**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.
8. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
9. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer: -**

The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

**Distribution of 2X1:**

The random variable 2X1 is obtained by multiplying X1 by 2. Since X1 follows a normal distribution, multiplying it by a constant scale the mean and variance of the distribution.

The mean of 2X1 is given by E(2X1) = 2E(X1) = 2μ.

The variance of 2X1 is given by Var(2X1) = (2^2)Var(X1) = 4σ^2.

Therefore, 2X1 follows a normal distribution with a mean of 2μ and a variance of 4σ^2, denoted as 2X1 ~ N(2μ, 4σ^2).

**Distribution of X1 + X2:**

The random variable X1 + X2 represents the sum of two independent normal random variables X1 and X2. The sum of independent normal variables is also a normal variable.

The mean of X1 + X2 is given by E(X1 + X2) = E(X1) + E(X2) = μ + μ = 2μ.

The variance of X1 + X2 is given by Var(X1 + X2) = Var(X1) + Var(X2) = σ^2 + σ^2 = 2σ^2.

Therefore, X1 + X2 follows a normal distribution with a mean of 2μ and a variance of 2σ^2, denoted as X1 + X2 ~ N(2μ, 2σ^2).

In summary:

2X1 follows a normal distribution with mean 2μ and variance 4σ^2, i.e., 2X1 ~ N(2μ, 4σ^2).

X1 + X2 follows a normal distribution with mean 2μ and variance 2σ^2, i.e., X1 + X2 ~ N(2μ, 2σ^2).

Both 2X1 and X1 + X2 are normally distributed, but they have different variances. The distribution of 2X1 has a larger variance (4σ^2) compared to the distribution of X1 + X2 (2σ^2).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
9. Specify the 5th percentile of profit (in Rupees) for the company
10. Which of the two divisions has a larger probability of making a loss in a given year?