WEEK 13: FINAL EXAM REVIEW PROBLEMS

1. Limits and continuity

(1) Find $\lim_{x\to 0} x^2 \sin(1/x)$.

(2) Find $\lim_{x\to\infty} \dfrac{\sin x}{x}$.

(3) Show that the equation

$$\sqrt[3]{x} = 1 - x$$

has a solution in the interval $(0, 1)$.

(4) Evaluate $\lim_{x\to\infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 - x}\right)$.

(5) Find $\lim_{x\to\infty} \dfrac{x^2 \sin x}{e^x}$.

(6) Either evaluate

$$\lim_{x\to\infty} \left(e^x + 1\right)^{\frac{1}{x}}$$

or state that the limit does not exist. Give reasons for your answer.

(7) Use l'Hospital's rule (or some other method) to find

$$\lim_{x\to 0} \frac{1 - \cos x}{\ln(1 + x^2)}.$$

2. Derivatives

(1) Differentiate $\tan^{-1}(x^4) + \cos^{-1}(e^x)$.

(2) Differentiate $\ln(x^2 + e^{\cos x})$ and state for which values of x the derivative is defined.

(3) Find the domain of the function

$$f(x) = \cos^1(\ln x) + e^{\sin x}.$$

Find the derivative of f and state for what values of x it is defined.

(4) Find the derivative of

$$F(x) \int_0^{x^2} e^{\sin t}\, dt.$$

(5) Suppose that a function f with domain $\mathbb{R}$ is defined by

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

(a) Explain why f is continuous on $\mathbb{R}$.

(b) Is f differentiable at 0? Give reasons for your answer.

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(6) The radius of a circle is measured to be $20mm$ to the nearest millimetre. Using this measurement, the area of the circle is then calculated to be $400\pi mm^2$. Use the differential approximation to estimate the maximum error for the calculated area of the circle.
(7) Differentiate $\cosh^2(3x-1)$.
(8) Compute the derivative of the function

$$A(t) = \int_1^{\cosh t} \sqrt{x^2-1}\,dx \qquad \text{for } t > 0.$$

Name any theorems that you use.

3. Integration

(1) Evaluate $\displaystyle\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$.
(2) Find $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$.
(3) Find $\displaystyle\int x(\ln x)^2\,dx$.
(4) Use the method of partial fractions to find $\displaystyle\int \frac{x+2}{x^2+x}\,dx$.
(5) Suppose that

$$g(x) = \frac{1}{x^2+2x+5}.$$

Calculate the average value of g over the interval $[1,3]$.
(6) Find

$$\int \tan^5 x \sec^8 x\,dx.$$

(7) Use the formula $\sin A \cos B = \frac{1}{2}\big(\sin(A+B) + \sin(A-B)\big)$ to find

$$\int \sin(3x)\cos(x)\,dx.$$

(8) Evaluate $\displaystyle\int_0^1 \frac{\tan^{-1} x}{x^2+1}\,dx$.

4. Inverse functions

(1) Find the domain of the function $f(x) = \tan(\cos^{-1}(x))$. Can you simplify it?
(2) Find the domain and range of the function

$$f(x) = \ln(\cos x).$$

(3) Find the domain and range of the function

$$f(x) = \sin^{-1}\left(\frac{x}{|x|}\right).$$

(4) If $\sinh x = \frac{3}{4}$, then find $\tanh x$.
(5) Consider the function f with domain $\mathbb{R}$ defined by $f(x) = x^5 + 5x^3 + 10x$.
   (a) Is the function one-to-one? Justify your answer.
   (b) Compute the derivative $f^{-1}(t)$ at $t = 0$.