★ Linear Algebra :-

* Matrix :-

A matrix represents a collection of number, arranged in an order of row and columns.

→ It is necessary to enclose the elements of a matrix in parantheses or Brackets.

Ex.

$$
\begin{array}{c c}
 & \begin{array}{ccc} \downarrow & \downarrow & \downarrow \text{ Columns} \end{array} \\
\begin{array}{c} \text{row}_1 \rightarrow \\ \text{row}_2 \rightarrow \\ \text{row}_3 \rightarrow \end{array} & \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}
\end{array}
$$

~~order~~ order of matrix = rows x columns
= 3x3 = 9

So what will $a_{31} = 4$

$\swarrow$ row $\searrow$ Columns

★ Trace of Matrix :- [tr(A)]

→ It is used only for squre matrix.

→ tr(A) = The sum of the diagonal elements of the matrix.

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Ex.

$$\begin{bmatrix} 1 & 2 & 4 \\ 8 & 3 & 9 \\ 3 & 4 & 2 \end{bmatrix} \quad tr(A) = 1+3+2 = 6$$

* **Square Matrix** :- When number of row and columns are same, it is called square matrix.

Ex.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}$$

row = 3 (3×3)
Columns = 3
So square Matrix

* **Diagonal Matrix** :- If we have values on the diagonal Entries, and zero on the rest, it is diagonal Mat.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

* **Scalar Matrix** :- A scalar matrix has equal diagonal entries and zero ~~off~~ on the rest.

$$A = \begin{bmatrix} 9 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

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(I) Symbol

* <u>Identity Matrix</u> :- The Identity matrix has 1 on the diagonal entries and 0 on the rest.

→ If you multiply any matrix with identity matrix, the result equals the original.

<u>Ex</u>

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* <u>The zero Matrix</u> :- The zero matrix has only zero on all values.

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

* <u>Equal Matrix</u> :- Matrices are equal if each element correspond.

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 1 \end{bmatrix}$$

* <u>Negative Matrix</u> :- If we find a negative matrix of any matrix

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than we will have to multiply the matrix by minus (−).

$$-\begin{bmatrix} -2 & 4 & -3 \\ -5 & 6 & 1 \\ 1 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -4 & 3 \\ 5 & -6 & -1 \\ -1 & 3 & -1 \end{bmatrix}$$

* Adding Matrices :- Both matrices ~~have~~ must have the same dimension.

⇒ We have to add same possition elements

Ex.

$$\underset{2\times3}{\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 5 \end{bmatrix}} + \underset{2\times3}{\begin{bmatrix} 1 & 3 & 8 \\ 2 & 1 & 5 \end{bmatrix}} = \begin{bmatrix} 3 & 4 & 11 \\ 6 & 3 & 10 \end{bmatrix}$$

* Subtracting Matrices :- Both matrices must have the same order/Dimension

→ We have to substract same Possition Values.

Ex.

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -4 & 2 \end{bmatrix}$$

* Scalar Multiplication :- ~~for mul~~

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* Scalar Multiplication → for Scalar Multiplication, we Just have multiply each number in the matrix with the scalar.

→ Scalar is a single number and on the other side, matrices have rows and Columns.

Ex.

$$\begin{bmatrix} 2 & 4 \\ 6 & 1 \end{bmatrix} \times \underbrace{2}_{\text{scalar}} = \begin{bmatrix} 4 & 8 \\ 12 & 2 \end{bmatrix}$$

* Transpose a Matrix: To transpose a matrix, means to replace rows with columns.

rows = columns
Columns = row

$$A = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} = A^T = \begin{bmatrix} 1 & 5 \\ 2 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 8 & 9 \\ 3 & 6 & 2 & 5 \\ 8 & 1 & 9 & 2 \\ 9 & 2 & 10 & 2 \end{bmatrix} = A^T = \begin{bmatrix} 1 & 3 & 8 & 9 \\ 3 & 6 & 1 & 2 \\ 8 & 2 & 9 & 10 \\ 9 & 5 & 2 & 2 \end{bmatrix}$$

1 Row × All Columns
2 Row × All Columns

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* Matrix Multiplication-: first of all, for matrix multiplication we to find both matrices order. than we have to find the resultant of the matrix.

Ex. $m \times n \times n \times p = m \times p$ (resultant)

than each of the row multiply by each of the Column.

Ex. So 1×3 by 3×1 gets a 1×1 result.

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1\times4 + 2\times5 + 3\times6 \end{bmatrix} = \begin{bmatrix} 32 \end{bmatrix}_{1\times1}$$

Ex. So here 3×1 by 1×3 result $= 3 \times 3$

$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4\times1 & 4\times2 & 4\times3 \\ 5\times1 & 5\times2 & 5\times3 \\ 6\times1 & 6\times2 & 6\times3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{bmatrix}$$

Ex. $\begin{bmatrix} 2 & 4 & -6 \\ 3 & 5 & 7 \\ 2 & 1 & 9 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & -2 \\ 6 & 4 \end{bmatrix}$ resultant order $= [3 \times 2]$

$$\begin{bmatrix} 4+4-36 & 2-8-24 \\ 6+5+42 & 3-10+28 \\ 4+1+54 & 2-2+36 \end{bmatrix} = \begin{bmatrix} -28 & -32 \\ 53 & 21 \\ 59 & 36 \end{bmatrix}$$ R

$|A|$

★ **Determinant** :- Determinant is a Scalar value that can be calculated from the elements of a square matrix.

$\det(A) = |A| \rightarrow$ Symbol

Ex.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2} \quad \text{so } \det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Ex.

$$A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \end{array} \qquad |A| = ?$$

$$|A| = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$|A| = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

Ex.

$$A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} 3 & 2 & 1 \\ 6 & 2 & 5 \\ 8 & 4 & 2 \end{bmatrix} \end{array} = \quad |A| = ?$$

$$|A| = 3(4-20) - 2(12-40) + 1(24-16)$$

$$|A| = -48 + 68 + 8$$

$$|A| = 28$$

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Adj A

★ **Adjoint of a Matrix**:- The adjoint of a matrix A, is the transpose of the co-factor matrix of A.

→ Denoted by = Adj A

→ To find Cofactor $C_{ij} = (-1)^{i+j}$

→ **co-factor**:- It is obtained by eliminating all the elements of the same row and column and calculating the determinant of the remaining element.

Ex. $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$ Adj A = ?

Because $(-1)^{i+j} = (-1)^{1+2}$
$(-1)^3 = -1$

$(-1)^{2+1}$
$= (-1)^3 = -1$

$A_{11} = + \begin{vmatrix} 3 & 4 \\ 4 & 3 \end{vmatrix} = -7$ $A_{12} = - \begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix} = 1$

$A_{13} = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 1$ $A_{21} = - \begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix} = 6$

$A_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 0$ $A_{23} = - \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = -2$

$A_{31} = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1$ $A_{32} = -1$, $A_{33} = 1$

So put the all values in matrix

Than

Co-factor ~~Adj A~~ matrix of A $= \begin{bmatrix} -7 & 1 & 1 \\ 6 & 0 & -2 \\ -1 & -1 & 1 \end{bmatrix}$

Transpose $m = \begin{bmatrix} -7 & 6 & -1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix} = \text{Adj } A$

★ Inverse of a Matrix :- (A^{-1})

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A) \qquad = \frac{\text{Adjoint of Matrix}}{\text{determinant}}$$

Where $|A| \neq 0$

Ex.

$$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \frac{1}{4 \times 6 - 7 \times 2} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

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Ex. 1

$$A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 6 & 2 & 4 \end{bmatrix} \end{array}$$ So find $A^{-1} = ?$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A \quad \text{---- (1)}$$

Adj A will be

$A_{11} = +(8-2) = 6$ | $A_{12} = -(18-6) = -12$ | $A_{13} = +(6-12) = -6$

$A_{21} = -(4-4) = 0$ | $A_{22} = +(4-12) = -8$ | $A_{23} = -(2-6) = 4$

$A_{31} = +(1-4) = -3$ | $A_{32} = -(1-6) = +5$ | $A_{33} = +(2-3) = -1$

So Co-factor $M = \begin{bmatrix} 6 & -12 & -6 \\ 0 & -8 & 4 \\ -3 & 5 & -1 \end{bmatrix}$ and $\text{Adj } A = \begin{bmatrix} 6 & 0 & -3 \\ -12 & -8 & 5 \\ -6 & 4 & -1 \end{bmatrix}$

now $|A| = 1(8-2) - 1(12-6) + 2(6-12)$

$|A| = 6 - 6 - 12 = -12$

Put the values in eq. (1) --

$$A^{-1} = -\frac{1}{12} \begin{bmatrix} 6 & 0 & -3 \\ -12 & -8 & 5 \\ -6 & 4 & -1 \end{bmatrix}$$