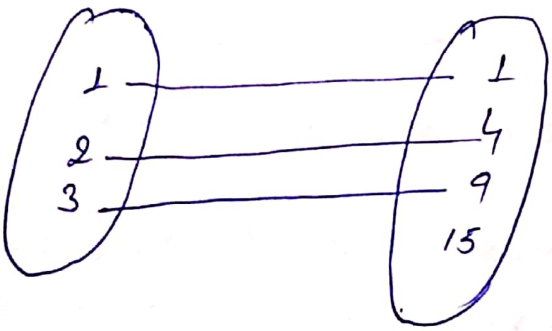
Ex =

① $A = \{1, 2, 3\}$ $\quad f(x) = x^2$

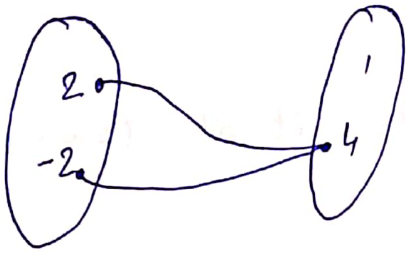
Input

$B = \{1, 4, 9, 15\}$

Output



② $A = \{2, -2\}$ $f(x) = x^2$ $B = \{1, 4\}$



Evaluating a Function

Domain (independent. variable)

$f(x) = x + 1$

Range (Dependent variable)

$f(1) = 1 + 1 = 2$

Ex – ① $f(x) = x^2 + x + 5$ $\boxed{f(-) = (-)^2 + (-) + 5}$

$f(5) = (5)^2 + (5) + 5$

$= 25 + 5 + 5 = \underline{35}$

$f(0) = (0)^2 + (0) + 5$

$= \underline{5}$

(ii) $f(x) = \dfrac{x^3 - 10x + x}{x^2 + 3}$ $\boxed{f(-) = \dfrac{(-)^3 - 10(-) + (-)}{(-)^2 + 3}}$

$f(-1) = \dfrac{(-1)^3 - 10(-1) + (-1)}{(-1)^2 + 3}$

$= \dfrac{-1 + 10 - 1}{4} = \dfrac{8}{4} = \underline{2}$

③ $f(x) = y : 3x^2 + x + 5x^3 - 1$

$f(h) : 3(h)^2 + h + 5(h)^3 - 1$

④ $f(x) : \sqrt{x+5}$ evalute it at $(5x+3+2)$

$f(x) : \sqrt{(\_) + 5}$

$f(5x+3) : \sqrt{(5x+5)+5}$

⑤ $f(y) = y+5$ evalatuate at $f(5) = ?$

$f(5) : 5+5 = \underline{10}$

Domain

Is the set of all the possible inputs that can go into the function and not break it.

① $f(x) = x+1$

$f(1) = 1+1 = 2$

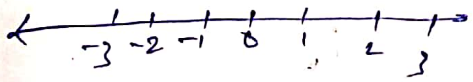
$f(10) = -10+1 = -9$

$f(0) = 0+1 = 1$

$f(1.5) : 1.5+1 = 2.5$

Input	output
1	2
3	4
-10	-9
0	1
1.5	2.5

[Domain is all the R numbers]



Real number

② $f(x) = \frac{1}{1-x}$

		Domain input	Range output
$f(0) = \frac{1}{1-0} = \frac{1}{1}$		0	1
$f(-1) = \frac{1}{1-(-1)} = \frac{1}{2}$		-1	½
$f(1) = \frac{1}{1-1} = \frac{1}{0}$ undifined			

$\boxed{\text{Domain is all the R except } 1}$

③ $f(x) = \frac{1}{x^2-x-2}$

	Domain	range
$f(1) = \frac{1}{1^2-1-2} = \frac{1}{-2}$	1	$-½$
$f(2) = \frac{1}{4-2-2} = \frac{1}{0}$ undifined	2	0
$f(-1) = \frac{1}{(-1)^2-(-1)-2} = \frac{1}{1+1-2} = \frac{1}{0}$	-1	0

$\boxed{\text{Domain is all the R except } (2, -1)}$

④ $f(x) = \sqrt{x-1}$

	Domain	Range
$f(1) = \sqrt{1-1} = 0$	1	0
$f(2) = \sqrt{2-1} = 1$	2	1
$\boxed{f(-1) = \sqrt{-1-1} = \sqrt{-2}}$ X		
$\boxed{f(-3) = \sqrt{-3-1} = \sqrt{-4}}$ X		

$\boxed{\text{Domain is all the R numbers except } x < 1}$

⑤ $A = \{(1,2), (5,8), (7,13)\}$ domain : $\{1, 5, 7\}$

$f(1) = 2$

$f(5) = 8$ $\boxed{\text{domain is only } 1, 5, 7\}}$

$f(7) = 13$

$f(10)$: undifined

Range

Is the set of all the possible outputs that can come out a function

$(3, 5)$, $(6, 11)$, $(7, 8)$

~~Range~~ Domain — Domain — Domain

Range — Range — Range

$f(x) = x + 1$

$f(x)$, y, $x+1$ ⇒ Range

Ex ① $f(x) = x^2$

Domain : R $(-4, 3/4, 0, 1, -1, 2)$

Range : $\boxed{\text{all } x,\ x \geq 0}$

$(16, \frac{9}{16}, 0, 1, 1, 4)$

Graph of functions

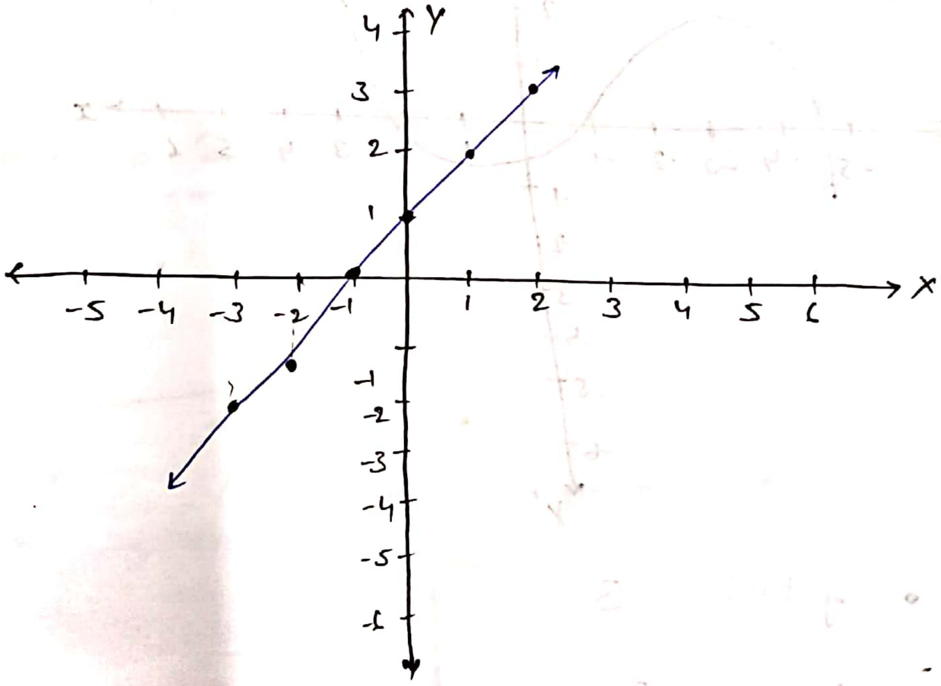
Picture (Image) Representation of a function

$f(x) = x+1$

Domain	Range
Input	outputs
x	$f(x)$

Input x	output $f(x)$
1	2
2	3
0	1
-1	0
-2	-1
-3	-2

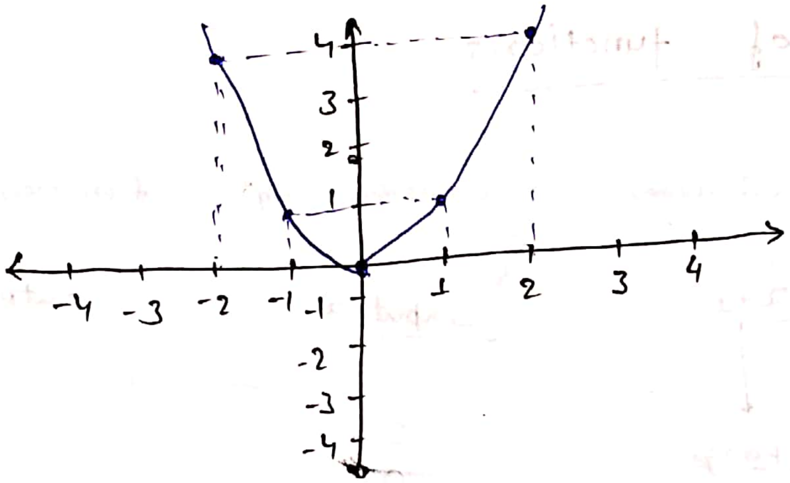
$(\underset{x}{1}, \underset{y}{2})$, $(\underset{x}{2}, \underset{y}{3})$, $(\underset{x}{0}, \underset{y}{1})$, $(\underset{x}{-1}, \underset{y}{0})$, $(\underset{x}{-2}, \underset{y}{-1})$, $(\underset{x}{-3}, \underset{y}{-2})$



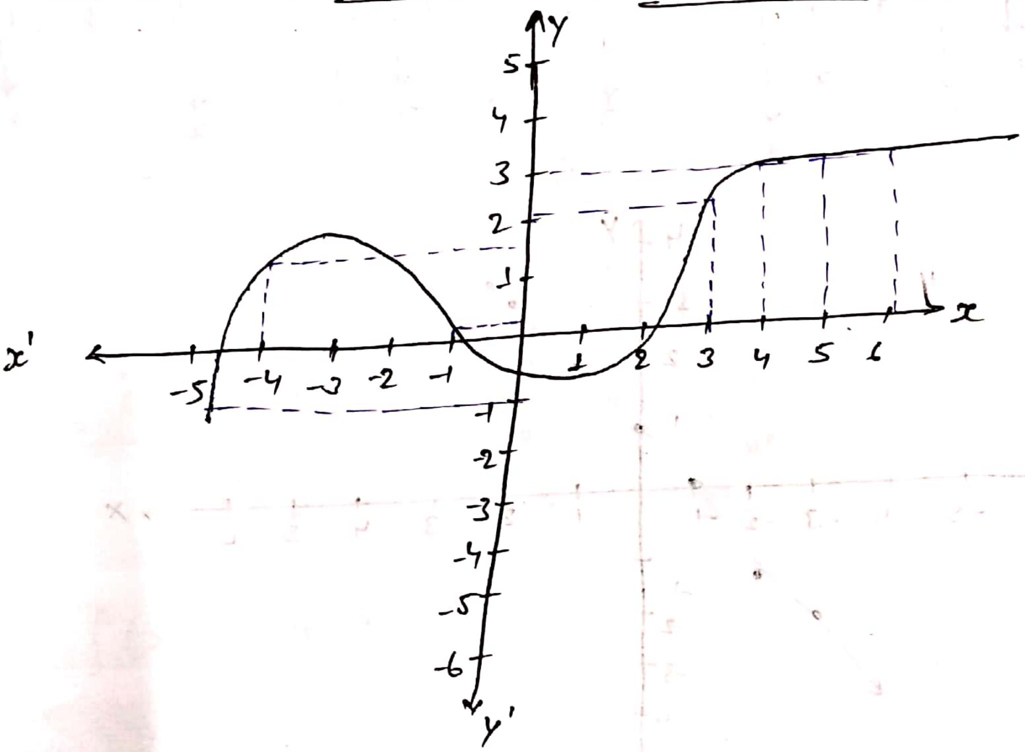
Ex-2

$f(x) = x^2$

Domain/ Input	Range / output
2	4
1	1
0	0
-1	1
-2	4



Extracting Information from a Graph



- $f(4) = 3$
- $f(3) = 2$
- when $y = 2$, find x?

  $x = 3$
- what is y when $x = 6$

  $y = 3$
- when $y = -1$, what x?

  $x = -4.6$

what is larger $f(5)$ or $f(6)$

$f(5) = 4$
$f(6) = 4$

$f(5) = f(6) = 4$

which one is larger $f(-1)$ or $f(-4)$

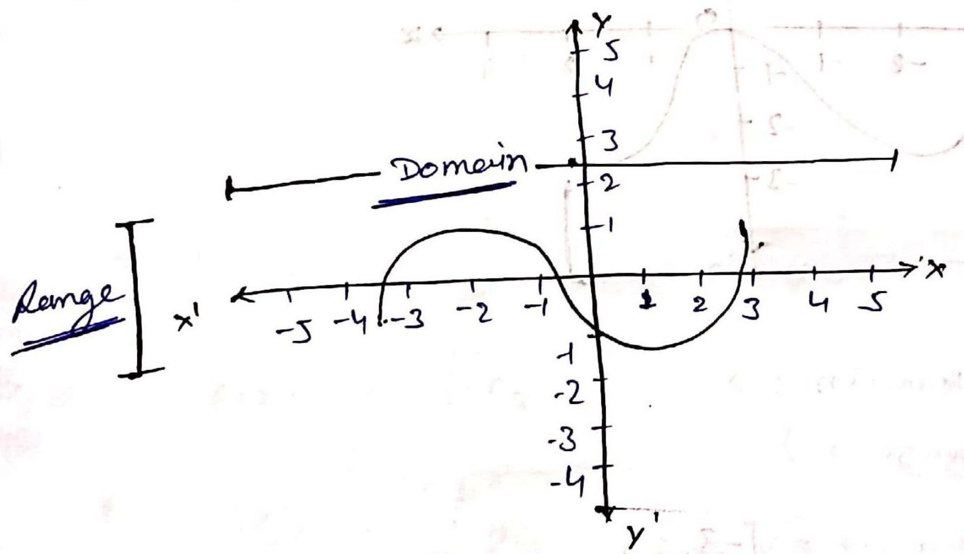
$f(-1) = 0.2$
$f(-4) = 1.5$

$f(-4) > f(-1)$
$1.5 > 0.2$

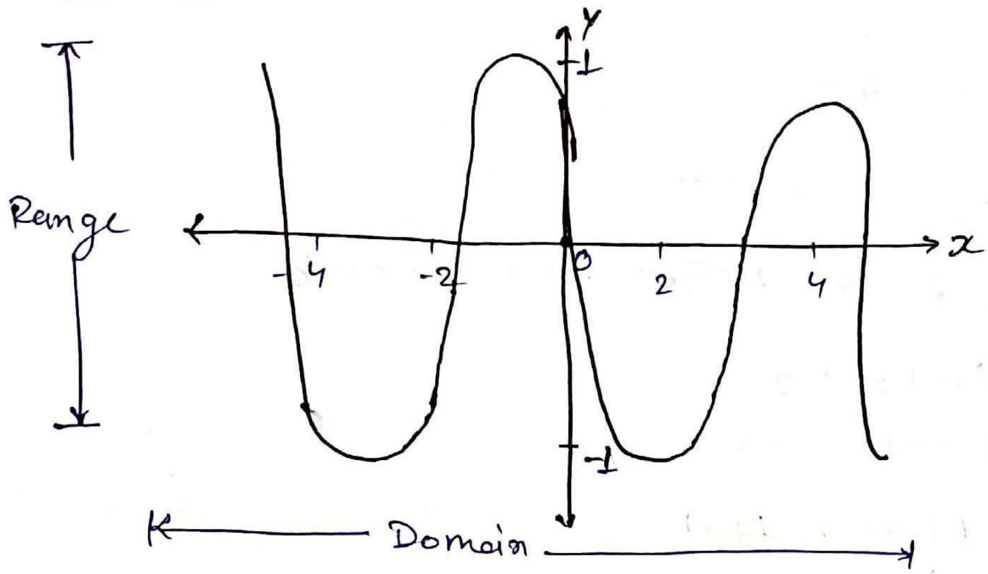
* find $f(x)$ when $x \geq 4$?

$\underline{f(x) = 4}$

Domain & Range from a Graph?



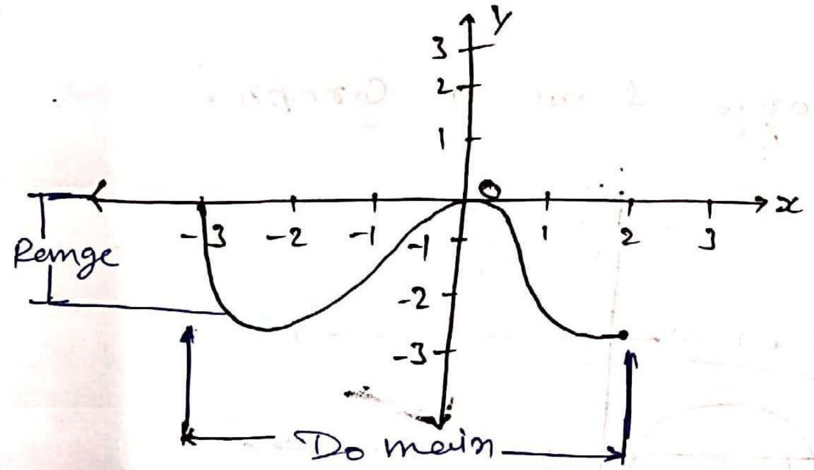
Ex) Graph of sin(x)



Range = y : [+1, -1]

Domain : R

Ex —



Domain = x

Range = y

Domain = [-3, 2]

Range = [0, -2.5]

Function Composition

$f(x) = x+1$ $\qquad g(x) = x^2$

$f(2) = 2+1 = 3$

$f(0) = 0+1 = 1$ $\qquad f\circ g(x) = f(g(x))$

$f(x^2) = x^2+1$ $\qquad \boxed{f(g(x))}$

Q) $f(x) = x^2$ $\qquad g(x) = x^3-5$

$f(g(x)) = (x^3-5)^2$

$g(f(x)) = (x^2)^3 - 5 = x^6 - 5$

Q) $f(x) = \sqrt{x}$ $\qquad g(x) = \sqrt{2-x-x^2}$

$f(g(x)) = \sqrt{\sqrt{2-x-x^2}}$

$g(f(x)) = \sqrt{2-\sqrt{x}-(\sqrt{x})^2}$

$f(f(x)) = \sqrt{\sqrt{x}}$

$g(g(x)) = \sqrt{2-(\sqrt{2-x-x^2})-(\sqrt{2-x-x^2})^2}$

Scanned with CamScanner

Function Combinations

$+ \quad - \quad * \quad \div$

$$\boxed{\begin{array}{ll}(i) & f(x) = x^2 \\ & g(x) = x+1\end{array}}$$

$$\boxed{\begin{array}{ll}(ii) & f(x) + g(x) = x^2 + (x+1) \\ & = x^2+x+1\end{array}}$$

$$\boxed{\begin{array}{ll}(iii) & f(x) - g(x) = x^2 - (x+1) \\ & = x^2 - x - 1\end{array}}$$

$$\boxed{\begin{array}{ll}(iv) & f(x) * g(x) = (x^2) * (x+1) \\ & = x^3 + x^2\end{array}}$$

$$\boxed{\begin{array}{ll}(v) & f(x) \div g(x) = x^2/x+1 \\ & \quad \to \neq 0\end{array}}$$

Examples

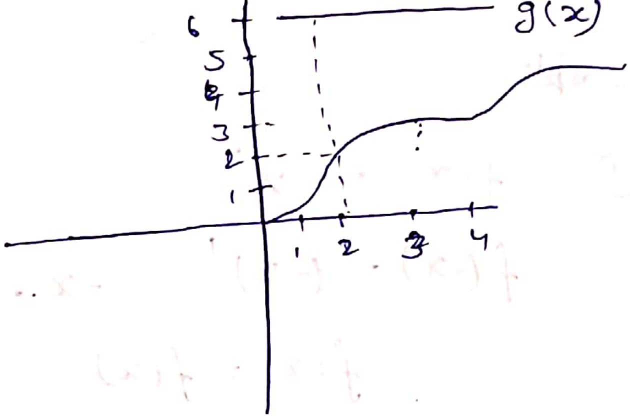
$f(2) = 4$

$g(2) = 3$

$f(x) + g(x) = x^2 + x + 1 = 2^2 + 2 + 1 = 7$

$f(x) * g(x) = x^3 + x^2 = 2^3 + 2^2 = 8 + 4 = 12$

Ex:-



$f(2) + g(2) = (f+g)(2)$

$2 + 6 = (2+2)(2)$

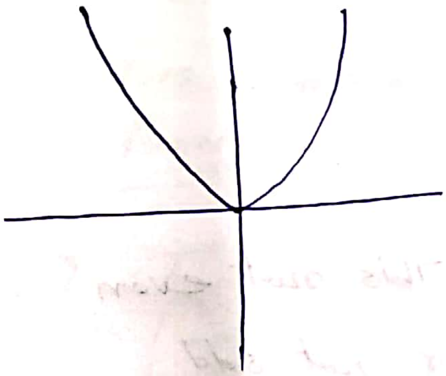
$\boxed{8 = 8}$

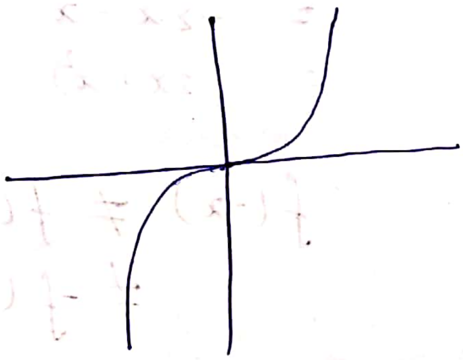
$f(3) * g(3) = (f*g)+3$

$2 * 6 = (3*3)+3$

$\boxed{12 = 9+3 = 12}$

Even & odd Functions





Even function :- $x^2, x^4, x^6, \ldots$

$f(x) = f(-x)$

odd functions :- $x^3, x^5, x^7 \ldots$

$f(x) = -f(x)$

Example

① $f(x) = x^2$

$f(-x) = (-x)^2 = -x \cdot -x = x^2$

$f(x) = f(x)$

This is even function

② $f(x) = x^5 + x$

$f(-x) = (-x)^5 + (-x)$

$= -x^5 - x = -(x^5 + x)$

$f(x) = -f(x)$

This is odd function

③ $f(x) = 2x - x^2$

$f(-x) = 2(-x) - (-x)^2$

$= -2x - x^2$

$= -(2x + x^2)$

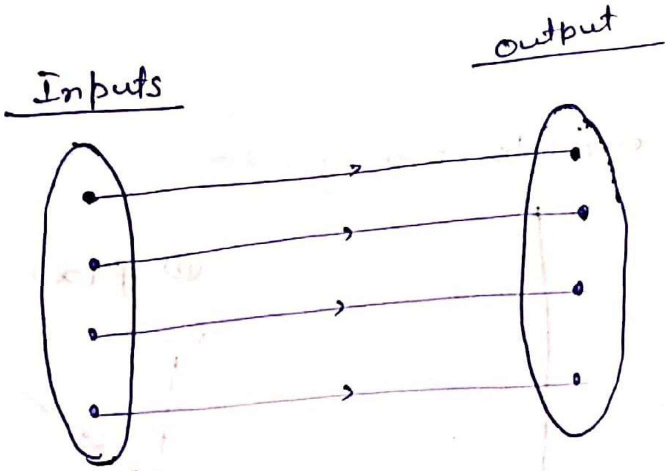
$f(-x) \neq f(x)$

$\neq -f(x)$

So, This not even or not odd function

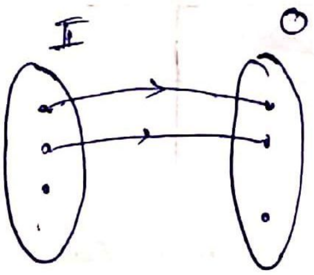
One - to - one functions (Injective function)

A function is one - to - one if no two elements in the domain have the same f(x)



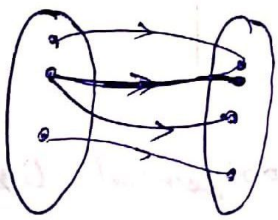
Ex-

(i)



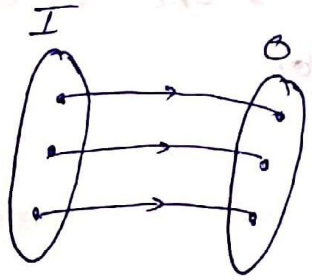
⇒ This is not a function

(ii)



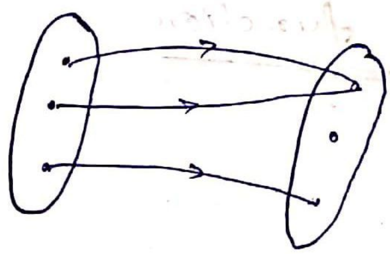
⇒ This is not a function

(iii)



⇒ This is function as well as one-to-one function

(iv)



⇒ This is function but not one - to - one function

(i) $f(x) = x^2$

$f(2) = 4$

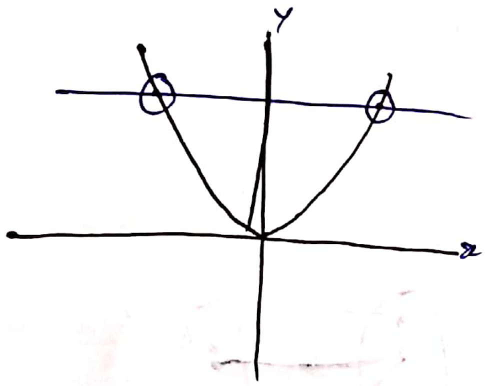
$f(-2) = 4$

$\boxed{\begin{array}{c} x = 2, -2 \\ f(x) = 4 \end{array}}$ This is not one-to-one function

(ii) $f(x+1)$

This is one-to-one function

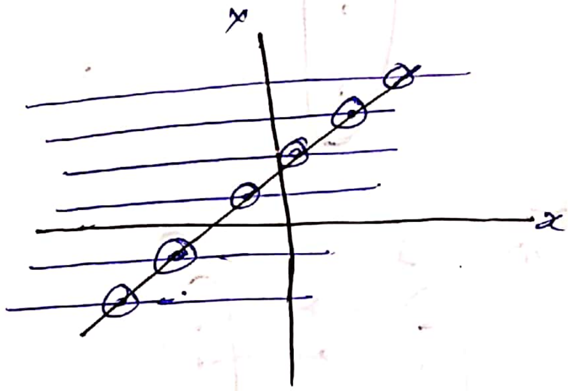
(i) $f(x) = x^2$



The horizontal line cut ~~two~~ times of graph so,

This not one-to-one function

(ii) $f(x) = x+1$



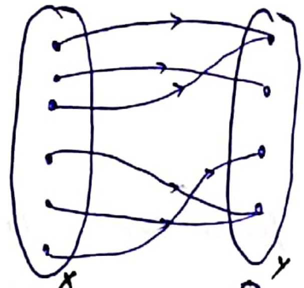
The horizontal line cut only one times of graph so,

This is one-to-one function

Onto function (surjective)

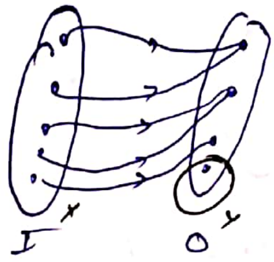
Domain Co-Domain

(i)



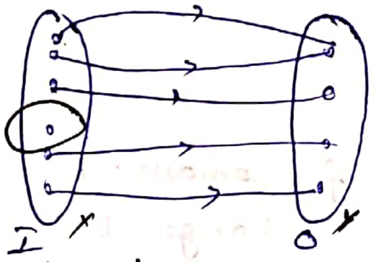
This is onto function

(ii)



This is not onto function

(iii)



This is still onto function

$$\boxed{F : X \to Y \text{ onto} \iff \forall y \in Y, \exists x \in X \text{ such that } F(x) = y}$$

It means,

If every element in Co-domain are Connected at least one in domain

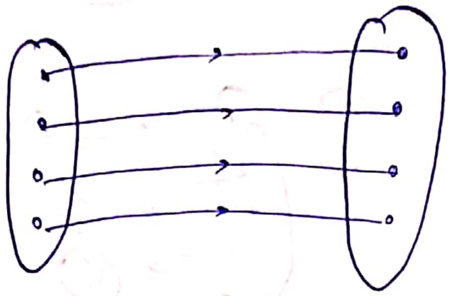
Ex – $x = \{1, 2, 3, 4\}$ $y = \{a, b, c\}$

$H : x \to y$

$H(1) \to a$
$H(2) \to a$
$H(3) \to b$
$H(4) \to c$

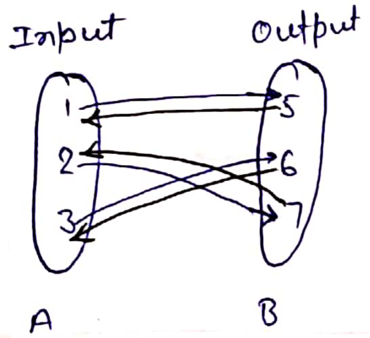
This is surjective function

Bijective



This is one-to-one function and onto function so,
This is also known as
Bijective function

Inverse Functions



if for f Domain $= A$
Range $= B$

for f' Domain $= B$
Range $= A$

Ex

$f(2) = 3$	$f(7) = -3$
$f'(3) = 2$	$f'(-3) = 7$

* Inverse function is applicable in only one-to-one function.

Ex — ①

① $f(x) = 3x - 2$

$y = 3x - 2$

$\Rightarrow y + 2 = 3x$

$\Rightarrow \dfrac{y+2}{3} = x$

$\Rightarrow \boxed{\dfrac{x+2}{3} = y} \Rightarrow f^{-1}(x)$

① $f(x) = y$

② solve for x

③ substitute

$y \Rightarrow x$

② $f(x) = \dfrac{2x+3}{x-1}$

$y = \dfrac{2x+3}{x-1}$

$\Rightarrow y(x-1) = 2x+3$

$\Rightarrow yx - y = 2x + 3$

$\Rightarrow yx - 2x = y + 3$

$\Rightarrow x(y-2) = y+3$

$\Rightarrow x = \dfrac{y+3}{y-2}$ Now, subsitute $y \Rightarrow x$

$\Rightarrow \boxed{y = \dfrac{x+3}{x-2}} \Rightarrow f^{-1}(x)$