Fourier Optics Classes 1-6: The Basics

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Prerequisites:

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astroconda with python 3 on your laptop

matrixDFT from https://github.com/mperrin/poppy/blob/master/poppy/matrixDFT.py
(there is a maintained version inside STScI, but we are only using this one
routine from poppy.  Get it any way that's convenient.)

Get the python sources utils.py and telclass.py.

Basic familiarity with complex numbers

Familiarity with DS9, image display stretches (linear, log,..) and matplotlib's
imshow

Time: 8 hours per week: preparatory work prior to the weekly class, and, post-
class, for expanding and writing up work.
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Course organization

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Pre-class work will get you acquianted with the ideas/questions and try out some
exercises.  It is not expected that you will "finish" the problems set for this
prep work, but the prep work is essential for the class.  At the start of the
class you will all discuss this prep work with each other, and to air your
confusion (or clarity) on the concepts and details.  This is a major vehicle of
learning in the class.

After the in-class discussion I will go over points that need clarification or
more in-depth understanding.  That, and setting the stage for the next pre-class
prep work will take the remaning part of the class.

After class you should finish the exercises associated with the prep work (e.g.
in an annotated Jupyter notebook).  This notebook will serve as your course
notes, and hopefully a valuable reference for you and a lasting legacy of the
course.
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Class Schedule

Date	Class #	Topics
Oct 24	#1	Intro, class software, EM waves, scalar field, far field, the Fraunhofer approximation behind Fourier optics. The Fourier transform definition.
Oct 31	#2	(The monochromatic plane wave, the single photon approach). Limits to discrete Fourier transforms: effects of sampling, and finite information input. Fourier transform properties (the main theorems), applied to telescopes/imaging (tilts, shifts). Simulating the image plane of a given telescope pupil. Pixel scale. Detector simulation. Polychromatic imaging. Equivalent Width Theorem.
Nov 7	#3	Matching theory and numerics, PSFs and imaging, asymptotic behavior, apodization, band-limited functions, Nyquist sampling and aliasing, imperfect images, tilt, speckles
Nov 14	#4	Wavefront sensing: focus sweeps, Gerchberg Saxton focus-diverse phase retrieval basics
Nov 21	-	The Lyot coronagraph, classical & band-limited. The Four Quadrant phase mask coronagraph. Sketch of Vortex coronagraph.
Nov 28	#5	
Dec 5	#6	
Dec 12	Back-up	

Book, articles, and links

1. The best textbook for the Fourier part is still R.N. Bracewell, **"The Fourier Transform and Its Applications"** (Third Edition, Mcgraw-Hill, 2000). This book is unfortunately out of print and has been so for some time. The remaining stock is expensive. The **pictorial dictionary of Fourier transforms** is on pp. 573-591.

2. "Introduction to Fourier Optics" by J. W. Goodman, (McGraw-Hill, second edition, 1996 (or any other edition)

3. "Principles of Optics" M. Born & E. Wolf, 1975 (Pergamon). A new edition of this ancient classic was announced a few years ago. Even the older editions have all the basic

material you are likely to need. This book is the bible for physical optics. I use the 7th edition.

4. "Interferometry and Synthesis in Radio Astronomy" A.R. Thompson, J.M. Moran, & G.W. Swenson, (Springer, 2017). This is the bible for the subject as applied to radio astronomy. PDF downloadable via the link above.

5. Fourier transforms page at NRAO From about 2/3 of the way down and on to the end this fairly short page the basic Fourier transform theorems are laid out well.

Class 1: Theoretical basis, numerical Fourier machinery

This is a hands-on course, but you should have some idea of the theoretical basis for numerical manipulations are, in case you need to delve deeper in the future. We will go quickly over the basics, and outline how we get to the stage of calculating the response of many optical systems to a point source or a sky scene.

We will cover the mathematical definition of the Fourier transform, and a couple of its fundamental properties.

Theoretical work

Look up **Fourier transforms** and **Fourier optics**, and [re]familiarize yourself with the key concepts/phrases.

The 1-dimensional Fourier transform. What sort of function (real, complex,. ...) is the function (e.g. f(x)) and its transform (F(k))? Are the domains of f and F (the spaces that x or k live in) real or complex? Are f and F, deep down, different descriptions of the same information?

Does the theoretical Fourier transform of a function lose informaton that was present in the original function?

Incoherent vs. **coherent** emission of light from distinct points an **extended object**. Does an atom on one side of a star emit spectral line radiation in phase (coherent with) another atom of the same element on the other side of the star, emitting light in the same spectral line? Do then two atoms communicate with each other to collaborate on releasing radiation coherent with each other? How is this different from a gas in a laser cavity?

Diffraction limit or **[angular] resolution** of a telescope, lambda/D

The Fraunhofer or **far field** situation, and the **Fresnel length** that governs when the far field limit calculations are valid. Fresnel length ~ D^2 / lambda. Parse out the meaning of this in terms of the angular resolution.

Describing light numerically/mathematically: a plane wave of monochromatic light (in a homogenous or non-dispersive medium or vacuum) is a propagating oscillation of the electric and magnetic fields. In (x,y,z) physical space z is the direction of propagation. (x,y) is the plane

transverse to the propagation direction. The wave can be expressed as the real part of a complex number that has a (real) amplitude A(x,y) and a unit-strength "phasor" exp(j(kz - wt + phi).

We average over many periods of the waves' oscillation in a typical optical or IR measurement. We also "follow a wave's crest", and ignore the (kz - wt) in the exponent. One resource is ApJ vol. 552 pp.397-408, 2001 May 1, Section 2.1 equation 1 and surrounding very short text.

What mathematical/numerical operation must you perform on a complex array describing the EM wave's "complex amplitude" to get a real array describing the **intensity** (brightness) that a CCD or IR array might detect?

The concept of the **pupil** (or aperture) plane of an optical system, and its illumination by **monochromatic light** from a distant star.

Describing the pupil quantitatively (eg. equation 2 of the above paper). If you create a numpy array that represents a circular mirror telescope without a secondary obstruction, what are the physical dimensions you assign (in your mind) to this in-memory array --- what physical thing does the numerical array span? What physical quantity is represented by a number (an element) in this numerical array?

Post-class-1 and pre-class-2 work

1. Become well-acquainted with the **definition of the convolution** of two functions.
2. Become familiar with the **Special functions** notation in the class1/BracewellIII_pp91_92SpecialFunctions.pdf file on our repository.
3. Familiarize yourself with the **basic Fourier theorems** in the [Fourier transforms page at NRAO](#).
4. Consider what effect a **lateral shift of the pupil** has on its Fourier transform, i.e. the image plane **complex amplitude**, and what effect it has on the **image plane intensity**. Write down a simple 1-D theoretical Fourier argument to support your conclusions.
5. Again, on purely theoretical grounds, what can you do to the pupil cpmplex amplitude to shift the image intensity array off the origin of the image plane?

Look at and run code/telclass.py

See how the matrixDFT object is instantiated, a pupil created, and an image complex amplitude ("field") and its intensity are calculated.

Look through the parameters of exer1() and its output in datadir/c1/ex1*.fits

Write up a short description of why the images appear as they do. Consider information content in the two domains, pupil and image.

Do the same with exer2(). How does this show the difference between a detector pixel and a pointwise value of an image?

After attempting or succeeding in answering questions 4 & 5 above, run exer3() and exer4() by uncommenting the call to class2() in the last line of telclass.py.