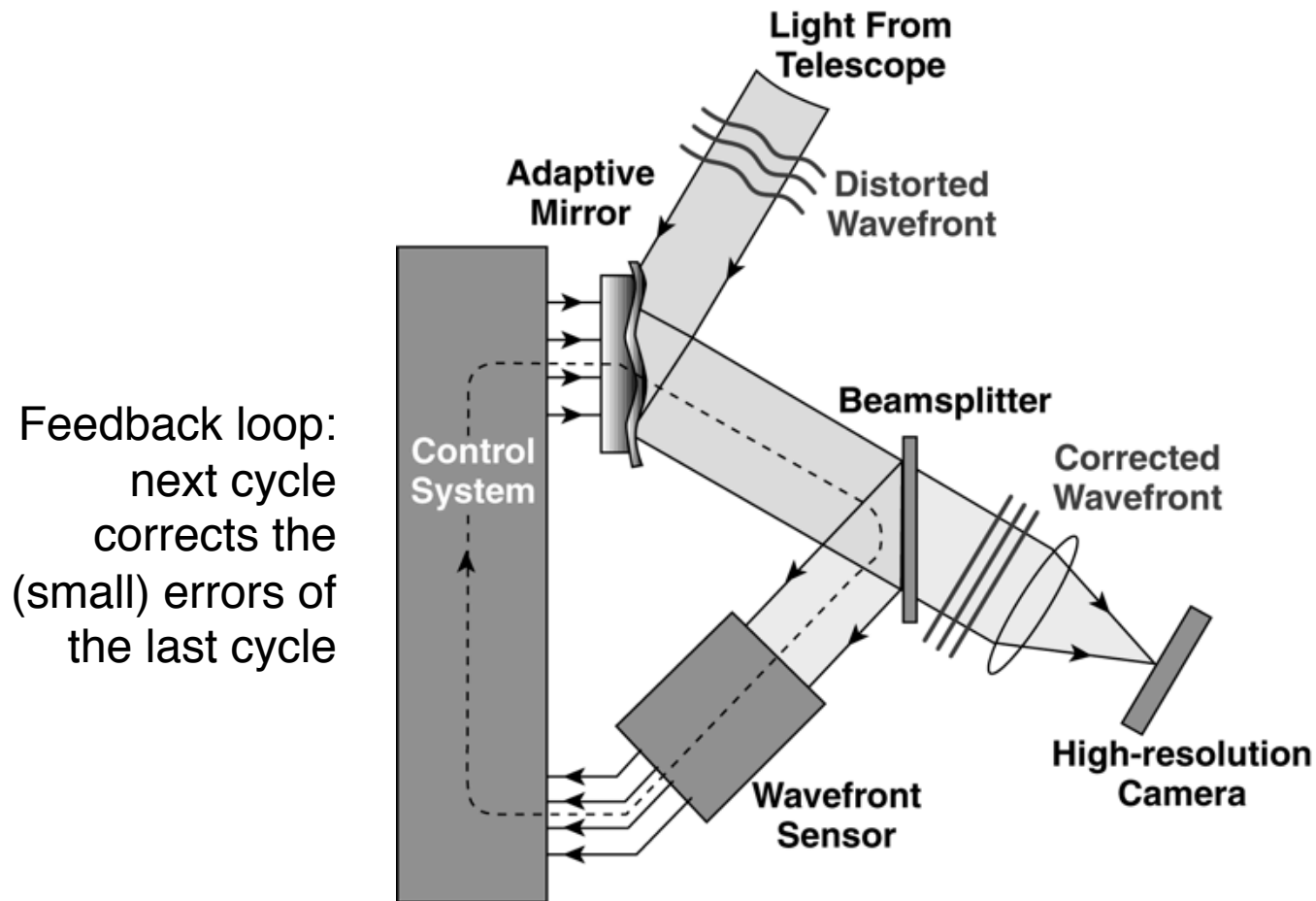


Schematic of adaptive optics system



**e.g. SHACK-HARTMANN WAVEFRONT SENSOR
or PYRAMID WAVEFRONT SENSOR**

Part II - PSF theory

Optical Path Difference

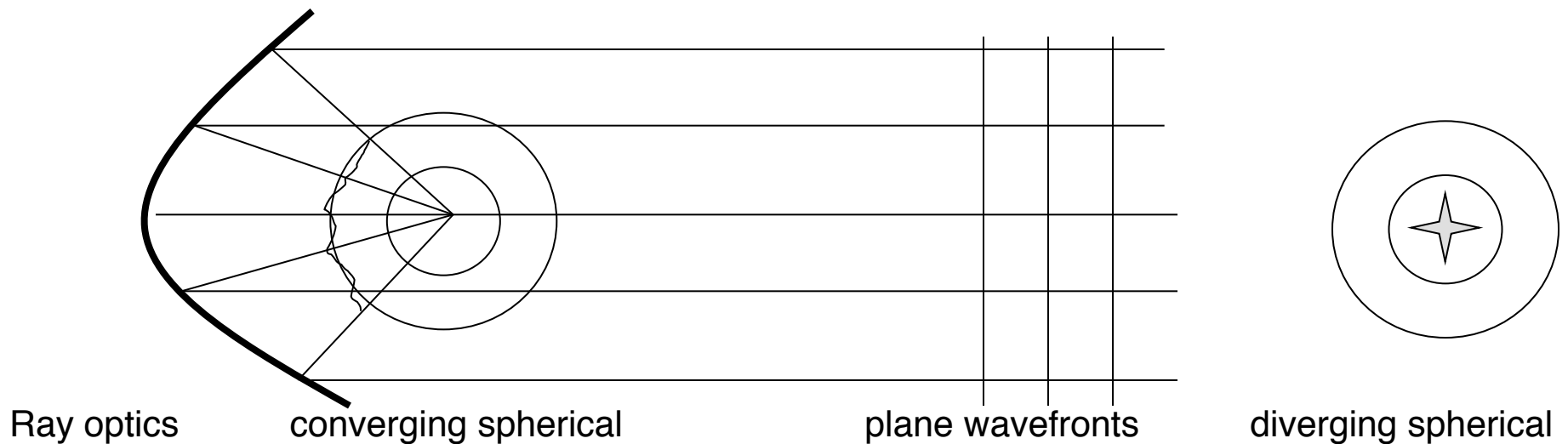
This is the deviation of the wavefront from 'perfect'... when talking of an image being formed by a converging wavefront,

THE DEVIATION OF THE WAVEFRONT FROM THE
PERFECT SPHERICAL CONVERGING WAVE

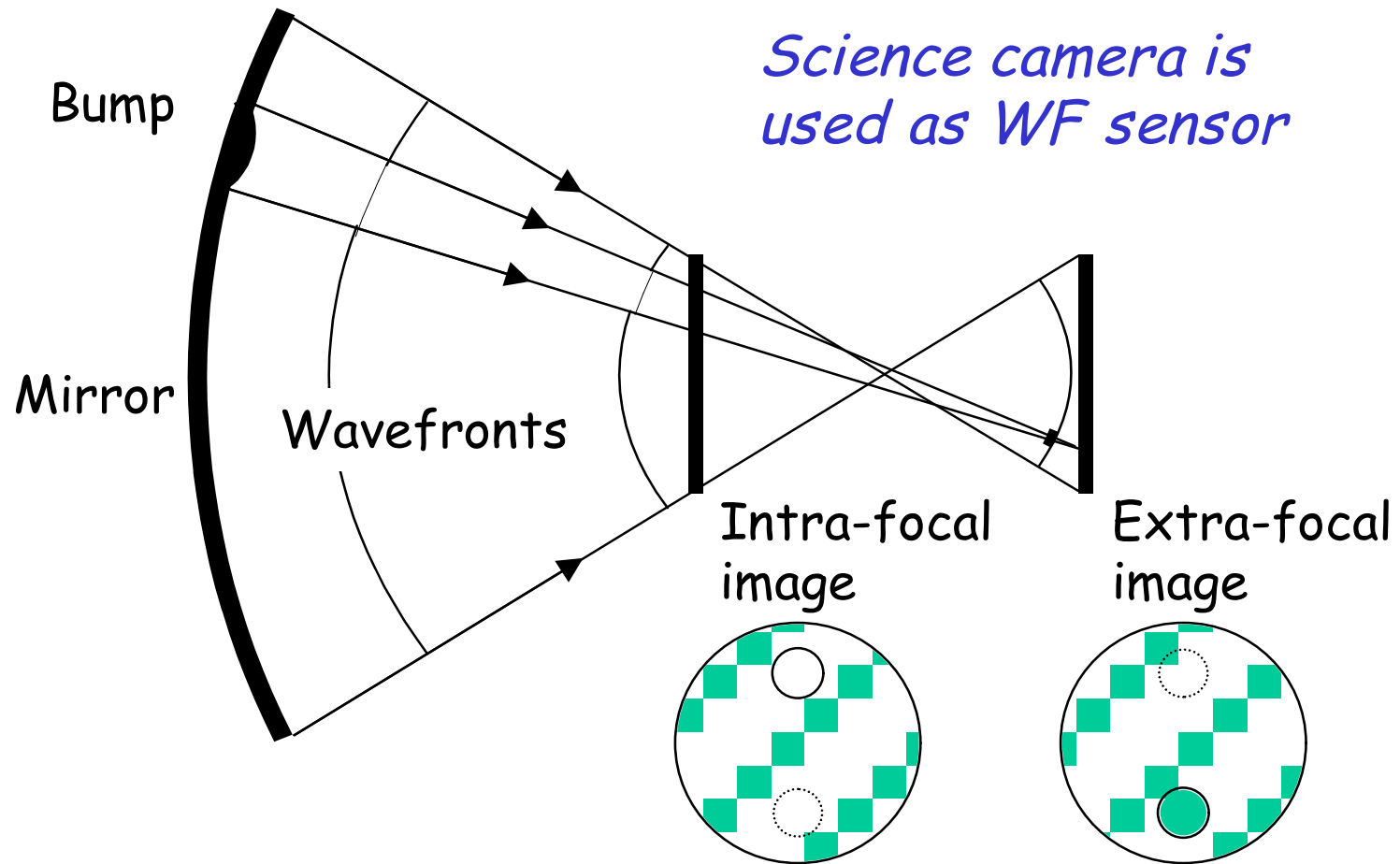
is the optical path difference.

In a collimated beam such as an interferometer, the deviation of a wavefront from the perfect, flat wavefront is the OPD.

OPD(x,y) is a real function in 'pupil space', dimensions of LENGTH usually
At wavelength it is expressed in RADIANS of PHASE: $\phi(x,y) = (2 \pi / \lambda) \text{OPD}(x,y)$



UNFOCUSSED IMAGES MEASURE THE MIRRORS



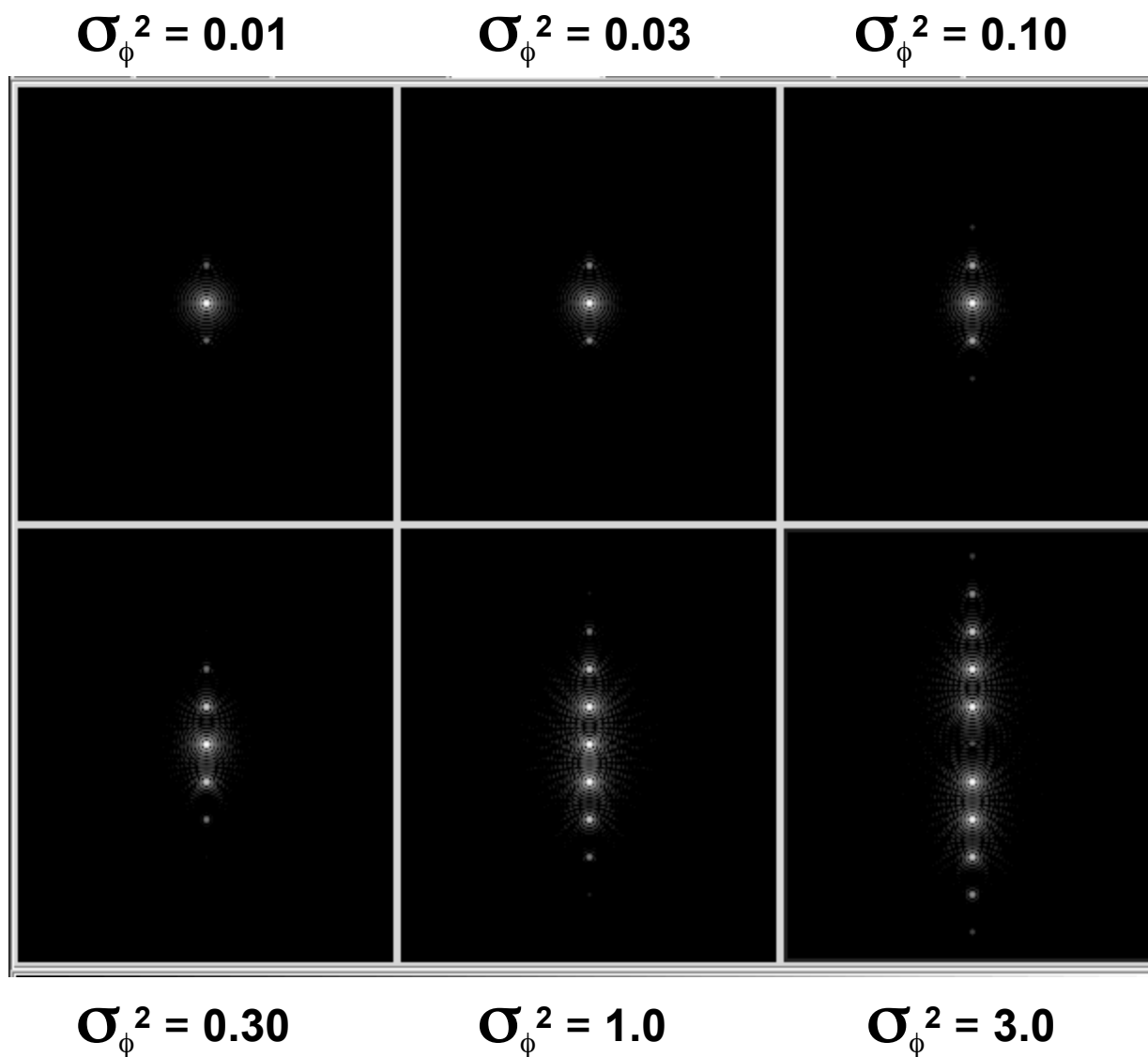
Phase aberrations in cycles per diameter

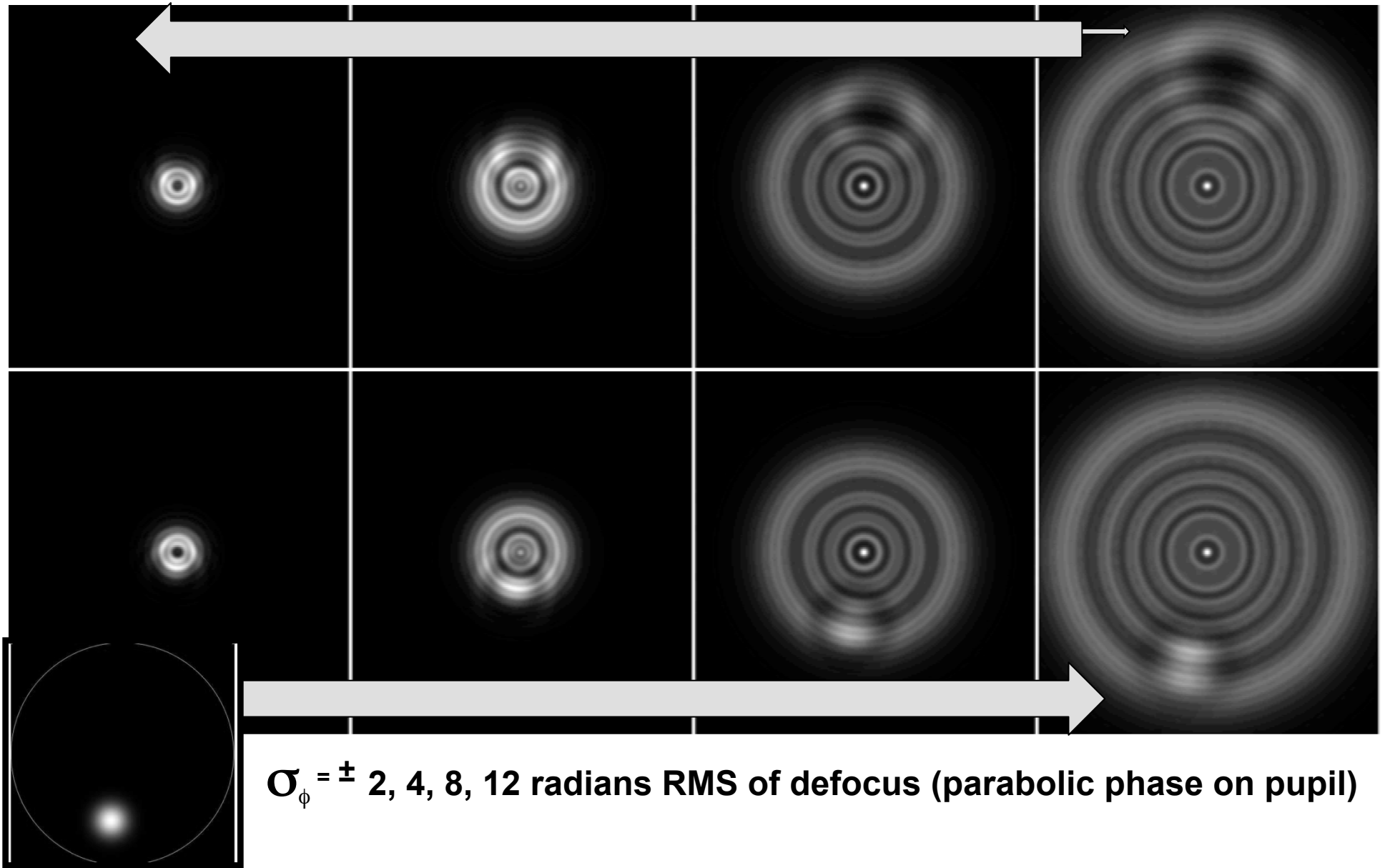
Think Fourier

Sine wave aberration is a pair of delta functions in its 'Fourier transform domain'

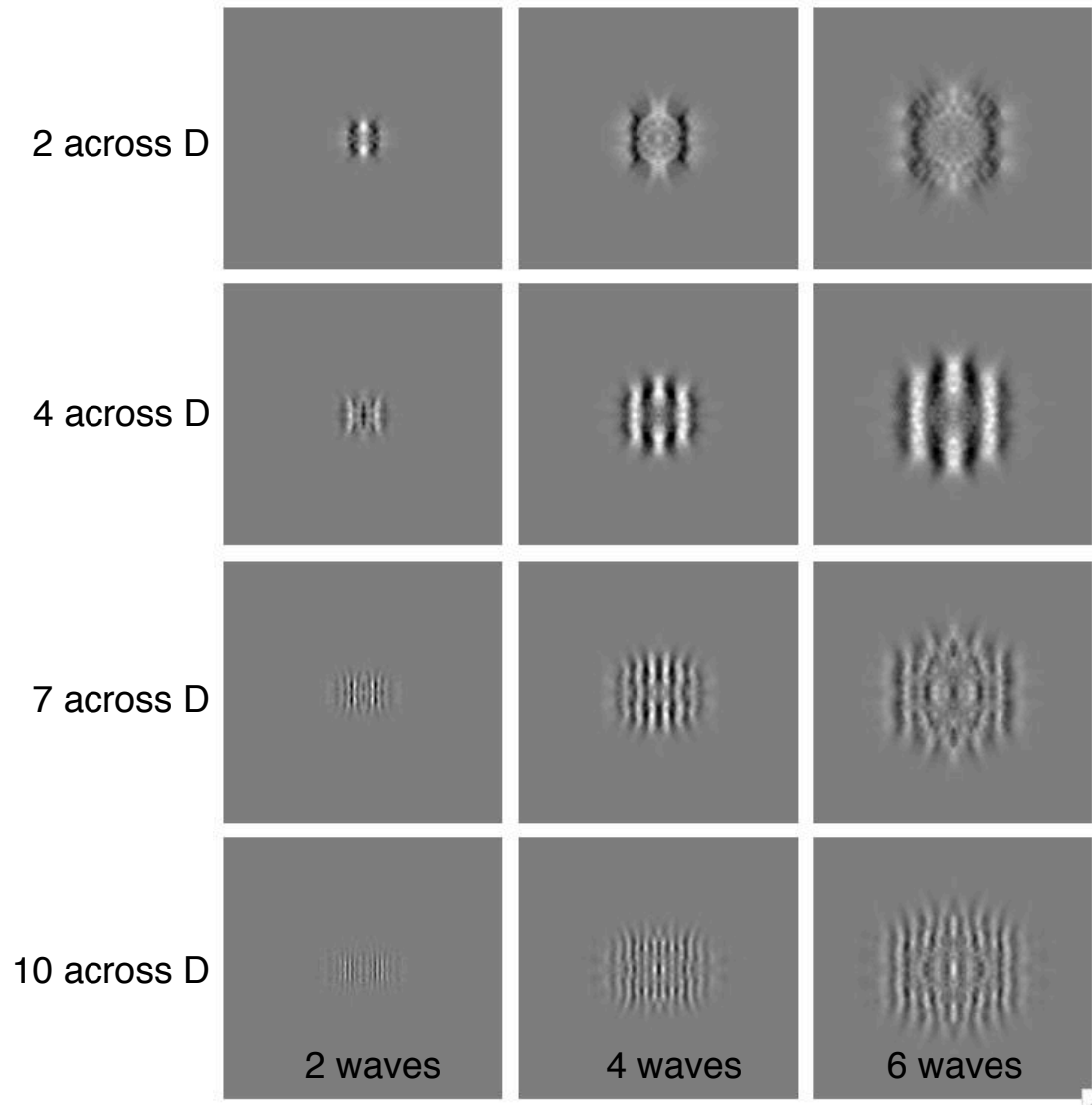
At small amplitudes this corresponds to pair of bright spots in the PSF:
pupil: $\exp(i\phi) \sim 1 + i\phi$
image: $\delta(0) + \text{FT}(\text{sine})$

As size of aberration increases, $\exp(i\phi)$ expansion gets higher order terms. Quadratic terms produce spots at twice the separation...





Choosing the amount of defocus



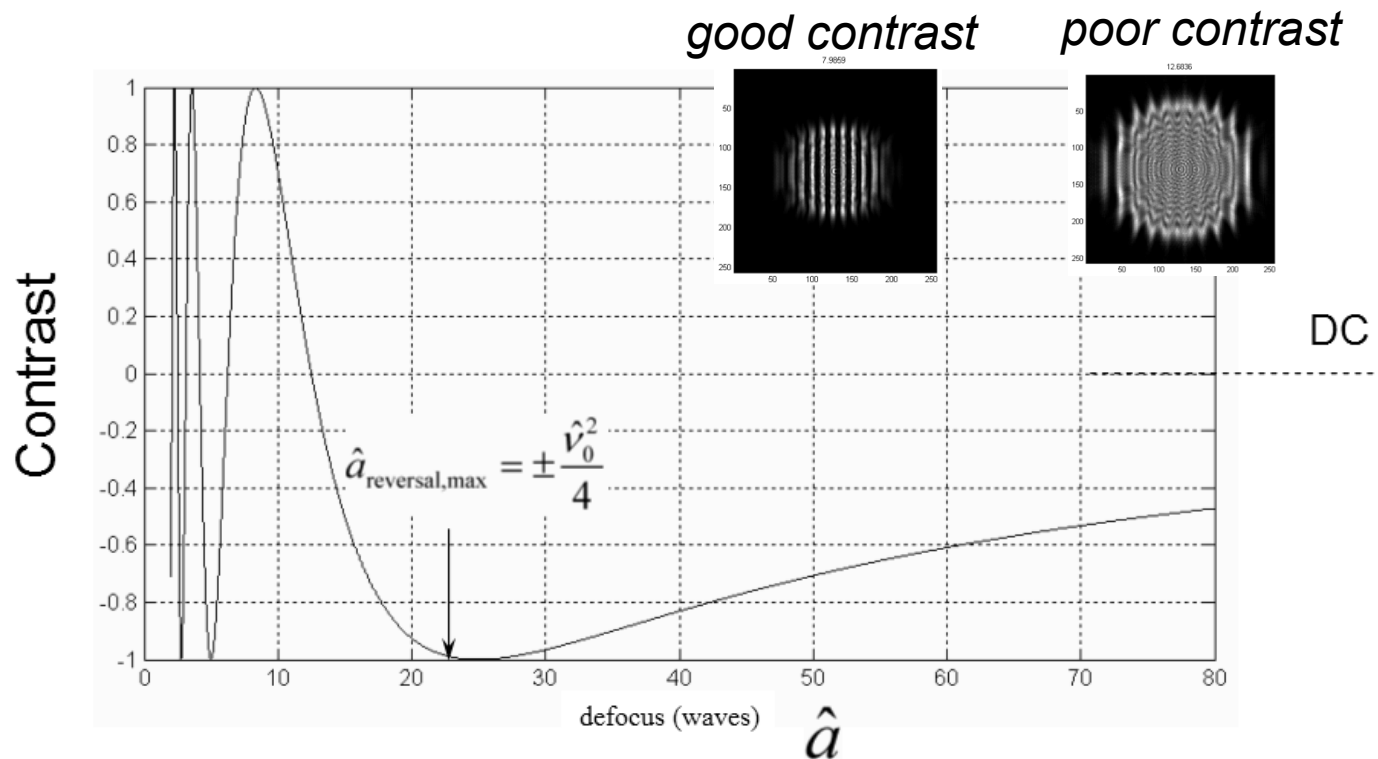
Numerical experiment

Place a sinusoidal phase aberration over the pupil and try three different amounts of defocus.

$$\begin{array}{c} \text{pre-focus image} \\ - \\ \text{rotated post-focus image} \\ = \\ \text{signal} \end{array}$$

What is the best defocus to use?

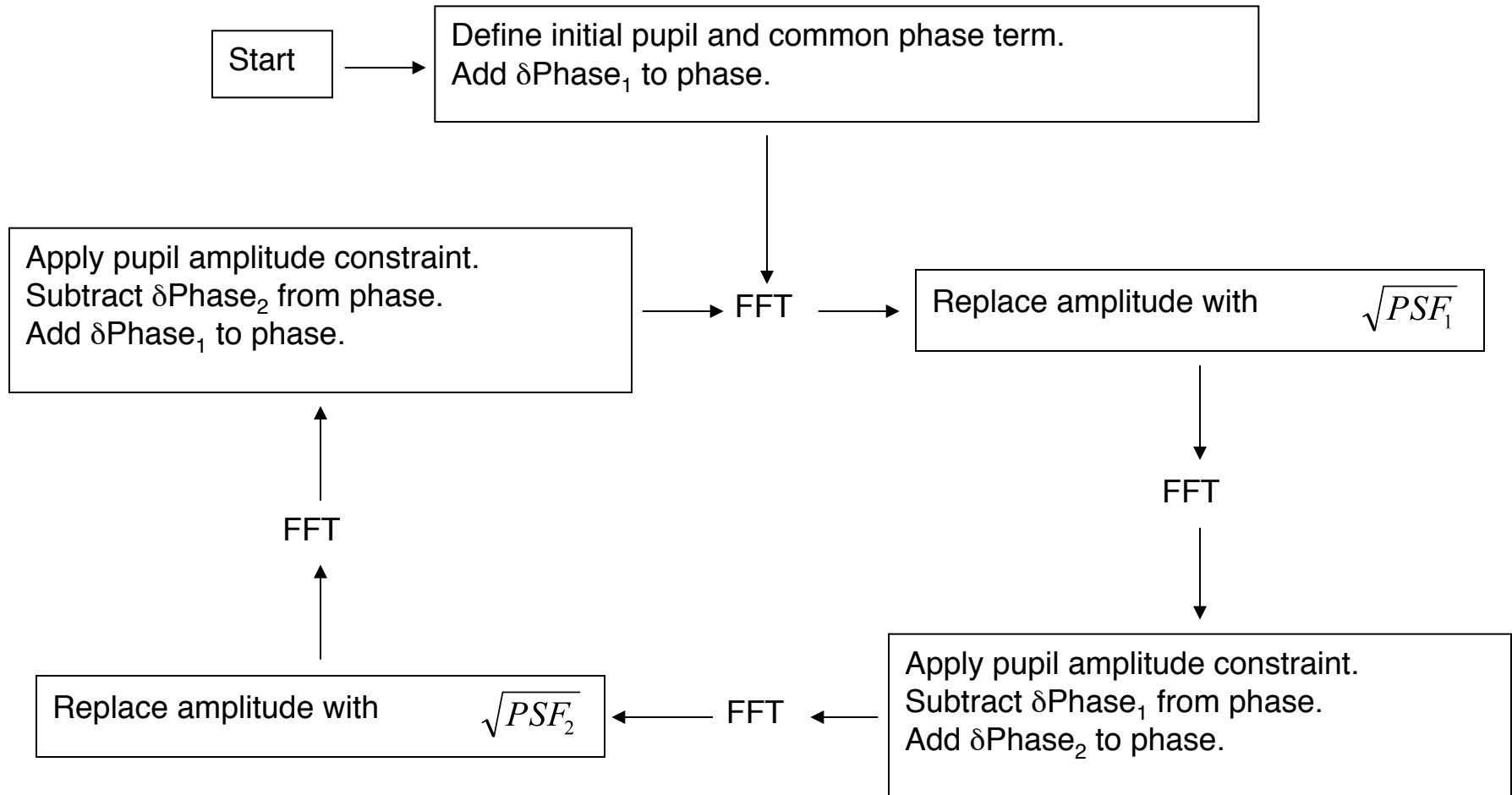
Signal strength for given spatial frequency of
aberration (number of ripples across mirror)
is periodic in 1/defocus



B. Dean, C. Bowers, "Diversity Selection for Phase-Diverse-Phase-Retrieval," JOSA, 20(8), 2003, pp. 1490-1504

Missel Algorithm

Goal: Derive high-resolution exit pupil amplitude and phase maps



Pupil amp constraint: Enforce amplitude, zero outside pupil, or free everywhere

Misell-Gerchberg-Saxton (MGS) algorithm

MGS in this case - a mapping from one guess at the phase, $f_1(x)$, to a better estimate, $f_2(x)$, using a known pupil function and image data $I(k)$

- Assume a pupil function $A(x)$ and a first-guess phase $f_1(x)$
- Calculate $a(k) = F[A(x) \exp \{i f_1(x)\}] = b(k) \exp \{i g(k)\}$ (b is real)
- Use measured data for intensity $I(k)$ - replace $b(k)$ with $\sqrt{I(k)}$
- Now we have $\sqrt{I(k)} \exp \{i g(k)\}$
- Back-transform it - we write this as $C(x) \exp \{i f_2(x)\}$
- This gives us our revised estimate of the phase, $f_2(x)$
- Now write the pupil field using known pupil A instead of C : $A(x) \exp \{i f_2(x)\}$
- And do the same operations to get the next estimate, $f_3(x)$, for the phase

Keep going till you are happy. It WILL converge but not necessarily to the right phase -

- Incorrect $A(x)$
- Improperly reduced data $I(k)$ (CR, flats, photon noise, real pixel response,...)
- Difference between FFT samples and physical pixels, *etc.*

Misell, D. L. 1972, J. Phys. D, 6, L6
Gerchberg, R. H. & Saxton, H. O. 1972, Optik, 35(2), 237