Fourier plane imaging

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10 November 2021

Abstract. Text.

Keywords: XXX, YYY, ZZZ

1. Introduction

2. Techniques

2.1. Aperture masking interferometry

Aperture masking interferometry (AMI), also known as sparse aperture masking (SAM) or non-redundant masking (NRM), is an observational technique that uses a physical mask with several holes (the so-called apertures) to divide a single-dish telescope into a multi-aperture interferometer. The key characteristic of the mask is that its Fourier coverage (or uv-coverage) is non-redundant. While redundant aperture masks exist and fall under the term AMI, only non-redundant (or sparse) masks fall under the terms SAM or NRM.

The light incident on each aperture k of an aperture mask can be described as a plane wave with electric field

$$E_k = E_{k,\mathrm{src}} e^{i\varphi_k}, \tag{1}$$

where $E_{k,\mathrm{src}}$ is the electric field of the source and φ_k is a phase shift due to atmospheric and instrumental phase errors. When the plane waves from two apertures m and n interfere, the complex visibility for the corresponding baseline is given by

$$V_{mn} \propto E_m E_n^* = E_{m,\mathrm{src}} E_{n,\mathrm{src}}^* e^{i(\varphi_m - \varphi_n)} \tag{2}$$

$$= V_{mn,\mathrm{src}} e^{i(\varphi_m - \varphi_n)}. \tag{3}$$

Hence, the atmospheric and instrumental phase errors introduce a phase shift in the complex visibility and the Fourier phase measured for the interferometric baseline mn is given by

$$\phi_{mn} = \phi_{mn,\mathrm{src}} + \varphi_m - \varphi_n. \tag{4}$$

The key is now to consider a closed triangle mnl of apertures and to sum the Fourier phase along it to obtain the closure-phase

$$\begin{aligned}\Phi_{mnl} &= \phi_{mn} + \phi_{nl} + \phi_{lm} &(5)\\ &= \phi_{mn,\mathrm{src}} + \varphi_m - \varphi_n + \phi_{nl,\mathrm{src}} + \varphi_n - \varphi_l + \phi_{lm,\mathrm{src}} + \varphi_l - \varphi_m &(6)\\ &= \phi_{mn,\mathrm{src}} + \phi_{nl,\mathrm{src}} + \phi_{lm,\mathrm{src}} &(7)\\ &= \Phi_{mnl,\mathrm{src}}, &(8)\end{aligned}$$

which is independent of the atmospheric and instrumental phase errors.

2.2. *Kernel-phase interferometry*

The complex visibility measured in the Fourier plane is now a superposition of phasors from multiple baselines, each with the same contribution from the source, but with a different phase shift from the atmospheric and instrumental phase errors. The phase of the complex visibility is thus given by

$$\begin{aligned}\arg\left(\sum_{K=1}^{R} V_{K,\mathrm{src}} e^{i\Delta\varphi_K}\right) &\approx \arg\left(\sum_{K=1}^{R} V_{K,\mathrm{src}}(1 + i\Delta\varphi_K)\right) &(9)\\ &= \arg\left(V_{1,\mathrm{src}} \sum_{K=1}^{R}(1 + i\Delta\varphi_K)\right) &(10)\\ &= \arg(V_{1,\mathrm{src}}) + \tan^{-1}\left(\frac{\sum_{K=1}^{R}\Delta\varphi_K}{\sum_{K=1}^{R} 1}\right) &(11)\\ &= \arg(V_{1,\mathrm{src}}) + \tan^{-1}\left(\frac{1}{R}\sum_{K=1}^{R}\Delta\varphi_K\right) &(12)\\ &\approx \arg(V_{1,\mathrm{src}}) + \frac{1}{R}\sum_{K=1}^{R}\Delta\varphi_K &(13)\end{aligned}$$

for $\Delta\varphi_K \ll 1$, that is in the high-Strehl regime where the pupil plane phase errors are small. Hence, in this regime that we call the kernel-phase approximation, the Fourier phase can be written as

$$\vec{\phi} = \vec{\phi}_{\mathrm{src}} + R^{-1} \cdot A \cdot \vec{\varphi}, \tag{14}$$

where R is a $N \times N$ matrix encoding the redundancy of each baseline and A is a $N \times M$ matrix containing only 0, 1, and -1 and encoding the aperture geometry, that is which aperture contributes to which baseline. Multiplication with $K \cdot R^{-1}$ from the left, where K is the kernel of A, yields

$$\begin{aligned}\vec{\Phi} &= K \cdot R^{-1} \cdot \vec{\phi} &(15)\\ &= K \cdot R^{-1} \cdot \vec{\phi}_{\mathrm{src}} + K \cdot A \cdot \vec{\varphi} &(16)\\ &= K \cdot R^{-1} \cdot \vec{\phi}_{\mathrm{src}} &(17)\\ &= \vec{\Phi}_{\mathrm{src}}, &(18)\end{aligned}$$

which is again independent of the atmospheric and instrumental phase errors.

2.3. Complex visibilities

The complex visibility of a binary with a contrast of $0 < c \leq 1$ and a separation vector of $\vec{\alpha}$, where the primary is normalized to a flux of 1, is given by

$$V_{\mathrm{bin}}(\vec{u}) = 1 + ce^{-2\pi i\vec{\alpha}\cdot\vec{u}}. \tag{19}$$

The amplitude of this expression is given by

$$\begin{aligned}
|V_{\mathrm{bin}}| &= \sqrt{(1 + c\cos x)^2 + (c\sin x)^2} && (20)\\
&= \sqrt{1 + 2c\cos x + c^2} && (21)\\
&= \sqrt{1 + 4c\cos^2\frac{x}{2} - 2c + c^2} && (22)\\
&\approx \sqrt{4\cos^2\frac{x}{2}} && (23)\\
&= 2\cos\frac{x}{2}, && (24)
\end{aligned}$$

where $x = 2\pi\vec{\alpha}\cdot\vec{u}$.