Non-redundant masking, kernel phases, and eigenphases in Fizeau interferometry

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April 2015 & 2017[1]

Introduction

Non-redundant masking (NRM - also called sparse aperture masking) has been used on ground-based near-IR telescopes, without adaptive optics at first, then with adaptive optics, to study structures at the scale of $\lambda/2D$ from a bright central object. We also include kernel phase (KP) data analysis of full pupil data in the same context; both fall under the umbrella of Fizeau interferometry. NRM and KP enable the highest angular resolution science possible given the telescope's diameter and operating wavelength. They can also provides precise information on a telescope's optical state.

These Fizeau methods enable a robust measurement and subsequent removal of instrumental effects from data, as long as wavefront aberrations do not change during an observation. Stability *during* an observation sets fundamental NRM and KP contrast limits. This kind of Fizeau imaging differs from coronagraphy, which requires exquisite wavefront quality to suppress diffraction from a bright point source so that a faint nearby source is not swamped by photon noise from the close-by star. NRM and KP cannot sidestep the photon noise from the bright source, but both methods can reach the photon noise floor.

Before the advent of adaptive optics on ground-based telescopes the need for stability meant that very short exposures were needed, to "freeze" atmospheric aberrations. Each frame could be processed to extract certain robust interferometric quantities, and these quantities could be averaged together for a single target. Target data could then have instrumental effects removed by using the corresponding averaged measurements from a point source calibrator.

With adaptive optics stabilizing the temporal evolution of the wavefont, longer exposures could be used. Ultimate stability is achieved from space-based platforms such as Hubble, where KP has been demonstrated. The *James Webb Space Telescope* (JWST) plans to utilize an NRM, but is also capable of providing data for KP analysis.

1 References yet to be done.

Non-redundant masking and kernel phase analysis

Interferometry with non-redundant baselines was initially developed for radio astronomy, and adapted to optical wavelengths. Today it is used in IR and optical bandpasses. A detailed description of NRM can be found in [...] and references therein. In brief, an N-hole pupil mask turns the extremely redundant full aperture of a telescope into a simpler interferometric array. Taking into account the geometry of the pupil (segmentation, central obscuration and spider vanes) as well as the spectral bandwidth, the mask is designed such that each baseline (a vector linking the centers of two holes) is unique: the mask is non-redundant in the sense that each spatial frequency is only sampled once

In a sense, NRM is a form of *imaging* with an unusual-looking PSF that displays multiple sharp peaks, whereas a traditional diffraction-limited PSFs is dominated by only one peak. However, the real virtues of the NRM PSF only appear after the image is Fourier transformed (e.g. Fig. 3 right panel): all spatial frequencies sampled by the mask appear as well separated peaks containing both amplitude and phase information. The advantages of this approach are that:

- The longest baseline provides a resolving power of $0.5\lambda/D$, compared to the traditionally accepted full aperture Rayleigh criterion limit of $1.22\lambda/D$.
- Unlike conventional PSF co-addition, dominated by a speckle noise floor, the **information** extracted from the NRM data **can be averaged** to reduce noise, even in the presence of slowly-varying speckles.
- Non-redundancy ensures that the complex visibilities can be used to form **closure phases**, an observable that **calibrates** wavefront residual errors as well as **non-common path errors** between the science and sensing arms of the instrument.
- The outer working angle (OWA) is set by the shortest baseline, typically $4\lambda/D$: the **NRM** search space nicely **complements** that of **coronagraphy**.

It may seem perverse to throw away most of a telescope's collecting area, but the 10 – 20% mask transmission price paid for NRM (comparable to most high performance coronagraphs' throughput) purchases not only a straightforward 2.44× gain in resolution but also a dramatic increase in signal to noise ratio. An odd beneficial side effect of NRM's reduced throughput and wide image wings is that targets too bright for full aperture imaging can become observable with an NRM in place. NRM is possible even with reduced image quality, as long as image stability during an exposure is maintained. At higher image quality, certainly for diffraction-limited imaging, full aperture images can be analyzed in similar ways: each image yields robust interferometric observables that, when calibrated, remove the effect of instrumental optical aberrations from science data.

Two hole interferometer in one dimension:

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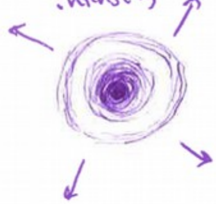
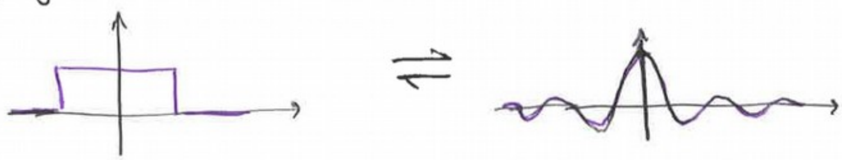
<u>Review</u>: Double slit / Two-hole interferometer (1-D)

2-D image intensity

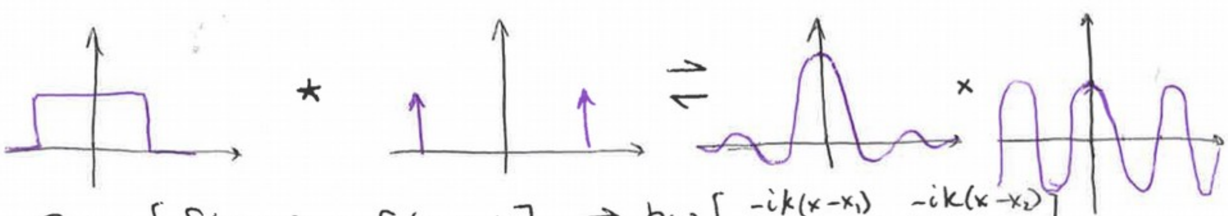


Single:



$P(x) = \Pi(x) \rightleftharpoons p(k) = \text{sinc}\, k$

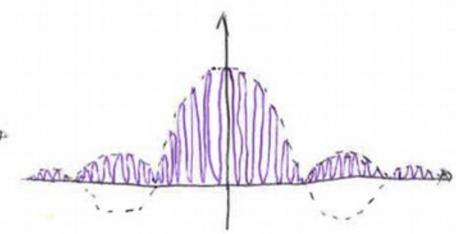
The two-hole is the convolution of the hole shape with delta functions at the hole locations



$$\underbrace{P(x) * \left[\delta(x-x_1) + \delta(x-x_2)\right] \rightleftharpoons p(k)\left[e^{-ik(x-x_1)} + e^{-ik(x-x_2)}\right]}$$

For example $x_1 = -x_2$ (symmetric) gives cosine transform

Image $\propto E^2$ ⟶



2-D intensity

$$\| p(k)\left[e^{-ik(x-x_1)} + e^{-ik(x-x_2)}\right]\|^2$$

$$= p^2(k)\left[e^{-ik(x-x_1)}e^{ik(x-x_2)} + e^{-ik(x-x_2)}e^{ik(x-x_2)} + e^{-ik(x-x_1)}e^{ik(x-x_2)} + e^{ik(x-x_1)}e^{-ik(x-x_2)}\right]$$

$= p^2(k)\left[2 + 2\cos(k(x_2-x_1))\right]$ All real, non-negative

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Complex visibility of a binary point source:

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Two-hole interferometer: binary source signal

From principles of Long Baseline Stellar Interferometry:

$$V_{binary} = e^{-2\pi i(u\alpha_1 + v\beta_1)} \frac{|V_1| + r|V_2|\, e^{-2\pi i(u\Delta\alpha + v\Delta\beta)}}{1+r} \qquad r = \text{contrast ratio}$$

$$\hookrightarrow \frac{1 + r e^{-2\pi i(u\Delta\alpha + v\Delta\beta)}}{1+r} \qquad (u, v) = \vec{x}_2 - \vec{x}_1$$

Notice that the signal is <u>baseline-dependent</u>

What can confuse this signal? Unknown phase errors.

$$\text{field:} \quad p(k)\left[e^{-i\phi_1} e^{-ik(x-x_1)} + e^{-i\phi_2} e^{-ik(x-x_2)} \right]$$

$$\text{psf:} \quad p^2(k)\left[2 + \underbrace{e^{i(k(x_2-x_1)+\phi_2-\phi_1)} + e^{-i(k(x_2-x_1)+\phi_2-\phi_1)}}_{2\cos(k(x_2-x_1)+\Delta\phi)} \right]$$

phase delay shifts fringes

<u>BUT</u> ϕ's don't care about baseline length!

↳ NRM can be useful for measuring hole-dependent errors!

②

Three hole interferometer:

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Suppose: three-hole interferometer

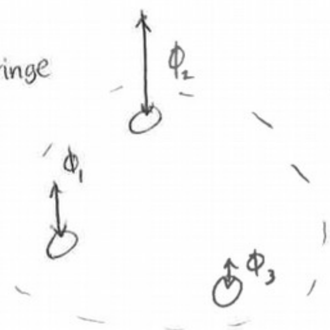
$$\text{pupil} = P(x) \ast [\delta(x-x_1) + \delta(x-x_2) + \delta(x-x_3)] \quad \text{in general}$$

$$\rightleftharpoons p(k)\left[e^{-ik\cdot(x-x_1)} + e^{-ik\cdot(x-x_2)} + e^{-ik(x-x_3)}\right]$$

$$\text{psf} = p^2(k)\left[3 + e^{ik(x_2-x_1)} + e^{-ik(x_2-x_1)} + e^{ik(x_3-x_2)} + e^{-ik(x_3-x_2)} + e^{+ik(x_1-x_3)} + e^{-ik(x_1-x_3)}\right]$$

Again, cosine fringes $\frac{N(N-1)}{2}$ unique fringe phases

Attach constant phases to each hole



For a point source, measured phases:

$\phi_2 - \phi_1$

$\phi_3 - \phi_2$

$\phi_1 - \phi_3$

in general how do we go from pupil phases to fringe phases?

Call this matrix "A"

$$\begin{bmatrix} & & \\ & ? & \\ & & \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} \phi_2 - \phi_1 \\ \phi_3 - \phi_2 \\ \phi_1 - \phi_3 \end{bmatrix}$$

If these are phase "errors" how can we get rid of them?

In other words, is there a matrix K such that:

$K \cdot A = 0$ (leaving only physical signal?)

③

Three hole interferometer closure phase:

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3-hole interferometer

closure phase $= \Phi_{12} + \Phi_{23} + \Phi_{31} = \phi_2 - \phi_1 + \phi_3 - \phi_2 + \phi_1 - \phi_3$

(hole phase errors only) $= \phi_2 - \phi_2 + \phi_1 - \phi_1 + \phi_3 - \phi_3 = 0$

Only 1 closure phase for three holes

$$\underbrace{\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}}_{K} \begin{bmatrix} \Phi_{12} \\ \Phi_{23} \\ \Phi_{13} \end{bmatrix} = 0 \qquad \binom{N}{3} \text{ closure phases}$$

in observations:

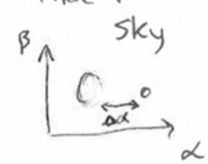
$$\Theta_{measured} = A \cdot \Phi_{holes} + \Theta_{physical}$$

$$K \cdot \Theta_{measured} = \underbrace{K \cdot A\, \Phi_{holes}}_{0} + K \cdot \Theta_{physical}$$

closure phases ------ measure physical structure

what does the physical signal look like?

$$V_{binary} = \frac{1 + r e^{-2\pi i (u \Delta\alpha + v \Delta\beta)}}{1 + r}$$



1-D in general, $r = 0.5$ example:

$$V_1 = \frac{1 + \frac{1}{2} e^{-2\pi i \Delta\alpha u_{12}}}{1 + \frac{1}{2}}$$

$$V_2 = \frac{1 + \frac{1}{2} e^{-2\pi i \Delta\alpha u_{23}}}{1 + \frac{1}{2}}$$

$$V_3 = \frac{1 + \frac{1}{2} e^{-2\pi i \Delta\alpha u_{31}}}{1 + \frac{1}{2}}$$

$$V_1 V_2 V_3 = \Big[1 + \tfrac{1}{2} e^{-2\pi i \Delta\alpha u_{12}} + \tfrac{1}{2} e^{-2\pi i \Delta\alpha u_{23}} + \tfrac{1}{2} e^{-2\pi i \Delta\alpha u_{31}} + \tfrac{1}{4} e^{-2\pi i \Delta\alpha u_{32}} + \tfrac{1}{4} e^{-2\pi i \Delta\alpha u_{21}} + \tfrac{1}{4} e^{-2\pi i \Delta\alpha u_{13}} + \tfrac{1}{8} \Big] \Big/ \left(1 + \tfrac{1}{2}\right)^3$$

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Redundant pupil:

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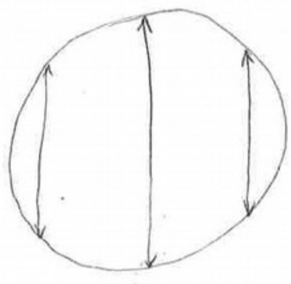
→ Redundant pupils?

?

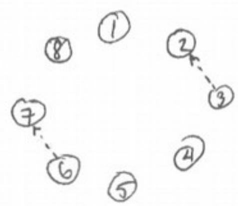
Goal: $\theta_{measured} = A \cdot \phi_{\text{"holes"}} + \theta_{physical}$

Example: partially redundant pupil – annulus



Doubly redundant in all except longest baselines

Break it into 8 supapertures



one row in "A":
to define baseline
" ↖ "

[0 -0.5 0.5 0 0 0.5 -0.5 0]

$$\theta_{measured} = A\,\phi_{subapertures} + \theta_{physical}$$

Choose K so $K \cdot A = 0$

$$K\theta_{measured} = \theta_{kernel} = K\,\theta_{physical}$$

For arbitrary pupil shapes kernel phase is a self-calibrating quantity.

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