Gerchberg-Saxton phase retrieval

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Wednesday, January 16 2019

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The GS algorithm starts with nominally in-focus image data, and a known pupil geometry (even a throughput map for an apodized pupil).

We seed it with an initial guess at the phase over pupil. It then steps between pupil and image planes, enforcing the known pupil and the known image intensity constraints (as appropriate). A GS iteration goes from one estimate of the pupil phase to the next, even if is sent through a forward transform to image, then an inverse transform back to pupil.

If you want a wider view of GS you can skim the very readable Section 3.1 of Osherovich's 2012 Ph.D. thesis for an overview of GS and three similar methods that Fienup developed. Osherovich describes Fienup's GS modification (the Hybrid Input-Output algorithm), which is more robust than GS. Figure 1.3 (p.7) of the thesis is amusing and quite educational.

A monochromatic, better-than-Nyquist image sampling version is presented in the class exercises. For undersampled broadband images, see the article by **Jim Fienup**.

Gerchberg-Saxton loop

- Initial data: Pupil transmission array $A(x)$, real
- Initial data: Image intensity array $I(k) >= 0$, $a(k) \equiv I(k)^{\frac{1}{2}}$
- Initial data: A guess at pupil phase, $\phi_0(x)$

From these initial data and guesses one iteration of the GS loop brings you back to the pupil plane, with a next guess at the phase. The loop always converges, though not always to the correct phase. It can stagnate or converge to a different phase. If the pupil has point symmetry there are two possible "correct" phase solutions.

Monitor the changes in the phase guess until they are small. Try different initial guesses, see if you get the same final phase after convergence. If so that's good.

Fig. 1 from Gerchberg & Saxton 1972

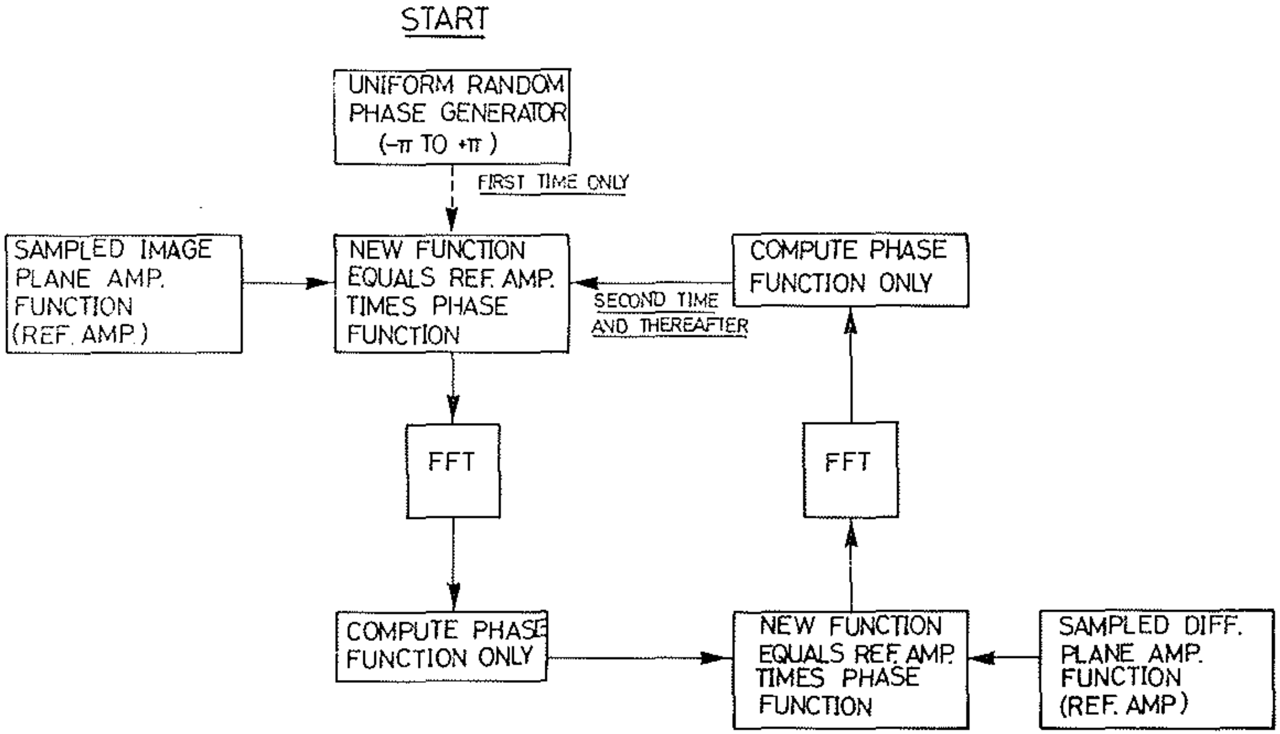

Fig. 1. Schematic drawing of phase determining algorithm.

GS loop flow in theory:

- Calculate image field: $a_0(k)\, e^{i\psi_0(k)} = \mathrm{FT}(A(x)\, e^{ik\phi_0(x)})$
- Enforce image intensity constraint: $a(k)\, e^{i\psi_0(k)}$
- Go back to pupil: $A_1(x)\, e^{i\phi_1(x)} = \mathrm{IFT}(a(k)\, e^{i\psi_0(k)})$
- Enforce pupil constraint: $A(x)\, e^{i\phi_1(x)}$
- Change in phase estimate: $E^2 = \dfrac{\int\int\; A(x)\,(\phi_1(x) - \phi_0(x))^2\; dx}{\int\int\; A(x)\; dx}$
- Are we done yet? Is $E^2 \leq E^2_{\text{requirement}}$?
- NO: go through another loop, calculate $\phi_2(x)$, ...
- YES: Done. Or retry with several different initial phase guesses to be sure.

Real details to think about:

- Is your image data noisy? Low-pass filter out noise?
- Got negative intensity data? Zero it out?
- Differencing phase arrays: Zero-mean the differences. De-tilt?
- Missing/bad pixel data: Filters out easily with finely sampled data. More of a problem with Nyquist and coarser data (*e.g.*, NIRCam at 2um)
- How will incorrect pupil geometry show? Incorrect pixel scale? Incorrect wavelength?
- Finite pupil support leads to infinite image extent. Think about spatial filtering in the GS loop context.

Final comments on GS:

Even in the absence of noise the GS algorithm can converge to a radically wrong answer. To be more confident of the answer we get from it, we can initialize our pupil phase guess with various random phase arrays over the pupil and see if one phase answer comes out most often.

We can also try to start it close to the final answer if we know something about the situation (e.g. the Hubble case during science ops, but not during the earliest days of unknown aberration, before HST spherical aberration was characterized).

Or we can 'parametrize' the way we allow the iterations to select the next estimate of the phase given our 'current' estimate of the phase over pupil. Only vary the lowest order few Zernikes of the phase, by filtering the suggested 'next iteration of the pupil phase' to only change focus, or astigmatism, or a few low order Zernikes, in the next iteration of the pupil phase.

MGS: TBD