Page No.
Date :

☆ **CALLCULUS** -: Calculus is the study of Differentiation and integration.

→ Calculus Explain the changes in values, on a small and large scale, related to any function.

Subtraction

☆ Differentiation / Differential Calculus [$f'(x)$] -:

→ Differential calculus is the rate of change of a variable or a quantity with respect to another variable / quantity.

$$\boxed{f'(x) = \frac{dy}{dx}}$$

→ rate of change of the y
→ with respect to x.

* Differential formulas:

① $\frac{dK}{dx} = 0$; K = Constant number

② $\frac{d(x)}{dx} = 1$

③ $\frac{d(Kx)}{dx} = K$; K = constant

Imp. ④ $\frac{d(x^n)}{dx} = n x^{n-1}$

Page No.
Date :

(Log) (e^x)

* Derivatives of Logarithmic and Exponential functions-!

① $\frac{d(e^x)}{dx} = e^x$

② $\frac{d(Ln(x))}{dx} = \frac{1}{x}$ | Ln = natural Log

③ $\frac{d(a^x)}{dx} = a^x \, Log\, a$

④ $\frac{d(x^x)}{dx} = x^x(1+Ln\,x)$

⑤ $\frac{d(Log_a x)}{dx} = \frac{1}{x} \times \frac{1}{Ln\,a}$

* Trignometric function-!

① $\frac{d(\sin x)}{dx} = \cos x$

② $\frac{d(\cos x)}{dx} = -\sin x$

③ $\frac{d(\tan x)}{dx} = \sec^2 x$

Page No.
Date :

(4) $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

(5) $\frac{d(\sec x)}{dx} = \sec x \cdot \tan x$

(6) $\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cdot \cot x$

* Inverse Trignometric functions:-

(1) $\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

(2) $\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$

(3) $\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$

(4) $\frac{d(\cot^{-1} x)}{dx} = \frac{-1}{1+x^2}$

* Sum (+) and difference (−) rule for Differentiation:-

→ When the function is the sum or difference of two functions, the derivative is the sum or Difference of derivative of each function.

→ In short you have ~~as~~ to differentiate both funtion and then add or difference.

If $f(x) = U(x) \pm V(x)$

then $f'(x) = U'(x) \pm V'(x)$

* **Product Rule :- (x)** :- for multiplication, we have to apply this formula :-

If $f(x) = u(x) \times v(x)$

Then
$$\boxed{f'(x) = u'(x) \times v(x) + u(x) \times v'(x)}$$

$u'(x)$ and $v'(x)$ = diffrentiation values

* **Quotient Rule** $(\div)$:-

If we have

$$f(x) = \frac{u(x)}{v(x)}$$

Page No.
Date :

derivative of the function can be expressed as:

$$\boxed{f'(x) = \frac{u'(x) \times v(x) - u(x) \times v'(x)}{[v(x)]^2}}$$

Ex $f(x) = 2x^3 - 4x^2 + x - 33$
$f'(x) = ?$

$\cancel{\frac{d}{dx}(2x^3 - 4x^2 + x - 33) = 2x(3x^2) - 4(x)}$

$f'(x) = 2\frac{d}{dx}(x^3) - 4\frac{d}{dx}(x^2) + 1\frac{d}{dx}(x) - \frac{d}{dx}(33)$

$= 2(3x^2) - 4(2x) + 1 - 0$

$= 6x^2 - 8x + 1$

$\because \frac{d(x^n)}{dx} = nx^{n-1}$

$\because \frac{d(x)}{dx} = 1$

$\because$ Constant $= 0$

$\boxed{\text{Ex. - 33}}$

Q. $f(x) = \frac{\sin x}{x}$ $\quad f'(x)$

(Quotient Rule)

$\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{\frac{d}{dx}(\sin x)x - \sin x\left(\frac{d}{dx}(x)\right)}{x^2}$

$= \frac{x\cos x - \sin x}{x^2}$

Page No.
Date :

(+)

★ Integration / Integral Calculus :- ($\int$)

→ The process of evaluating the area under a curve or a function is called integral calculus.

or

→ The process of finding the anti-derivative of a function.

✱ List of Integral formulas :-

| c = Constant

① $\int 1\, dx = x + c$

② $\int a\, dx = ax + c$

~~③ $\int x^n dx =$~~

Imp ③ $\int x^n dx = \frac{(x^{n+1})}{n+1} + c \quad ; n \neq 1$

④ $\int \sin x\, dx = -\cos x + c$

⑤ $\int \cos x\, dx = \sin x + c$

⑥ $\int \sec^2 x\, dx = \tan x + c$

⑦ $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

⑧ $\int \sec x\, dx =$

Page No.
Date :

(8) $\int \sec x \times (\tan x)\, dx = \sec x + c$

(9) $\int \operatorname{cosec} x \times (\cot x)\, dx = -\operatorname{cosec} x + c$

(10) $\int \left(\frac{1}{x}\right) dx = \ln|x| + c$

(11) $\int e^x dx = e^x + c$

$\ln$ = natural Log

$|x|$ = mode of x

(12) $\int a^x dx = \frac{(a^x)}{\ln a} + c \quad ; \; a > 0 \; ; \; a \neq 1$

(13) $\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + c$

(14) $\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + c$

(15) $\int \frac{1}{|x| \sqrt{(x^2-1)}}\, dx = \sec^{-1} x + c$

(16) $\int \sin^n (x) dx = \frac{-1}{n} \sin^{n-1}(x) \cos(x) + \frac{n-1}{n} \int \sin^{n-2}(x)\, dx$

(17) $\int \cos^n (x) dx = \frac{1}{n} \cos^{n-1}(x) \sin(x) + \frac{n-1}{n} \int \cos^{n-2}(x)\, dx$

(12) $\int \tan^n (x)\, dx = \frac{1}{n-1} \tan^{n-1}(x) - \int \tan^{n-2}(x)\, dx$

Page No.
Date :

(13) $\int \sec^n(x)\,dx = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx$

(14) $\int \operatorname{cosec}^n(x)\,dx = \frac{-1}{n-1}\csc^{n-2}(x)\cot(x) + \frac{n-2}{n-1}\int \csc^{n-2}(x)\,dx$

Ex. $\int_0^3 x^3\,dx$ - find integral

$\left[\frac{x^4}{4}\right]_0^3$

$\because \int x^n dx = \frac{(x^{n+1})}{n+1}$

apply limits

$\left(\frac{3^4}{4}\right) - \left(\frac{0^4}{4}\right)$

we don't add c because we have limit.

$= 20.25$

Ex. $\int (x^2 - 1)(4+3x)\,dx$

mutiply

$= \int (4x^2 + 3x^3 - 3x - 4)\,dx$

now integrate

$= 4\left(\frac{x^3}{3}\right) + 3\left(\frac{x^4}{4}\right) - 3\left(\frac{x^2}{2}\right) - 4x + c$

Page No.
Date :

★ Integration By Part formula:-

$$\boxed{\int uv\,dx = u\int v\,dx - \int \left(\underbrace{u'}_{\text{Diff. of } u}\int v\,dx\right) dx}$$

ILATE Rule:- For identify the function that comes first (u) and the second (v), we use ILATE Rule.

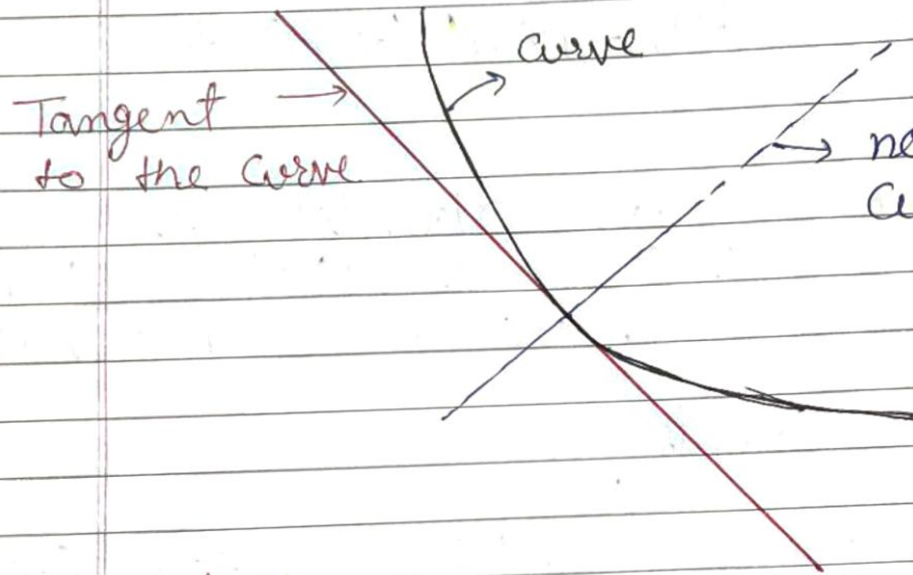
ILATE stands for:-

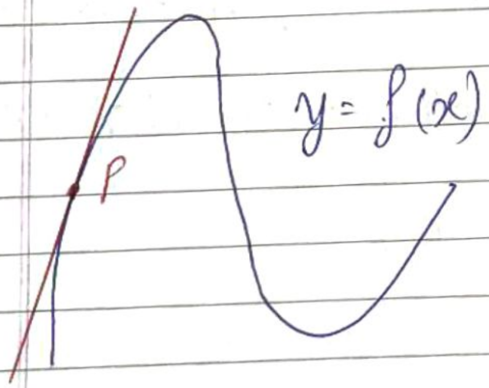
1. I :- Inverse Trignometric function. (A^{-1})
2. L :- Logarithmic functions : $\ln x$, $\log 5(x)$, etc.
3. A :- Algebric function:-
4. T :- Trignometric fn. ($\sin x$, $\cos x$, $\tan x$ etc.)
5. E :- Exponential fn. (e^x)

(Ex. SS.)

★ Tangents and normals:- A Tangent to a curve is a line the touches the curve at one point and has the same slope as the curve at that point.

→ A normal to a curve is a line perpendicular to a tangent to the curve.





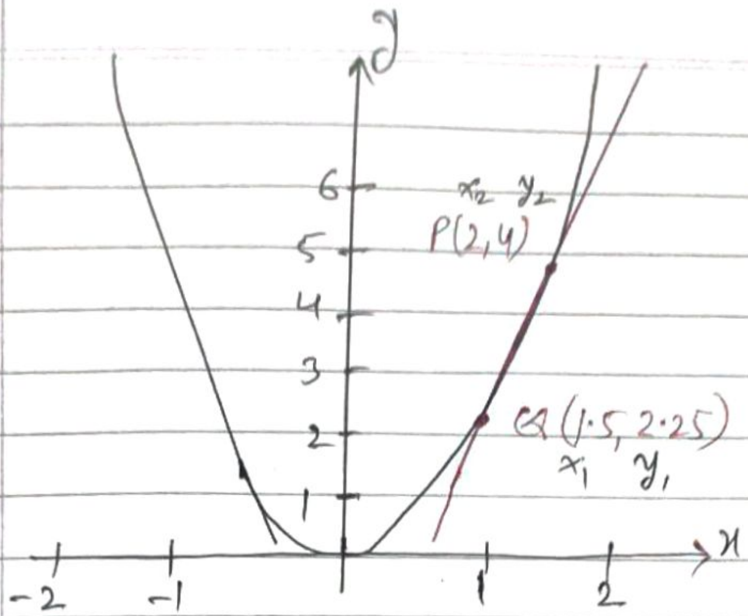
$y = f(x)$ The slope of curve $y = f(x)$

Tangent = P

$$\text{Slope } (m) = \frac{\text{Change in } y}{\text{change in } x}$$

Imp.

$$\boxed{\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}}$$

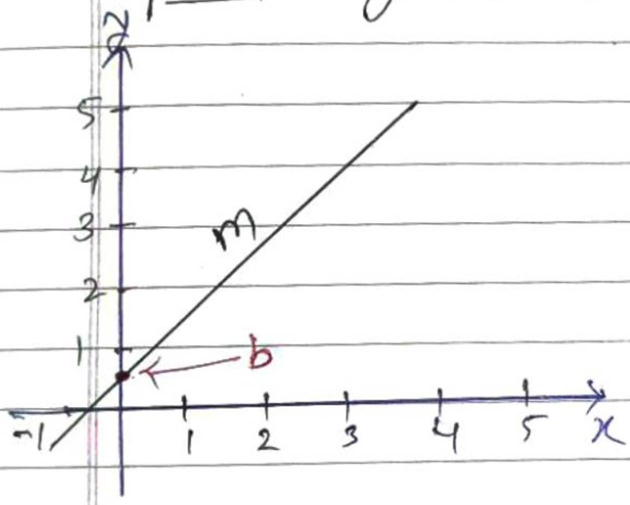


$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 2.25}{2 - 1.5}$$

$$= 3.5$$

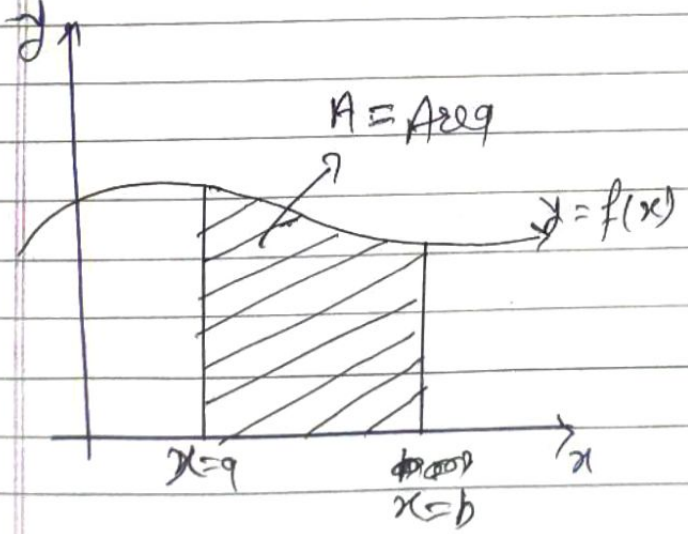
★ Equation of a straight line:-



$$\boxed{y = mx + b}$$

m = slope
b = Intercept (where slope cut on y axis

★ Application of integration:-



To find Area ~~between~~ under curve, Calculated using integration with given limits.